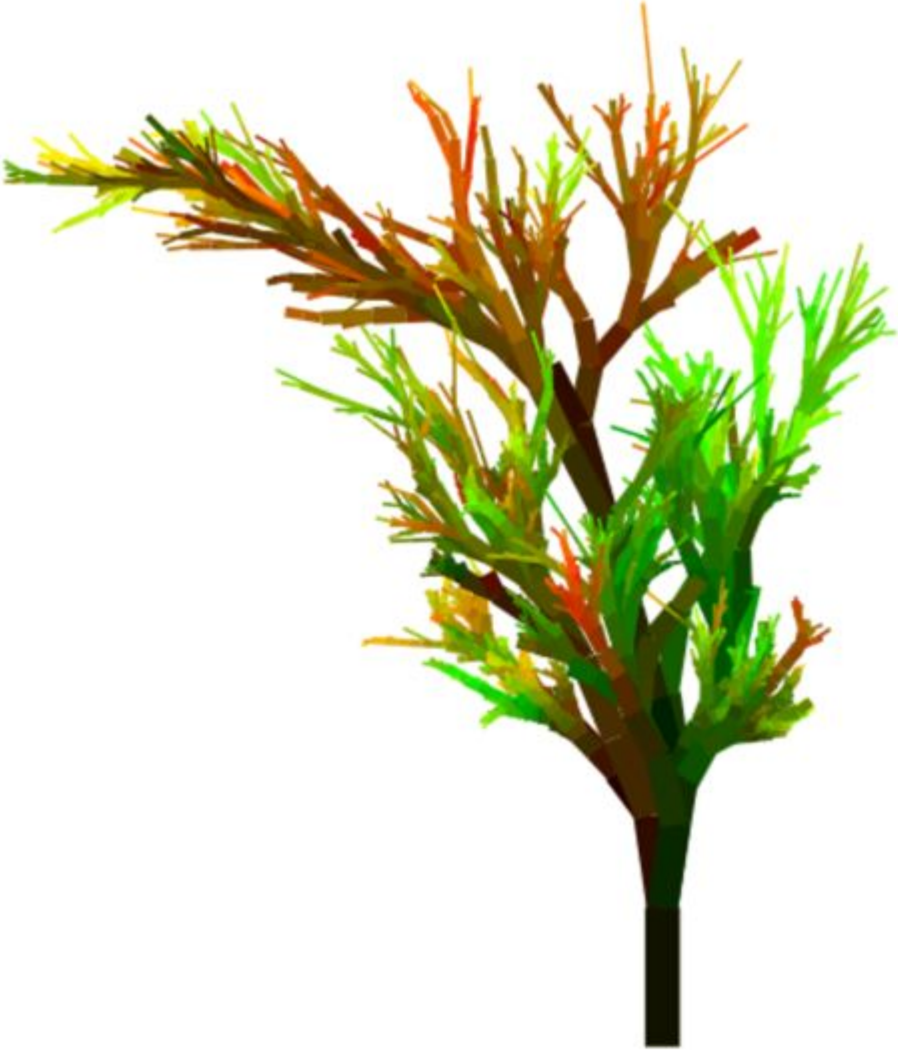
CS61B

Lecture 25: Advanced Trees

- Tree Traversals
- Level Order Traversal
- Range Finding
- Spatial (a.k.a. Geometric) Trees
- Tree Iterators (Extra)

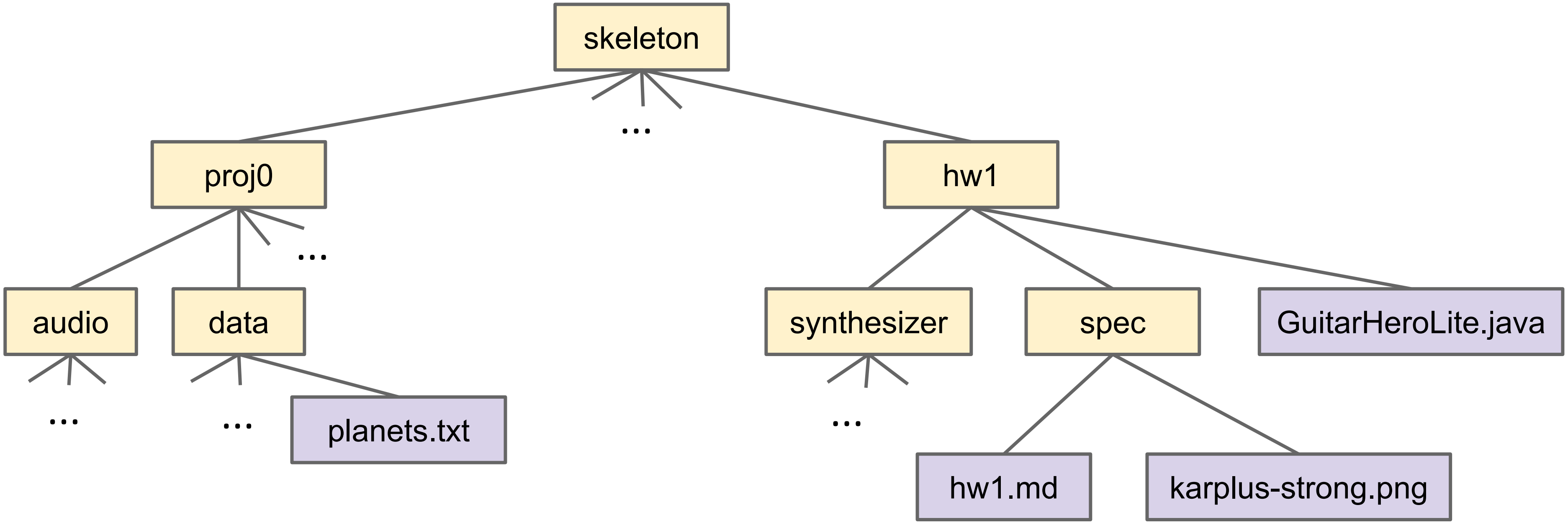


Traversals

Rooted Trees

We’ve used BSTS to build Maps and Sets, and Heaps to build a PQ.

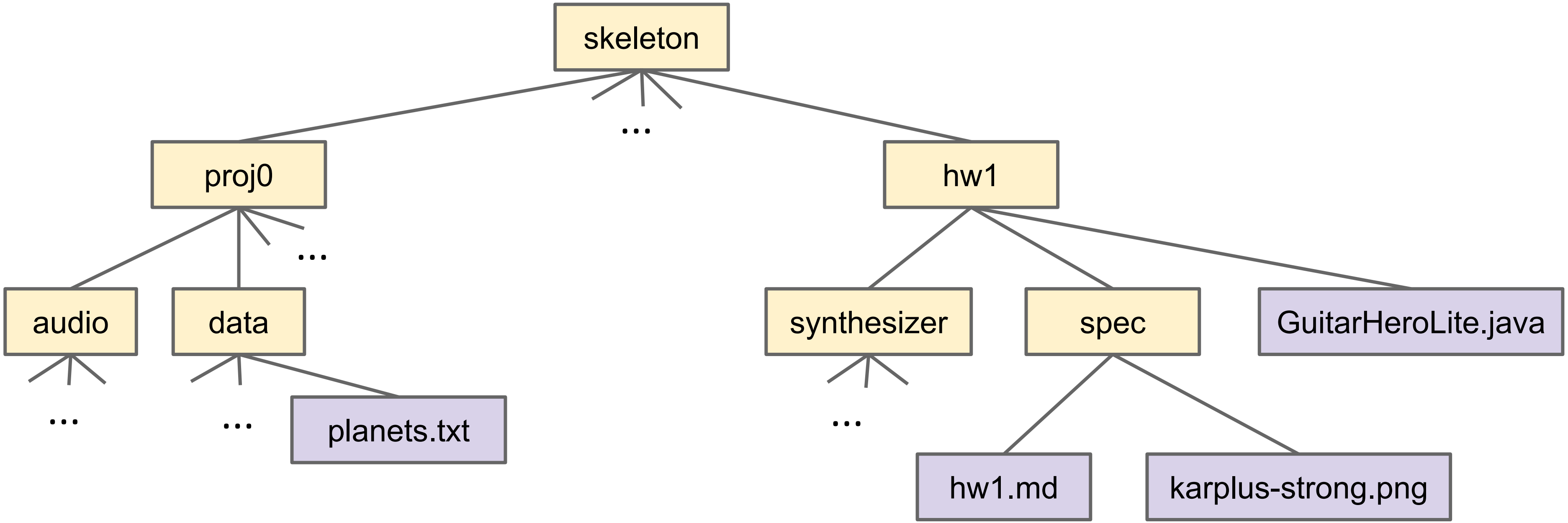
… but trees are a more general concept.



