WRITE FIRST NAME, LAST NAME, AND ID NUMBER ("MATRICOLA") BELOW AND READ ALL INSTRUCTIONS BEFORE STARTING WITH THE EXAM! TIME: 2.5 hours.

FIRST NAME: .....

LAST NAME: .....

ID NUMBER: .....

INSTRUCTIONS

- solutions to exercises must be in the appropriate spaces, that is:
  - Exercise 1: pag. 1, 2, 3
  - Exercise 2: pag. 4, 5, 6
  - Exercise 3: pag. 7, 8, 9
  - Exercise 4: pag. 10, 11, 12

  Solutions written outside the appropriate spaces (including other papersheets) will not be considered.
- the use of notes, books, or any other material is forbidden and will make your exam invalid;
- electronic devices (smartphones, calculators, etc.) must be turned off; their use will make your exam invalid;
- this booklet must be returned in its entirety.

Exercise 1 [8 points]

1. With reference to the binary classification problem, introduce the concepts of model class, loss function, empirical risk and (expected) risk.
2. In the context above, provide the formulation of PAC learning.
3. Discuss the role that the model class complexity plays in determining sample complexity.

[Solution: Exercise 1]

[Solution: Exercise 1]

[Solution: Exercise 1]

Exercise 2 [8 points]

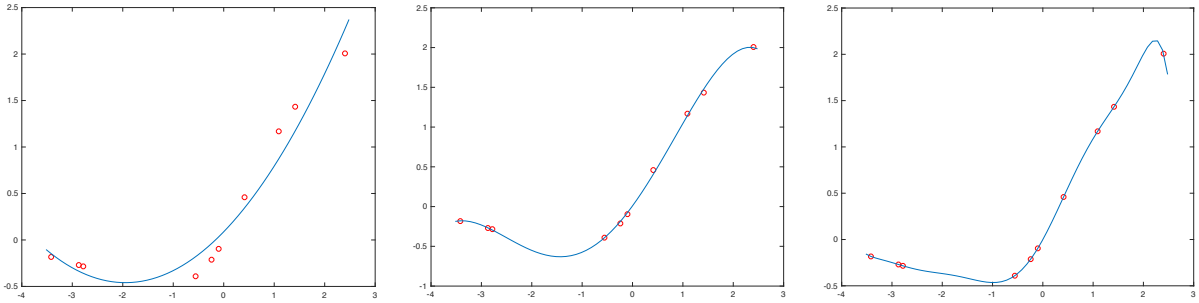
With reference to the regression problem:

1. Formulate the problem of estimating a function $h(x) : \mathbb{R}^d \to \mathbb{R}$ under the squared loss

2. Assume you have to choose among the three model classes $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$; let

$$\hat{h}_i := \arg\min_{h \in \mathcal{H}_i} L_S(h)$$

be as in the figure below. Which of the three model classes would you immediately discard and how would you choose between the other two?



$\hat{h}_1 \in \mathcal{H}_1$ $\hat{h}_2 \in \mathcal{H}_2$ $\hat{h}_3 \in \mathcal{H}_3$

[Solution: Exercise 2]

[Solution: Exercise 2]

[Solution: Exercise 2]

Exercise 3 [8 points]

1. Describe hard SVM for binary classification, highlighting how it is different from "standard" linear methods (e.g., the perceptron).

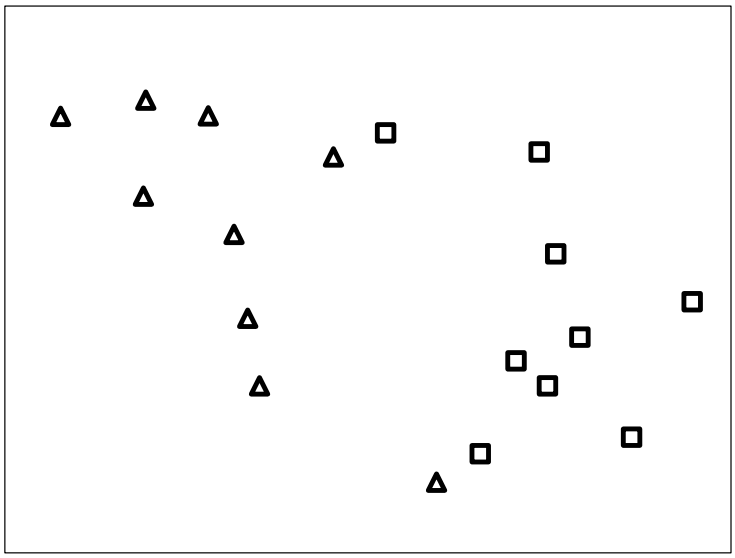
2. Describe the difference between hard SVM and soft SVM, using the objective function of soft SVM for the comparison.

3. The following figure shows an input for soft SVM for binary classification on the data points (in $\mathbb{R}^2$) in the figure, where the class of each point is represented by its shape (triangle or square). Let λ be the (regularization) parameter for the objective function of soft SVM. Draw in the figure below (approximate) solutions for the soft SVM when:

   (a) the value of λ is ≈ 0;

   (b) the value of λ is high.

   Explain the reasoning you followed to derive the solution.



[Solution: Exercise 3]

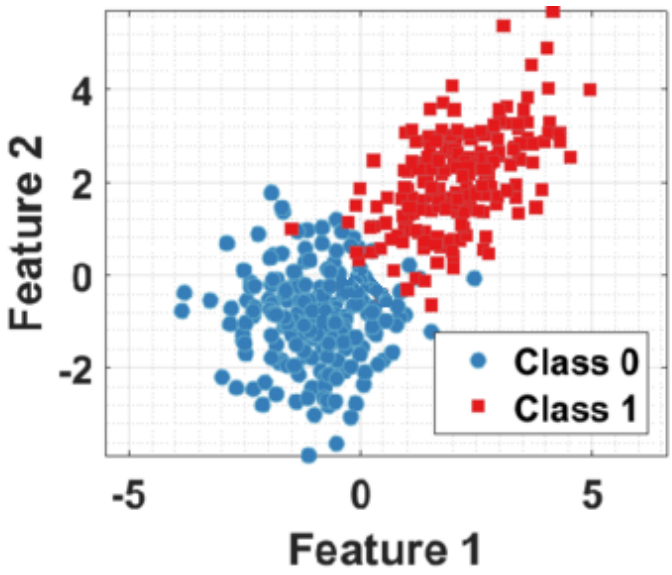
[Solution: Exercise 3]

[Solution: Exercise 3]

Exercise 4 [8 points]

Reducing the dimensionality of data can help machine learning tools to achieve better performances. Various techniques for this task have been proposed.

- Present a dimensionality reduction technique a briefly explain how it works.
- Assume you need to classify the data in the Figure using a machine learning algorithm. Firstly describe what happens when you apply your dimensionality reduction technique to the data in the Figure in order to convert them from the bi-dimensional representation in the Figure to a one dimensional representation (you can use a drawing to show the transformation).
- Does your dimensionality reduction approach allow to use a simpler classifier in order to recognize the two classes? Explain how could you classify the data before and after applying your dimensionality reduction tool.



[Solution: Exercise 4]

[Solution: Exercise 4]

[Solution: Exercise 4]