Within-Country Leakage due to the Exemption of Small Emitters from Emissions Pricing*

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Abstract

I show how the exemption of small-scale emitting firms from emissions pricing results in within-country emissions leakage — an emissions price increase for the regulated firms prompts an increase in the emissions of the unregulated. I use a heterogeneous firm model in which a fixed share of firms is subject to emissions pricing. The firms at the lower part of the productivity distribution benefit from being exempted, such that the higher the emissions price, the more and dirtier firms can survive in the domestic market. Leakage is stronger if firms are exempted only if they emit less than a fixed threshold (as for the EU Emission Trading System) because some firms strategically bunch below the threshold, making the emissions price an even weaker tool to reduce total emissions. In environments with low social costs of emission or high fixed regulatory costs, an exemption may be justified; over time, however, the criteria for exemptions should be adjusted accordingly.

Keywords: Asymmetric Emissions Pricing, Emissions Leakage, Heterogeneous Firms, Monopolistic Competition

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1. Introduction

Leakage refers to a phenomenon in which a tightening of environmental policies results in emission reductions for the targeted subjects, but increases the emissions outside of the regulatory scope. Leakage is commonly analysed as a problem across countries (Copeland and Taylor, 1995; Balistreri and Rutherford, 2012; Richter, Kreickemeier and Egger, 2021). As I show in this paper, it can also occur within a country when a number of firms are exempted from emissions pricing. In particular, I look into the implications of the exemption on emissions, whether it can actually "rescue" some firms from exiting the market, and whether it is welfare-maximising.

My research is motivated by the European Union (EU) Emission Trading System (ETS) and similar regulations,[1] which allow small-scale emitting instalments to be exempted from participation in the carbon pricing system in some sectors (European Commission, 2021*b*). The reasoning behind the regulation is that smaller emitters suffered from transaction costs of the first versions of the EU ETS relatively more than large-scale firms. Those transaction costs are "mainly administrative in nature" according to Jaraitė, Convery and Di Maria (2010) and include e.g. monitoring, reporting, and verification of emissions (Heindl, 2017). Article 27 of the European Commission (2009) states that small emitters can opt-out from the EU ETS if they introduce equivalent measures. Small emitters are installations that emit less than 25 000 tons CO_2 equivalent per year.[2]

To analyse the effect of the exemption regulation, I introduce discontinuous emissions pricing in a model with firm heterogeneity in productivity and monopolistic competition à la Melitz (2003). Firms generate greenhouse gas emissions in their production along the lines of Copeland and Taylor (2004). The model features emissions pricing - but to capture the exemption, I add the novel feature of only applying it to the most productive firms. In my baseline model, a fixed share of active firms is regulated. I furthermore examine the alternative of regulating firms above an emissions threshold and two extensions (free allocation and international trade). Finally, I analyse the role of the emissions price and the exemption scope for welfare under those different model assumptions.

I obtain five results: First, I find that an exemption of firms from emissions pricing results in within-country emissions leakage - for an increase in the emissions price, the aggregate emissions of the regulated firms decrease, but the ones of the unregulated firms go up. The effect is caused by the emissions price affecting firms' profits asymmetrically along the productivity distribution. If the price increases, the regulated, more productive firms decrease their emissions intensity, leading to a reduction in their aggregate emissions. On the

[1] For example, Singapore and Chile have an emissions threshold for their carbon tax (NCCS Singapore, 2022; ICAP, 2022), and most jurisdictions of Canada have pollution levies applying only to "large industrial emitters" (Sawyer, Stiebert, Gignac, Campney and Beugin, 2021)

[2] If they are combustion installations, they additionally need a thermal rated input under 35MW.

other hand, the non-regulated firms do not change their emissions intensity, but rather can increase their sales volume given an increased price level - they now benefit relatively more from being exempted than before. As a result, the cutoff productivity at which firms break even is pushed down so that less productive - and by definition dirtier - firms are able to survive.

Second, within-country leakage is higher when the exemption is not determined with a fixed share of unregulated firms, but rather with an emissions threshold (as in the EU ETS): Firms may decide to abate more such that they can self-select into the group of non-regulated firms. The shadow value of being exempted from emissions pricing goes up when the emissions price increases: More firms than before choose to be exempted. Hence, the emissions of the unregulated firms increase, making the emissions price an even less efficient tool to drive down aggregate emissions.

Third, the emissions price is more effective when it can target a higher market share, which is the case if the most productive firms have the option to export their goods abroad. Similarly, if the emissions pricing is complemented with free allocation of emissions rights, firms are incentivised to stay regulated; hence, it limits the output losses from emissions pricing as it shifts more resources to the more efficient firms. As both extensions demonstrate positive impacts on welfare, they result in the recommendation of a higher optimal emissions price and a greater proportion of regulated firms.

Fourth, allowing firms to bunch at an emissions threshold is mostly negative for welfare because it distorts optimal production levels. If the threshold is based on firms' emissions, the bunching is at least benefiting welfare from the emissions perspective as aggregate social costs of emissions are reduced. Free allocation in this environment has different implications than in the one with a fixed share of regulated firms: If the emissions reductions resulting from bunching cannot be realised because firms are incentivised to stay regulated because of free permits, the negative welfare implications of the emissions threshold are intensified.

Finally, the trade-off between saving bureaucratic effort and reducing total emissions may justify an exemption of small firms from emissions pricing: Increasing the coverage beyond the optimal level would result in less output to be consumed due to increased fixed costs of the regulation, whereas regulating fewer firms increases total emissions. If the social costs of emissions are low, or monitoring, reporting, and verification of emissions are associated with high costs, it may be reasonable to exempt some firms. When a fixed share of firms is regulated, the resource reallocation towards more productive firms induced by free allocation or trade makes justifying the same level of exemption more difficult. In conclusion, it is crucial to reassess the scope of the exemption as the fixed costs of regulation and the emissions costs for welfare evolve over time.

With these theoretical insights, I contribute to a growing literature analysing the effects of different emissions pricing designs. I am to my knowledge one of the first ones to look at exemptions from emissions pricing within one industry based on firm heterogeneity in terms of productivity, whereas other contributions to this literature look at exemptions of whole industries or sectors. Those studies show empirically that sectoral exemptions are costly and often do not reduce across-country leakage which is one of the reasons why they exist (Clò, 2010; Böhringer and Rutherford, 1997; Martin, Muûls, de Preux and Wagner, 2014).

My research studies within-country leakage and relates the exemption from emissions pricing to firm selection within an industry - by doing so, it also fits into the literature on size-dependent policies. Similarly to Guner, Ventura and Xu (2008) and Becker and Henderson (2000), I show that with an endogenous firm size distribution, a size-dependent policy results in more smaller establishments. In my model, the change in the firm size distribution is not only captured by firms bunching at the threshold (changing output for existing firms) but also additional entry at the lower end of the productivity distribution due to revenue shifting towards unregulated firms. The change in the size distribution is one channel of emissions leakage in my model; additionally, Becker, Pasurka Jr and Shadbegian (2013) mention different abatement intensities across firm sizes, so that they may contribute to leakage which Qi, Tang and Xi (2021) attribute to the fact that "larger firms are more likely to use clean technology but face higher distortions".

Similarly, in my model, abatement per output is higher for regulated firms as they face higher variable costs of emissions. Also in the work of Kaplow (2019) on optimal exemptions, regulation imposes different variable and fixed costs, but additionally, regulated firms have to adopt a different production technology. The two technologies have a different, but fixed per-unit harm such that there are no differences in energy efficiency within type - it allows for addressing the externality perfectly with output taxes. Contrary to Kaplow (2019), my model allows for an endogenous firm survival threshold, equilibrium (price level) effects, and heterogeneous energy intensities within types; hence, it enables a market share (revenue) shifting mechanism due to heterogeneous prices: unregulated firms produce more because they are relatively better off than before the regulation. Chen, Chen, Liu, Suárez Serrato and Xu (2021) show a similar mechanism: They use Chinese firm-level data in a Differences-in-Differences framework to show that the Chinese Energy Conservation Program made regulated firms (the 1 000 largest firms) shift 40% of their production loss to unregulated firms within their conglomerate instead of raising their energy efficiency. They support their findings by analysing the regulation in a heterogeneous firm model in which each conglomerate is characterised by its firm with the highest productivity. As in their work, I also base my exemption of firms from emissions pricing on the productivity of a firm but do not consider within-conglomerate effects - an additional force to the within-country leakage I present in this paper.

In its setup, my work is closest to Anouliès (2017) and Richter et al. (2021) as they both study emissions pricing in a heterogeneous firm model. Anouliès (2017) uses a cap-and-trade system, but the free permits are distributed via output-based allocation, whereas I use grandparenting as an allocation mechanism for free permits in one of my model extensions. While most of the existing literature looking at emission permit allocation schemes and firm selection has studied a set-up in autarky (Anouliès, 2017; Konishi and Tarui, 2015; Dardati and Saygili, 2020), I consider international trade as an extension to the model, similar to Richter et al. (2021) who look at an emissions tax in an open economy. None of the latter studies allows for discontinuous emissions pricing. Furthermore, an exemption via an emissions threshold allows me to look how strategic considerations of firms come into play and affect firm selection: Firms might not only "make strategic behavioural changes" in their output level (Branger, Ponssard, Sartor and Sato, 2015), but also in their choice of inputs when being confronted with discontinuous thresholds in an emissions pricing system.

In my setup, the firms with the lowest productivity are the ones with the highest emissions-intensity. Empirical studies using data from India (Barrows and Ollivier, 2018), China (Cao, Qiu and Zhou, 2016), or the US (Shapiro and Walker, 2018) show that there is indeed a negative relationship between emission intensity and productivity.[3] Other literature mainly finds a negative relation between export and CO_2 emission intensity (Richter and Schiersch, 2017; Forslid, Okubo and Ulltveit-Moe, 2018; Cui, Lapan and Moschini, 2016) which is consistent with my model findings. Exporters produce on a larger scale which allows them to spread the fixed costs of abatement investment (Forslid et al., 2018), such that there is evidence that being able to produce on a larger scale via a higher productivity is negatively related to emission intensity.

The other closely related line of research analyses the effect of different emission permit allocation mechanisms on firm selection; in my work, I focus on the exemption regulation and look at free allocation in an extension. Whereas for auctioned permits, firms have to buy all required permits via a common auction platform (for the EU ETS, the European Energy Exchange (EEX) in Leipzig (European Commission, 2021*a*)), free allocation via grandparenting or output-based allocation provides participating firms with free permits. As Fischer and Fox (2007) mention in their work, the number of free permits via grandparenting "is often determined by historical emissions or market shares, but the key aspect is that the allocation does not vary with changes in circumstances". On the contrary, the number of permits of output-based allocation is updated over time according to firms' production, such that effectively, firms' current output is subsidised by the prospect of increased free permits in the future. Similar to a carbon tax, the price of permits in an ETS is supposed to incentivise firms to reduce their emissions; e.g. via changing the scale of production, the technique, or the composition of produced goods (Grossman and Krueger, 1991). Konishi and Tarui (2015) find that neither the auctioned emissions trading nor a free distribution of permits under output-based allocation change the productivity level at which firms break even - only grandparenting of permits does so. Contrarily, Anouliès (2017) finds that the higher the ratio of free permits via output-based allocation to auctioned permits, the more reallocation occurs as less productive firms have to leave the market, affecting firm selection. Dardati and Saygili (2020) find that grandparenting as well as a uniform free allocation induce excessive firm entry and also a higher mass of incumbents. While the results of those studies depend on the model specifications, they all underline

[3] A summary can be found in Copeland, Shapiro and Taylor (2021).

one of my most important findings: Only if emissions pricing is asymmetric along the productivity distribution, it affects firm selection.

My paper is a first step towards understanding how the exemption of small-scale emitting firms from emissions pricing can create within-country leakage and affect the productivity level at which firms break even. It will neither discuss consumption-generated pollution, domestic outsourcing, across-country leakage via offshoring, nor scale effects of emission abatement investment (Forslid et al., 2018). In my model, firms face a static profit maximisation problem such that any dynamic considerations like investment and R&D in green technologies are not considered.

The rest of the paper is organised as follows. In Section 2, I set out my baseline model and look into firm behaviour, firm participation in equilibrium, and the implications of the exemption on aggregate emissions. Section 3 covers two extensions to my baseline model, namely free allocation of emission rights and international trade, and discusses an emissions threshold as an alternative to determine an exemption. Section 4 conducts a welfare analysis by examining the role of the emissions price and the scope of the exemption for welfare in the context of different model assumptions. Section 5 concludes.

2. The Model

In this section, I first describe the environment of my baseline model before looking into firm behaviour. Afterwards, I discuss the role of the exemption for firm participation in equilibrium and for aggregate emissions, concluding with the optimal share of exemption.

2.1. Model Environment

My baseline model is a one-period version of the model by Melitz (2003), similar to Richter et al. (2021). The model features a country with ex-ante homogeneous firms, producing only for the domestic market. Before entering the market, each firm has to pay an entry fee f_e, which is e.g. bureaucratic work by registering business and hiring workers. Entry requires labour such that these costs are in terms of wages w, hence wf_e. After having paid the entry fee, firms draw a productivity ϕ from a Pareto distribution over the support $[1, \infty)$ with pdf $g(\phi) = \theta\phi^{-(\theta+1)}$ and cdf $G(\phi) = 1 - \phi^{-\theta}$. Hence, firms are heterogeneous in their ϕ. The inverse degree of productivity dispersion is captured in θ, the shape parameter; such that for a high θ, firms' productivity is quite homogeneous.

After having received its productivity, the firm will stay in the market to produce or will exit right away if the productivity is so low that it would incur losses otherwise. The critical threshold is ϕ_d; a firm with ϕ_d is the marginal firm and will make zero profit in the domestic market (d). Firms with $\phi < \phi_d$ exit.

The market is characterised by monopolistic competition in one sector: Firms produce the variety ω (of a set of all varieties Ω) facing the households'

CES utility, interpreted as one final good Q for simplicity:

$$U = Q = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} \mathbf{d}\omega \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with $1 < \sigma < \infty$ and $\theta > \sigma > 1$ such that the average output per firm is finite. If $\sigma \to \infty$, goods are perfect substitutes.

Firms produce $q(\phi) = \phi(1-a(\phi))l(\phi)$ as in Copeland and Taylor (2004), with $\phi a(\phi)l(\phi)$ being the emissions abatement efforts undertaken, using $l(\phi)$ units of labour. Abatement $a(\phi)$ reduces the emissions each firm generates, which are: $e(\phi) = (1 - a(\phi))^{1/\alpha} l(\phi)$. Hence, emissions are proportional to labour input.

The production function can be rewritten in a Cobb-Douglas format:

$$q(\phi) = \phi e(\phi)^{\alpha} l(\phi)^{1-\alpha}, \tag{2}$$

in which $\alpha \in (0, 1)$ determines the output elasticity with respect to emissions for $a(\phi) > 0$.

By making the production a linear function of productivity, I will show later in (4) that the profit-maximising emissions intensity of the firm is also increasing in productivity. This feature is crucial in my model as it implies that the firms at the lowest part of the productivity distribution are also the dirtiest ones. The labour market clears and wages are set as numeraire. I set $w = \$89\,872$ such that all prices in my model are in terms of 2020 USD.[4]

The emissions pricing can consist of the price for an emission permit or an emission tax. As in the EU ETS, it does not apply to small emitters which is motivated by the transaction costs implied by participating in the ETS, denoted by f_r.

On their website, the European Commission (2021*b*) states that "[p]articipation in the EU ETS is mandatory for companies in these sectors[[5]], but in some sectors, only installations above a certain size are included, certain small installations can be excluded if governments put in place fiscal or other measures that will cut their emissions by an equivalent amount [...]"[6]. As described above, small installations are those which emit less than 25 000 tons CO_2

[4] More details on the parameterisation can be found in Section 2.5.

[5] "[C]arbon dioxide (CO_2) from electricity and heat generation, energy-intensive industry sectors including oil refineries, steel works, and production of iron, aluminium, metals, cement, lime, glass, ceramics, pulp, paper, cardboard, acids and bulk organic chemicals, commercial aviation within the European Economic Area; nitrous oxide (N2O) from production of nitric, adipic and glyoxylic acids and glyoxal; perfluorocarbons (PFCs) from production of aluminium."

[6] In most sectors, there is a minimum production capacity for installations to be regulated (European Commission, 2003), determining the majority of exempted instalments. Additionally, an exemption based on emissions can be granted by the Member State when equivalent measures are in place (European Commission, 2009). Those are on national level, need to be approved by the European Commission, and are for example the national CO_2 law (Liechtenstein), reported emission reductions (Germany, Spain), or some other form of CO_2 levy (Germany, Slovenia, Spain).

equivalent per year (Article 27, European Commission (2009)). The threshold is the same in Chile (ICAP, 2022) and Singapore (NCCS Singapore, 2022), and similar in most jurisdictions of Canada (50 000 tCO_2e, Sawyer et al. (2021)). For simplicity, I treat installations/facilities as firms.

In my model, only sufficiently productive firms (with $\phi \geq \phi_r$) are regulated and have to pay an emissions price p_r.[7] The threshold ϕ_r follows from the share of active firms being regulated, b, which is set by the regulating authority; hence, $\phi_r > \phi_d$ per definition. All firms also have to pay some costs indirectly associated with the emission intensive input, c_e, e.g. consisting of energy costs. Hence, the cost per unit of emission generated, $h(\phi)$, is:

$$h(\phi) = \begin{cases} c_e + p_r & \text{if } \phi \geq \phi_r \\ c_e & \text{if } \phi < \phi_r \end{cases}.$$

Assumption 1. For simplicity, let the regulator define ϕ_r by exempting the lower share $1 - b$ of the active firms; that is:

$$b = \int_{\phi_r}^{\infty} \frac{g(\phi)}{1 - G(\phi_d)} \mathbf{d}\phi = \left(\frac{\phi_d}{\phi_r}\right)^{\theta},$$

which yields: $\phi_r = \phi_d \; b^{-1/\theta}$. In my baseline model, b is exogenous and ϕ_r is defined as above.

As I will show later in (4), the emissions are an increasing function of productivity. Without regulation, they are continuously increasing such that a regulation based on firms' productivity is a sufficient simplification in the model. The simplifying assumption of a predefined b and the alternative specification of an emissions threshold will be discussed in detail in Section 3.3.

The non-regulated firms have different fixed costs of production as they are not facing the transaction costs of the emissions pricing, f_r. The fixed (overhead) costs are:

$$f(\phi) = \begin{cases} f_y + f_r & \text{if } \phi \geq \phi_r \\ f_y & \text{if } \phi < \phi_r \end{cases}.$$

Additionally, I assume that the bureaucratic effort for being regulated. e.g. emissions' monitoring, reporting, and verification of the emissions, is smaller than the business-as-usual effort f_y, that is $f_y > f_r$.

2.2. Firm Behaviour

Firms are facing the demand of households who will choose the consumption of each good $q(\omega)$ by minimising their expenditure given their utility constraint.

[7] I will disregard any complementary policies for unregulated firms.

The resulting demand for every variety is decreasing in its price, according to:

$$q(\omega) = QP^{\sigma} p(\omega)^{-\sigma} = RP^{\sigma-1} p(\omega)^{-\sigma}. \tag{3}$$

The revenue of each firm is $r(\omega) = p(\omega)\ q(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma}$, taking the aggregate price level $P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}\mathbf{d}\omega\right]^{\frac{1}{1-\sigma}}$ and aggregate revenue $R = \int_{\omega\in\Omega} r(\omega)\mathbf{d}\omega = PQ$ as given (with Q as the composite of final goods, see (1)). As the only source of heterogeneity is a firm's productivity, I use ϕ instead of ω to index a firm from now on.

In the following, I assume that the economy is closed and firms can only deliver to domestic consumers. With $h(\phi)$ as the price per unit of emissions and w as wage, the total production costs of every firm are $TC(\phi) = wf(\phi) + wl(\phi) + h(\phi)e(\phi) = wf(\phi) + k(\phi)q(\phi)$, given fixed overhead costs $f(\phi)$ (overhead labour) and variable unit costs $k(\phi) = w\frac{l(\phi)}{q(\phi)} + h(\phi)\frac{e(\phi)}{q(\phi)}$. The firm's cost minimisation subject to the production function yields the following optimal inputs:

$$e(\phi) = \frac{q(\phi)}{\phi}\left[\frac{1-\alpha}{\alpha}\frac{h(\phi)}{w}\right]^{\alpha-1}, \tag{4}$$

$$l(\phi) = \frac{q(\phi)}{\phi}\left[\frac{1-\alpha}{\alpha}\frac{h(\phi)}{w}\right]^{\alpha}. \tag{5}$$

The emissions and labour intensity, $\frac{e(\phi)}{q(\phi)}$ and $\frac{l(\phi)}{q(\phi)}$, are decreasing in a firm's productivity. For both, the decrease is discontinuous at the regulating threshold ϕ_r, as shown for the emissions intensity in Figure 1.[8] The firms just below and above ϕ_r differ in their optimal emissions intensity: The regulated one has to face an additional emissions price such that their emissions intensity is lower and abatement is higher.