Rooted Trees

Given such a tree, find how much disk space all the files use.

- What one might call "tree iteration" is usually called "tree traversal."
- Unlike lists, there are many natural orderings.



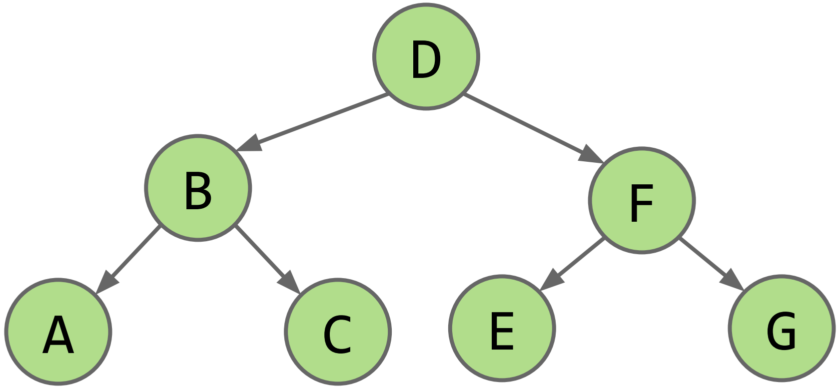
Tree Traversal

Level Order

- Traverse top-to-bottom, left-to-right (like reading in English):
- We say that the nodes are ‘visited’ in the given order.

Depth First Traversals

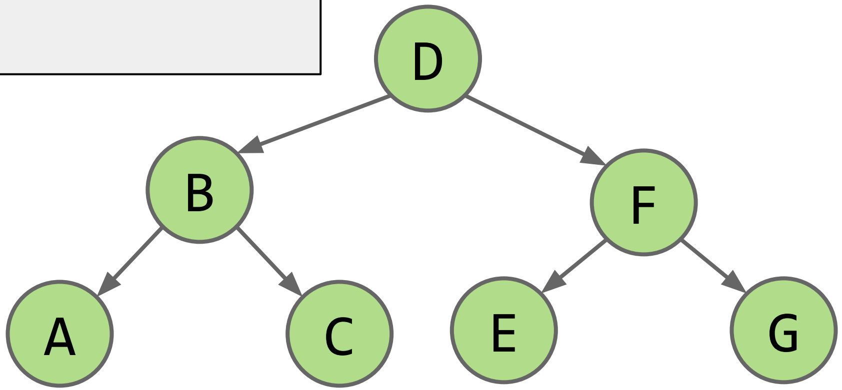
- Preorder, Inorder, Postorder
- Basic (rough) idea: Traverse “deep nodes” (e.g. A) before shallow ones (e.g. F).



Depth First Traversals

Preorder: “Visit” a node, then traverse its children: D B A C F E G

```
preOrder(BSTNode x) {
    if (x == null) return;
    print(x.key)
    preOrder(x.left)
    preOrder(x.right)
}
```



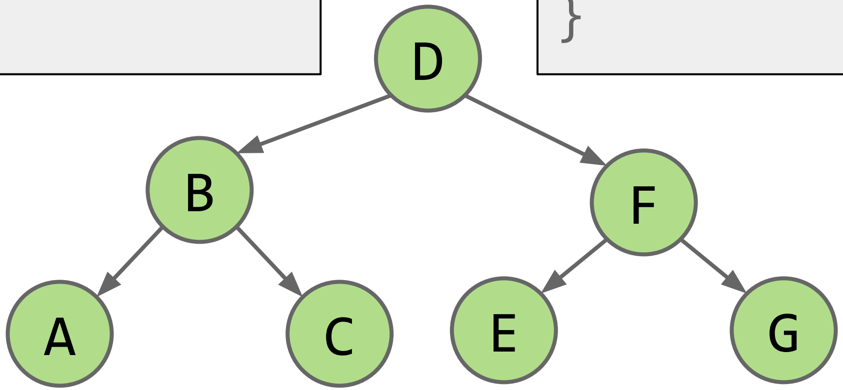
Depth First Traversals

Preorder traversal: "Visit" a node, then traverse its children: DBACFEG

Inorder traversal: Traverse left child, visit, then traverse right child: A B C D E F G

```
preOrder(BSTNode x) {
    if (x == null) return;
    print(x.key)
    preOrder(x.left)
    preOrder(x.right)
}
```

```
inOrder(BSTNode x) {
    if (x == null) return;
    inOrder(x.left)
    print(x.key)
    inOrder(x.right)
}
```



Depth First Traversals http://yellkey.com/top

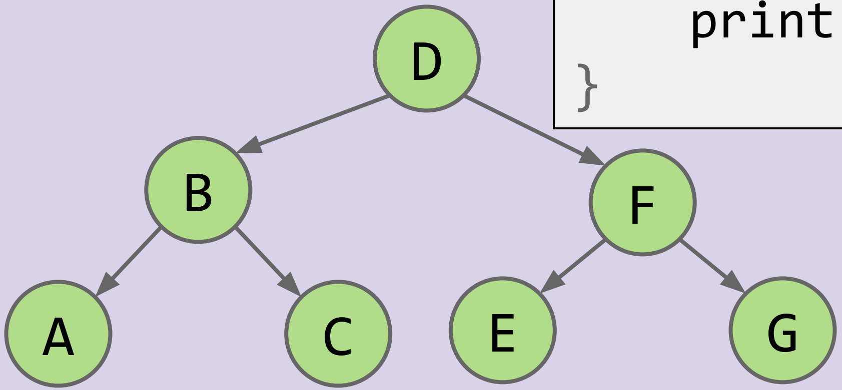
Preorder traversal: "Visit" a node, then traverse its children: DBACFEG

Inorder traversal: Traverse left child, visit, traverse right child: ABCDEFG

Postorder traversal: Traverse left, traverse right, then visit: ???????

```
postOrder(BSTNode x) {
    if (x == null) return;
    postOrder(x.left)
    postOrder(x.right)
    print(x.key)
}
```

1. DBACEFG
2. GFEDCBA
3. GEFCABD
4. ACBEGFD
5. ACBFEGD
6. Other



Depth First Traversals

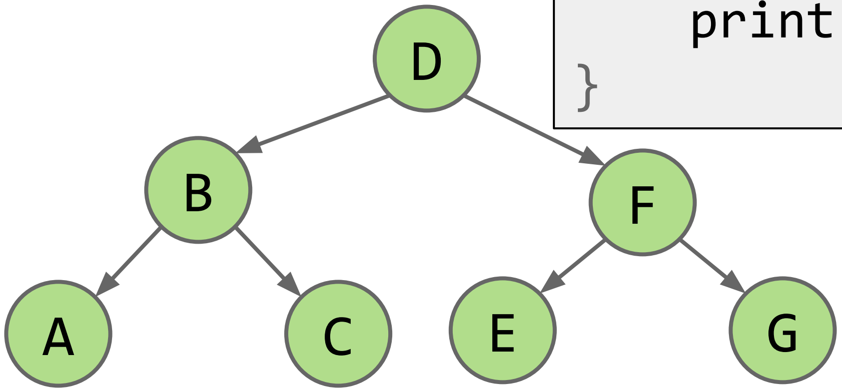
Preorder traversal: "Visit" a node, then traverse its children: DBACFEG

Inorder traversal: Traverse left child, visit, traverse right child: ABCDEFG

Postorder traversal: Traverse left, traverse right, then visit: ACBEGFD

1. DBACEFG
2. GFEDCBA
3. GEFCABD
4. **ACBEGFD**
5. ACBFEGD
6. Other

```
postOrder(BSTNode x) {
    if (x == null) return;
    postOrder(x.left)
    postOrder(x.right)
    print(x.key)
}
```

