

Machine Learning

Computer Engineering

Fabio Vandin

October 2nd, 2023

A Formal Model (Statistical Learning)

We have a *learner* (us, or the machine) has access to:

① **Domain set** \mathcal{X} : set of all possible objects to make predictions about

- domain point $\mathbf{x} \in \mathcal{X} = \text{instance}$, usually represented by a vector of features
- \mathcal{X} is the *instance space*

$$\mathcal{X}_1 = \{ \text{all graduates in CS} \}$$
$$\vec{x} \in \mathcal{X}_1, \vec{x} \in \mathbb{R}^k$$

② **Label set** \mathcal{Y} : set of possible labels.

- often two labels, e.g. $\{-1, +1\}$ or $\{0, 1\}$

③ **Training data** $S = ((x_1, y_1), \dots, (x_m, y_m))$: finite sequence of labeled domain points, i.e. pairs in $\mathcal{X} \times \mathcal{Y}$

- this is the learner's **input**
- S : *training example* or *training set*

$$\mathcal{Y} = \{ \text{F UoW, NOT F UO} \}$$
$$= \{ +1, -1 \}$$

Training data S : 20 (vectors, labels)

\downarrow

$\begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix}^{\text{age}} \vdots$

A Formal Model

- ④ **Learner's output** h : prediction rule $h : \mathcal{X} \rightarrow \mathcal{Y}$
 - also called *predictor*, *hypothesis*, or *classifier*
 - $A(S)$: prediction rule produced by learning algorithm A when training set S is given to it
 - sometimes \hat{f} used instead of h

A Formal Model

- ④ **Learner's output** h : prediction rule $h: \mathcal{X} \rightarrow \mathcal{Y}$
 - also called *predictor*, *hypothesis*, or *classifier*
 - $A(S)$: prediction rule produced by learning algorithm A when training set S is given to it
 - sometimes \hat{f} used instead of h
- ⑤ **Data-generation model**: instances are generated by some probability distribution and labeled according to a function
 - \mathcal{D} : probability distribution over \mathcal{X} (**NOT KNOWN TO THE LEARNER!**)
 - labeling function $f: \mathcal{X} \rightarrow \mathcal{Y}$ (**NOT KNOWN TO THE LEARNER!**)
 - label y_i of instance x_i : $y_i = f(x_i)$, for all $i = 1, \dots, m$
 - each point in training set S : first sample x_i according to \mathcal{D} , then label it as $y_i = f(x_i)$
- ⑥ **Measures of success**: *error of a classifier* = probability it does not predict the correct label on a random data point generate by distribution \mathcal{D}