The abatement per firm is then only depending on the output elasticities and the input prices:

$$\begin{aligned} a(\phi) &= 1 - \left(\frac{e(\phi)}{l(\phi)}\right)^{\alpha} = 1 - \left(\frac{\alpha}{1-\alpha}\frac{w}{h(\phi)}\right)^{\alpha} \\ &= \begin{cases} 1 - \left(\frac{\alpha}{1-\alpha}\frac{w}{c_e+p_r}\right)^{\alpha} & \text{if } \phi \geq \phi_r \\ 1 - \left(\frac{\alpha}{1-\alpha}\frac{w}{c_e}\right)^{\alpha} & \text{if } \phi < \phi_r \end{cases}. \end{aligned} \tag{6}$$

Given that regulated firms have to pay more for every unit emitted, their abatement per unit produced is higher.

By using the variable unit production costs $k(\phi)$, the profit is $\pi(\phi) = [p(\phi) - k(\phi)]\, q(\phi) - wf(\phi)$ with $k(\phi) = \frac{1}{\phi}\ \frac{h(\phi)^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$.

[8] See Section 2.5 for details on the parameterisation.

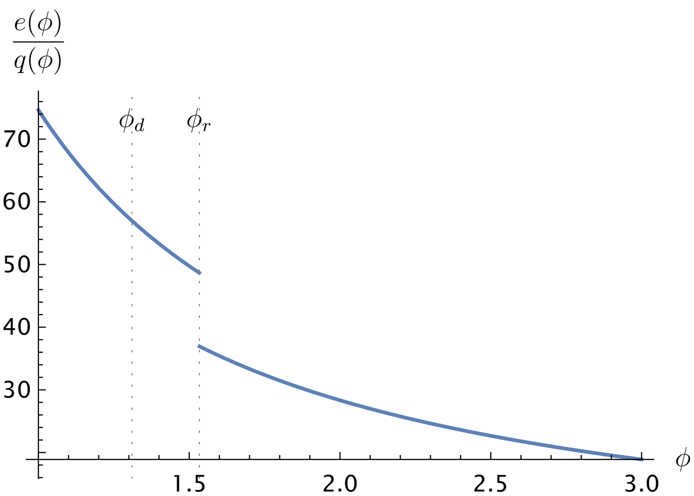

Figure 1: Optimal firm emission intensity along the productivity distribution

To maximise profit, the firm will choose the following price:

$$p(\phi) = \frac{\sigma}{\sigma - 1} \, \frac{1}{\phi} \, \frac{h(\phi)^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}.$$

Hence, a firm's optimal markup is a constant fraction of the price. It originates from monopolistically competitive firms combined with the iso-elastic demand function (Melitz, 2003). A firm's profit is

$$\pi(\phi) = \frac{1}{\sigma} r(\phi) - w f(\phi). \tag{7}$$

Overall, more productive firms have lower prices, higher output, higher revenue (as $\sigma > 1$) and higher profits.[9] Despite decreasing emissions intensity for higher productivity, the quantity produced increases in productivity which is the dominating effect: Emissions are increasing in productivity as visible in Figure 2.[10] The exemption regulation causes the difference in variable costs along the productivity distribution, such that the increase is discontinuous and emissions jump at the regulating threshold ϕ_r.[11] The discontinuity stems from the fixed b; the more realistic scenario with an emissions threshold will be discussed in Section 3.3.

[9]The other respective expressions can be found in Appendix A.1.

[10]The emissions in my model are tons of CO_2e as the model is parametrised for the US manufacturing industry and its CO_2e emissions.

[11]For further calculations, the ratios between two firms with different productivities are needed which can be found in Appendix A.2.

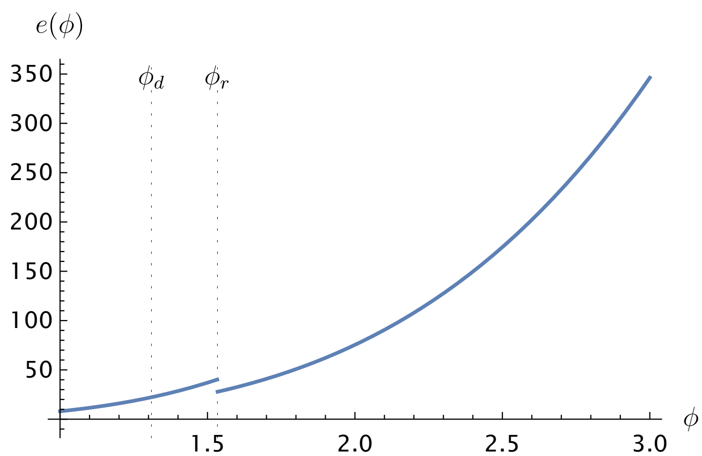

Figure 2: Profit-maximising firm emissions along the productivity distribution, in thousand tons of CO_2e

2.3. Equilibrium

2.3.1. Zero Cutoff Profit Condition

After having paid the (overhead) fixed entry costs f_e, some firms draw a low ϕ, such that they would incur losses when they start producing for the domestic market. Hence, they exit right after obtaining ϕ. Let the threshold for zero profits be ϕ_d which is defined via:

$$\pi(\phi_d) = \frac{r(\phi_d)}{\sigma} - wf_y \ = 0 \tag{8}$$

The probability to be able to stay in the market after entry is $\mathcal{P}(\phi \geq \phi_d) = 1 - G(\phi_d)$, such that the distribution of firms which stay in the market has the following pdf:

$$\mu(\phi) = \begin{cases} \frac{g(\phi)}{1-G(\phi_d)} = \frac{\theta}{\phi}\left(\frac{\phi_d}{\phi}\right)^{\theta} & \text{if } \phi \geq \phi_d \\ 0 & \text{if } \phi < \phi_d \end{cases}.$$

For simplicity, I assume that b (the share of regulated firms) is always chosen such that no firm being regulated by emissions pricing ($\phi \geq \phi_r$) has to leave the market because of too high production costs, that is

$$\pi(\phi_r) = \frac{r(\phi_r)}{\sigma} - w(f_y + f_r) \ > 0, \qquad \forall b \in (0,1).$$

The profit per firm can be seen in Figure 3: Below ϕ_d, firms would incur losses; above, they can produce profitably. At the regulatory threshold ϕ_r, the profits jump down due to the increased fixed and variable costs, but remain positive by assumption (see above).

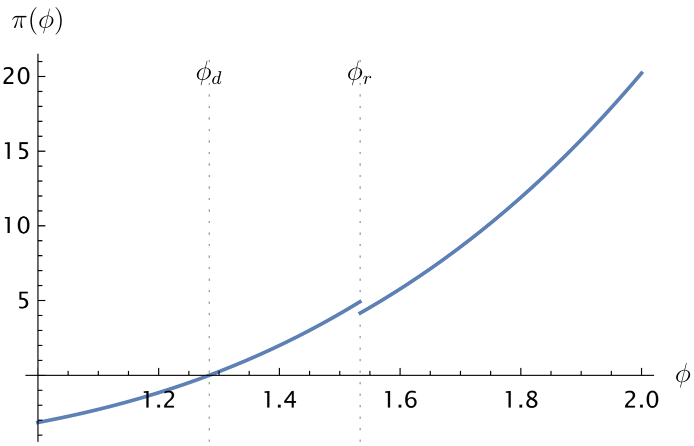

Figure 3: Firm profits along the productivity distribution, in million USD

Using (A.3), (A.4), and imposing that profits are zero at the participation threshold, (8), the average profit can be written as follows, constituting the zero cutoff profit (ZCP) condition:[12]

$$\begin{aligned}\bar{\pi}(\phi_d) &= \int_{\phi_d}^{\phi_r} \pi(\phi)\mu(\phi)\ \mathbf{d}\phi + \int_{\phi_r}^{\infty} \pi(\phi)\mu(\phi)\ \mathbf{d}\phi \\ &= wf_y\left[\frac{\theta}{\theta-\sigma+1}\eta(p_r) - 1\right] - bwf_r, \qquad \forall b \in [0,1),\end{aligned} \tag{ZCP}$$

with

$$\eta(p_r) = \left(1 - b^{\frac{\theta-\sigma+1}{\theta}}\left[1 - \left(\frac{c_e}{p_r + c_e}\right)^{\alpha(\sigma-1)}\right]\right) \leq 1,$$

and $\frac{\partial \bar{\pi}}{\partial \eta(p_r)} > 0$, $\frac{\partial \eta(p_r)}{\partial b} < 0$, $\frac{\partial \eta(p_r)}{\partial p_r} < 0$.

Equation (ZCP) above shows that the average profit depends negatively both on the emissions price, p_r, and on the share of regulated firms, b. In particular, the larger the wedge between emissions price p_r and indirect emission cost c_e, the lower the average profit $\bar{\pi}$. If the emissions price is higher, then the regulated, top-productive firms experience a reduction of their profits, shifting average profits down (following from (7)). Hence, the effect of the emissions price is stronger when more firms are regulated.

With the emissions pricing exemption, only the regulated firms have to pay for the regulatory transaction costs, adding an extra advantage for the non-regulated firms. The effects of b on the average profits are hence stronger with

[12]The derivation can be found in Appendix A.3.

positive regulation fixed costs, f_r. They also depend on $\frac{\partial p_r}{\partial b} \geq 0$:

$$
\begin{aligned}
\frac{\partial \eta(p_r)}{\partial b} = & -\frac{\theta-\sigma+1}{\theta} b^{\frac{-\sigma+1}{\theta}} \left[1-\left(\frac{c_e}{p_r+c_e}\right)^{\alpha(\sigma-1)}\right] \\
& -\alpha(\sigma-1) b^{\frac{\theta-\sigma+1}{\theta}} \frac{c_e^{\alpha(\sigma-1)}}{(p_r+c_e)^{\alpha(\sigma-1)+1}} \frac{\partial p_r}{\partial b}.
\end{aligned}
$$

With an ETS, the derivative $\frac{\partial p_r}{\partial b}$ is positive: If more firms are regulated, the increase in demand increases the emissions price, strengthening the effect of b on average profits. With a emissions tax, p_r would be exogenous and the derivative $\frac{\partial p_r}{\partial b}$ is zero.

2.3.2. Free Entry Condition

Firms only pay the entry fee f_e (so employ labour for entering the market) if the expected profit covers it. Free entry (FE) then results in the following condition:

$$
\begin{aligned}
& w f_e = [1-G(\phi_d)]\, \bar{\pi}(\phi_d), \\
\implies \quad & \bar{\pi}(\phi_d) = w f_e \phi_d^{\theta}.
\end{aligned}
\tag{FE}
$$

2.3.3. Firm Participation Threshold

Equations (ZCP) and (FE) determine a unique solution for $\bar{\pi}$ and ϕ_d as in Melitz (2003), which is depicted in Figure 4. Solving for the firm participation threshold yields:

$$
\phi_d = \left[\frac{f_y}{f_e}\left(\frac{\theta}{\theta-\sigma+1}\eta(p_r)-1\right) - b\frac{f_r}{f_e}\right]^{\frac{1}{\theta}}, \qquad \forall b \in [0,1). \tag{9}
$$

With the regulation discontinuity ($b \in (0,1)$), the cutoff value ϕ_d for firm selection depends both negatively on the emissions pricing coverage b and on the emissions price, p_r. The threshold increases in the fixed cost of production, f_y, and decreases both in the entry fixed cost, f_e, and in the bureaucratic costs of emissions pricing, f_r.

Figure 4 shows that a higher firm participation threshold requires higher average profits for active firms to satisfy the free entry condition (FE), hence the FE has a positive slope.

Apart from the entry fixed costs, a change in the other parameters affects firm selection via the ZCP: As shown before (see (ZCP)), the average profits decrease for a higher emissions price or a higher b, that is, the more firms are falling under the regulation. As visible in Figure 4, the downshift of the ZCP results in a looser firm selection: The cutoff productivity to break even is lower; dirtier firms can survive in the market.

Those effects stem from the asymmetry of the effects of regulation across the size distribution of firms: The dirtier, more unproductive, small-scale emitting firms have a relative benefit, which makes survival at the lower part of the

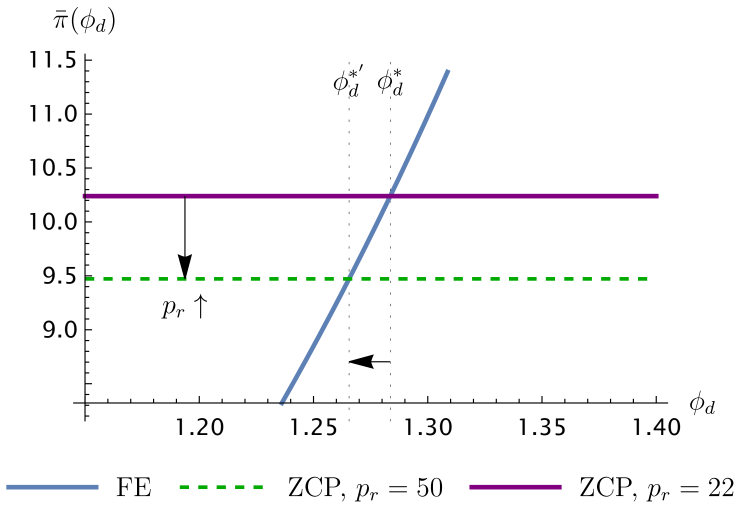

Figure 4: Average profits (in million USD) and the domestic cutoff value in equilibrium when the emissions price changes from 22 to 50 (in USD)

productivity distribution easier. This relative advantage increases if the cost advantage is high or the share of unregulated firms is small.

Going back to Figure 4: Why can a firm with productivity $\phi < \phi_d^*$ and $\phi > \phi_d^{*'}$ now survive in the market, when its input costs and end-user prices have not changed? The answer lies in the market structure: The quantity produced depends on how the other firms in the market behave. The overall price level increases (see later in (13)) when p_r increases, such that the products of the dirtier firms are relatively cheaper and the firm can sell more. Given constant markups, the firm can now break even and survive in the market.

Proposition 1. *Exempting smaller firms from emissions pricing benefits the lower part of the productivity distribution relatively more such that the firm participation threshold moves down, allowing more unproductive firms to stay in the market.*

In the following, I look at the maximum and minimum of coverage: With $b = 1$, all firms pay f_r in their fixed costs and $c_e + p_r$ for their variable emissions input; with $b = 0$, no firm pays f_r and they all only pay c_e per unit of emissions. The ZCP condition is:

$$\bar{\pi}(\phi_d) = wf(\phi_d)\frac{\sigma - 1}{\theta - \sigma + 1}, \qquad \text{for } b = 0 \vee b = 1.$$

The domestic threshold is:

$$\phi_d = \left[\frac{f(\phi_d)}{f_e} \frac{\sigma - 1}{\theta - \sigma + 1} \right]^{\frac{1}{\theta}}, \qquad \text{for } b = 0 \vee b = 1, \tag{10}$$

$$\text{with } f(\phi_d) = \begin{cases} f_y & \text{if } b = 0 \\ f_y + f_r & \text{if } b = 1 \end{cases}.$$

Hence, the threshold does not depend on the emissions price if $b = 0$ or $b = 1$; in line with the results of Richter et al. (2021), and the results for auctioned permits of Anouliès (2017) and Konishi and Tarui (2015). That is, firm selection is not affected if emissions pricing is perfectly symmetric along the productivity distribution.

Proposition 2. *If and only if profits along the firms' productivity distribution are affected asymmetrically, emissions pricing has an effect on firm selection.*

2.4. Aggregate Outcomes

The previous subsections have analysed the effect of the discontinuous emissions pricing on firms' individual decisions and on firm selection in equilibrium. This subsection derives and presents the results for aggregate variables.

First of all, the labour market clears; that is:

$$L = L_e + L_f + L_q = M\phi_d^\theta f_e + M(f_y + bf_r) + M\bar{l}(\phi_d), \tag{11}$$

where L_e is entry-fixed, L_f production-fixed, and L_q variable production labour input, and M is the number of active firms in the market. The latter can be derived using the average labour employed per firm for variable production $\bar{l}(\phi_d)$[13], such that:

$$M = \frac{L}{f_y \eta(p_r)} \frac{1 + \theta - \sigma}{\theta(\sigma - \sigma\alpha + \alpha)}, \qquad \forall b \in [0, 1). \tag{12}$$

Compared to the case without the discontinuity ($\eta(p_r) = 1$), the mass of firms is scaled with $\frac{1}{\eta(p_r)} > 1$ as more entrants are able to survive with an exemption in place. The lower survival threshold means a lower average employment of labour. The effect is stronger for a higher choice of b and a higher emissions price p_r.

If $b = 1$ or $b = 0$, the mass of firms does not depend on wages or the price of emissions. One critical assumption is that the fixed costs are not associated with emissions; otherwise, the result would change as in Richter et al. (2021).

[13] The derivation can be found in Appendix A.4.

The price level can be expressed as follows:[14]

$$P = \left(\frac{L}{f_y}\ \frac{1}{\sigma - \sigma\alpha + \alpha}\right)^{\frac{1}{1-\sigma}}\ \frac{\sigma}{\sigma - 1}\frac{c_e^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\ \phi_d^{-1}, \qquad \forall b \in [0,1). \tag{13}$$

The price level is determined by the costs associated indirectly with emissions, c_e, and production fixed costs f_y, which both apply to all firms. Additionally, it indirectly depends positively on the emissions price and the share of regulated firms via the productivity cutoff ϕ_d.

Furthermore, the equilibrium of the model requires the aggregate resource constraint to be satisfied: The representative household can only spend as much money on the final good as high as his/her income is:

$$R = wL + p_r E_r + c_e(E_r + E_{nr}), \tag{14}$$

where E_r and E_{nr} are the aggregate emissions of regulated and unregulated firms, respectively. Aggregate revenue has to equal income of the representative household: a combination of labour income (inelastically provided), the income generated by the emissions pricing (assuming that it is paid to consumers in a lump-sum manner) plus the income from holding shares of the energy firms.[15] The household furthermore receives the profits of all firms and has to pay the entrance fees: It follows from the free-entry condition (FE) that those two terms cancel out.

A change in the emissions price does not affect the aggregate revenue ($\frac{\partial R}{\partial p_r} = 0$ [16]), but induces an increase in the aggregate price level as regulated firms increase their goods' prices. Both effects lead to a shifting of market shares from the regulated firms to the unregulated ones, see Figure 5.

This shift affects the aggregate emissions of the regulated firms, which are:[17]

$$E_r = \frac{\left[\frac{c_e}{p_r + c_e}\right]^{\alpha(\sigma-1)+1} b^{\frac{\theta-\sigma+1}{\theta}}}{\eta(p_r)}\ \frac{wL}{c_e}\frac{\alpha(\sigma-1)}{\sigma - \alpha(\sigma-1)}, \qquad \forall b \in [0,1). \tag{15}$$

They are decreasing in the emissions price as regulated firms will abate more (composition effect) when emissions are getting more expensive and cut back on production (scale effect), see Figure 6. Furthermore, if more firms are regulated (b increases), the level of regulated firms' emissions is higher.

Despite the revenue shifting, the total emissions are also decreasing in the

[14]The derivation can be found in Appendix A.5.

[15]For simplicity, I assume that the energy firms are owned by households and do not have costs for providing energy, but charge c_e for every unit, e.g. because of a scarcity rent.

[16]The derivation can be found in Appendix A.6.

[17]The derivation can be found in Appendix A.7.