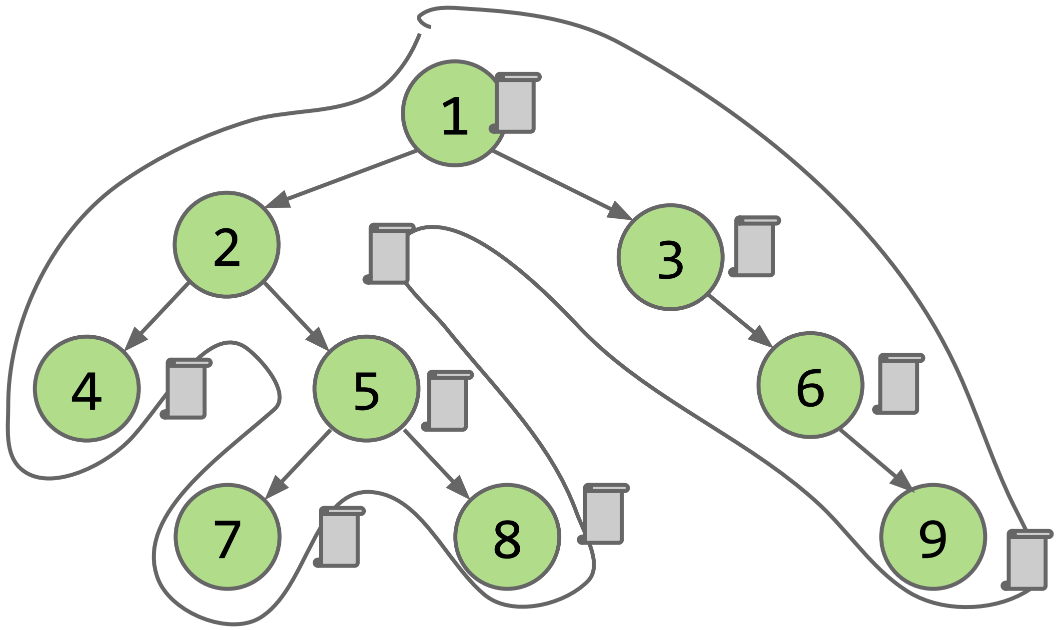


A Weird Trick

- Preorder traversal: We walk the graph, from top going counter-clockwise. Shout every time we pass the LEFT of a node.
- Inorder traversal: Shout when you cross the bottom.
- Postorder traversal: Shout when you cross the right.

Example: Post-Order Traversal

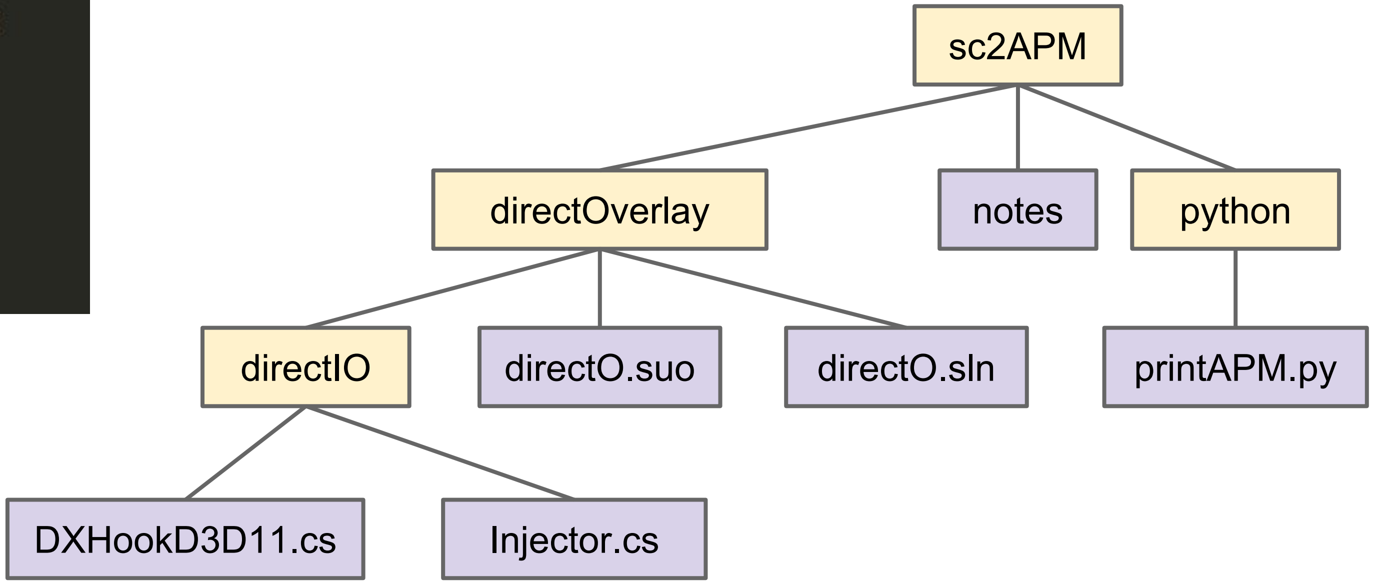
- 4 7 8 5 2 9 6 3 1



What Good Are All These Traversals?

Example: Preorder Traversal for printing directory listing:

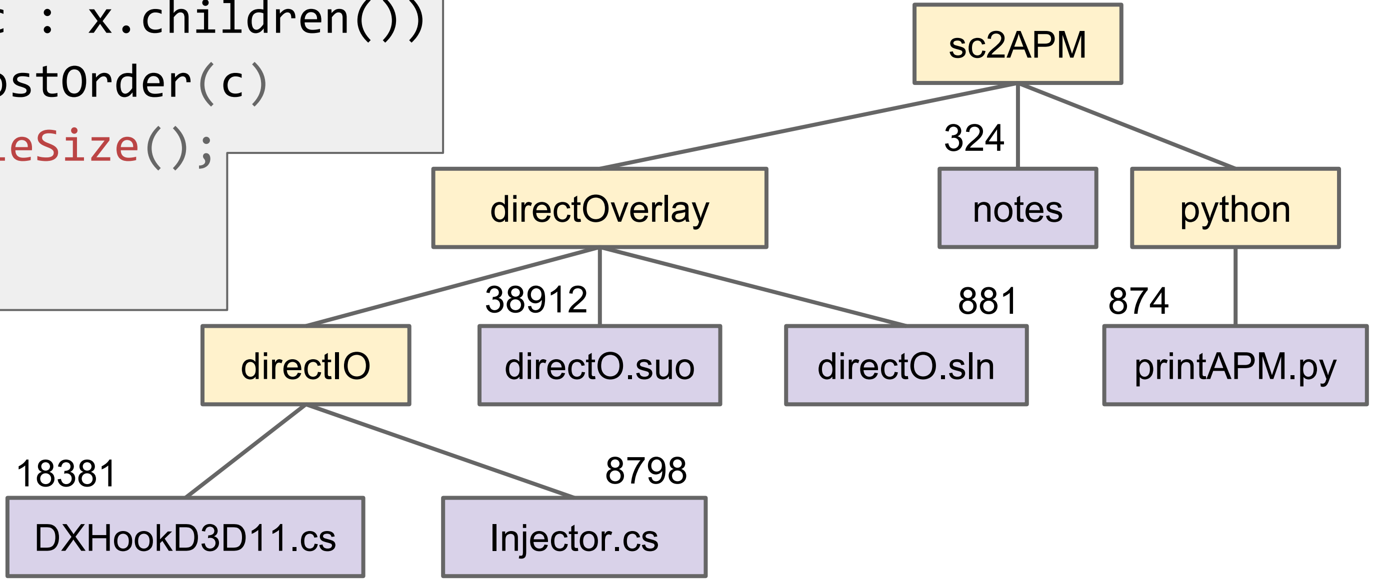
```
sc2APM/
  directOverlay/
    directIO/
      DXHookD3D11.cs
      Injector.cs
    directO.suo
    directO.sln
  notes
  python/
    printAPM.py
```



What Good Are All These Traversals?

Example: Postorder Traversal for gathering file sizes.

