

# Machine Learning

## Linear Models

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# Polynomial Models

Consider a regression problem.

linear in the parameters

Can we as hypothesis set the set of polynomials of degree  $r$  with the tools we have already developed for linear regression?

Assume  $X = \mathbb{R}$

polynomial of degree  $r$ :  $\sum_{i=0}^r w_i \cdot x^i = w_0 \cdot 1 + w_1 \cdot x + w_2 x^2 + \dots + w_r x^r$

Given  $x \in \mathbb{R}$ , compute the following vector: (feature expansion)

$\vec{x}^1 = \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \\ \vdots \\ x^r \end{bmatrix}$ ,  $\vec{w} = [w_0, w_1, \dots, w_r]^T \Rightarrow \langle \vec{w}, \vec{x}^1 \rangle = w_0 \cdot 1 + w_1 x + \dots + w_r x^r$

$\Rightarrow$  the hypothesis class of linear models for  $\vec{x}^1$  corresponds to the hypothesis class of polynomial models of degree  $r$  for  $x$ .

Consider  $X = \mathbb{R}^d$ ,  $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$ . You can use the following feature expansion:

$$\vec{x}' = [1, x_1, x_2, \dots, x_d, x_1^2, x_2^2, \dots, x_d^2, \dots, x_1^r, x_2^r, \dots, x_d^r]^T$$

⇒ use linear models for  $\vec{x}'$ .

Different feature expansion:

$$\vec{x} \in \mathbb{R}^3, \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, r=2$$

$$\vec{x}' = [1, x_1, x_2, x_3, x_1^2, x_2^2, x_3^2, x_1 \cdot x_2, x_1 \cdot x_3, x_2 \cdot x_3]$$

⇒ build a linear model for  $\vec{x}'$

## Feature normalization

Given the training set, we have "normalized" each feature  $x_i$ ,  $i=1, \dots, d$  so that:

- the average of each feature across the training set is 0
- the standard deviation of each feature is 1

Data normalization is important:

- stability of the computation
- interpretability of linear models (weight is high  $\Rightarrow$  feature is important)

If you build a model using normalized data  
 $\Rightarrow$  the same normalization function must be applied to the data on which you make predictions.