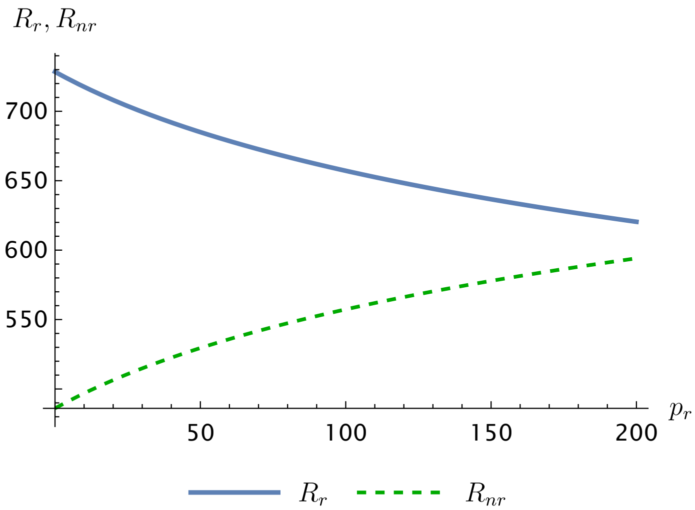

Figure 5: Revenue shifting for an increase in emissions price, here illustrated for $b = 0.2$, in billion USD

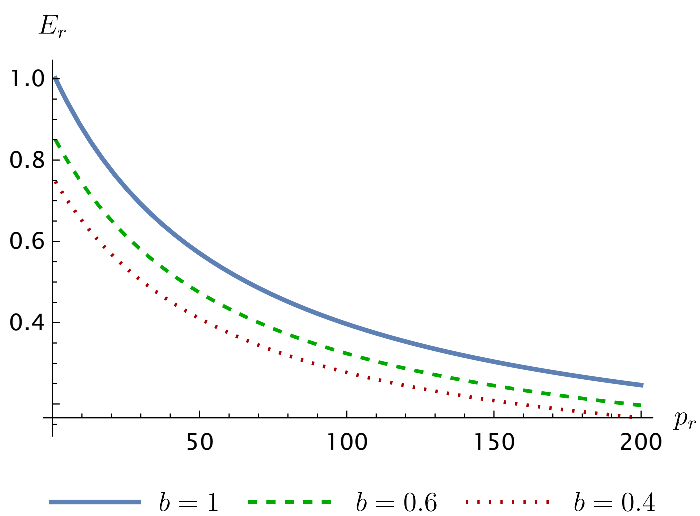

Figure 6: The negative relationship between emissions price and aggregate emissions for different levels of coverage, in billion tons of CO_2e

emissions price:

$$E = \underbrace{\frac{\left(1 - b^{\frac{\theta-\sigma+1}{\theta}}\right) + \left[\frac{c_e}{p_r + c_e}\right]^{\alpha(\sigma-1)+1} b^{\frac{\theta-\sigma+1}{\theta}}}{\eta(p_r)}}_{\leq 1} \frac{\alpha(\sigma-1)}{\sigma - \alpha(\sigma-1)} \frac{wL}{c_e}, \quad \forall b \in [0,1). \tag{16}$$

The effect of the emissions price is incorporated in the first fraction (recall (ZCP) for $\eta(p_r)$) and is stronger if all firms are regulated ($b = 1$) and not existent if no firm is regulated ($b = 0$).

When an exemption regulation is present, within-country leakage arises - that is, the aggregate emissions of unregulated firms increase in the emissions price:

$$E_{nr} = \frac{\left(1 - b^{\frac{\theta-\sigma+1}{\theta}}\right)}{\eta(p_r)} \frac{\alpha(\sigma-1)}{\sigma - \alpha(\sigma-1)} \frac{wL}{c_e}, \qquad \forall b \in [0,1), \tag{17}$$

$$\text{such that: } \frac{\partial E_{nr}}{\partial p_r} > 0.$$

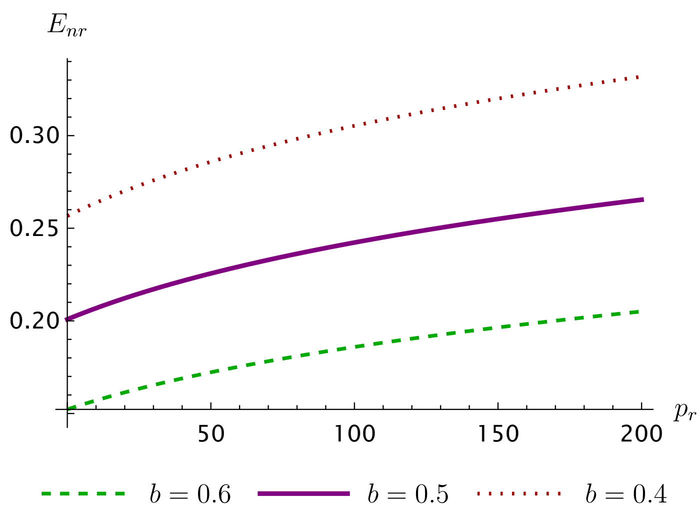

Figure 7: Within-country leakage: positive relationship between unregulated firms' emissions and the emissions price for different levels of coverage, in billion tons of CO_2e

Generally, the aggregate emissions of unregulated firms are higher if more firms are exempted from the emissions pricing (low b), see Figure 7. Additionally, the higher the emissions price, that is, the stricter environmental regulation, the higher the emissions of unregulated firms because their relative advantage of being exempted increases. For an increase in p_r, the regulated firms will abate more than before and cut their production. That is, in the domain of regulated firms, the scale effect and the within-firm composition effect (as defined by Grossman and Krueger (1991)) both reduce emissions. Given that the productivity of every firm is assumed to be fixed, the technique effect is absent in the model. For the non-regulated firms, the within-firm composition (hence, the labour share dedicated to abatement and production, see (5) and (6)) stays the same, but they can increase production due to an increased price level (scale effect). Additionally, the within-industry composition effect (more low-productivity firms can survive) drives up the aggregate emissions of the unregulated firms. Having an exemption for smaller firms thus jeopardises reaching emission goals.

PROPOSITION 3. *With the emissions pricing exemption regulation, the aggregate emissions for non-regulated firms increase for an increase in the emissions price, leading to within-country leakage.*

2.5. Parameterisation

Parameter	L	w	b	α	σ	θ	c_e	p_r
Value	12 790 000	\$89 872	0.375	0.0678	4.76	5.51	\$64	\$22

Parameter	f_y	f_e	f_r	f_x	τ	$\underline{e}$	s	SCE
Value	\$5 174 708/w	$0.5f_y$	\$35 392/w	f_y	1.3	25 000t CO_2e	0.1	\$185

Table 1: Parameter choices; details and sources can be found in Table A.1 in Appendix A.8

For the graphical illustrations in this paper, I parametrised my model to the US manufacturing industry with a hypothetical emissions pricing system similar to the EU ETS, see Table 1.[18]

3. Extensions and an Alternative Exemption Criterion

3.1. Free Allocation

In this section, I extend the baseline model with free allocation of permits and international trade as they both represent environments in which emissions pricing is embedded in reality. Furthermore, I examine the impact of an alternative criterion for exempting firms from emissions pricing, namely an emissions threshold which is a common tool to determine exemption in most emissions pricing systems.

The EU ETS does not only have auctioning of permits but also grants firms free permits with a free allocation scheme.[19] For my setup, the free allocation

[18]The figures in my paper were created with Mathematica. The code is available upon request.

[19]Also some emission tax systems have some "free" emissions; e.g. the Dutch carbon dioxide emission tax only applies to emissions above a benchmark which makes it analogous to free permits (Janssen, Peelen, Bevers and De Booij, 2020).

rule follows a measure for production capacity of the firm and a benchmark for emissions intensity, which will be denoted by z in the following. The number of free permits (or "free" emissions) per firm is $\hat{e}(\phi) = z\tilde{q}(\phi)$, so each firm gets the benchmark emission intensity times its own output capacity when not regulated, denoted by $\tilde{q}(\phi)$. Contrary to the output-based allocation rule in Anouliès (2017), the free allocation considered here does not depend on the output of the previous period, but on a measure of capacity. The regulating institution can perfectly estimate the equilibrium output the respective firm would have without regulation. The individual firm does not take its influence on the benchmark into account which is due to the existence of a continuum of firms. When a firm exits the market directly after entry, it won't get any free permits.

The benchmark z is the arithmetic average emission intensity of the s percent best-performing firms in terms of emission intensity per period. It is a simplification of the 54 benchmarks of the EU ETS which are calculated following the top 10 percent most efficient firms per sector. In the following, I assume $b > s$, so the benchmark firms are a subset of the regulated ones. The productivity of the marginal firm, ϕ_s, is determined as follows:

$$\begin{aligned} s &= \int_{\phi_s}^{\infty} \frac{g(\phi)}{1 - G(\phi_d)} \mathbf{d}\phi = \left(\frac{\phi_d}{\phi_s}\right)^{\theta}, \\ \implies \quad \phi_s &= \phi_d \; s^{-1/\theta}. \end{aligned}$$

Hence, the benchmark is:

$$z = \int_{\phi_s}^{\infty} \frac{e(\phi)}{q(\phi)} \frac{g(\phi)}{1 - G(\phi_s)} \mathbf{d}\phi = \frac{\theta}{1+\theta} \left[\frac{\alpha}{1-\alpha} \frac{w}{p_r + c_e}\right]^{1-\alpha} s^{\frac{1}{\theta}} \phi_d^{-1}.$$

The benchmark is decreasing in the (domestic) productivity cutoff and the price of emissions, and increasing in the share of firms considered (s) and wages. Anouliès (2017) furthermore defines the firm with the benchmark emission intensity as the firm with productivity ϕ_e, that is: $z = \frac{e(\phi_e)}{q(\phi_e)}$. A firm with $\phi > \phi_e$ is net seller, one with $\phi_r \leq \phi < \phi_e$ net buyer of permits; hence the allocation scheme is regressive, the top of the productivity distribution (cleaner firms) benefit more. The asymmetry introduced should push the domestic threshold up and put dirtier firms out of the market, as it is also shown by similar allocation schemes in the work of Anouliès (2017), Konishi and Tarui (2015), and Dardati and Saygili (2020).

When free allocation is introduced, regulated firms additionally see positive income from those free permits $\hat{e}(\phi)$ on their profit calculations, that is:

$$\pi(\phi|\phi > \phi_r) = \frac{r(\phi)}{\sigma} - w(f_y + f_r) + p_r \hat{e}(\phi).$$

An important distinction to output-based allocation as in Anouliès (2017) is that the used free allocation based on potential production capacity does not alter the optimal input decision of a regulated firm as it takes the free permits as a reduction of its fixed costs, but not as an implicit subsidy of increasing one's production.

In equilibrium, the domestic production cutoff is implicitly defined via the ZCP condition from above[20] and the free entry condition (FE):

$$\phi_d = \left[\frac{f_y}{f_e} \left(\frac{\theta}{\theta - \sigma + 1} \eta(p_r) - 1 \right) - b \frac{f_r}{f_e} + \underbrace{\frac{b^{\frac{\theta-\sigma}{\theta}} s^{\frac{1}{\theta}}}{w f_e} \rho(p_r)}_{\text{free allocation effect}} \right]^{\frac{1}{\theta}}, \tag{18}$$

where $\rho(p_r)$ is defined in Appendix A.12 and satisfies $\frac{\partial \rho(p_r)}{\partial p_r} > 0$.

Free allocation increases the break-even productivity threshold, compensating for the exemption regulation's negative effects on firm selection (compare with (9)). When p_r increases, $\eta(p_r)$ decreases, putting downward pressure on ϕ_d. At the same time, the value of the free permits increases and $\rho(p_r)$ goes up, such that especially net sellers of the permits benefit from the price increase, whereas net buyers mostly see their input costs increase. In particular, the more energy-efficient a firm, the more it is positively affected by the price increase. Given the assumptions made, the more energy-efficient firms are those with higher productivity.

Due to the redistributional consequences, the effect of an emissions price increase on ϕ_d is not monotonic when free allocation is present, as shown in Figure 8. Whereas the lower part of the productivity distribution sees a rising advantage of being exempted from the ETS with and without free allocation as the emissions price rises, the top part of the distribution (first) benefits from being able to sell the leftover permits from free allocation for a higher price. At some level of the emissions price, the negative effect of higher input prices dominates and the survival threshold is also decreasing in the emissions price.

The Cobb-Douglas specification of the production function and the fact that free allocation does not alter the input decisions of firms, combined with a fixed share of regulated firms, imply that within-country leakage is not different from the case without free allocation.[21] Apart from leakage, resources are shifted towards more efficient firms with free allocation, such that the emissions price has a stronger effect on reducing aggregate emissions, which is discussed in Section 4.

[20]See Appendix A.12 for derivation and effects of an increase of p_r.

[21]Emissions leakage can be summarised by measuring it as "the increase in emissions in non-taxing regions as a percentage of the reduction in emissions in the taxing region" (Kortum and Weisbach, 2021). In the given context, the leakage rate would express the increase in emissions of exempted firms in terms of decreased emissions of regulated firms when the emissions price changes: $LR = -\frac{dE_{nr}/dp_r}{dE_r/dp_r}$.

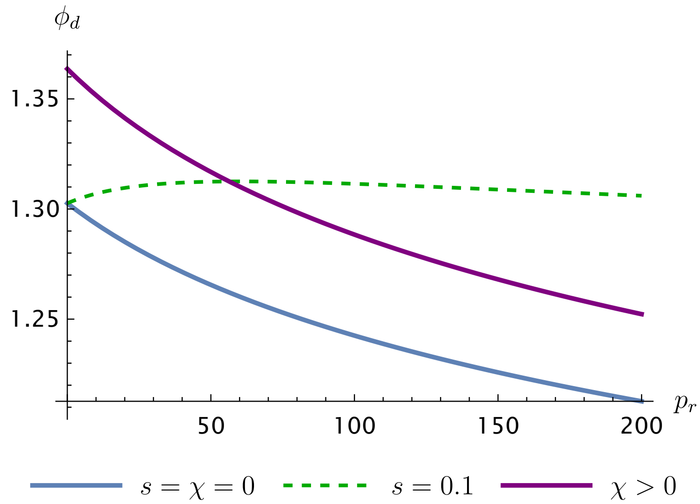

Figure 8: Survival threshold without free allocation/trade ($s = \chi = 0$), then adding free allocation based on the emission intensity of the cleanest 10 % of firms ($s = 0.1$) or trade ($\chi > 0$)

3.2. International Trade

Within-country leakage may also depend on international trade which is another asymmetry used in Melitz (2003) models: It benefits the firms at the top of the productivity distribution as they can make additional profits by selling products abroad. This asymmetric effect of international trade shifts the productivity threshold for domestic production upwards, hence inducing less productive, dirtier firms to exit (Richter et al., 2021).

As in Balistreri and Rutherford (2012), I assume that two large, open economies can trade goods with each other ("subglobal entities"). The home country introduces carbon pricing with an exemption of small emitters. Households in both economies still have the same CES utility function and hence, the same demand function as in Subsection 2.2. In the following, I will only focus on the domestic effects in the baseline model (based on Assumption 1) and will not discuss across-country leakage.

If a firm wants to export, it faces the fixed costs of exporting, wf_x. Additionally, goods export is hindered by iceberg costs $\tau \geq 1$ which could be a tariff rate or variable transportation costs. All firms have to decide whether to produce for the domestic and/or the foreign market. As every firm which ships one unit of the final good to the foreign country can only sell $\frac{1}{\tau}$ of the final good abroad, the exporting price needs to be higher, namely

$$p_x(\phi) = \tau \; p_d(\phi) = \tau \frac{\sigma}{\sigma - 1} \; \frac{1}{\phi} \; \frac{h(\phi)^\alpha w^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}.$$

The revenue in the foreign country is:

$$r_x(\phi) = p_x(\phi)\ q_x(\phi) = R_j P_j^{\sigma-1} \left(\tau \frac{\sigma}{\sigma-1}\ \frac{1}{\phi}\ \frac{h(\phi)^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \right)^{1-\sigma},$$

denoting the home country with i and the foreign one with j.

As in the autarky case, every firm decides after entry whether to produce or not. Now, it faces the decision for two markets, hence, there are two ZCP thresholds:

$$\pi_d(\phi_d) = \frac{r_d(\phi_d)}{\sigma} - wf_y = 0,$$
$$\pi_x(\phi_x) = \frac{r_x(\phi_x)}{\sigma} - wf_x = 0.$$

One important assumption is that firms which export always also sell domestically, but not always the other way around ($\phi_x > \phi_d$), and that small emitters (the targeted exempted group) are not exporting ($\phi_x > \phi_r$). It is fulfilled whenever f_x and b are large enough.

For simplicity, I also assume that the effect of the change in emissions price affects the domestic survival threshold more strongly in the home country (i) than abroad (j), so that the effect on the price level in the home country is also stronger than that in the foreign one. Furthermore, I assume that the changes in aggregate revenues abroad are negligible.[22] Richter et al. (2021) show that they are actually negative.

For the ZCP condition, the average profit is needed. It depends on the probability of being an exporting firm given already supplying the domestic market:

$$\begin{aligned}\chi(p_r) = \mathcal{P}(\phi \geq \phi_x | \phi \geq \phi_d) &= \frac{1-G(\phi_x)}{1-G(\phi_d)} = \left(\frac{\phi_d}{\phi_x}\right)^{\theta} \\ &= \left(\frac{P_j}{\tau P_i}\right)^{\theta} \left(\frac{f_y}{f_x}\frac{R_j}{R_i}\right)^{\frac{\theta}{\sigma-1}} \left[\frac{c_e}{c_e+p_r}\right]^{\alpha\theta}. \end{aligned} \tag{19}$$

As in autarky, the domestic threshold depends on the emissions price (as the small emitters have a relative advantage), but given that none of the small emitters are exporting (given the assumptions made), the ratio of both thresholds depends on this relative advantage. If the emissions price increases, the regulated firms (hence, all exporting firms) have higher costs and will increase prices. Exporting goods without incurring losses becomes more difficult: ϕ_x increases. At the same time, the relative advantage for unregulated firms increases, such that dirtier firms can survive: ϕ_d decreases. The chance of being able to export when a firm is already active in the domestic market therefore

[22] That is, $\frac{\partial(P_i/P_j)}{\partial p_r} > 0$ and $\frac{\partial R_j}{\partial p_r} = 0$. In Appendix A.6, I show that also $\frac{\partial R_i}{\partial p_r} = 0$.

drops. That is, $\frac{\partial \chi(p_r)}{\partial p_r} < 0$.

The average profit in the trade equilibrium is derived in Appendix A.13; together with the FE condition (FE), it defines the following survival threshold:

$$\phi_d = \left[\frac{f_y}{f_e} \left(\frac{\theta}{\theta - \sigma + 1} \eta(p_r) - 1 \right) - b \frac{f_r}{f_e} + \underbrace{\chi(p_r) \frac{\sigma - 1}{\theta - \sigma + 1} \frac{f_x}{f_e}}_{\text{trade effect}} \right]^{\frac{1}{\theta}},$$

with $\frac{\partial \chi(p_r)}{\partial p_r} < 0$.

The cutoff for serving the domestic market is, as in Melitz (2003), higher under international trade than in autarky (recall (9)) - irrespective of any regulation, see Figure 8. The difference is bigger, the higher the chance to export given being active in the domestic market ($\chi(p_r)$).

The now higher productivity cutoff reacts more strongly to a change in the emissions price given that more profit is concentrated at the top of the productivity distribution due to exporting. As p_r rises, not only does the negative impact on $\eta(p_r)$ drive down the productivity cutoff but also the negative impact on the chance of exporting, $\chi(p_r)$. The result of Melitz (2003) helps to understand this finding: Having a positive chance to export when being active in the domestic market ($\chi(p_r) > 0$) pushes the least productive firms out of the market. It stems from the most productive firms being able to expand their production, ship products to foreign countries, and hence, increase their profits. The average profits in the market go up given this advantageous position of the top-productive firms, the ZCP moves up, and the cutoff for domestic production moves up. Coming back to emissions pricing: Crucially, when the emissions price rises, fewer firms find themselves being able to compete with prices abroad. Hence, the positive effect of trade on the threshold decreases. The negative effect of p_r on $\chi(p_r)$ additionally implies that non-regulated emissions increase stronger in p_r.[23] Nevertheless, as resources are allocated towards the most efficient firms who can export their goods, trade results in the highest emission reductions for an increase in p_r compared to other scenarios, which will be discussed in the welfare analysis in Section 4.

3.3. Exemption based on an Emissions Threshold

In the baseline model, I have assumed that the share of regulated firms b is fixed and set by the regulator; the assumption can be justified when the regulator looks at the firm distribution and wants to give firms of smaller size some advantage. Smaller firms are disproportionally affected by the variable part of emissions pricing under the assumption that they are dirtier, and by the fixed part (transaction costs), such that the regulator exempts the smallest share $(1 - b)$ of active firms.

[23] For details, see Appendix A.14.

With this assumption, the firms at the regulation threshold ϕ_r emit as follows:

$$\lim_{\phi \downarrow \phi_r} e(\phi) = \left(\frac{c_e}{p_r + c_e} \right)^{1-\alpha+\alpha\sigma} \lim_{\phi \uparrow \phi_r} e(\phi) \tag{20}$$

As visible in Figure 2, critically, the emissions when being regulated are lower than being unregulated at the regulation threshold because unregulated firms do not have to pay the emissions price.

Assuming that the regulator perfectly knows the productivity distribution of active firms is a rather unrealistic scenario. Hence, the participation threshold of most emissions pricing systems is an emissions threshold, e.g. 25 000 tons CO_2 equivalent per year (EU ETS, article 27, European Commission (2009)).