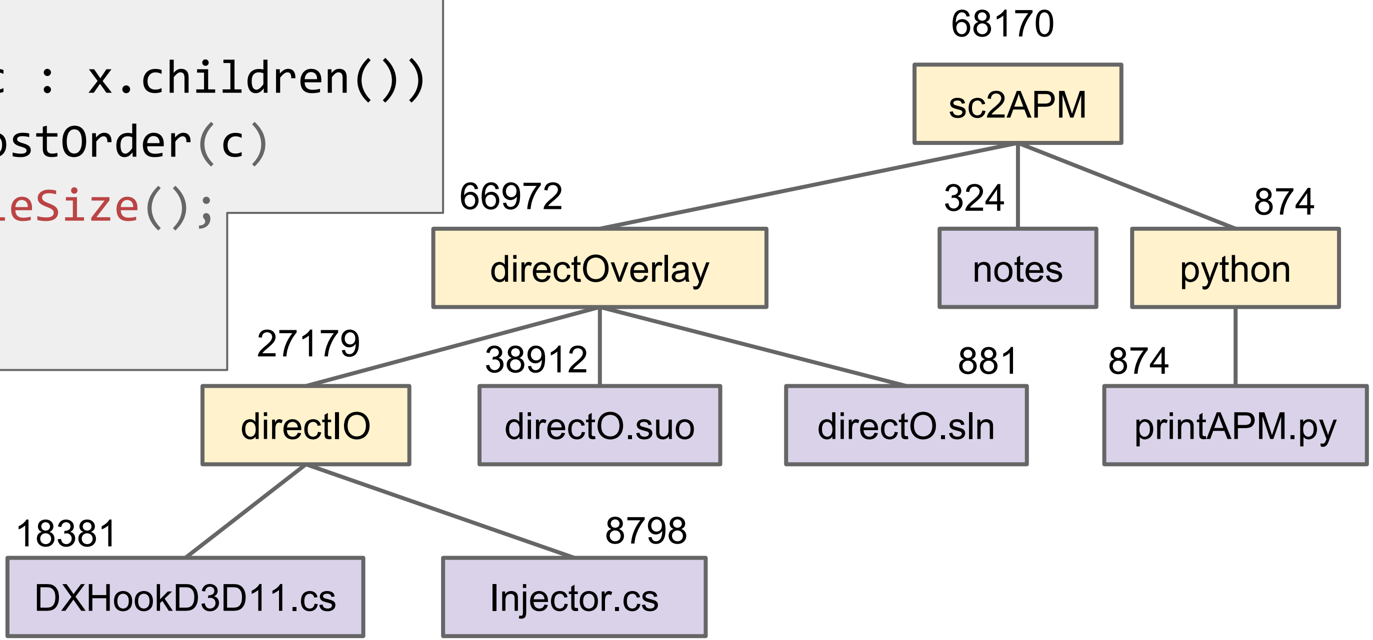
```
postOrder(BSTNode x) {
    if (x == null) return 0;
    int total = 0;
    for (BSTNode c : x.children())
       total += postOrder(c)
    total += x.fileSize();
    return total;
}
```



What Good Are All These Traversals?

Example: Postorder Traversal for gathering file sizes.

```
postOrder(BSTNode x) {
    if (x == null) return 0;
    int total = 0;
    for (BSTNode c : x.children())
        total += postOrder(c)
    total += x.fileSize();
    return total;
}
```



Visitor Pattern (Patterns)

When writing general tree traversal code. Avoid rewriting traversal for every task of interest (print, sum filesizes, etc.) by using the Visitor pattern.

```java
void preorderTraverse(Tree<Label> T, Action<Label> whatToDo) {
   if (T == null) { return; }
   whatToDo.visit(T); /* before we hard coded a print */
   preorderTraverse(T.left, whatToDo);
   preorderTraverse(T.right, whatToDo);
}
```

```java
interface Action<Label> {
   void visit(Tree<Label> T);
}
```

```java
class FindPig implements Action<String> {
   boolean found = false;
   @Override
   void visit(Tree<String> T) {
      if ("pig".equals(T.label))
         { found = true; }
   }
```

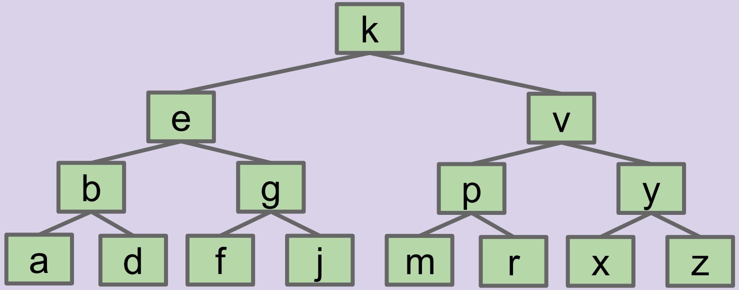
```java
preorderTraverse(someTree, new FindPig());
```

The real visitor pattern is more complex.

Preorder Traversal Runtime: http://yellkey.com/most

What is the runtime of a preorder traversal in terms of N, the number of nodes? (in code below, assume the visit action takes constant time)

1. $\Theta(1)$
2. $\Theta(\log N)$
3. $\Theta(N)$
4. $\Theta(N \log N)$
5. $\Theta(2^N)$




```
void preorderTraverse(Tree<Label> T, Action<Label> whatToDo) {
   if (T == null) { return; }
   whatToDo.visit(T);
   preorderTraverse(T.left, whatToDo);
   preorderTraverse(T.right, whatToDo);
}
```

Preorder Traversal Runtime

What is the runtime of a preorder traversal in terms of N, the number of nodes? (in code below, assume the visit action takes constant time)

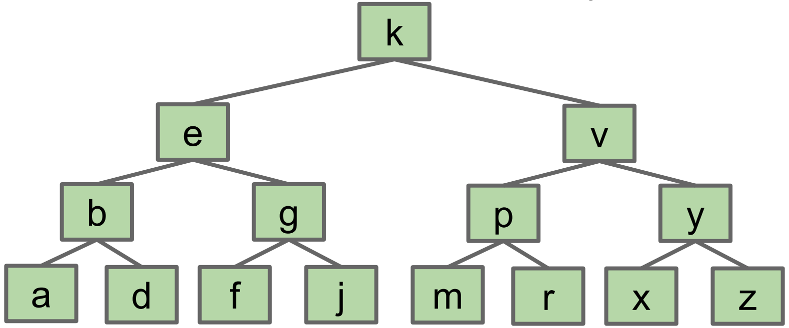
3. Θ(N) : Every node visited exactly once. Constant work per visit.

Runtime is exponential in height of the tree, not number of items.

- $\Theta(2^H)$, but $H = \Theta(\log N)$
- This is not a proof of runtime, but rather a response to a possible objection.

N = 15

H = 3



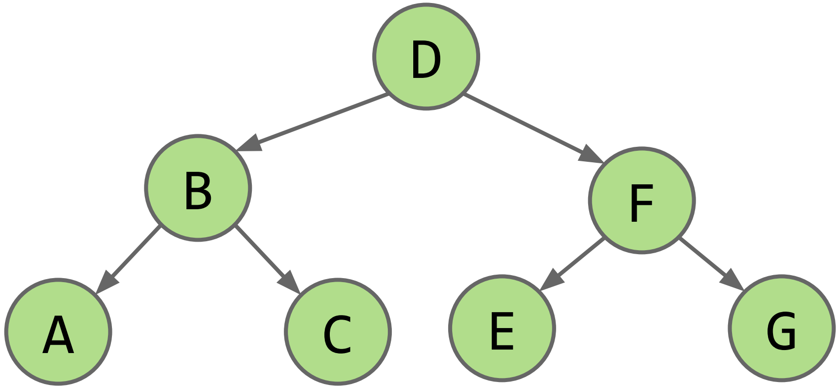
Level Order Traversal

Tree Traversal: Level Order Traversal

The Level Order Traversal is the result of reading the nodes "like a book", one level at a time.

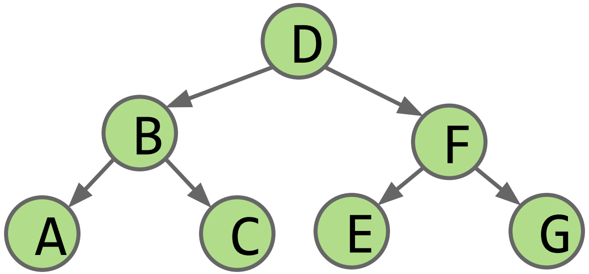
How would we implement a level order traversal?

