State

1. Fill out the timing diagram for the circuit below:

```
        +---+      +---+      +---+
   IN-|D Q|-s0-|D Q|-s1-|D Q|--Out
        +-^-+      +-^-+      +-^-+
         |          |          |
   CLK--+-------+-------+

   clk  ___|‾|_|‾|_|‾|_|‾|_|‾|_|‾|_
    in  _____|‾|_|‾‾‾‾‾|_____
    s0  ▓▓|_____|‾‾‾‾‾‾‾|_
    s1  ▓▓▓▓▓|_____|‾‾‾
   out  ▓▓▓▓▓▓▓▓▓|_____|‾
```
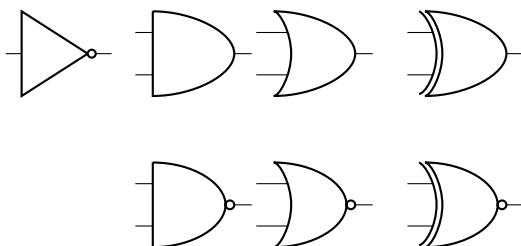
2. Fill out the timing diagram for the circuit below:

```
        +---+      +---+
   A--|D Q|-R1-|D Q|-R2--
        +-^-+      +-^-+
         |          |
   CLK--+---|>o--+

   clk  ___|‾‾|___|‾‾|___|‾‾|___|‾‾
   !clk ‾|__|‾‾|___|‾‾|___|‾‾|__|‾
    A   _|‾|_|‾‾‾‾‾|_____
    R1  ▓▓|_|‾‾‾‾‾‾‾‾‾‾‾|_____
    R2  ▓▓▓▓▓▓|‾‾‾‾‾‾‾‾‾‾‾‾‾|__
```
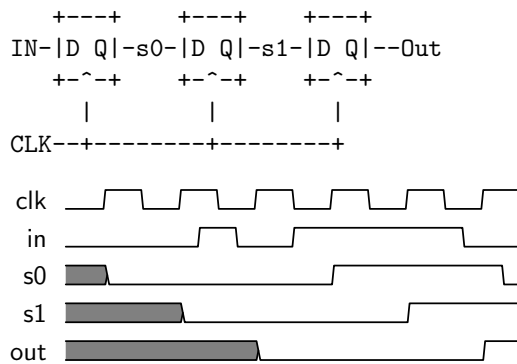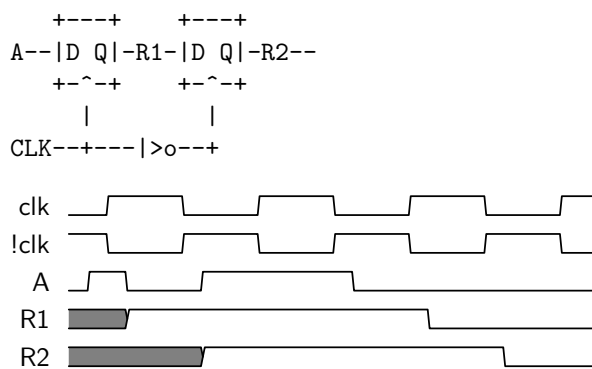
Logic Gates

1. Label the following logic gates:

> **Solution:** not, and, or, xor, nand, nor, xnor

2. Convert the following to boolean expressions:
   (a) NAND

Solution: $\bar{A}\bar{B} + \bar{A}B + A\bar{B}$ or $\bar{A} + \bar{B}$

(b) XOR

Solution: $\bar{A}B + A\bar{B}$ or $(\bar{A} + \bar{B})(A + B)$

(c) XNOR

Solution: $\bar{A}\bar{B} + AB$ or $(A + \bar{B})(\bar{A} + B)$

3. Create an AND gate using only NAND gates.

Solution:

4. How many different two-input logic gates can there be? How many n-input logic gates?

> **Solution:** A truth table with n inputs has 2^n rows. Each logic gate has a 0 or a 1 at each of these rows. Imagining a function as a 2^n-bit number, we count 2^{2^n} total functions, or 16 in the case of n = 2.

Boolean Logic

$1 + A = 1$ $A + \bar{A} = 1$ $A + AB = A$ $(A + B)(A + C) = A + BC$
$0B = 0$ $B\bar{B} = 0$ $A + \bar{A}B = A + B$
DeMorgan's Law: $\overline{AB} = \bar{A} + \bar{B}$ $\overline{A + B} = \bar{A}\bar{B}$

1. Minimize the following boolean expressions:

   (a) Standard: $(A + B)(A + \bar{B})C$

   > **Solution:**
   > $$(AA + A\bar{B} + AB + B\bar{B})C = (A + A(\bar{B} + B))C = AC \tag{1}$$

   (b) Grouping & Extra Terms: $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + A\bar{B}C$

   > **Solution:**
   > $$
   > \begin{aligned}
   > \bar{A}\bar{C}(\bar{B} + B) + A\bar{C}(B + \bar{B}) + AC(B + \bar{B}) &= \bar{A}\bar{C} + A\bar{C} + AC \tag{2}\\
   > &= \bar{A}\bar{C} + A\bar{C} + A\bar{C} + AC \tag{3}\\
   > &= (\bar{A} + A)\bar{C} + A(\bar{C} + C) \tag{4}\\
   > &= A + \bar{C} \tag{5}
   > \end{aligned}
   > $$

   (c) DeMorgan's: $\overline{A(\bar{B}\bar{C} + BC)}$

   > **Solution:**
   > $$
   > \begin{aligned}
   > \overline{A(\bar{B}\bar{C} + BC)} &= \bar{A} + \overline{\bar{B}\bar{C} + BC} \tag{6}\\
   > &= \bar{A} + \overline{\bar{B}\bar{C}}\,\overline{BC} \tag{7}\\
   > &= \bar{A} + (B + C)(\bar{B} + \bar{C}) \tag{8}\\
   > &= \bar{A} + B\bar{C} + \bar{B}C \tag{9}
   > \end{aligned}
   > $$