Setting an emissions threshold implies that regulated firms assign a shadow value to being unregulated: some firms have the choice of abating more to fall under the exemption or setting their emissions according to the optimal emission function (see (4)) but paying for emissions pricing. I will show in this subsection that an emissions threshold will make sure that there is no firm right under the regulation threshold having higher emissions than the counterpart right above it - as it was in the specifications before (recall Figure 2 or (20)). Instead of Assumption 1, I now impose:

Assumption 2. Let the regulator fix an emissions threshold $\underline{e}$ instead of the share of regulated firms b.

Now, there exists a ϕ_r at which firms are indifferent between reducing emissions, such that they fall right under the exemption threshold and do not have to pay for emissions pricing, and emitting according to the optimal emission function plus paying the emissions price. That is:

$$\lim_{\phi \downarrow \phi_r} \pi(\phi) = \lim_{\phi \uparrow \phi_r} \pi(\phi). \tag{21}$$

Firms with a productivity directly under ϕ_r will emit exactly the maximum amount possible to be still exempted from the emissions pricing whereas the firm above sets it according to the optimal emission function as before (see (4)):[24]

$$\begin{aligned} &\lim_{\phi \uparrow \phi_r} e(\phi) = \underline{e}, \\ &\lim_{\phi \downarrow \phi_r} e(\phi) = \frac{q(\phi_r)}{\phi_r} \left[\frac{1-\alpha}{\alpha} \frac{c_e + p_r}{w} \right]^{\alpha-1}. \end{aligned}$$

Below ϕ_r, firms set their emissions rather to the threshold value $\underline{e}$ to not be reg-

[24] With a fixed $\bar{q}$, the implications are similar to the case with $\underline{e}$, described in this section: For firms with $\phi_o \leq \phi \leq \phi_r$, the shadow value of being exempted is large enough to produce exactly $\bar{q}$ and not to be regulated.

ulated, but only until the profit of setting the emissions lower than the threshold value is higher. That is, there exists a ϕ_o with $\phi_o < \phi_r$ at which firms revert back to setting emissions according to the optimal emission function:

$$\underline{e} = e(\phi_o) = \frac{q(\phi_o)}{\phi_o} \left[\frac{1-\alpha}{\alpha} \frac{c_e}{w} \right]^{\alpha - 1}. \tag{22}$$

Between ϕ_o and ϕ_r, the shadow value of not being regulated is high enough to perform relatively higher abatement efforts. When minimising costs given the production function and the emissions constraint ($e(\phi) \leq \underline{e}$), those firms choose $l(\phi)$ accordingly, such that their optimal inputs are:

$$\begin{aligned} e(\phi | \phi_o \leq \phi < \phi_r) &= \underline{e}, \\ l(\phi | \phi_o \leq \phi < \phi_r) &= \left(\frac{q(\phi)}{\phi} \right)^{\frac{1}{1-\alpha}} \underline{e}^{\frac{\alpha}{\alpha - 1}}. \end{aligned}$$

This medium-productivity group of firms hence will also choose a different quantity-price combination to offer their products;[25] their price and quantity do not depend on the emissions price, but on the regulating emissions threshold $\underline{e}$. For simplicity, I set $f_r = 0$ in the following.

The indifference condition for profits at ϕ_r (21) contains the following elements:

$$\lim_{\phi \downarrow \phi_r} \pi(\phi) = \frac{1}{\sigma} R P^{\sigma - 1} \left(\frac{\sigma}{\sigma - 1} \frac{1}{\phi_r} \frac{(c_e + p_r)^\alpha w^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \right)^{1-\sigma} - w f_y, \tag{23}$$

$$\begin{aligned} \lim_{\phi \uparrow \phi_r} \pi(\phi) = &\left(\phi_r^{\sigma - 1} \frac{\underline{e}^{(\sigma-1)\alpha}}{w^{(1-\alpha)(\sigma-1)}} R P^{\sigma - 1} \right)^{\frac{1}{1-\alpha+\sigma\alpha}} \left\{ \left[\frac{(1-\alpha)(\sigma-1)}{\sigma} \right]^{\frac{(\sigma-1)(1-\alpha)}{1-\alpha+\sigma\alpha}} \right. \\ &\left. - \left[\frac{(1-\alpha)(\sigma-1)}{\sigma} \right]^{\frac{\sigma}{1-\alpha+\sigma\alpha}} \right\} - c_e \underline{e} - w f_y. \end{aligned} \tag{24}$$

Given the costs for the emissions, I can only implicitly solve for ϕ_r, see Appendix A.10. The implicit solution is sufficient to see the positive effect of an increase in the emissions price p_r on the threshold ϕ_r. If the emissions price goes up, the profits when being regulated go down such that the shadow value of being unregulated is relatively higher: more firms will abate more to fall in the exemption by reaching $\underline{e}$. Hence, for a tighter environmental policy, more of the targeted firms (who naturally would emit more than $\underline{e}$) are exempted by the regulation: ϕ_r moves up to ϕ'_r as in Figure 9.

[25] See Appendix A.9.

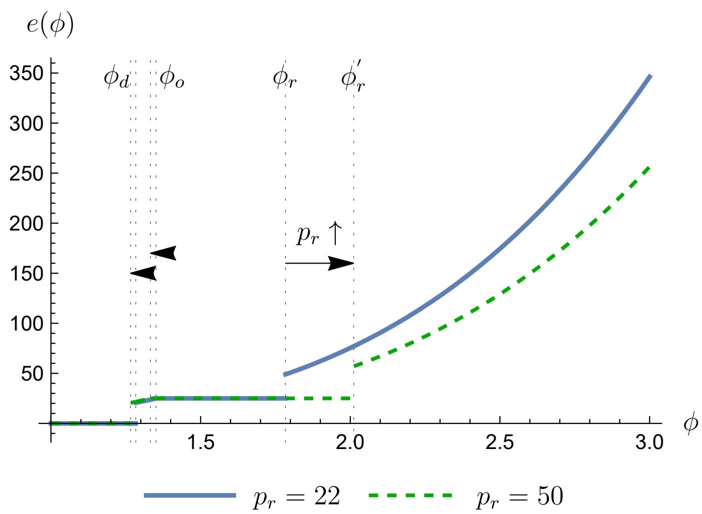

Figure 9: Emissions per firm with an emissions threshold for regulation and a change in emissions prices from 22 to 50, in thousand tons of CO_2e

The threshold ϕ_o can be derived from (A.1) and (22) as follows:

$$\phi_o = \left(\frac{RP^{\sigma-1}}{\underline{e}} \left[\frac{1-\alpha}{w}\right]^{(1-\alpha)(\sigma-1)} \left[\frac{\alpha}{c_e}\right]^{1-\alpha+\sigma\alpha} \left[\frac{\sigma-1}{\sigma}\right]^{\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{25}$$

Note that ϕ_o is not directly influenced by the emissions price; only via a change in the price level, the emissions price can alter ϕ_o.

Within-country leakage depends on how strong the thresholds react to a change in p_r. The first effect originates from a decreasing ϕ_d: The average profits consists of the weighted averages of the low-, medium-, and high-productivity firms. They need to decrease when p_r goes up,[26] such that the ZCP is shifted down (recall Figure 4). The free-entry condition (FE) ensures that the threshold ϕ_d at which firms can survive in the domestic market moves down such that more low-productive, dirty firms can survive. The second effect stems from an increasing ϕ_r: The emissions price increase makes it attractive for more firms than before to fall right under the regulation threshold $\underline{e}$ and abate more to "save" themselves from increased costs. Overall, more firms than before are unregulated as ϕ_d decreases and ϕ_r increases. The tighter the environmental policy, the broader the scope of firms falling outside of the policy.

Proposition 4. *If $\underline{e}$ is exogenous instead of b, within-country leakage is higher because some firms strategically abate more to fall in the exemption scope.*

[26] See Appendix A.11.

4. Welfare Analysis

4.1. The Role of the Emissions Price

I now present a comprehensive welfare analysis: In particular, I focus on five different scenarios, as shown in Table 2, to explore how changes in the emissions price and regulatory coverage impact welfare differently depending on the way the exemption is determined and whether trade or free allocation are present.

In this section, it is crucial to underscore that the welfare results presented are contingent upon the specific parameterisation employed. Variations in the model parameters may lead to different outcomes in the analysis unless explicitly highlighted.

The welfare analysis is divided into two parts. First, I analyse the effects of a change in the emissions price on welfare: The emissions price is the key player to drive down aggregate emissions. Second, I investigate whether the exemption of small emitters is welfare-maximising. In particular, I discuss the crucial role of the social costs of emissions and the fixed cost of regulation in determining the optimal scope of the exemption.

Abbreviation	**Fixed Exemption Criterion**	**Extension**
Share	b share of firms regulated	—
ShareFree	b share of firms regulated	Free permits for regulated
ShareTrade	b share of firms regulated	Exporting goods possible
Emis	$\underline{e}$ emissions threshold	—
EmisFree	$\underline{e}$ emissions threshold	Free permits for regulated

Table 2: Different scenarios considered in the welfare analysis

The exemption from emissions pricing was based on two different criteria in the previous sections. Whereas an emissions threshold offers firms the option to self-select into the exemption from regulation, the fixed share means that firms learn their regulatory status after entry based on their drawn productivity.[27] Depending on the industry, strategically bunching below the emissions threshold might be challenging due to long-term capital investments, financial or technological constraints, or other frictions. Therefore, the welfare analysis examines both the fixed share and the fixed emissions threshold as exemption criteria. Given the nature of a fixed share, some firms close to the regulatory threshold might switch status when the firm survival threshold changes.[28] I show in Appendix A.15 that the effect does not drive my main results by running the counterfactual exercise of a productivity threshold.

[27] In most sectors regulated by the EU ETS, there is an additional minimum production capacity for installations to be regulated (European Commission, 2003), determining the majority of exempted instalments. Similar to regulating only the most productive firms, a minimum production capacity does not allow for exempting oneself from regulation after the setup of a plant (which can be understood as a draw of productivity).

[28] For example, if the survival threshold decreases, some previously unregulated firms will now belong to the top b% productive firms, making them subject to regulation.

Welfare is measured in terms of overall output produced (equal to utility, recall (1)) minus the social costs of total emissions (in utils by scaling it with the price level):

$$W = Q - \frac{SCE}{P}E, \tag{26}$$

in which SCE represents the social costs of one unit of emissions.[29]

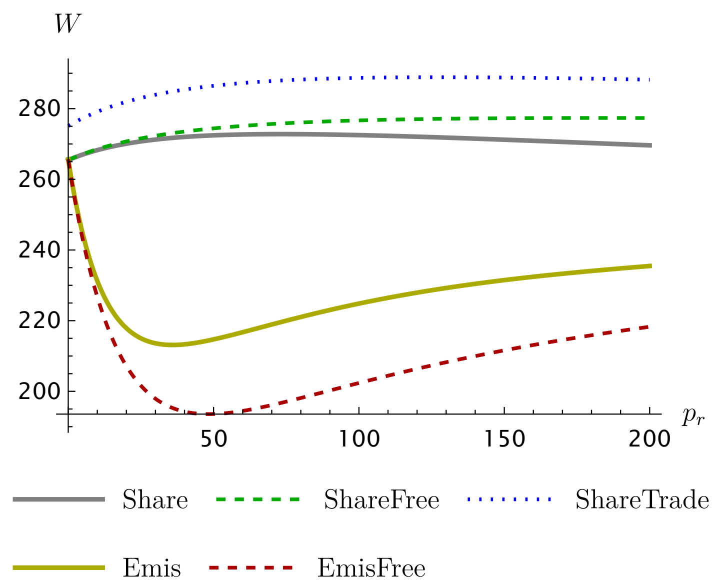

Figure 10: Welfare along different emissions prices, in million utils

For the non-self-selection scenarios (Share-, see Figure 10), welfare first increases in the emissions price because emissions are being driven down. In the baseline scenario (no extensions to Share), the optimal emissions price is lower than in the trade and free permit scenarios. That is, with any higher price, the benefit from fewer emissions starts to be lower than the loss in the output of the regulated firms which are more efficient in producing than their unregulated counterparts.[30] Given that trade and free allocation both benefit the cleaner, bigger firms, the adverse effects of a higher emissions price on efficiency are only dominating the emission reductions at a higher level of p_r.[31] In general, welfare is always higher with trade or free allocation, as I show in Appendix A.19.

[29] SCE is set to \$185 per ton of CO_2e, see the parameterisation in Section 2.5.

[30] Given the assumptions of the model, cross-country leakage is not accounted for, which might change the level of optimal emissions price.

[31] With the given specifications, the optimal price is ca. \$70 for Share, \$180 for ShareFree, and ca. \$120 with ShareTrade.

As soon as firms have the option to self-select into the exemption by bunching below an emissions threshold (Emis-)[32], the welfare function looks quite different: At small emissions prices, welfare is declining in the emissions price as more dirty firms start surviving,[33] charging relatively high prices while having a small production scale with high emission intensity. At the same time, the firms which decide to bunch below the emissions threshold are also artificially lowering their output below their production capacity. The emission reductions induced by those bunching firms cannot make up for their production cuts, such that welfare decreases. If regulated firms receive free permits (EmisFree), being exempted is no longer as attractive; an increase in the emissions price will incentivise fewer firms to reduce their emissions to just below the threshold level. The emissions price cannot work as well to decrease aggregate emissions. Furthermore, welfare suffers from efficiency losses at the lower part of the productivity distribution as even dirtier firms can survive (see Figure A.3 in the Appendix).[34] Under the current parameterisation, EmisFree results in being the least attractive scenario as free allocation intensifies the negative welfare implications from allowing firms to distort their production by bunching at a threshold.[35]

Interestingly, the welfare differences between scenarios are mainly driven by output differences (see Figure A.5 in the Appendix). If the social costs of emissions were higher, the emissions effect at some point would dominate given that it represents the weight of aggregate emissions in the welfare function. I discuss the implications of the level of SCE in the next subsection. Figure 11 shows the aggregate emissions along different levels of the emissions price. Allowing firms to self-select into the exemption (Emis) lets aggregate emissions decline very strongly for the first increases of p_r. This effect is much weaker when firms have the incentive to stay regulated by receiving free permits (EmisFree). The emissions price is most effective as an emission reduction tool when the model includes international trade as it implies a larger share of output being produced by regulated, bigger firms. Free allocation in the baseline model (ShareFree) performs better in efficiency terms than no free allocation (Share),[36] so that welfare is higher despite the same emission reductions.

Proposition 5. *Exempting firms reduces the effectiveness of emissions pricing*

[32]All those scenarios were plotted with the help of interpolation due to solving the equilibria in discrete terms for tractability, see Appendix A.21.

[33]See Appendix A.16 for elaboration, plus Figure A.4.

[34]With an emissions threshold plus free permits, more firms incorporate the emissions price in their final good prices (as more firms are regulated than without permits), giving an upward drive to the price level, whereas at the same time, the dirtiest firms in the market benefit even more from being exempted, also pushing the price level up with their very inefficient production.

[35]This result holds under the current parameterisation for a subset of possible policy choices; I elaborate in Appendix A.19 that if the emission threshold becomes nonbinding ($\underline{e} \leq \underline{e}_{min}$), the welfare results coincide with those when $b = 1$ in the fixed share scenario, such that welfare is higher under free allocation.

[36]Again see Figure A.5 in the Appendix.

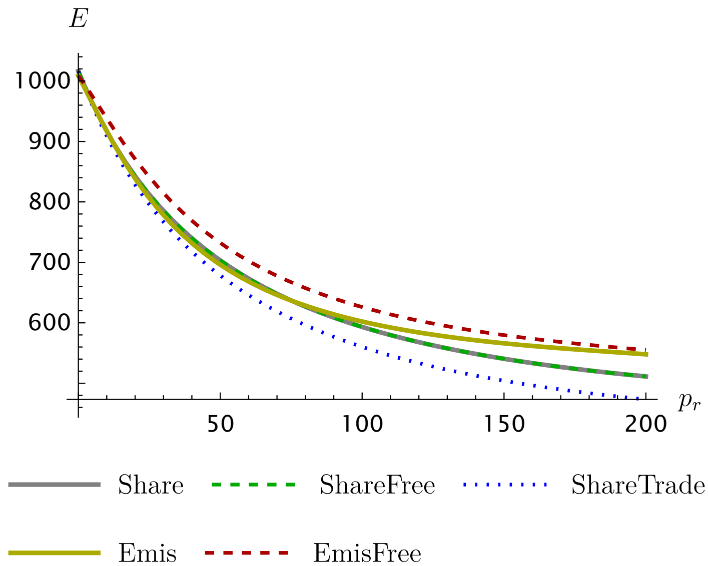

Figure 11: Aggregate Emissions for different scenarios along different emissions prices, in million tons of CO_2e

due to within-country leakage; however, emissions pricing is more effective when it targets a higher market share, which is the case when the most efficient firms export their goods abroad.

4.2. The Role of the Scope of the Exemption

If firms cannot self-select into the exemption from emissions pricing, regulating more firms increases welfare. For different shares of regulation, b, free allocation of permits has a slightly larger welfare increase for a higher share of regulated firms than the baseline scenario Share (see Figure 12). Free permits work as a subsidy to the cleanest firms, hence enabling a bigger share of production with a lower emission intensity, working against the negative effect of the exemption. At the same time, as b rises, more firms are regulated but receive the subsidy as well, which compared to baseline or trade, stabilises the survival threshold (see Figure A.6 in the Appendix).

Interestingly, the opposite result is obtained in the case of a fixed emissions threshold: As shown in Figure 13, making the scope of the exemption more lenient (by having a higher threshold) results in higher welfare for the given social costs of carbon (which function as a weight between the importance of aggregate output vs. aggregate emissions). The result seems to contradict the one derived from the baseline model (see Figure 12 above) – it is driven by

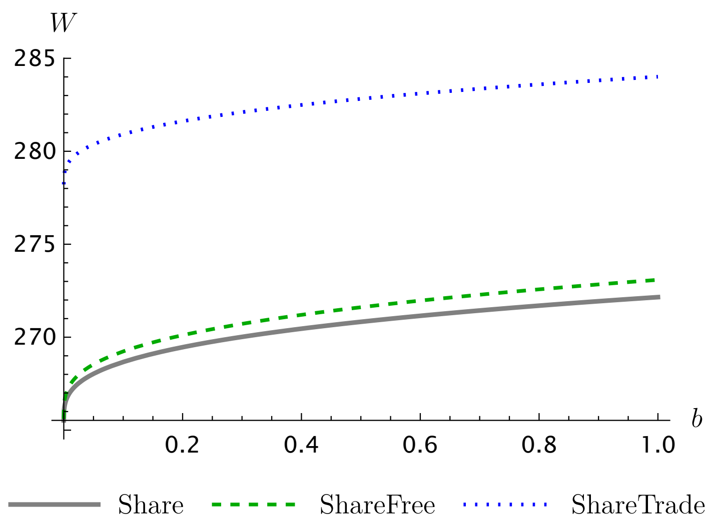

Figure 12: Welfare for different scenarios along different shares of firms being regulated, in million utils

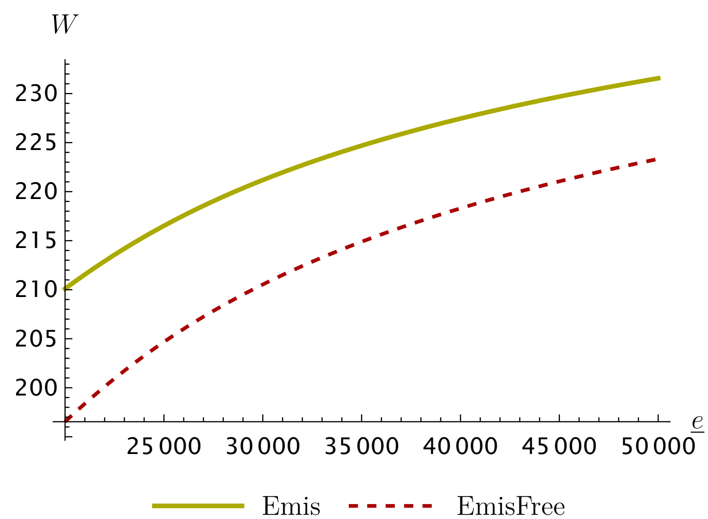

Figure 13: Welfare for different scenarios along different emissions thresholds for regulation, in million utils

the different effects on aggregate output.[37] Exempting an increasing number of

[37]See Figures A.7 and A.8 in the Appendix for output and emissions.

firms by increasing the threshold results in higher welfare, as the output loss from bunching at the threshold is more limited, the fewer firms are targeted by the regulation.[38] As before, the social costs of emissions dictate the tradeoff between benefits and damages of emissions, and hence, the curvature of the welfare function in the threshold level: Only for some higher levels of social costs of emissions, the welfare function decreases in the threshold level - with current parameterisation, a threshold level of 25 000tCO_2e (the emissions threshold in the EU ETS) would be welfare-maximising for SCE of \$767 without, and SCE of \$911 with free allocation.[39]

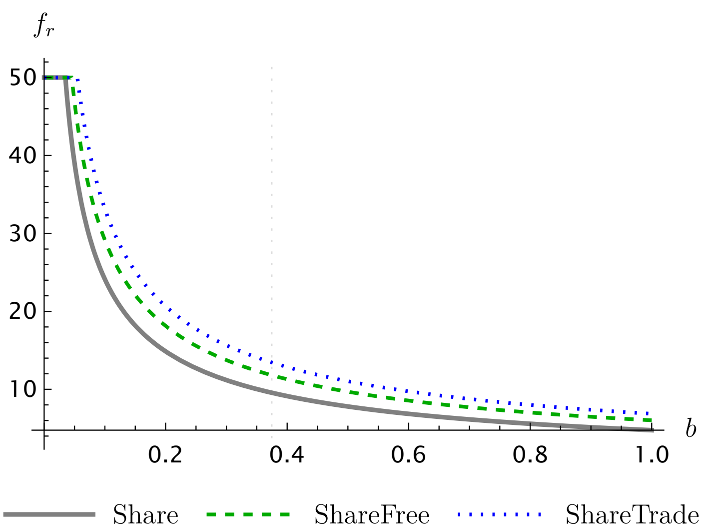

Figure 14: Fixed regulatory costs which make different levels of coverage of firms welfare-maximising, in number of employees (with $b = 0.375$ indicated)

Figure 12 showed that when firms cannot self-select into regulation, it is optimal to regulate all firms given the current parameterisation, see Section 2.5. In this case, not only can lower social costs of emissions justify an exemption from emissions pricing because a lower weight is given to emission reductions, but also higher regulatory fixed costs which crowd out production - increasingly so the more firms are regulated as shown in Figure 14.[40] Assuming the EU chose the appropriate share of exempted firms, the exemption (at $b = 0.375$) is welfare-maximising at fixed costs for monitoring, reporting, and verification of

[38]I show in Appendix A.20 that for some low (high) levels of $\underline{e}$, the emissions threshold is (in the limit) non-binding and no active firm can bunch below the threshold. Without the inefficiency resulting from bunching, welfare results are (in the limit) equivalent to the case with a fixed $b = 1$ ($b = 0$) (recall Figure 12).