- Level order: D B F A C E G
- Goal: Visit nodes on 0th level, then 1st level, then 2nd level, etc.



Level-Order Traversal through Iterative Deepening

```
public void levelOrder(Tree T, Action toDo) {
   for (int i = 0; i < T.height(); i += 1) {
      visitLevel(T, i, toDo);
   }
}
```



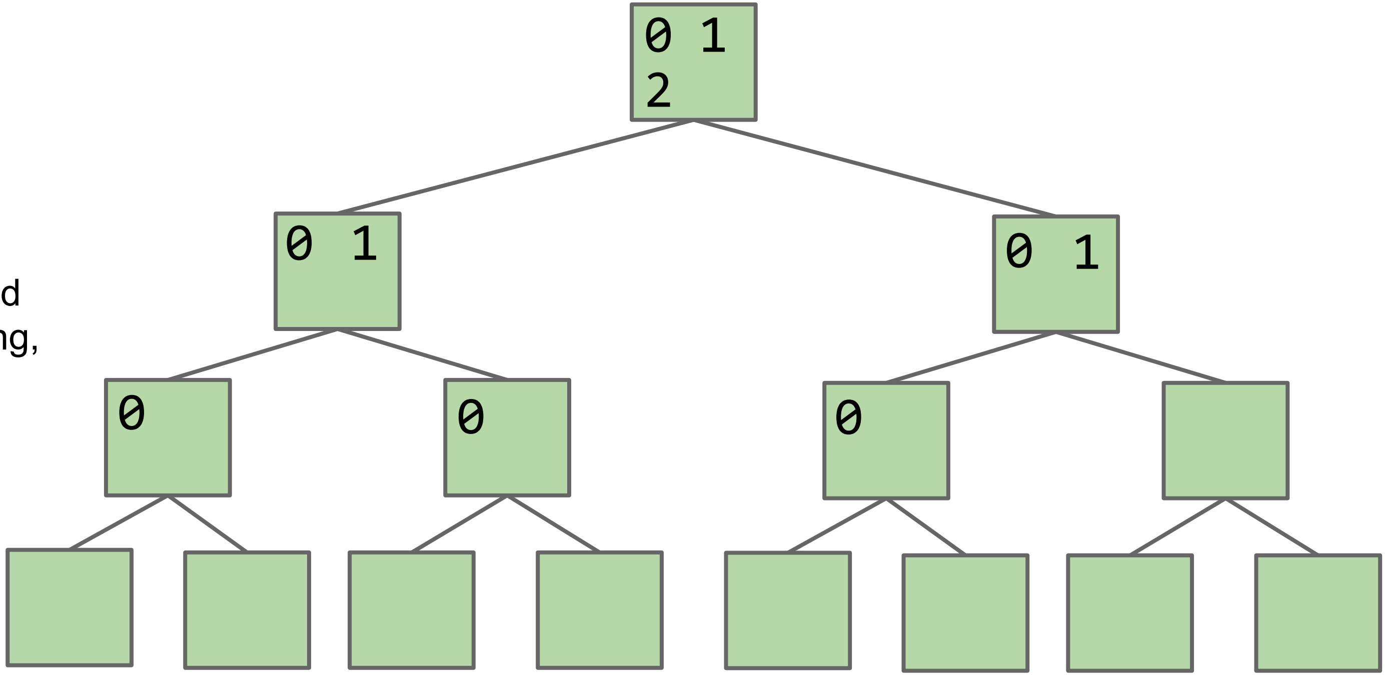
Run visitLevel H times, one for each level.

```
public void visitLevel(Tree T, int level, Action toDo) {
   if (T == null)
      { return; }
   if (lev == 0)
      { toDo.visit(T.key); }
   else {
      visitLevel(T.left(), lev - 1, toDo);
      visitLevel(T.right(), lev - 1, toDo);
   }
}
```

The strategy described on this slide is called “Iterative Deepening”.

Level-Order Traversal through Iterative Deepening

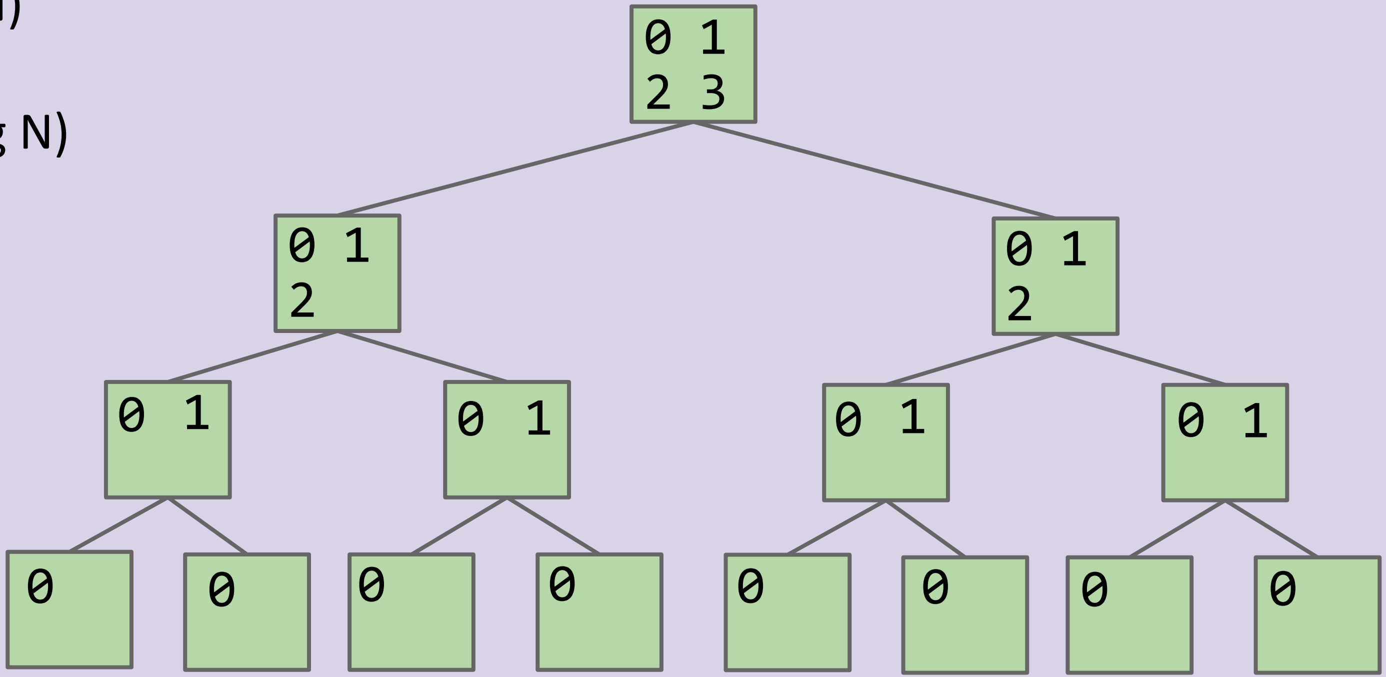
Partially completed
Iterative Deepening,



Iterative Deepening Runtime: http://yellkey.com/nor

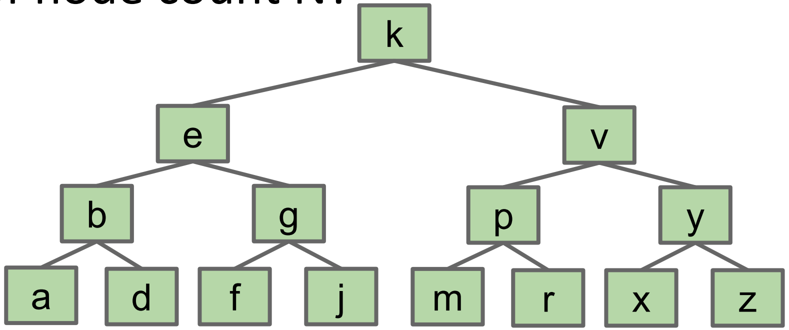
What is the runtime to complete iterative deepening on a **complete** tree (as shown below) as a function of node count N?

