Quantifying Health

Correlation vs Collinearity vs Multicollinearity

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Here's a table that summarizes the differences between correlation, collinearity and multicollinearity:

	Correlation	Collinearity	Multicollinearity
Definition	Correlation refers to the linear relationship between 2 variables	Collinearity refers to a problem when running a regression model where 2 or more independent variables (a.k.a. predictors) have a strong linear relationship	Multicollinearity is a special case of collinearity where a strong linear relationship exists between 3 or more independent variables even if no pair of variables has a high correlation
Number of variables involved	2	2 or more	3 or more
Can be evaluated using	Correlation coefficient	Correlation matrix or VIF	VIF

In this article, we're going to discuss correlation, collinearity and multicollinearity in the context of linear regression:

$$Y = \beta_0 + \beta_1 \times X_1 + \beta_2 \times X_2 + ... + \varepsilon$$

One important assumption of linear regression is that a linear relationship should exist between each predictor X_i and the outcome Y. So, a strong correlation between these variables is considered a good thing.

However, when correlation exists between the predictors, we can no longer determine the effect of 1 while holding the other constant because the 2 variables change together. The result is that their coefficients will become less exact and less interpretable.

The strong correlation between 2 independent variables will cause a problem when interpreting the linear model and this problem is referred to as *collinearity*.

In fact, collinearity is a more general term that also covers cases where 2 or more independent variables are linearly related to each other. So collinearity can exist either because a pair of predictors are correlated or because 3 or more predictors are linearly related to each other. This last case is sometimes referred to as *multicollinearity*.

Note that because multicollinearity is a special case of collinearity, some textbooks refer to both situations as collinearity such as: *Regression Modeling Strategies* by Frank Harrell and *Clinical Prediction Models* by Ewout Steyerberg. Others, such as *An Introduction to Statistical Learning* by Gareth James et al. prefer to make that distinction.

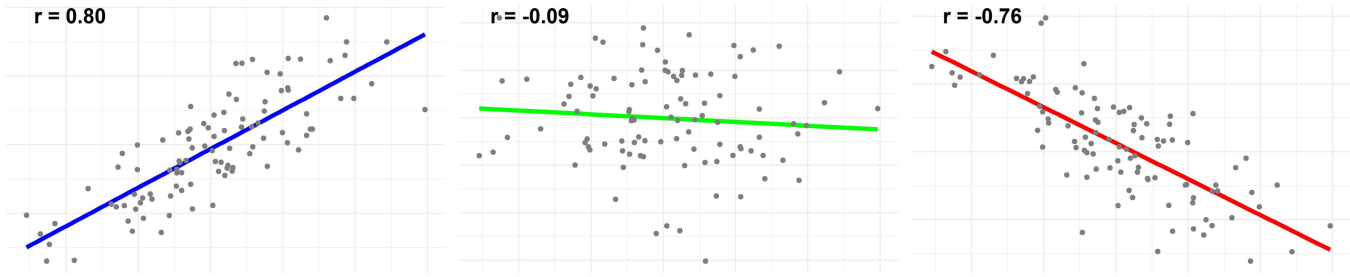
Correlation coefficient, correlation matrix and VIF

The correlation coefficient r can help us quantify the linear relationship between 2 variables.

r is a number between -1 and 1 ($-1 \leq r \leq 1$):

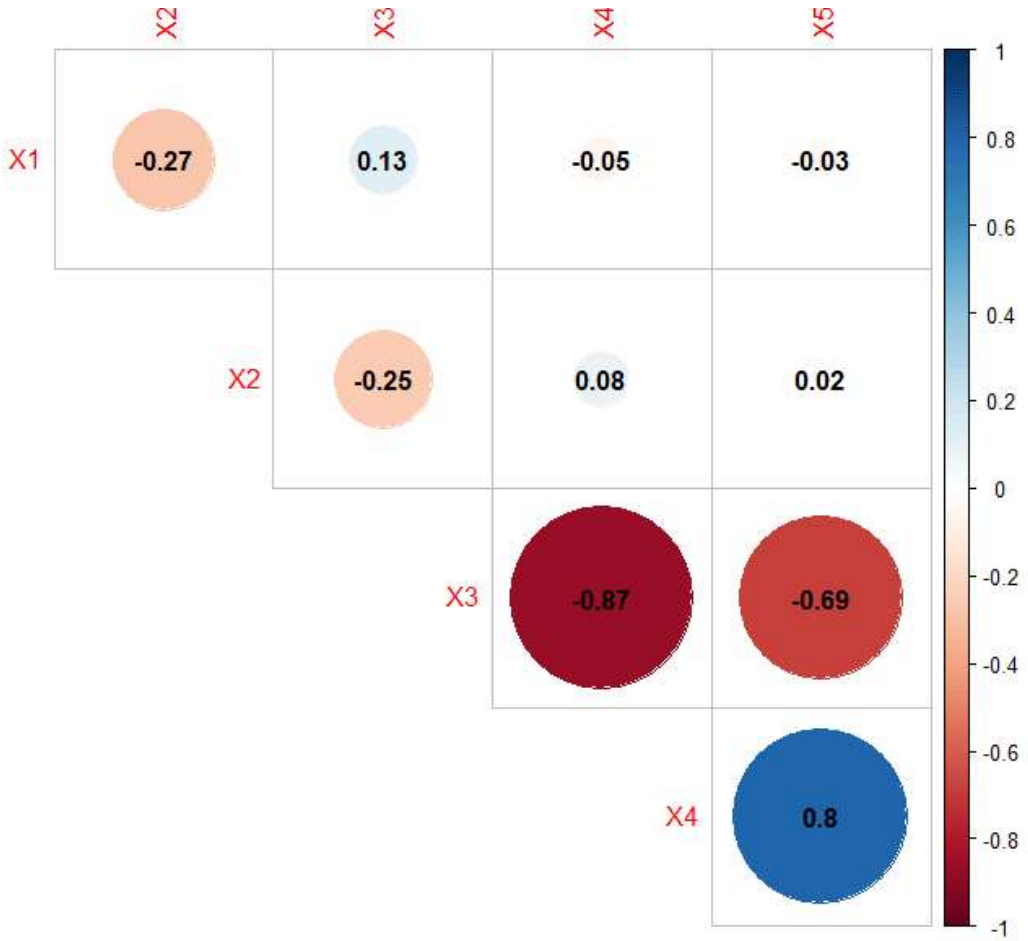
- **A value of r close to -1**: means that there is negative correlation between the variables (when one increases the other decreases and vice versa)
- **A value of r close to 0**: indicates that the 2 variables are not correlated (no linear relationship exists between them)
- **A value of r close to 1**: indicates a positive linear relationship between the 2 variables (when one increases, the other does)