[39]See Figure A.9 in the Appendix.

[40]For the social costs of emissions justifying an exemption in the Share cases, please see Figure A.10 in the Appendix.

emissions of 9.55 (which corresponds to ca. 10 persons employed per year).[41] If regulated firms receive the free permits in a lump sum manner as it is in ShareFree, then the free permits work as an opposite force to the fixed costs of regulation. In the trade and free allocation scenarios, the market consists of larger, cleaner firms than in the baseline – given their size, those firms are less prone to changes in the fixed costs of regulation given their larger profit scale. Additionally, in the two extension cases, fewer firms are active in the market such that the aggregate spending on regulatory fixed costs is lower. Hence, the fixed costs would need to be even larger to justify the exemption (11.80 for ShareFree and 13.39 for ShareTrade).[42]

The analyses above showed the trade-off between saving bureaucratic effort induced by an emissions pricing system and reducing total emissions. The magnitudes of both effects dictate which one is dominating the effect of the exemption on welfare, quantified by the fixed costs of regulation and the social costs of emissions. In practice, this trade-off can be different when the total emissions are restricted to a budget. In Appendix A.18, I show that exempting firms from emissions pricing can be welfare-improving for a lower level of bureaucratic costs when total emissions are bounded to a target. It is due to the emissions price strongly increasing if fewer firms are regulated that can make emission reductions happen, such that a wider exemption of firms comes at strong efficiency losses for the regulated, cleanest firms.

Proposition 6. *The welfare-maximising scope of the exemption from emissions pricing hinges on the size of the fixed costs associated with the regulation, as well as the social costs of emissions.*

Overall, this welfare analysis has shown that exempting some firms from emissions pricing can be welfare-maximising: The exemption leads to a reduced aggregate bureaucratic effort, thus limiting potential output losses. On the other hand, the emissions of the exempted firms can increase in the emissions price (within-country leakage), with this effect being even stronger when the exemption is determined with an emissions threshold: The option of escaping regulation makes the emissions price an even less effective tool to drive down aggregate emissions. Making emissions pricing more ambitious over time will therefore only achieve net zero emissions if the threshold to be regulated is adjusted over time.[43]

[41] In my chosen parameterisation, the fixed costs of regulation correspond to 0.39 employees per year.

[42] For Emis and EmisFree, the fixed regulatory costs are positive for welfare as they work as an emissions price themselves by inducing emission reductions: Firms reduce their emissions to fall below the threshold (see Section Appendix A.17 for more).

[43] The goals of most emissions pricing systems are to eventually reduce emissions to zero (e.g. "the European Green Deal [...] endorsed the objective of achieving a climate-neutral EU by 2050, in line with the objectives of the Paris Agreement" (European Union, 2020)).

5. Conclusion

In this paper, I have analysed how the exemption of small emitters from emissions pricing leads to within-country emissions leakage in a heterogeneous firm model. Firms differ in their productivity levels and can choose the scale of output and input combination of emissions and labour for their production. Mandatory emissions prices increase the costs of firms: They can counteract by dedicating more effort to emission abatement (such that they change their input composition in production) or by reducing the scale of production to decrease their emissions.

When an emissions pricing system exempts small emitters such that the rather unproductive firms do not have to participate, the firms at the lower part of the productivity distribution receive a relative cost advantage from the exemption. A higher level of productivity implies a lower emission intensity, such that dirtier firms are more likely to survive in the domestic market. Compared to no exemption, more and dirtier firms are active in the market.

A tightening of environmental policy by increasing the emissions price has asymmetric effects on profits along the productivity distribution. The exempted, dirtier firms benefit from the price level increase by receiving a larger market share, resulting in emissions leakage to the unregulated part of the distribution: The aggregate emissions of regulated firms decrease, but the ones of unregulated firms increase.

The effect is stronger if regulation only exempts firms which emit less than a fixed threshold, instead of a fixed share of firms: It allows some firms to strategically choose to be exempted by abating more and reducing their emissions to a level just below the regulatory threshold. When the emissions price rises, the shadow value of falling below the regulation threshold increases such that more firms decide to increase their abatement effort to fall in the exempted group. Hence, the leakage effect is stronger: Not only are dirtier firms active in the market, but also do more firms fall into the unregulated scope.

Overall, the emissions price is not as effective in driving down total emissions when some firms are exempted, which comes at the social costs of emissions. The issue is aggravated when some firms have the option to self-select into exemption by emitting less than a specific threshold. The emissions price is more effective when it can target a higher market share, which is the case if the most efficient firms have the option to export their goods abroad. A higher emissions price is beneficial for welfare by reducing total emissions; on the other hand, it results in output distortions when some firms reduce their production to fall below an emissions threshold or more market share is shifted towards less efficient firms. If emissions pricing is complemented with free allocation, regulated firms above (below) an emissions benchmark can buy (sell) lacking (excess) emissions rights. The lump-sum payments of free permits incentivise firms to stay regulated: If a fixed share of firms is regulated, free allocation limits the efficiency losses as it shifts more resources to the more efficient firms. Under a regime with an emissions threshold, fewer firms will cut their emissions to be exempted if being regulated comes with free permits. Nevertheless, the bunching of some firms below the threshold continues to yield adverse output effects.

The trade-off between saving bureaucratic effort and reducing total emissions may justify an exemption of small firms from emissions pricing: Regulating more firms than optimal would result in less output to be consumed due to increased bureaucratic costs. Regulating fewer firms, on the other hand, increases total emissions. Consequently, the appropriate extent of firm coverage under emissions pricing hinges on the size of the fixed costs associated with the regulation, as well as the social costs of emissions.

Efficient emissions pricing systems are crucial for achieving net zero emissions. In situations where the social costs of emissions are low or monitoring, reporting, and verification of emissions is associated with high costs, it may be reasonable to exempt some firms. However, the problem of within-country leakage stemming from the regulation, along with welfare cost estimates for every additional unit emitted increasing over time (Tol, 2023) and with advancements in emissions accounting technology highlight the need to revise exemption criteria.

In my current setup, I can only look at within-country leakage within one market. Future research could study within-country leakage also between different sectors with one common emissions cap, e.g. by extending my model by allowing for multiple sectors, as in Shapiro and Walker (2018), or a second sector with homogeneous goods, as in Richter et al. (2021) who base it on Grossman and Krueger (1991) and Antweiler, Copeland and Taylor (2001). For quantification it could furthermore be interesting to add a regulation threshold in a CGE modeling framework with emissions pricing, as in Balistreri and Rutherford (2012) or Böhringer, Rutherford and Schneider (2021).

References

Anouliès, Lisa. 2017. "Heterogeneous firms and the environment: a cap-and-trade program." *Journal of Environmental Economics and Management*, 84: 84–101.

Antweiler, Werner, Brian R Copeland, and M Scott Taylor. 2001. "Is free trade good for the environment?" *American Economic Review*, 91(4): 877–908.

Balistreri, Edward J, and Thomas F Rutherford. 2012. "Subglobal carbon policy and the competitive selection of heterogeneous firms." *Energy Economics*, 34: S190–S197.

Barrows, Geoffrey, and Hélène Ollivier. 2018. "Cleaner firms or cleaner products? How product mix shapes emission intensity from manufacturing." *Journal of Environmental Economics and Management*, 88: 134–158.

Becker, Randy A, Carl Pasurka Jr, and Ronald J Shadbegian. 2013. "Do environmental regulations disproportionately affect small businesses? Evidence from the Pollution Abatement Costs and Expenditures survey." *Journal of Environmental Economics and Management*, 66(3): 523–538.

Becker, Randy, and Vernon Henderson. 2000. "Effects of air quality regulations on polluting industries." *Journal of political Economy*, 108(2): 379–421.

Böhringer, Christoph, Thomas F Rutherford, and Jan Schneider. 2021. "The incidence of CO2 emissions pricing under alternative international market responses: A computable general equilibrium analysis for Germany." *Energy Economics*, 101: 105404.

Branger, Frédéric, Jean-Pierre Ponssard, Oliver Sartor, and Misato Sato. 2015. "EU ETS, free allocations, and activity level thresholds: the devil lies in the details." *Journal of the Association of Environmental and Resource Economists*, 2(3): 401–437.

Böhringer, Christoph, and Thomas F. Rutherford. 1997. "Carbon Taxes with Exemptions in an Open Economy: A General Equilibrium Analysis of the German Tax Initiative." *Journal of Environmental Economics and Management*, 32(2): 189–203.

Cao, Jing, Larry D Qiu, and Mohan Zhou. 2016. "Who invests more in advanced abatement technology? Theory and evidence." *Canadian Journal of Economics/Revue canadienne d'économique*, 49(2): 637–662.

Chen, Qiaoyi, Zhao Chen, Zhikuo Liu, Juan Carlos Suárez Serrato, and Daniel Xu. 2021. "Regulating Conglomerates in China: Evidence from an Energy Conservation Program." National Bureau of Economic Research Working Paper 29066.

Clò, Stefano. 2010. "Grandfathering, auctioning and Carbon Leakage: Assessing the inconsistencies of the new ETS Directive." *Energy Policy*, 38(5): 2420–2430.

Copeland, Brian R, and M Scott Taylor. 1995. "Trade and transboundary pollution." *American Economic Review*, 716–737.

Copeland, Brian R, and M Scott Taylor. 2004. "Trade, growth, and the environment." *Journal of Economic Literature*, 42(1): 7–71.

Copeland, Brian R, Joseph S Shapiro, and M. Scott Taylor. 2021. "Globalization and the Environment." National Bureau of Economic Research Working Paper 28797.

Coria, Jessica, and Jūratė Jaraitė. 2019. "Transaction Costs of Upstream Versus Downstream Pricing of CO_2 Emissions." *Environmental and Resource Economics*, 72(4): 965–1001.

Cui, Jingbo, Harvey Lapan, and GianCarlo Moschini. 2016. "Productivity, export, and environmental performance: air pollutants in the United States." *American Journal of Agricultural Economics*, 98(2): 447–467.

Dardati, Evangelina, and Meryem Saygili. 2020. "Aggregate impacts of cap-and-trade programs with heterogeneous firms." *Energy Economics*, 92: 104924.

European Commission. 2003. "Directive 2003/87/EC of the European Parliament and of the Council of 13 October 2003 establishing a system for greenhouse gas emission allowance trading within the Union and amending Council Directive 96/61/EC." *Official Journal of the European Union*, 50(275): 32–46.

European Commission. 2009. "Directive 2009/29/EC of the European Parliament and of the Council of 23 April 2009 amending Directive 2003/87/EC so as to improve and extend the greenhouse gas emission allowance trading scheme of the Community." *Official Journal of the European Union*, 5.

European Commission. 2021*a*. "Auctioning." *https://ec.europa.eu/clima/policies/ets/auctioning_en*. Accessed: August 12, 2021.

European Commission. 2021*b*. "EU Emissions Trading System (EU ETS)." *https://ec.europa.eu/clima/policies/ets_en#tab-0-0*. Accessed: June 06, 2021.

European Commission. 2021*c*. "Update of benchmark values for the years 2021 – 2025 of phase 4 of the EU ETS." *https://climate.ec.europa.eu/system/files/2021-10/policy_ets_allowances_bm_curve_factsheets_en.pdf*, Updated final version issued on 12 October 2021.

European Union. 2020. "Long-term low greenhouse gas emission development strategies (LT-LEDS)." European Union Report.

Exchange Rates UK. n.d.. "EUR USD Spot Exchange Rates History 2013 (2009)." *https://www.exchangerates.org.uk/EUR-USD-spot-exchange-rates-history-2013.html*, Accessed: May 1, 2023.

Fischer, Carolyn, and Alan K Fox. 2007. "Output-based allocation of emissions permits for mitigating tax and trade interactions." *Land Economics*, 83(4): 575–599.

Forslid, Rikard, Toshihiro Okubo, and Karen Helene Ulltveit-Moe. 2018. "Why are firms that export cleaner? International trade, abatement and environmental emissions." *Journal of Environmental Economics and Management*, 91: 166–183.

Grossman, Gene M, and Alan B Krueger. 1991. "Environmental Impacts of a North American Free Trade Agreement." National Bureau of Economic Research Working Paper 3914.

Guner, Nezih, Gustavo Ventura, and Yi Xu. 2008. "Macroeconomic implications of size-dependent policies." *Review of economic Dynamics*, 11(4): 721–744.

Heindl, Peter. 2017. "The impact of administrative transaction costs in the EU emissions trading system." *Climate Policy*, 17(3): 314–329.

ICAP. 2022. "ETS Detailed Information: Chile." https://icapcarbonaction.com/en/ets/chile. Accessed: August 18, 2022.

Janssen, Laura, Jan Jakob Peelen, Jurjen Bevers, and Thomas De Booij. 2020. "The Dutch carbon dioxide emission tax." https://www.dentons.com/en/insights/alerts/2020/november/26/the-dutch-carbon-emission-tax. Accessed: January 27, 2022.

Jaraitė, Jūratė, Frank Convery, and Corrado Di Maria. 2010. "Transaction costs for firms in the EU ETS: lessons from Ireland." *Climate Policy*, 10(2): 190–215.

Kaplow, Louis. 2019. "Optimal regulation with exemptions." *International Journal of Industrial Organization*, 66: 1–39.

Konishi, Yoshifumi, and Nori Tarui. 2015. "Emissions trading, firm heterogeneity, and intra-industry reallocations in the long run." *Journal of the Association of Environmental and Resource Economists*, 2(1): 1–42.

Kortum, Samuel S, and David A Weisbach. 2021. "Optimal unilateral carbon policy." *Cowles Foundation Discussion Paper*, No. 2311.

LaPlue, Lawrence D. 2019. "The environmental effects of trade within and across sectors." *Journal of Environmental Economics and Management*, 94: 118–139.

Martin, Ralf, Mirabelle Muûls, Laure B. de Preux, and Ulrich J. Wagner. 2014. "On the empirical content of carbon leakage criteria in the EU Emissions Trading Scheme." *Ecological Economics*, 105: 78–88.

Melitz, Marc J. 2003. "The impact of trade on intra-industry reallocations and aggregate industry productivity." *Econometrica*, 71(6): 1695–1725.

National Association of Manufacturers. 2022. "2022 United States Manufacturing Facts." https://www.nam.org/state-manufacturing-data/2022-united-states-manufacturing-facts/, Accessed: May 1, 2023.

National Institute of Standards and Technology. 2023. "Total U.S. Manufacturing: Manufacturing Industry Statistics." https://www.nist.gov/el/applied-economics-office/manufacturing/total-us-manufacturing/manufacturing-economy/total-us, Accessed: May 1, 2023.

NCCS Singapore. 2022. "Carbon Tax." *https://www.nccs.gov.sg/faqs/carbon-tax/*. Accessed: August 18, 2022.

Qi, Ji, Xin Tang, and Xican Xi. 2021. "The size distribution of firms and industrial water pollution: a quantitative analysis of China." *American Economic Journal: Macroeconomics*, 13(1): 151–83.

Rennert, Kevin, Frank Errickson, Brian C Prest, Lisa Rennels, Richard G Newell, William Pizer, Cora Kingdon, Jordan Wingenroth, Roger Cooke, Bryan Parthum, et al. 2022. "Comprehensive evidence implies a higher social cost of CO2." *Nature*, 610(7933): 687–692.

Richter, Philipp M, and Alexander Schiersch. 2017. "CO2 emission intensity and exporting: Evidence from firm-level data." *European Economic Review*, 98: 373–391.

Richter, Philipp M, Udo Kreickemeier, and Hartmut Egger. 2021. "Environmental Policy and Firm Selection in the Open Economy." *Journal of the Association of Environmental and Resource Economists*, 8(4): 413–430.

Sawyer, D., S. Stiebert, R. Gignac, A. Campney, and D. Beugin. 2021. "2020 Expert Assessment of Carbon Pricing Systems." *Canadian Institute for Climate Choices.*

Shapiro, Joseph S, and Reed Walker. 2018. "Why is pollution from US manufacturing declining? The roles of environmental regulation, productivity, and trade." *American Economic Review*, 108(12): 3814–54.

The Boyd Company, Inc. 2021. "Comparative Advanced Manufacturing Operating Costs." *https://www.southdundas.com/sites/3/files/2021-10/Comparative-Advanced-Manufacturing-Facility-Operating-Costs-Report.pdf*, Accessed: April 28, 2023.

Tol, Richard S. J. 2023. "Social cost of carbon estimates have increased over time." *Nature Climate Change.*

Trading Economics. 2023. "EU Carbon Emissions Allowances Prices." *https://tradingeconomics.com/commodity/carbon*, Accessed: May 1, 2023.

U.S. Bureau of Labor Statistics. 2023. "Employment, Hours, and Earnings from the Current Employment Statistics survey (National)." *U.S. Department of Labor.*

U.S. Bureau of Labor Statistics. n.d.. "Consumer Price Index Inflation Calculator." *https://www.bls.gov/data/inflation_calculator.htm*, Accessed: May 1, 2023.

U.S. Energy Information Administration. 2022*a*. "Annual Energy Outlook 2022." *https://www.eia.gov/outlooks/aeo/*, Accessed on April 28, 2023.

U.S. Energy Information Administration. 2022*b*. "U.S. Energy-Related Carbon Dioxide Emissions, 2021." *https://www.eia.gov/environment/emissions/carbon/pdf/2021_co2analysis.pdf*, Accessed on April 28, 2023.