1. Θ(log N)
2. Θ(N)
3. Θ(N log N)
4. $\Theta(N^2)$
5. $\Theta(2^N)$



Preorder Traversal and Prefix Expressions

What is the runtime to complete iterative deepening on a **complete** tree (as shown below) as a function of node count N?



1. **Θ(N)**

Top level considered: 1

Then top two levels considered: 1 + 2 = 3

Then top three levels considered: 1 + 2 + 4 = 7

Then top four: 1 + 2 + 4 + 8 = 15

Top H levels: $2^1+2^2+2^3+...+2^H$ - H = **Θ(N)**

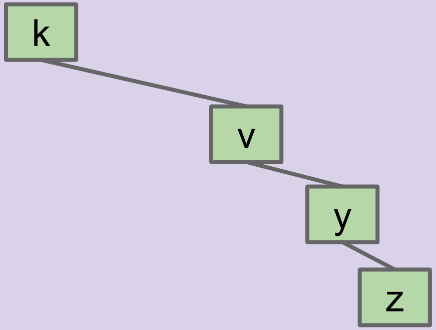
Note: Exact sum doesn't matter, the order of growth (and hence the pattern) is what is important.

Interesting aside: Much harder to solve as "4 visits at level 0" then "6 visits at level 1", etc.

Iterative Deepening Runtime: http://yellkey.com/work

What is the runtime for iterative deepening on a “spindly” tree?

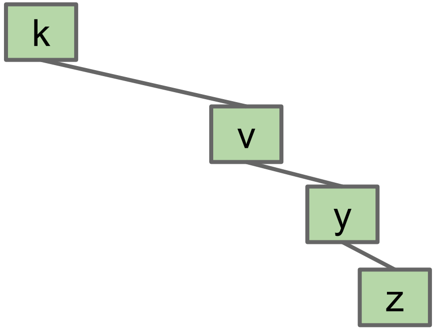
1. $\Theta(\log N)$
2. $\Theta(N)$
3. $\Theta(N \log N)$
4. $\Theta(N^2)$
5. $\Theta(2^N)$



Iterative Deepening Runtime

What is the runtime for iterative deepening on a “spindly” tree?

4. **$\Theta(N^2)$**



Top level considered: 1

Then top two levels: 1 + 1 = 2

Then top three levels: 1 + 1 + 1 = 3

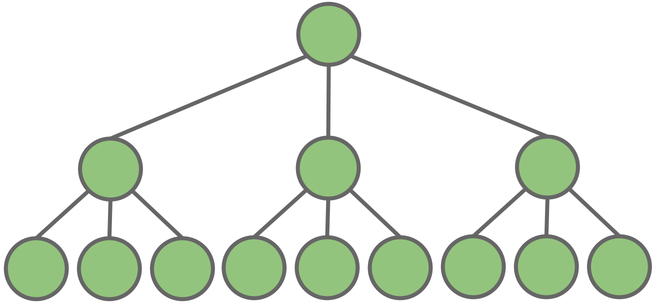
Top H levels: H

Total: $1 + 2 + 3 + \ldots + H = H^2$

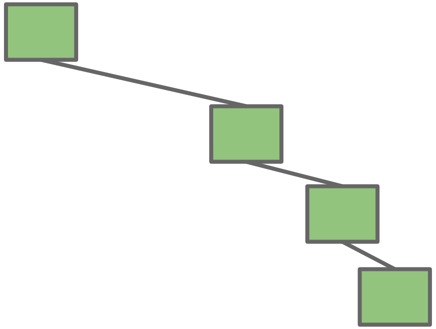
H = N - 1, so **$\Theta(N^2)$**

Tree Height

For algorithms whose runtime depends on height, difference between bushy tree and spindly tree can be huge!



$H = \Theta(\log(N))$



$H = \Theta(N)$

Iterative deepening runtimes: $\Theta(N)$ vs. $\Theta(N^2)$

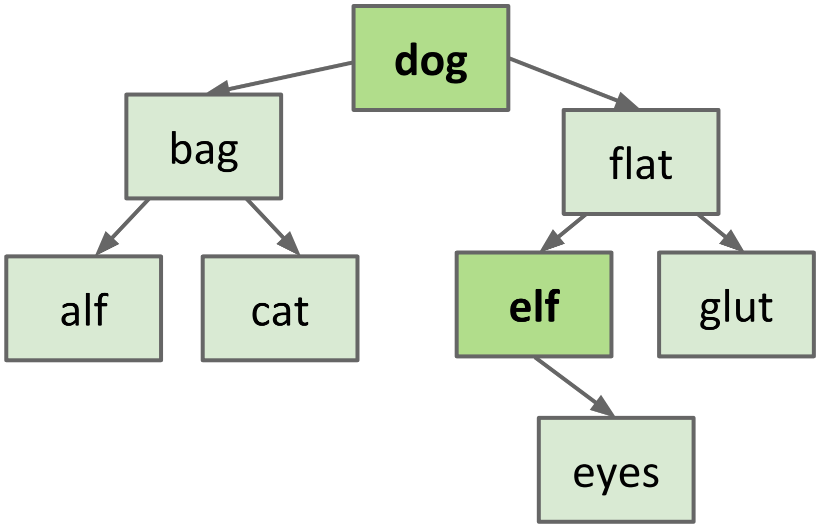
- Note: No simple mapping from height to runtime.
- Extra for experts: Write a better level order Traversal algorithm.

Range Finding

Geometric Search

Suppose we want an operation that returns all items in a range:

- `public Set<Label> findInRange(Tree T, Label min, Label max)`



Example:

- `findInRange(T, “dog”, “elves”)`
- Should return:
  - `{“dog”, “elf”}`

Geometric Search

Easy approach, just do a traversal of the whole tree, and use visitor pattern to collect matching items.

```
class rangeFind implements Action<String> {
    private Label min, max;
    public Set<Label> inRange;
    public rangeFind(Label min, Label max) {
        this.min = min; this.max = max;
        inRange = new HashSet<Label>();
    }

    void action(Tree<Label> T) {
        if (T.label ⩽ max && T.label ⩾ min) {
          inRange.add(T.label);
        }
    }
}
```

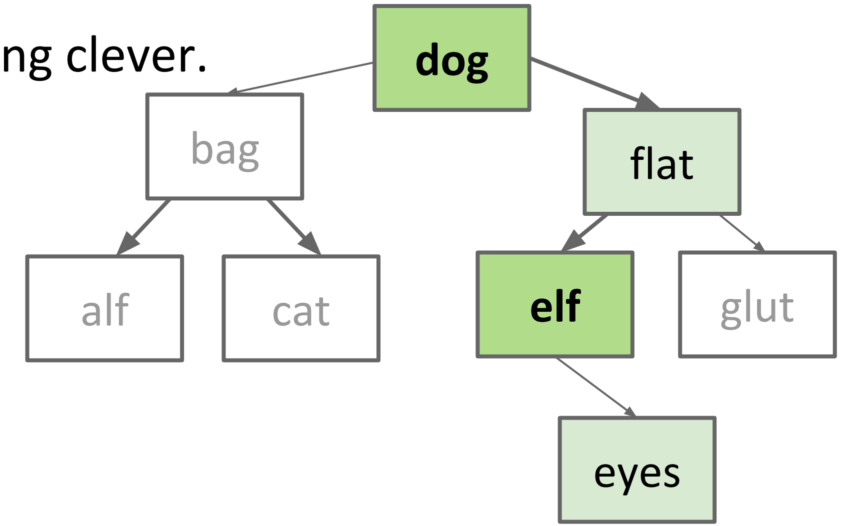
Runtime is Θ(N)

Geometric Search

Suppose we want an operation that returns all items in a range:

- `public Set<Label> findInRange(Tree T, Label min, Label max)`

Can avoid need to traverse entire tree by being clever.



Example:

- `findInRange(T, “dog”, “elves”)`
- No need to look:
  - Left of dog.
  - Right of flat.