Here are 3 plots to visualize the relationship between 2 variables with different correlation coefficients. The first was drawn with a coefficient r of 0.80, the second -0.09 and the third -0.76:



When we have a linear model with multiple predictors (X_1, X_2, X_3, ...), we can compute the correlation coefficient for each pair and put it in a matrix.

This correlation matrix can help us identify collinearity. Here's an example:



The correlation matrix above shows signs of collinearity as the absolute value of the correlation coefficients between X_3-X_4 and X_4-X_5 are above 0.7 [source].

However, because collinearity can also occur between 3 variables or more, EVEN when no pair of variables is highly correlated (a situation often referred to as "multicollinearity"), the correlation matrix cannot be used to detect all cases of collinearity.

This is where the variance inflation factor (VIF) comes to the rescue.

Here's the formula for calculating VIF for the variable X_1 in the model:

$$VIF_1 = \frac{1}{1 - R^2}$$

In this equation, R^2 is the coefficient of determination from the linear regression model which has:

- X_1 as dependent variable
- X_2, X_3, X_4, ... as independent variables

i.e. R^2 comes from the following linear regression model:

$X_1 = \beta_0 + \beta_1 \times X_2 + \beta_2 \times X_3 + \beta_3 \times X_4 + ... + \varepsilon$

Because R^2 is a number between 0 and 1:

- When R^2 is close to 1 (X_2, X_3, X_4, ... are highly predictive of X_1): the VIF will be very large
- When R^2 is close to 0 (X_2, X_3, X_4, ... are not related to X_1): the VIF will be close to 1

As a rule of thumb, a VIF > 10 is a sign of multicollinearity [source: *Regression Methods in Biostatistics*, Vittinghoff et al.]. However, this is somewhat an oversimplification. For more details on how to choose a threshold to detect multicollinearity and how to interpret a VIF value, I suggest my other article: What is an Acceptable Value for VIF?

Example: multicollinearity without correlation between any pair of predictors

Here's the R code if you want to follow along:

```
library(corrplot)
library(regclass)

# First define the predictors such that x5 is "slightly" related to all of
the others
set.seed(1)
x1 = rnorm(100)
x2 = rnorm(100)
x3 = rnorm(100)
x4 = rnorm(100)
x5 = 0.1*x1 + 0.1*x2 + 0.1*x3 + 0.1*x4 + rnorm(100)*0.03

# y will be our depedent variable
y = rnorm(100)

# pack all the variables into a data frame
df = data.frame(X1=x1, X2=x2, X3=x3, X4=x4, X5=x5, Y=y)

# plot the correlation matrix
corrplot(cor(df[,c("X1", "X2", "X3", "X4", "X5")]), diag = FALSE,
type="upper",addCoef.col = "black")

# then take a look at the VIF of each predictor
VIF(lm(Y ~ X1 + X2 + X3 + X4 + X5, data=df))
```

After running the code above, we get the following correlation matrix:

As you can see, the correlation matrix shows no sign of pairwise collinearity as all correlation coefficients are below 0.7.

However, looking at the VIF of each variable:

```
      X1       X2       X3       X4       X5
8.662448 9.599640 10.272285 9.574024 34.629869
```

We see that 2 of them have a VIF > 10 signaling a multicollinearity problem.

Further reading

- Variables to Include in a Regression Model
- 7 Tricks to Get Statistically Significant p-Values
- Residual Standard Deviation/Error: Guide for Beginners
- Understand the F-statistic in Linear Regression
- P-value: A Simple Explanation for Non-Statisticians

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