Appendix A. Details and Derivations for Selected Equations

Appendix A.1. Firm Profit, Demand, and Revenue

The demand every firm is facing is

$$q(\phi) = RP^{\sigma-1} \left(\frac{\sigma}{\sigma-1} \, \frac{1}{\phi} \, \frac{h(\phi)^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \right)^{-\sigma}, \tag{A.1}$$

and the corresponding revenue

$$r(\phi) = RP^{\sigma-1} \left(\frac{\sigma}{\sigma-1} \, \frac{1}{\phi} \, \frac{h(\phi)^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \right)^{1-\sigma}. \tag{A.2}$$

Appendix A.2. Input, Revenue, and Output Ratios for different Productivities

For the equilibrium calculations, the ratios between two firms with different productivities ϕ_1 and ϕ_2 are helpful. Critically, the simple ratios do hold if and only if ϕ_1 and ϕ_2 are both facing the same emissions price $h(\phi)$. That is:

$$\begin{aligned}
&\text{if } \phi_1, \phi_2 < \phi_r \text{ or } \phi_1, \phi_2 \geq \phi_r, \text{ then:} \\
&\frac{p(\phi_1)}{p(\phi_2)} = \frac{\phi_2}{\phi_1}, \\
&\frac{q(\phi_1)}{q(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{\sigma}, \\
&\frac{r(\phi_1)}{r(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{\sigma-1} = \frac{e(\phi_1)}{e(\phi_2)} = \frac{l(\phi_1)}{l(\phi_2)}.
\end{aligned} \tag{A.3}$$

Additionally, the ratio of revenues at the threshold ϕ_r, following from (A.2), is:

$$\lim_{\phi \downarrow \phi_r} r(\phi) = \left(\frac{c_e + p_r}{c_e}\right)^{\alpha(1-\sigma)} \lim_{\phi \uparrow \phi_r} r(\phi) \qquad \forall b \in (0,1). \tag{A.4}$$

The first term on the right hand side determines the discontinuity at the threshold, analogous to the one for emissions in Figure 2.

Appendix A.3. Average Profits

The average profits (ZCP) can be derived as follows:

$$
\begin{aligned}
\bar{\pi}(\phi_d) &= \int_{\phi_d}^{\phi_r} \pi(\phi)\mu(\phi)\ \mathbf{d}\phi + \int_{\phi_r}^{\infty} \pi(\phi)\mu(\phi)\ \mathbf{d}\phi \\
&= \int_{\phi_d}^{\phi_r} \left(\frac{r(\phi)}{\sigma}\right)\mu(\phi)\ \mathbf{d}\phi + \int_{\phi_r}^{\infty} \left(\frac{r(\phi)}{\sigma}\right)\mu(\phi)\ \mathbf{d}\phi - w(f_y + bf_r) \\
&= \frac{1}{\sigma}\Big(\int_{\phi_d}^{\phi_r} wf_y\sigma\left(\frac{\phi}{\phi_d}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi \\
&\quad + \int_{\phi_r}^{\infty} \lim_{\phi\downarrow\phi_r} r(\phi)\left(\frac{\phi}{\phi_r}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi\Big) - w(f_y + bf_r) \\
&= \frac{1}{\sigma}\Big(\int_{\phi_d}^{\phi_r} wf_y\sigma\left(\frac{\phi}{\phi_d}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi \\
&\quad + \int_{\phi_r}^{\infty} \lim_{\phi\uparrow\phi_r} r(\phi)\left[\frac{c_e}{p_r + c_e}\right]^{\alpha(\sigma-1)}\left(\frac{\phi}{\phi_r}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi\Big) - w(f_y + bf_r) \\
&= \frac{1}{\sigma}\Big(\int_{\phi_d}^{\phi_r} wf_y\sigma\left(\frac{\phi}{\phi_d}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi \\
&\quad + \int_{\phi_r}^{\infty} wf_y\sigma\left(\frac{\phi_r}{\phi_d}\right)^{\sigma-1}\left[\frac{c_e}{p_r + c_e}\right]^{\alpha(\sigma-1)}\left(\frac{\phi}{\phi_r}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi\Big) - w(f_y + bf_r) \\
&= wf_y\Big(\int_{\phi_d}^{\phi_r}\left(\frac{\phi}{\phi_d}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi \\
&\quad + \left[\frac{c_e}{p_r + c_e}\right]^{\alpha(\sigma-1)}\int_{\phi_r}^{\infty}\left(\frac{\phi}{\phi_d}\right)^{\sigma-1}\mu(\phi)\ \mathbf{d}\phi - 1\Big) - bwf_r \\
&= wf_y\left[\frac{\theta}{\theta - \sigma + 1}\eta(p_r) - 1\right] - \underbrace{bwf_r}_{\text{bureaucratic effort for ETS}}, \qquad \forall b \in [0, 1),
\end{aligned}
$$

with

$$
\eta(p_r) = \left(1 - b^{\frac{\theta-\sigma+1}{\theta}}\left[1 - \underbrace{\left(\frac{c_e}{p_r + c_e}\right)^{\alpha(\sigma-1)}}_{<1}\right]\right) \leq 1.
$$

Appendix A.4. Average Labour Input in Production

The average labour employed per firm for variable production follows:

$$
\bar{l}(\phi_d) = \int_{\phi_d}^{\infty} l(\phi)\mu(\phi)\ \mathbf{d}\phi.
$$

The labour input per firm can be derived via (8) and the fact that the revenue at the cutoff consists of fixed cost and total variable costs: $r(\phi_d) = wf_y +$

$q(\phi_d)k(\phi_d) = \frac{r(\phi_d)}{\sigma} + wl(\phi_d) + h(\phi_d)e(\phi_d)$. Furthermore, the Cobb-Douglas specification implies that $wl(\phi_d) = (1-\alpha)\left[q(\phi_d)k(\phi_d)\right]$. By also using the ratios from (A.3), I can write the labour input of regulated and unregulated firms as follows:

$$l(\phi|\phi_d \leq \phi < \phi_r) = \left(\frac{\phi}{\phi_d}\right)^{\sigma-1}(1-\alpha)(\sigma-1)f_y,$$
$$l(\phi|\phi_r \leq \phi) = \left(\frac{\phi}{\phi_d}\right)^{\sigma-1}(1-\alpha)(\sigma-1)f_y\left[\frac{c_e}{p_r+c_e}\right]^{\alpha(\sigma-1)}.$$

Hence the average input of labour:

$$\bar{l} = \frac{(1-\alpha)(\sigma-1)f_y\theta}{1+\theta-\sigma}\ \eta(p_r), \qquad \forall b \in [0,1).$$

Appendix A.5. Price Level

The price level (13) can be derived as follows:

$$
\begin{aligned}
P &= \left[\int_{\phi_d}^{\infty} M p(\phi)^{1-\sigma} \mu(\phi)\ \mathbf{d}\phi\right]^{\frac{1}{1-\sigma}}, \\
P^{1-\sigma} &= M\Big[\int_{\phi_d}^{\phi_r} p(\phi)^{1-\sigma} \mu(\phi)\ \mathbf{d}\phi + \int_{\phi_r}^{\infty} p(\phi)^{1-\sigma} \mu(\phi)\ \mathbf{d}\phi\Big] \\
&= M\Big[\int_{\phi_d}^{\phi_r} p(\phi_d)^{1-\sigma} \left(\frac{\phi}{\phi_d}\right)^{\sigma-1} \mu(\phi)\ \mathbf{d}\phi \\
&\quad + \int_{\phi_r}^{\infty} \lim_{\phi\downarrow\phi_r} p(\phi)^{1-\sigma} \left(\frac{\phi}{\phi_r}\right)^{\sigma-1} \mu(\phi)\ \mathbf{d}\phi\Big] \\
&= M\Big[\int_{\phi_d}^{\phi_r} p(\phi_d)^{1-\sigma} \left(\frac{\phi}{\phi_d}\right)^{\sigma-1} \mu(\phi)\ \mathbf{d}\phi \\
&\quad + \int_{\phi_r}^{\infty} \lim_{\phi\uparrow\phi_r} p(\phi)^{1-\sigma} \left[\frac{p_r + c_e}{c_e}\right]^{\alpha(1-\sigma)} \left(\frac{\phi}{\phi_r}\right)^{\sigma-1} \mu(\phi)\ \mathbf{d}\phi\Big] \\
&= M p(\phi_d)^{1-\sigma}\Big[\int_{\phi_d}^{\phi_r} \left(\frac{\phi}{\phi_d}\right)^{\sigma-1} \mu(\phi)\ \mathbf{d}\phi \\
&\quad + \int_{\phi_r}^{\infty} \left[\frac{p_r + c_e}{c_e}\right]^{\alpha(1-\sigma)} \left(\frac{\phi}{\phi_d}\right)^{\sigma-1} \mu(\phi)\ \mathbf{d}\phi\Big] \\
&= M\eta(p_r)\frac{\theta}{\theta-\sigma+1} p(\phi_d)^{1-\sigma} \\
&= \eta(p_r)\frac{\theta}{\theta-\sigma+1}\,\frac{L}{f_y \eta(p_r)}\ \frac{1+\theta-\sigma}{\theta(\sigma-\sigma\alpha+\alpha)}\left(\frac{\sigma}{\sigma-1}\ \phi_d^{-1}\frac{c_e^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{1-\sigma} \\
&= \frac{L}{f_y}\ \frac{1}{\sigma-\sigma\alpha+\alpha}\left(\frac{\sigma}{\sigma-1}\ \phi_d^{-1}\frac{c_e^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{1-\sigma}, \\
P &= \left(\frac{L}{f_y}\ \frac{1}{\sigma-\sigma\alpha+\alpha}\right)^{\frac{1}{1-\sigma}}\ \frac{\sigma}{\sigma-1}\frac{c_e^{\alpha} w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\ \phi_d^{-1}, \qquad \forall b \in [0,1).
\end{aligned}
$$

Appendix A.6. Constancy of the Aggregate Revenue

This paragraph shows that the aggregate revenue (14) does not change in the emissions price: $\frac{\partial R}{\partial p_r} = 0$. Given that labour supply L is exogenous and the wage w is a numeraire, the only endogenous change in aggregate revenue (14) could stem from the revenue generated from emissions. With the Cobb-Douglas production function, the expenditure shares for both emissions and labour as a variable part of production are fixed for every firm, such that the ratio of emissions expenditure to labour expenditure follows the ratio of both output elasticities:

$$\frac{\alpha}{1-\alpha} = \frac{h(\phi)\ e(\phi)}{w\ l(\phi)}.$$

In aggregate, this relationship is:[44]

$$\frac{\alpha(\sigma-1)}{\sigma-\alpha(\sigma-1)}=\frac{c_e(E_r+E_{nr})+p_rE_r}{w\ L}.$$

As α and σ are fixed, the aggregate emissions revenue $c_e(E_r+E_{nr})+p_rE_r$ cannot change for a change in p_r; hence, $\frac{\partial R}{\partial p_r}=0$. This is true for all scenarios in which a fixed share b of firms is regulated.

Appendix A.7. Aggregate Emissions

By using the average emissions of the regulated firms, the aggregate emissions for regulated firms (15) can be derived as follows:

$$E_r=bM\frac{1}{1-G(\phi_r)}\int_{\phi_r}^{\infty}e(\phi)g(\phi)\mathbf{d}\phi.$$

The emissions input of unregulated and regulated firms can be derived analogously to the labour input (see Appendix A.4):

$$\begin{aligned}e(\phi|\phi_d\leq\phi<\phi_r)&=\left(\frac{\phi}{\phi_d}\right)^{\sigma-1}\alpha\frac{w}{c_e}(\sigma-1)f_y,\\ e(\phi|\phi_r\leq\phi)&=\left(\frac{\phi}{\phi_d}\right)^{\sigma-1}\alpha\frac{w}{c_e}(\sigma-1)f_y\left[\frac{c_e}{p_r+c_e}\right]^{\alpha(\sigma-1)+1}.\end{aligned}$$

such that:

$$E_r=\frac{\left[\frac{c_e}{p_r+c_e}\right]^{\alpha(\sigma-1)+1}b^{\frac{\theta-\sigma+1}{\theta}}}{\eta(p_r)}\ \frac{wL}{c_e}\frac{\alpha(\sigma-1)}{\sigma-\alpha(\sigma-1)},\qquad\forall b\in[0,1).$$

The aggregate emissions for all firms are defined as $E=M\bar{e}(\phi_d)$. The average emissions are as follows:

$$\begin{aligned}\bar{e}(\phi_d)&=\int_{\phi_d}^{\phi_r}e(\phi)\mu(\phi)\ \mathbf{d}\phi+\int_{\phi_r}^{\infty}e(\phi)\mu(\phi)\ \mathbf{d}\phi\\ &=\frac{\alpha(\sigma-1)f_y\theta}{1+\theta-\sigma}\ \frac{w}{c_e}\left(\left(1-b^{\frac{\theta-\sigma+1}{\theta}}\right)+\left[\frac{c_e}{p_r+c_e}\right]^{\alpha(\sigma-1)+1}b^{\frac{\theta-\sigma+1}{\theta}}\right),\quad\forall b\in[0,1).\end{aligned}$$

[44] Using the condition for labour market clearing, (11).

Appendix A.8. Parameter Choices with Sources

Parameter	Description	Value	Source
L	Labour supply	12 790 000	All employees, manufacturing, seasonally adjusted, Jan 2020 (U.S. Bureau of Labor Statistics, 2023)
w	wage	\$89 872	average annual wage per manufacturing worker; \$42.78 per hour and 40.4 hours worked per week, Jan 2020 (U.S. Bureau of Labor Statistics, 2023)
b	Share of regulated firms	0.375	With a threshold of 25 000tCO_2e (reported in 2009) and free allocation $s = 0.1$: 66.5% of emissions regulated which would need b= 37.5% in the model with a fixed share of firms regulated
α	energy share in production	0.0678	Exploiting Cobb-Douglas expenditure shares: \$89 290mn Expenditures for Purchased Energy Sources, 2018, without electricity (U.S. Energy Information Administration, 2022*a*) (inflation-adjusted \$91 404)
σ	elasticity of substitution	4.76	Shapiro and Walker (2018)
θ	firms' productivity homogeneity (Pareto shape)	5.51	Shapiro and Walker (2018): $\vartheta = 5.14$ if $\varphi^{1-\alpha} = \phi$ and $\varphi \sim P(1, \vartheta)$, then$\phi \sim P(1, \frac{\vartheta}{1-\alpha})$; hence: $\theta = \frac{\vartheta}{1-\alpha} = \frac{5.14}{1-0.0678} = 5.51$
c_e	emissions-related energy costs	\$64	Energy Expenditure associated with one metric ton of CO_2e: 1 433 MMT CO_2e for 2019 (U.S. Energy Information Administration, 2022*b*); \$91 404 Expenditures for Purchased Energy Sources without electricity (see above for α)

Parameter	**Description**	**Value**	**Source**
p_r	emissions price	\$22	Average EU ETS Permit Price in 2009 (according to emission threshold reported in 2009): 13.19€ (Trading Economics, 2023) (EXR 1.39 \$/€ (Exchange Rates UK, n.d.), then US inflation-adjusted)
f_y	operating fixed costs	\$5 174 708 / w	Annual operating fixed costs per plant for 2016: \$4 826 820 (The Boyd Company, Inc., 2021) (inflation-adjusted)
f_e	entry fixed costs	$0.5 f_y$	Entry cost per plant, over 40 years; LaPlue (2019) uses $f_e = 20 f_y$ with a probability of firm death of 0.025 which corresponds to an expected age of 40y per plant
f_r	regulation fixed costs	\$35 392 / w	26 500€ annual EU ETS MRV costs, Coria and Jaraitė (2019) for 2013: EXR 1.33\$/€ (Exchange Rates UK, n.d.) (inflation-adjusted)
f_x	export fixed costs	f_y	Fixed costs of exporting, via LaPlue (2019): $f_x = f_y$

Parameter	**Description**	**Value**	**Source**
τ	iceberg trade costs	1.3	Variable costs of exporting, via LaPlue (2019): range from 1.1 to 1.5; fits US manufacturing industry as it corresponds to range from 51% to 10% of output exported in my model; use average of 1.3. Data: 56% of manufacturing output exported ($2 497bn Output in 2021, $1 399bn Exports (National Association of Manufacturers, 2022)) vs. 10.6% of manufacturing goods imported in 2020 (National Institute of Standards and Technology, 2023). Model requires international goods market clearing.
$\underline{e}$	emission threshold	25 000t CO_2e	Threshold of 25 000t CO_2e (European Commission, 2009)
s	benchmarking share for free allocation	0.1	54 benchmarks for determining the average performance of the s% most efficient installations in 2007/2008 (European Commission, 2021*c*)
SCE	Social Costs of Emissions per ton of CO_2e	$185	Social Cost of Carbon: $185 per ton of CO_2 (Rennert et al., 2022)

Table A.1: Parameter choices with sources. All values are chosen such that one period represents one year and the Jan 2020 USD value for the average plant in manufacturing in the United States, assuming that values in the time interval of 2009-2020 can be representative. Inflation-adjustments via U.S. Bureau of Labor Statistics (n.d.).

Appendix A.9. Price, Quantity, and Revenue of Medium-Productivity Firms for an Emissions Threshold

The price, quantity, and revenue chosen by medium-productivity firms for a fixed $\underline{e}$ are as follows:

$$p(\phi|\phi_o \leq \phi < \phi_r) = \left\{ \left[\frac{\sigma}{(1-\alpha)(\sigma-1)} w \right]^{1-\alpha} \frac{(RP^{\sigma-1})^{\alpha}}{\phi \ \underline{e}^{\alpha}} \right\}^{\frac{1}{1-\alpha+\sigma\alpha}},$$

$$q(\phi|\phi_o \leq \phi < \phi_r) = \left\{ \left[\frac{\sigma}{(1-\alpha)(\sigma-1)} w \right]^{-\sigma(1-\alpha)} \frac{(RP^{\sigma-1})^{1-\alpha}}{\phi^{-\sigma} \ \underline{e}^{-\sigma\alpha}} \right\}^{\frac{1}{1-\alpha+\sigma\alpha}},$$

$$r(\phi|\phi_o \leq \phi < \phi_r) = \left\{ \left[\frac{\sigma}{(1-\alpha)(\sigma-1)} w \right]^{(1-\sigma)(1-\alpha)} \frac{RP^{\sigma-1}}{\phi^{1-\sigma} \ \underline{e}^{(1-\sigma)\alpha}} \right\}^{\frac{1}{1-\alpha+\sigma\alpha}}.$$

Appendix A.10. Implicit Solution for the Regulation Threshold

When the emissions pricing is based on $\underline{e}$, the regulating threshold ϕ_r can be implicitly solved for:

$$\begin{aligned}
&\phi_r^{\sigma-1} \ \frac{1}{\sigma} RP^{\sigma-1} \left(\frac{\sigma-1}{\sigma} \ \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(c_e+p_r)^{\alpha} w^{1-\alpha}} \right)^{\sigma-1} \\
=&\phi_r^{\frac{\sigma-1}{1-\alpha+\sigma\alpha}} \left(\frac{\underline{e}^{(\sigma-1)\alpha}}{w^{(1-\alpha)(\sigma-1)}} \ RP^{\sigma-1} \right)^{\frac{1}{1-\alpha+\sigma\alpha}} \left\{ \left[\frac{(1-\alpha)(\sigma-1)}{\sigma} \right]^{\frac{(\sigma-1)(1-\alpha)}{1-\alpha+\sigma\alpha}} \right. \\
&\left. \quad - \left[\frac{(1-\alpha)(\sigma-1)}{\sigma} \right]^{\frac{\sigma}{1-\alpha+\sigma\alpha}} \right\} - c_e \underline{e}. \qquad \text{(A.5)}
\end{aligned}$$

Appendix A.11. Average Profits with an Emissions Threshold

With a threshold $\underline{e}$, the average profits consist out of the following parts:

$$\bar{\pi} = \int_{\phi_d}^{\phi_o} \pi(\phi)\mu(\phi) \ \mathbf{d}\phi + \int_{\phi_o}^{\phi_r} \pi(\phi)\mu(\phi) \ \mathbf{d}\phi + \int_{\phi_r}^{\infty} \pi(\phi)\mu(\phi) \ \mathbf{d}\phi.$$

First of all, for an increase in the emissions price, the profits of the high-productivity firms decrease. Secondly, the change in ϕ_r lets some firms switch to emitting less such that they fall in the exemption (are now incorporated in the group "medium-productivity"). Those switching firms also see a decrease of their profit calculations (otherwise, they would have emitted $\underline{e}$ already before). Overall, the threshold ϕ_d needs to decrease as average profits drop, pushing the ZCP line down (following from the free entry condition, (FE)). The negative effect on ϕ_d is dampened because all profits are increasing in the price level P when ϕ_d goes down (see (7), (23), and (24), and (25)).