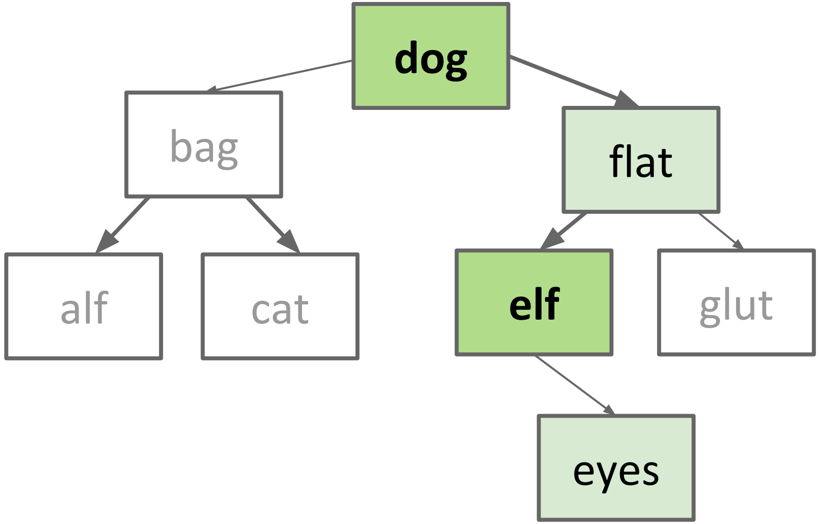
Nodes inspected: dog, flat, elf, eyes
Nodes matching: dog, elf

Pruning and findInRange Runtime

Suppose we want an operation that returns all items in a range:

- `public Set<Label> findInRange(Tree T, Label min, Label max)`

Pruning: Restricting our search to only nodes that might contain the answers we seek.



Nodes inspected: dog, flat, elf, eyes
Nodes matching: dog, elf

Pruning and findInRange Runtime

Suppose we want an operation that returns all items in a range:

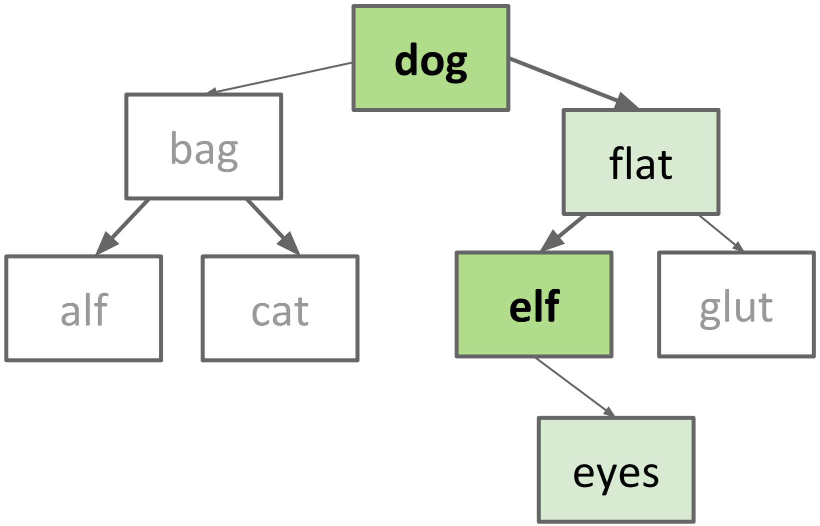
- `public Set<Label> findInRange(Tree T, Label min, Label max)`

Pruning: Restricting our search to only nodes that might contain the answers we seek.

Runtime for our search: $\Theta(\log N + R)$

- N: Total number of items in tree.
- R: Number of matches.

See study guide A-level problems for proof.



Nodes inspected: dog, flat, elf, eyes
Nodes matching: dog, elf

Spatial Trees

2D Range Finding

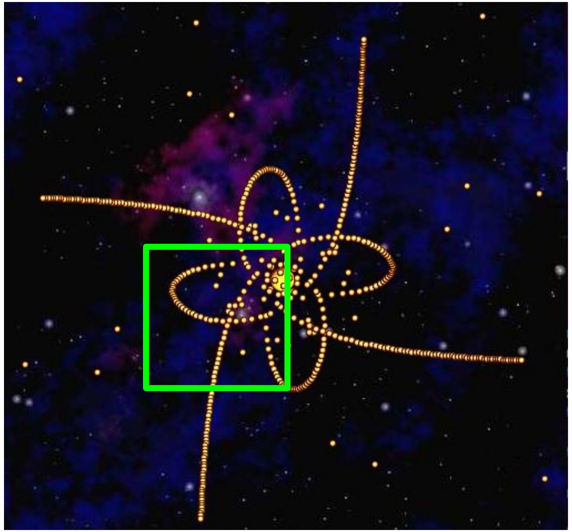
Suppose we want to do range finding on Planets in space.

- Query: How many objects are in the highlighted rectangle?

Could iterate through all objects in Θ(N) time.

- But could we do some sort of tree + pruning?

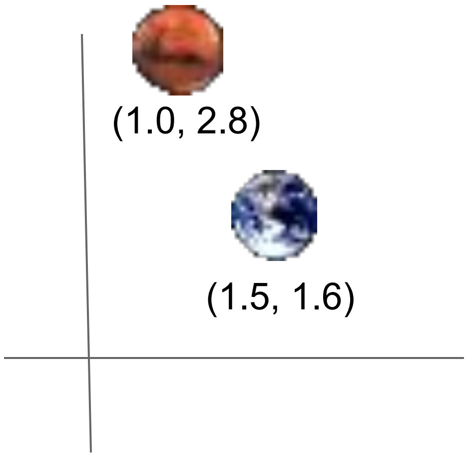
Pruning implies we need some kind of tree, but ...



Building Trees of Two Dimensional Data

So far, we've only considered one dimensional data.

- There exists a total order!
  - 5 < 10
  - "alf" < "elf"

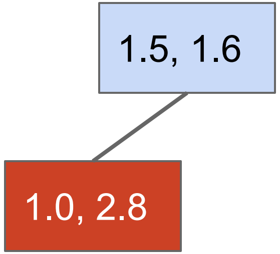


Some data is two dimensional, e.g. the location of Planets.

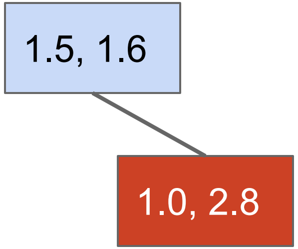
- earth.xPos = 1.5, earth.yPos = 1.6
- mars.xPos = 1.0, mars.yPos = 2.8

If we're comparing by location:

- In xPos, Mars < Earth
- In yPos, Mars > Earth



X-Based Tree



Y-Based Tree

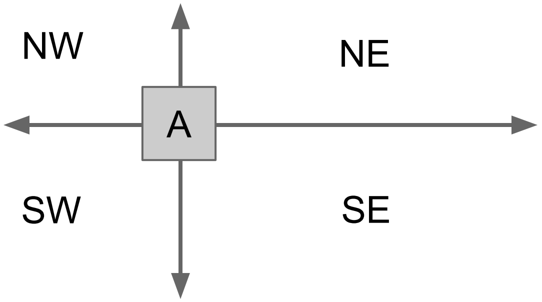
Handling Multidimensional Data: Quadtrees

Quadtrees:

- Divide and conquer by splitting 2D space into four quadrants.
    - Store items into appropriate quadrant.
    - Repeat recursively if more than one item in a quadrant.

Definition, quadtree is either:

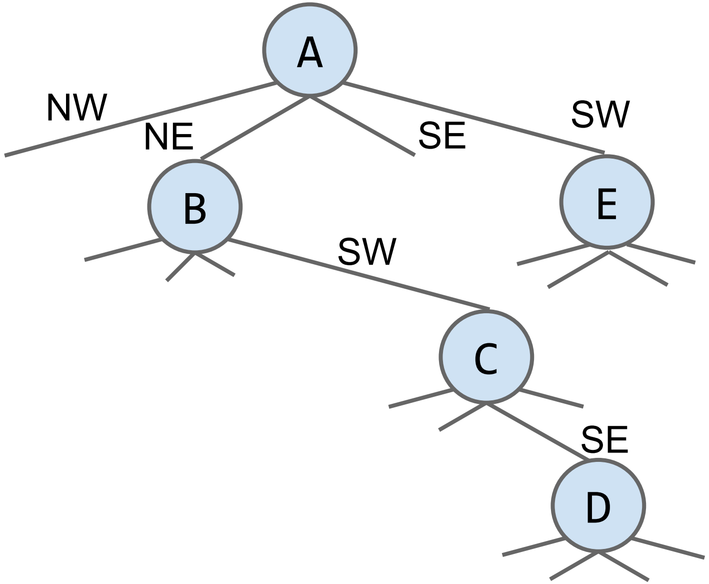
- Empty
- A ‘root’ item at some position (x, y) AND four quadtrees that are northwest, northeast, southwest, southeast of (x, y)
- Use TWO compares to decide which direction to go.

