Discussion 14: ECC/Parity, RAID

Hamming ECC

Recall the basic structure of a Hamming code. Given bits 1, . . . , m, the bit at position 2n is parity for all the bits with a 1 in position n. For example, the first bit is chosen such that the sum of all odd-numbered bits is even.

Bit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Data	P1	P2	D1	P4	D2	D3	D4	P8	D5	D6	D7	D8	D9	D10	D11
P1	X		X		X		X		X		X		X		X
P2		X	X			X	X			X	X			X	X
P4				X	X	X	X					X	X	X	X
P8								X	X	X	X	X	X	X	X

1. How many bits do we need to add to 0011_2 to allow single error correction?
   Parity Bits: 3
2. Which locations in 0011_2 would parity bits be included?
   Using P for parity bits: $PP0P011_2$
3. Which bits does each parity bit cover in 0011_2?
   Parity bit #1: 1, 3, 5, 7
   Parity bit #2: 2, 3, 6, 7
   Parity bit #3: 4, 5, 6, 7
4. Write the completed coded representation for 0011_2 to enable single error correction.
   1000011_2
5. How can we enable an additional double error detection on top of this?
   Add an additional parity bit over the entire sequence.
6. Find the original bits given the following SEC Hamming Code: 0110111_2
   Parity group 1: error
   Parity group 2: okay
   Parity group 4: error
   Incorrect bit: 1 + 4 = 5, change bit 5 from 1 to 0: 0110011_2
   $0110011_2 \rightarrow 1011_2$
7. Find the original bits given the following SEC Hamming Code: 1001000_2
   Parity group 1: error
   Parity group 2: okay
   Parity group 4: error
   Incorrect bit: 1 + 4 = 5, change bit 5 from 1 to 0: 1001100_2
   $1001100_2 \rightarrow 0100_2$
8. Find the original bits given the following SEC Hamming Code: 010011010000110_2
   Parity group 1: okay
   Parity group 2: error
   Parity group 4: okay
   Parity group 8: error

Incorrect bit: 2 + 8 = 10, change bit 10 from 0 to 1: 010011010100110_2
$0100110\underline{1}0100110_2 \rightarrow 01100100110_2$

RAID

Fill out the following table:

	Configuration	Pro / Good for…	Con / Bad for…
RAID 0	Data disks without check information	No overhead Fast read / write	Reliability
RAID 1	Mirrored Disks: Extra copy of disks	Fast read / write Fast recovery	High overhead → Expensive
RAID 2	Hamming ECC: One check disk per parity group	Smaller overhead	Redundant check disks
RAID 3	Single check disk for error correction (Disk controllers can detect failures) Transfer units are spread over all disks in a group (bit interleaving)	Smallest check information overhead	Read all data disks for small accesses to detect errors
RAID 4	Transfer units = a sector within a single disk Errors are detected within a single transfer unit Independent reads/writes per disks	Higher throughput of small reads	Still slow small writes (A single check disk is a bottleneck)
RAID 5	Check information is distributed across all disks in a group.	Higher throughput of small writes	

Small accesses = an access to a single disk in a group