Appendix A.12. Average Profits with Free Allocation

With free allocation, the average profits are:

$$\bar{\pi}(\phi_d) = \int_{\phi_d}^{\phi_r} \pi(\phi)\mu(\phi)\ \mathbf{d}\phi + \int_{\phi_r}^{\infty} \pi(\phi)\mu(\phi)\ \mathbf{d}\phi$$
$$= wf_y \left[\frac{\theta}{\theta-\sigma+1}\eta(p_r) - 1\right] - bwf_r + \underbrace{\rho(p_r)\ b^{\frac{\theta-\sigma}{\theta}} s^{\frac{1}{\theta}}}_{\text{income from free permits}},$$

with

$$\eta(p_r) = \left(1 - b^{\frac{\theta-\sigma+1}{\theta}}\left[1 - \underbrace{\left(\frac{c_e}{p_r+c_e}\right)}_{<1}^{\alpha(\sigma-1)}\right]\right),$$

$$\rho(p_r) = R(\phi_d P)^{\sigma-1}\frac{\theta^2}{(1+\theta)(\theta-\sigma)}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} p_r \frac{\alpha^{1-\alpha+\alpha\sigma}}{(p_r+c_e)^{1-\alpha}\ c_e^{\alpha\sigma}}\left(\frac{1-\alpha}{w}\right)^{(\sigma-1)(1-\alpha)},$$

such that

$$\frac{\partial\bar{\pi}(\phi_d)}{\partial\eta(p_r)} > 0 \quad \text{with} \quad \frac{\partial\eta(p_r)}{\partial p_r} < 0,$$
$$\frac{\partial\bar{\pi}(\phi_d)}{\partial\rho(p_r)} > 0 \quad \text{with} \quad \frac{\partial\rho(p_r)}{\partial p_r} > 0 \quad \text{given} \quad \frac{\partial(\phi_d P)}{\partial p_r} = 0.$$

Given that $\frac{\partial\bar{\pi}(\phi_d)}{\partial\rho(p_r)} > 0$ and $\frac{\partial\rho(p_r)}{\partial p_r} > 0$, the average profits are now not only decreasing in the emissions price via the exemption regulation (compare with (ZCP)), but also increasing in p_r via free allocation as it benefits mostly large, clean firms. The higher s, that is, the more firms are net sellers of permits, the stronger the second effect resulting from free allocation.

Appendix A.13. Average Profits with International Trade

In an open economy, the average profits of the domestic firms can be derived as follows:

$$\bar{\pi} = \bar{\pi}_d(\phi_d) + \chi(p_r)\bar{\pi}_x(\phi_x),$$
$$\bar{\pi}_x(\phi_x) = \frac{\sigma-1}{1+\theta-\sigma}wf_x,$$
$$\bar{\pi} = wf_y\left[\frac{\theta}{\theta-\sigma+1}\eta(p_r) - 1\right] - bwf_r + \underbrace{\chi(p_r)\frac{\sigma-1}{1+\theta-\sigma}wf_x}_{\text{income from exporting}}.$$

Appendix A.14. Aggregate Emissions with International Trade

The emissions of the regulated firms are:

$$E_r = \frac{f_y \left[\frac{c_e}{p_r+c_e}\right]^{\alpha(\sigma-1)} b^{\frac{\theta-\sigma+1}{\theta}} + f_x\chi(p_r)}{f_y\eta(p_r) + f_x\chi(p_r)} \frac{\alpha(\sigma-1)}{\sigma-\sigma\alpha+\alpha} \frac{wL}{p_r+c_e}. \tag{A.6}$$

As in (15), the aggregate emissions of the regulated depend negatively on the emissions price. For the unregulated firms, they are:

$$E_{nr} = \frac{\left(1-b^{\frac{\theta-\sigma+1}{\theta}}\right) f_y}{f_y\eta(p_r) + f_x\chi(p_r)} \frac{\alpha(\sigma-1)}{\sigma-\alpha(\sigma-1)} \frac{wL}{c_e}.$$

Compared to no trade ($\chi(p_r) = 0$, see (17)), within-country emissions leakage is now higher for a change in p_r: both $\eta(p_r)$ and $\chi(p_r)$ are decreasing in the emissions price. The argument is similar to the one for ϕ_d and E_r: The positive effect of trade on the firm participation thresholds shrinks when the exports of the most productive firms go down: Their upward force on the productivity threshold goes down such that ϕ_d decreases to a larger extent for a change in emissions price. The effect on the aggregate emissions of the unregulated firms is hence stronger.

Appendix A.15. Exemption based on a Productivity Threshold

Instead of Assumption 1 (fixed share of firms regulated) or 2 (emissions threshold), I now set the productivity threshold exogenously, that is:

Assumption 3. The regulator decides on a minimum productivity ϕ_r to be regulated.

When the regulation with the fixed productivity threshold ϕ_r is introduced, ϕ_d will move down (as shown in (9) for $b = 0$ vs. $b > 0$).

Figure A.1 depicts the thresholds ϕ_d and ϕ_r and shows how they change upon an increase in the emissions price. Critically, the two thresholds ϕ_d and ϕ_r do not move correspondingly when ϕ_r is fixed; only ϕ_d goes down, such that the share of regulated firms, b, drops. The emissions of unregulated firms necessarily increase, see (17).

Nevertheless, these effects do not drive my main results in the welfare analysis, as one can see in Figure A.2: the welfare function is not significantly different for a fixed share of regulated firms or a fixed productivity threshold. For completeness, all other figures in the Appendix for analysing aggregate outcomes also include the productivity threshold.

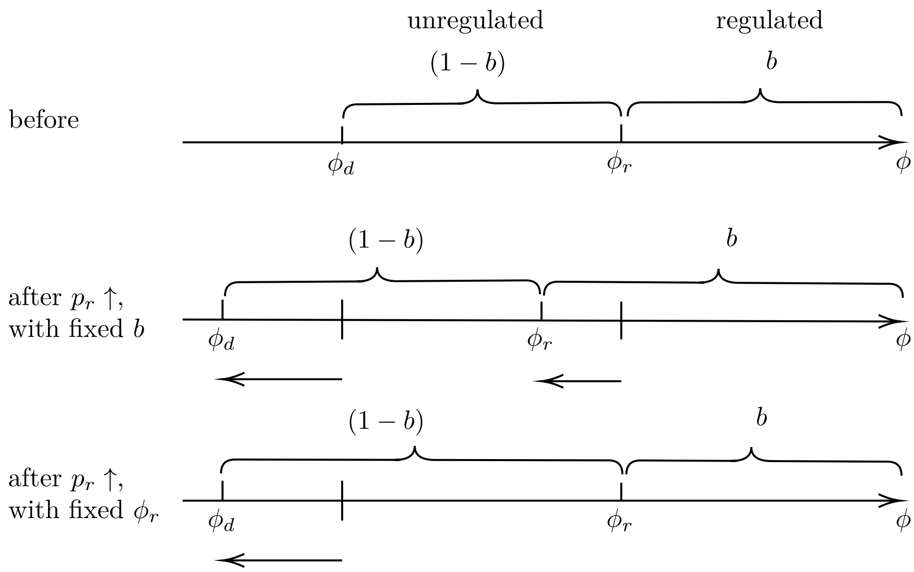

Figure A.1: Illustration of a change in thresholds for an increase in the emissions price

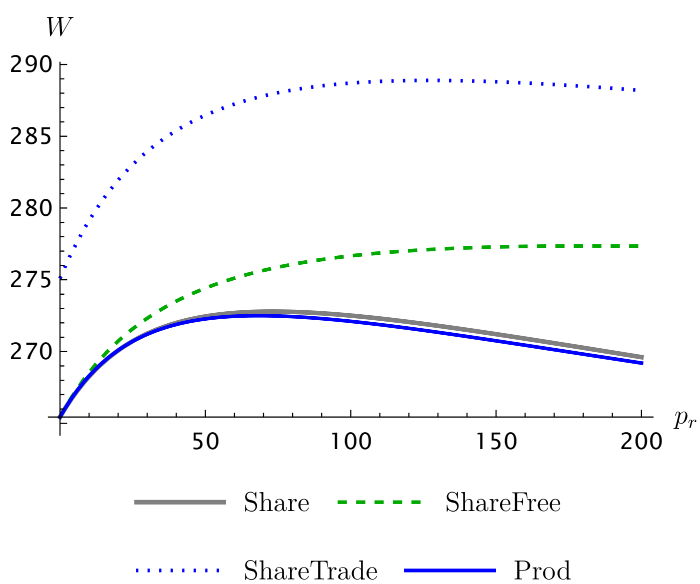

Figure A.2: Welfare similar as in Figure 10, but now with "Prod" for a productivity threshold, in million utils

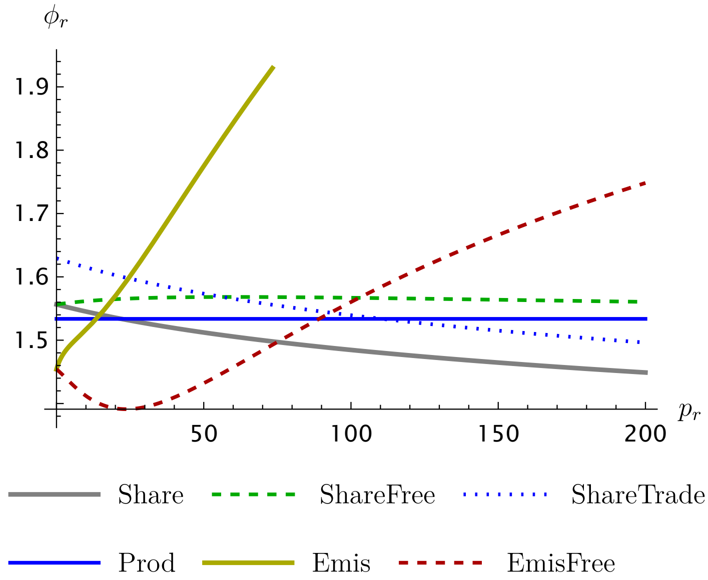

Figure A.3: Regulatory thresholds for different scenarios along the emissions price

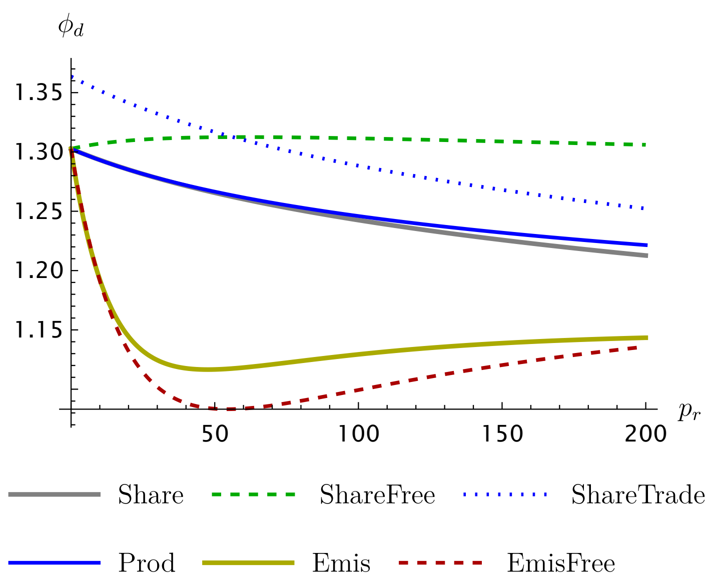

Figure A.4: Survival thresholds for different scenarios along the emissions price

Appendix A.16. Firm Selection into Survival and Regulation along Different Emissions Prices

Whereas the self-selection scenarios (with an emissions threshold) have fewer firms being regulated as the emissions price increases, see Figure A.3,[45] the survival threshold is always lower than in the scenarios without self-selection (Share- & Prod) for any non-zero emissions price (see Figure A.4). The positive effect on the survival chances of the dirtiest firms is even stronger now because the firms who bunch below the threshold are restricting their production (and hence increase their higher prices) such that the other exempted firms benefit even more from being not regulated. Hence, with an emissions threshold, some of the dirtiest firms can survive. Both developments (in ϕ_r and ϕ_d) taken together imply that the absolute number of unregulated firms increases the higher p_r, whereas the unregulated firms consist of some of the dirtiest, most unproductive firms possible. Those firms are charging relatively high prices and are using a lot of energy for the output they are producing. Hence, it comes at the cost of relatively little production combined with lower emission reductions overall.

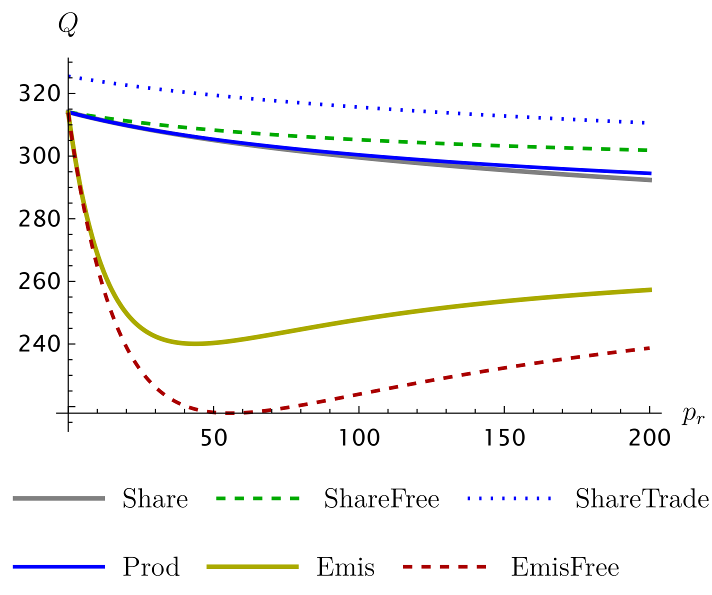

Figure A.5: Aggregate Output for different scenarios along different emissions prices, in million utils

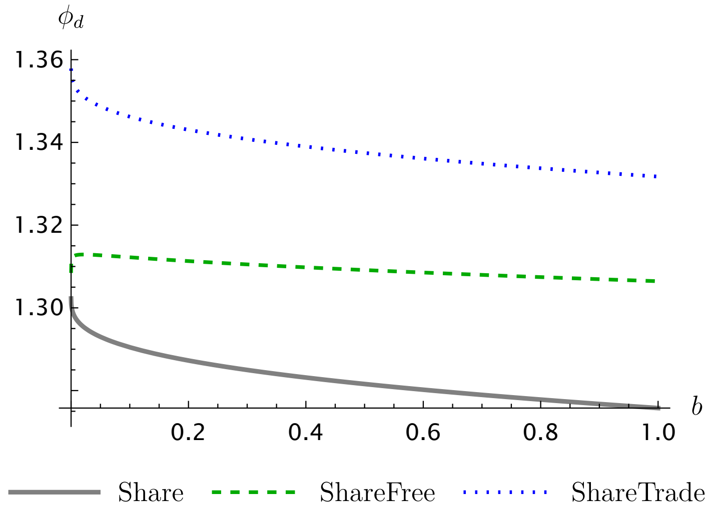

Figure A.6: Survival threshold for different shares of regulated firms

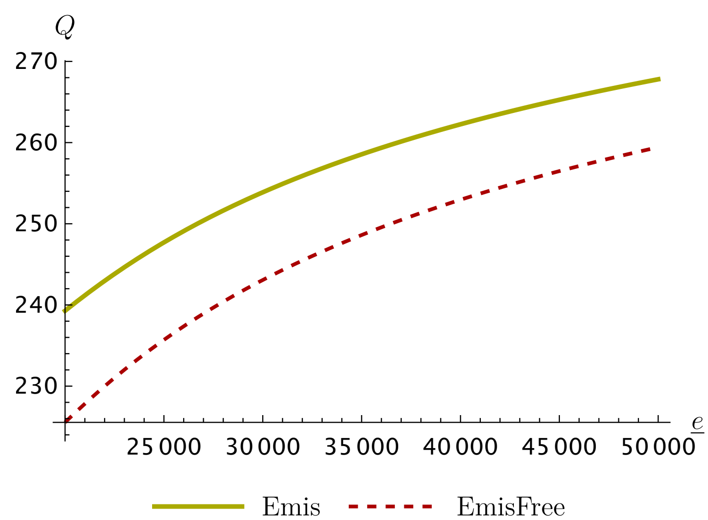

Figure A.7: Output along different emissions thresholds for regulation, in million utils

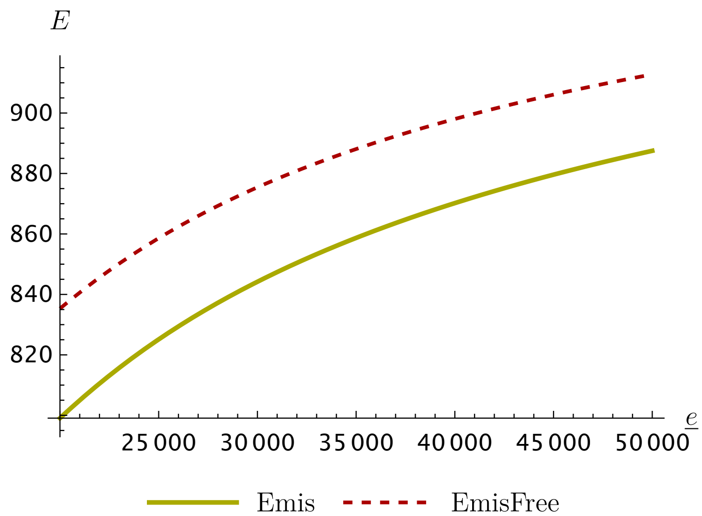

Figure A.8: Emissions along different emissions thresholds for regulation, in million tons of CO_2e

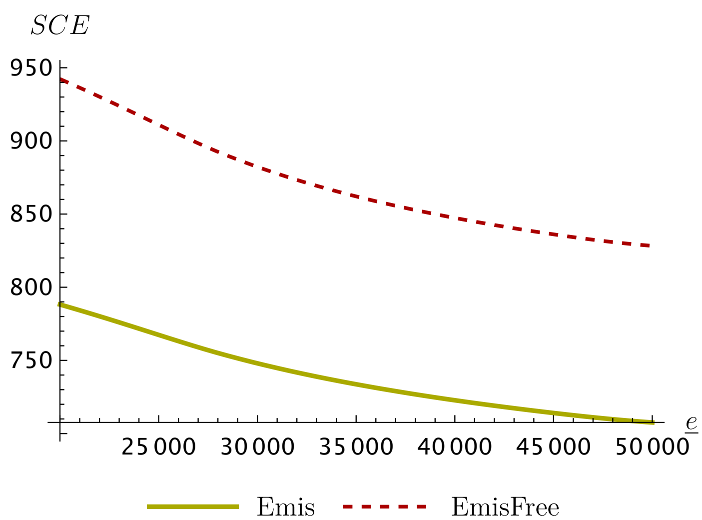

Figure A.9: Social Costs of Emissions needed for different emissions thresholds to be welfare-maximising, in USD

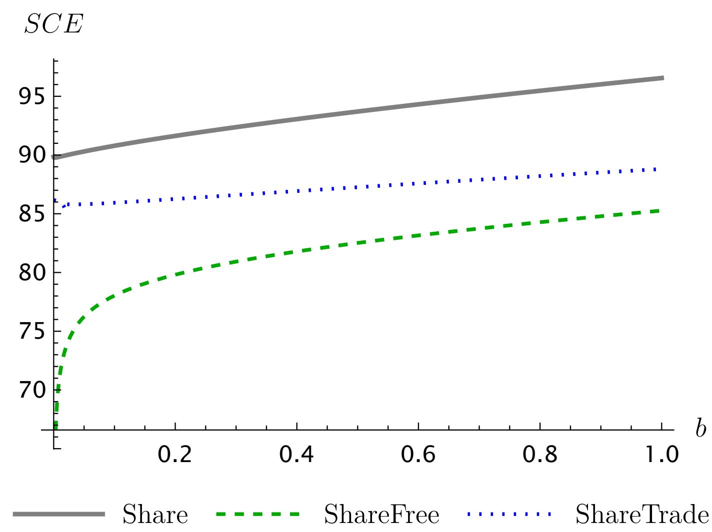

Figure A.10: Social Costs of Emissions needed for different levels of coverage b to be welfare-maximising, in USD

Appendix A.17. The Role of the Fixed Costs of Regulation for Welfare

Welfare is most strongly decreasing in the fixed costs of the emission regulation in the baseline model (Share), making it more likely that an exemption of firms can be welfare maximising, see Figure A.11. Welfare is always higher with either free allocation or trade. Given that with trade, more output falls under regulation (because the biggest firms now also have revenue abroad), the average price for emissions is higher, pushing emissions down - which has a positive effect on welfare. If regulated firms receive the free permits in a lump sum manner as it is in ShareFree, then the free permits work as an opposite force to the fixed costs of regulation. In the trade and free allocation scenarios, the market consists of larger, cleaner firms than in the baseline – given their size, those firms are less prone to changes in the fixed costs of regulation given their larger profit scale. Interestingly, an increase in fixed costs implies an increase in welfare when firms have the option to bunch under the threshold, especially if there is a distribution of free permits. Higher fixed costs increase the incentive to bunch under the threshold, such that the number of unregulated firms increases (less emissions price pass-through to the price level) – the fixed regulatory costs in this case work as an emissions price themselves by inducing emission reductions (to fall under the threshold), see Figure A.14. At the same time, rather inefficient firms have it a bit harder to survive as they share the benefit of being

[45] The free permits make it more attractive to be regulated only for the lower levels of emissions prices; after $p_r \approx 25$, firms start actively selecting into the exemption again.