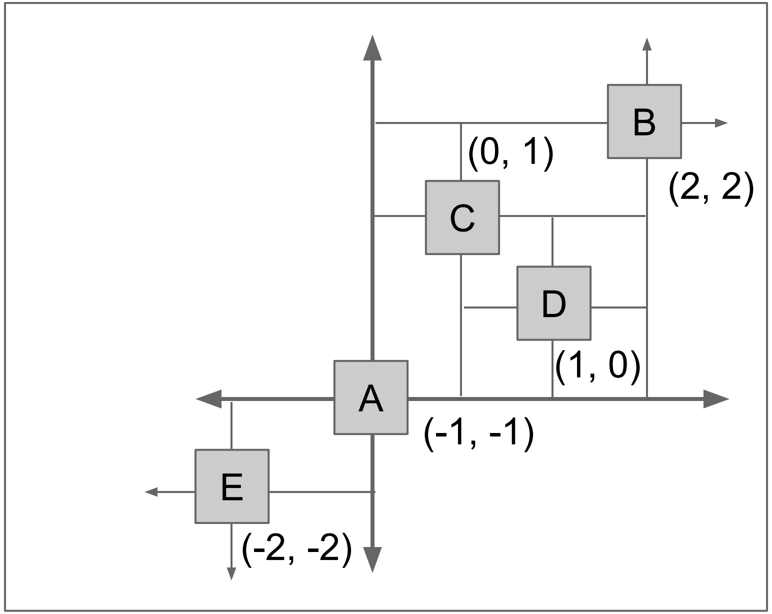


Quadtree Demo

Below: Quadtree Representation of 5 objects in 2D space.

- Demo: [Link]

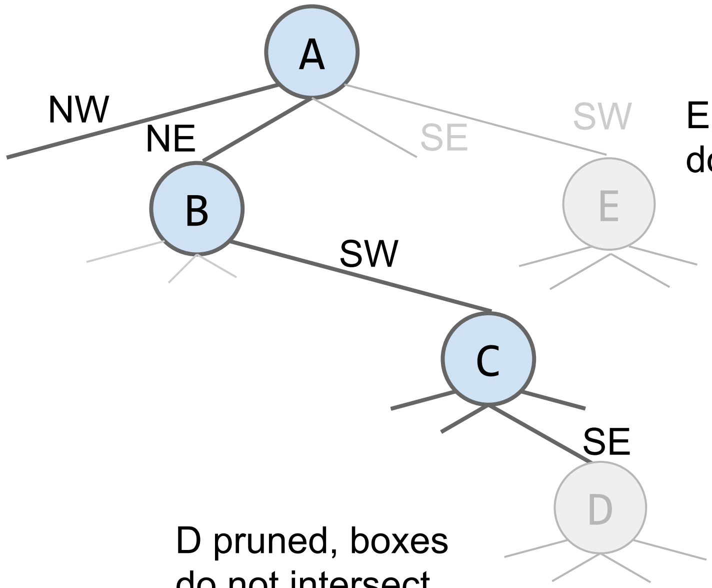




Quadtree Demo

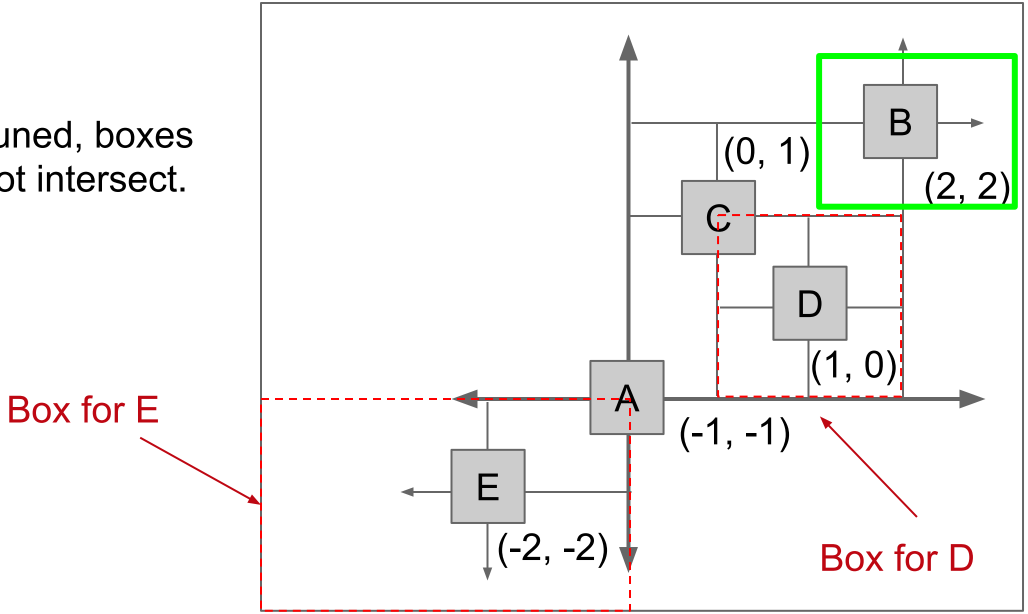
Quadtrees allow us to prune when performing a rectangle search.

- Basic rule: Prune a branch if the search rectangle doesn't overlap a quadrant of potential interest.



E pruned, boxes do not intersect.

D pruned, boxes do not intersect.



Only item that intersects box is B.

Optional: Tree Iterators

Iterators

Suppose we want to iterate through a tree using the : operator.

How can we adapt our traversal code to implement next() and hasNext()?

```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
    }
}
```

Iteration: The Obvious Way

One approach: Create an action class that puts visited item in a list.

```
public class ListBuilder<Label> implements Action<Label> {
    public List<Label> L = new ArrayList<Label>();
    public void action (Tree<Label> T) {
        L.add(T.label());
    }
}
```

```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
    }
}
```

Iteration: The Obvious Way

One approach: Create an action class that puts visited item in a list.

- iterator method creates such a list and returns an iterator to it.
- What's the downside of this solution?

```
public class ListBuilder<Label> implements Action<Label> {
    public List<Label> L = new ArrayList<Label>();
    public void action (Tree<Label> T) {
        L.add(T.label());
    }
}
```

```
public Iterator<Label> jankyIterator(Tree<Label> T) {
    ListBuilder<Label> lb = new ListBuilder<Label>();
    T.preorderTraverse(T, lb);
    return lb.L.iterator();
}
```

Iteration: Space-saving Approach

Tricky question: How could convert our recursive traversal into iterative code using a stack?

```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
    }
}
```

Observation: Each call to preorderTraverse is the equivalent of putting a call on the call stack.

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```
public void preorderTraverseIterative(Tree<Label> T, Action<Label> whatToDo)
{
    Stack<Tree<Label>> s = new Stack<Tree<Label>>();
    s.push(T);
    while (!s.isEmpty()) {
        Tree<Label> node = s.pop();
        if (node == null)
            continue;

        whatToDo.action (node);
        for (int i = node.numChildren()-1; i >= 0; i -= 1)
            s.push(node.child(i));
    }
}
```

Iteration: Space-saving Approach

Use our stack-based approach, but use next() instead of looping.

```
private class preorderIterator implements Iterator<Label>{
    Stack<Tree<Label>> s = new Stack<Tree<Label>>();
    public preorderIterator() {
        s.push(Tree.this); /* new syntax, Tree.this is parent tree */
    }
    public boolean hasNext() {
        return (!s.isEmpty());
    }
    public Label next() {
        Tree<Label> node = s.pop();
        for (int i = node.numChildren()-1; i >= 0; i -= 1)
            s.push(node.child(i));
        return node.label;
    }
}
```

Citations

Title figure: A thing I made (one of the first Java programs I wrote during my teaching career)

Pruning image:
https://res.cloudinary.com/dc8hy36qb/image/upload/v1435213404/Fruit-Tree-Pruning-Methods_o7ieen_atkmmq.jpg

Jonathan Shewchuk: Nice intuitive use cases for various traversals.