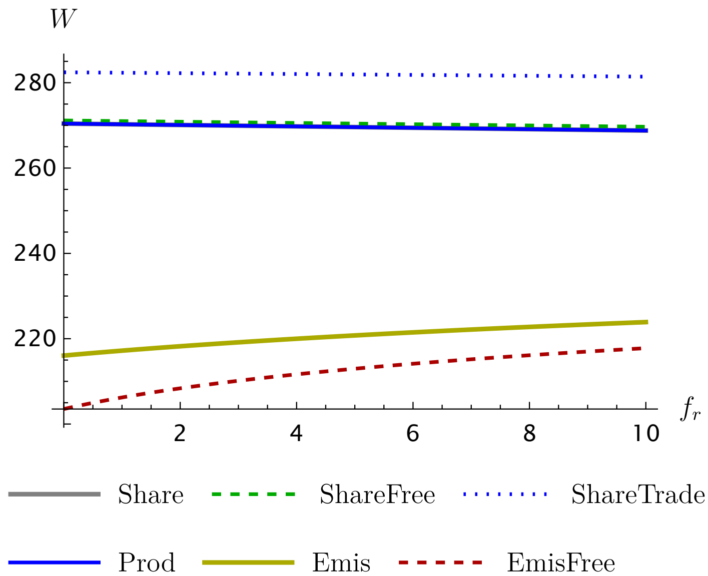

Figure A.11: Welfare for different scenarios along different levels of regulatory fixed costs, in million utils

exempted with an increasing number of other firms. Both effects (see Figure A.12 and Figure A.13) let welfare increase in f_r due to efficiency and emission reduction gains.

The results of this subsection suggest an ambiguous relationship between regulatory costs and optimal thresholds of exemption, similar to Kaplow (2019). The scenarios in Figure A.11 show the efficiency losses for regulated firms when regulatory costs increase. Figure A.14 highlights the second channel coming in: Additional emission reductions from more firms which bunch under the threshold.

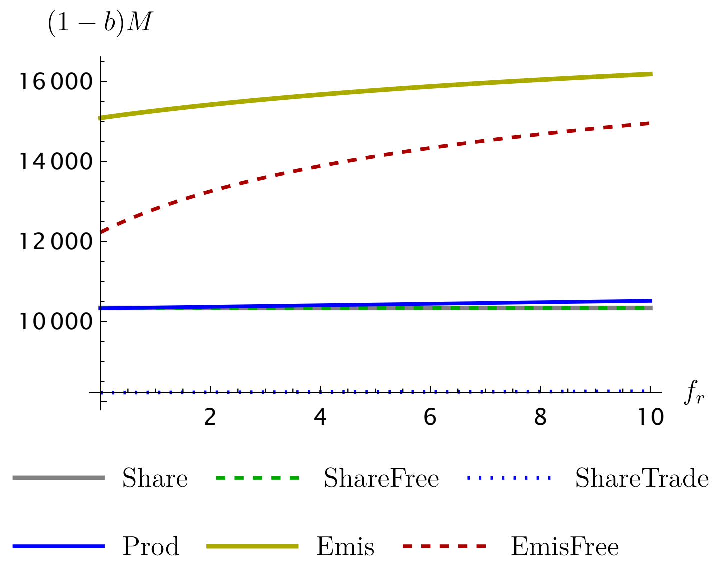

Figure A.12: Number of unregulated firms for different levels of fixed costs of regulation

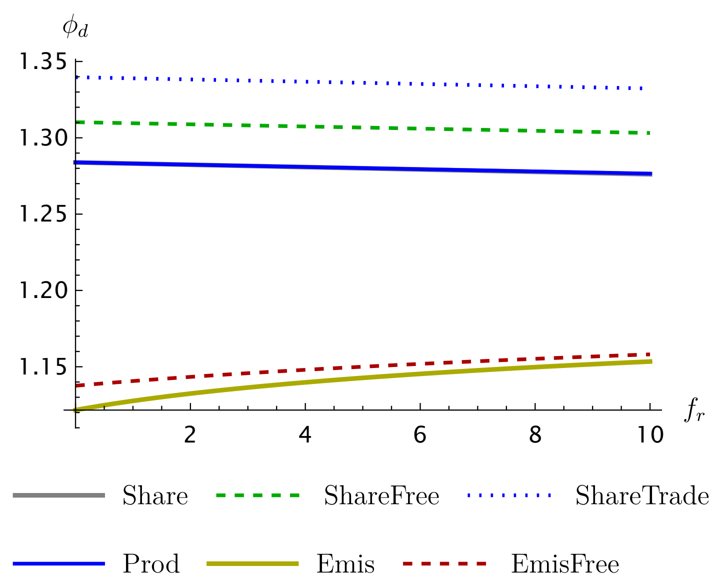

Figure A.13: Survival Threshold for different levels of regulatory fixed costs

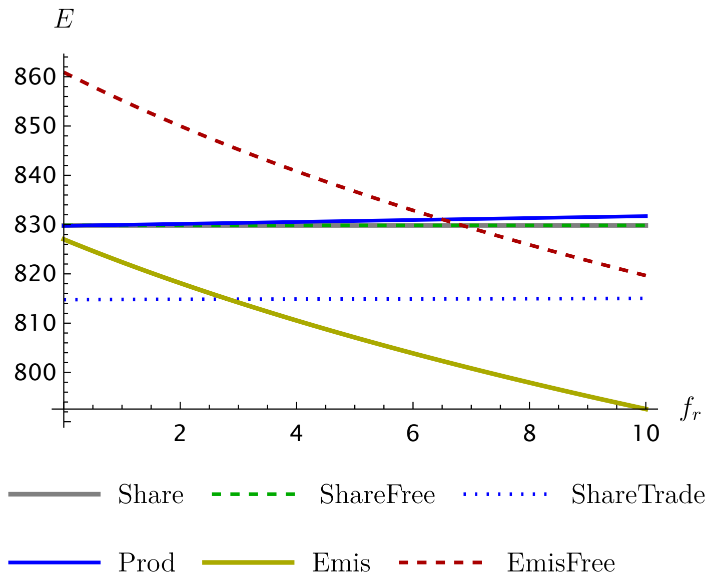

Figure A.14: Aggregate Emissions along different levels of regulatory fixed costs, in million tons of CO_2e

Appendix A.18. Optimal Share of Exemption with an Emission Target

The welfare analysis has highlighted the reason why a full regulation might not be optimal: the burden of additional bureaucracy. When regulated, firms have to monitor, report, and verify their emissions - additional fixed expenses besides paying per unit emitted. Some of the smallest firms would not survive with the increased costs, such that the regulation would crowd out production.

The trade-off between emission reduction and administrative costs is intensified by setting an overall emissions target. An emission reduction goal like the one of the Paris Agreement can make changes to the regulatory scope b problematic - a reduction in the share of firms being regulated requires an increase in the emissions price of the regulated firms, in order to meet the target.

With a binding emissions target $\hat{E}$, the relationship between emissions price p_r and b cannot be solved analytically - the numerical solution is depicted in Figure A.15. Reducing the number of firms being regulated needs to be accompanied by an increase of the emissions price p_r such that the regulated firms decrease their emissions even more than before. There exists some low $\underline{b}$ such that for all $b \leq \underline{b}$, no price p_r exists which could keep total emissions E below the emissions target $\hat{E}$ - the few regulated firms cannot reduce their emissions further to meet the needed emissions abatement.

With a fixed emissions target $\hat{E}$, there exists an optimal b^*. Increasing the scope of regulated firms beyond b^* (for $b^* < 1$) will raise the number of firms which have to pay the variable and fixed costs of the emissions pricing - the

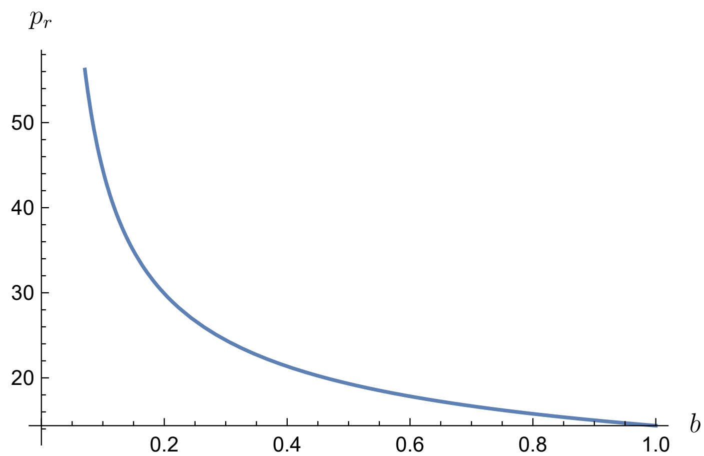

Figure A.15: Emissions price and coverage with a binding emissions target of 830 MMT CO_2e (representative for total emissions with $b = 0.375$)

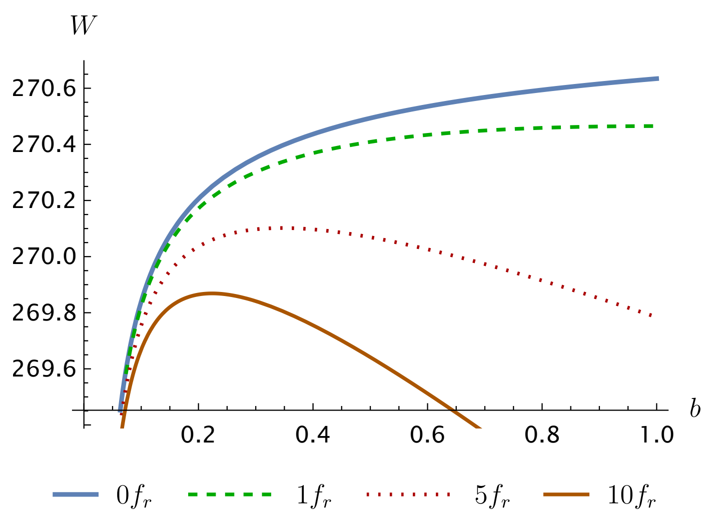

Figure A.16: Welfare along different levels of coverage, for different levels of fixed costs of the emissions pricing (indicated with factor for f_r), with a binding emissions target, in million utils price level P will go up. At the same time, the total production will decrease such that the utility from consumption of the final good Q goes down (recall the utility function (1)). This crowding out of production is stronger with higher bureaucratic costs of the emissions pricing, f_r, such that for a higher f_r, the optimal coverage b^* is lower. Combined with the empirical findings for the costs for monitoring, verification, reporting of emissions by Coria and Jaraitė (2019), my model suggests that more firms should be exempted in a system with emission permits than in one with an emissions tax.

The second argument for regulating the optimal share b^* of firms is that any lower b comes with an increase in p_r. Abatement of emissions is now more concentrated among fewer firms - which is less efficient due to increasing returns to scale of emissions in production (see (2)). Hence, the relationship between p_r and b is convex (recall Figure A.15: p_r needs to increase stronger the lower b) - causing the price level P to increase and total output Q to decrease. With both effects, the welfare function is concave in b: Moving away from the optimal b^* comes with efficiency losses and decreases welfare, as shown in Figure A.16 above.[46]

Appendix A.19. Analytical Welfare Comparisons with a Fixed Share of Firms Regulated

Welfare is calculated as in (26), namely as aggregate production and social cost of emissions measured in utils. For analytical comparisons of different extensions, it is easier to look at welfare in the following form:

$$W = [R \; - \; SCE \; * \; E] * \frac{1}{P}.$$

When a fixed share b of active firms is regulated, firms will spend the Cob-Douglas share α on emissions and $(1-\alpha)$ on labour expenses, see Appendix A.6. Hence, the aggregate revenue will be the same across all scenarios. The welfare differences stem only from differences in aggregate emissions E or in price level P, with the latter being inversely related to differences in output Q. The hump-shape of the welfare function in Figure 10 results from the effect of the emissions first dominating, and then being overtaken by the adverse effect on price level.

Welfare will always be higher with free allocation: Firstly, the aggregate emissions will be the same due to the fixed share of regulated firms and the Cobb-Douglas shares, as argued above for R:

$$E^{\text{ShareFree}} = E^{\text{Share}}.$$

Secondly, the price level will only differ in terms of the productivity cutoff ϕ_d,

[46] For the illustration, I used an emissions target of 830 MMT CO_2e which is representative of the Share and ShareFree total emissions with $b = 0.375$.

see (13):

$$P^{\text{ShareFree}} < P^{\text{Share}} = P^{\text{ShareFree}} * \frac{\phi_d^{\text{ShareFree}}}{\phi_d^{\text{Share}}},$$
$$\text{as} \quad \phi_d^{\text{ShareFree}} > \phi_d^{\text{Share}}, \ \text{see (10) and (18).}$$

Hence, it needs to be true that:[47]

$$W^{\text{ShareFree}} > W^{\text{Share}}.$$

For the trade extension, both E and P will differ from the baseline scenario. The aggregate emissions with trade are the following:

$$E^{\text{ShareTrade}} = \frac{\left(1 - b^{\frac{\theta-\sigma+1}{\theta}} \left[1 - \left(\frac{c_e}{p_r+c_e}\right)^{\alpha(\sigma-1)+1}\right]\right) f_y + \frac{c_e}{p_r+c_e} f_x \chi(p_r)}{f_y \eta(p_r) + f_x \chi(p_r)} \, \frac{\alpha(\sigma-1)}{\sigma - \alpha(\sigma-1)} \, \frac{wL}{c_e}.$$

Comparing them with the baseline scenario (see (16)), results in the following condition:

$$\frac{c_e}{p_r + c_e} < 1,$$
$$\implies E^{\text{ShareTrade}} < E^{\text{Share}}.$$

The effect driving the difference comes from two channels: The non-regulated firms' emission are lower under trade because these firms are more productive on average. For regulated firms' emissions, the same holds, but additionally, they on average scale up production because their most productive firms export goods; hence, the cleanest firms make up a bigger share of aggregate emissions.

From (19) and the related simplifying assumptions[48], the following needs to be true:

$$P_i(p_r) = \chi(p_r)^{-\frac{1}{\theta}} \frac{P_j}{\tau} \left(\frac{f_y}{f_x} \frac{R_j}{R_i}\right)^{\frac{1}{\sigma-1}} \left[\frac{c_e}{c_e + p_r}\right]^{\alpha}.$$

Although both $P_i(p_r)$ and $\chi(p_r)$ are endogenously determined, a change in an exogenously given variable like b can change the chance to export $\chi(p_r)$ only via the price level $P_i(p_r)$, such that we can conclude that an increase in $\chi(p_r)$ needs to be mirrored by a decrease in $P_i(p_r)$. Hence, it needs to be true that $P^{\text{ShareTrade}} < P^{\text{Share}}$.

For $P^{\text{ShareTrade}} < P^{\text{Share}}$ and $E^{\text{ShareTrade}} < E^{\text{Share}}$, welfare is always higher

[47] When comparing the different scenarios, I assume $s > 0$ and $p_r > 0$ throughout.

[48] These are that the foreign price level does not react more strongly to changes in the emission pricing than the domestic one, $\frac{\partial(P_i/P_j)}{\partial b} > 0$, and that changes in foreign aggregate revenue are negligible, $\frac{\partial R_j}{\partial b} = 0$.

under the trade extension: $W^{\text{ShareTrade}} > W^{\text{Share}}$.

Appendix A.20. Nonbinding Emissions Threshold

It is hard to make analytical welfare comparisons between the fixed share of firms regulated and an emissions threshold. As visible in Figure 11, aggregate emissions may be higher or lower depending on the exemption criteria, depending on the setting of b or $\underline{e}$. For an emissions price of $p_r = \$22$ (see Appendix A.8) and free allocation, the same relative amount of emissions is regulated (imposed by parameterisation), namely 66.5%. At this emission price, welfare is lower with the emissions threshold because a share of the unregulated emissions originates from firms bunching below the threshold, distorting their production.

For scenarios with the emissions threshold, making analytical statements proves very difficult. The discontinuities in price setting and input choices along the firm productivity distribution (at the thresholds), together with thresholds themselves depending on the aggregate outcomes, result in an analytically non-tractable problem when trying to see the effect of free allocation.[49]

Still, it is possible to make statements about the extreme situation with all firms or no firms being regulated. Comparing welfare with a fixed share of firms regulated vs. an emissions threshold, we know that if the latter threshold is chosen too low, all active firms will be regulated because bunching would incur losses. Therefore, it must be true that the welfare of some $\underline{e} \leq \underline{e}_{min}$ coincides with that when $b = 1$ in the fixed share scenario, that is, when all firms are regulated. On the other hand, when $\underline{e} \to \infty$, the welfare is approaching that of $b = 0$ because only the infinitely productive firm will be regulated.

In Figure 13, one can see that under current parameterisation, for $\underline{e} \geq \underline{e}_{min}^{\text{EmisFree}} \geq \underline{e}_{min}^{\text{Emis}}$, the welfare is greater without free allocation of emission rights:

$$W^{\text{Emis}} > W^{\text{EmisFree}}, \text{ for } \underline{e} \geq \underline{e}_{min}^{\text{EmisFree}} \geq \underline{e}_{min}^{\text{Emis}}.$$

For the upper corner solution, $\underline{e} \to \infty$, the welfare is approaching that of $b = 0$ (no firm regulated) such that free allocation cannot play a role anymore:

$$\lim_{\underline{e} \uparrow \infty} W^{\text{EmisFree}} = \lim_{\underline{e} \uparrow \infty} W^{\text{Emis}} = W^{\text{Share}}|_{b=0} = W^{\text{ShareFree}}|_{b=0}.$$

These corner solutions can also be found for certain values of other variables, for example, also for $p_r \to \infty$, no firm will be regulated anymore.

There exists a lower limit of the emissions threshold to be an effective policy tool so that small emitters are exempted from regulation. Below this limit, the following relationship must hold, as shown in the previous subsection:

$$W^{\text{Emis}} = W^{\text{Share}} < W^{\text{EmisFree}} = W^{\text{ShareFree}}, \text{ for } \underline{e} \leq \underline{e}_{min}^{\text{Emis}} \leq \underline{e}_{min}^{\text{EmisFree}}.$$

[49] See Appendix A.21: Solving for Equilibria with an Emissions Threshold.

The lower limit $\underline{e}_{min}$ is the emission level of the marginal firm with $\phi_o = \phi_r$ that is bunching at this threshold: $\underline{e}_{min} = e(\phi_o|\phi_o = \phi_r)$. I can show that $\frac{\partial \phi_r}{\partial s} < 0$ (see further below), that is, that the regulation threshold is decreasing in free allocation - fewer firms will bunch in the case that being regulated not only means higher costs but also a lump-sum payment of free emission rights. Hence, using (2) and (4), it needs to be true that:

$$\underline{e}_{min}^{\text{EmisFree}} > \underline{e}_{min}^{\text{Emis}}, \quad \text{iff } R^{\text{EmisFree}} \left(P^{\text{EmisFree}}\right)^{\sigma-1} > R^{\text{Emis}} \left(P^{\text{Emis}}\right)^{\sigma-1}.$$

With the current parameterisation, these conditions are met.[50]

For $\frac{\partial \phi_r}{\partial s} < 0$ to be true, ϕ_r must be smaller with free allocation; see Figure A.3. Following (A.5):

$$\lim_{\phi \downarrow \phi_r} \pi(\phi) = \lim_{\phi \uparrow \phi_r} \pi(\phi).$$

With free allocation, the left-hand side ($\lim_{\phi \downarrow \phi_r} \pi(\phi)$) changes as firms receive a lump-sum payment of permits ($p_r z \tilde{q}(\phi)$):

$$\phi_r^{\sigma-1} \frac{1}{\sigma} R P^{\sigma-1} \left(\frac{\sigma - 1}{\sigma} \, \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(c_e + p_r)^{\alpha} w^{1-\alpha}} \right)^{\sigma-1} + p_r z \tilde{q}(\phi) = \lim_{\phi \uparrow \phi_r} \pi(\phi),$$

where z is emission-intensity benchmark for free allocation (depending on the s percent best-performing firms) and $\tilde{q}(\phi)$ is the output capacity when not regulated (see Section 3.1).

With this additional term for profits when regulated, profits from "daily business" (profits minus lump-sum payments from free allocation) do not need to be as high anymore to meet the profits of the marginal bunching firm ($\lim_{\phi \uparrow \phi_r} \pi(\phi)$). It must be true that:

$$\phi_r^{\text{Emis}} > \phi_r^{\text{EmisFree}}.$$

Appendix A.21. Solving for Equilibria with an Emissions Threshold

The following steps describe how I numerically solved for the endogenous variables in case of a varying p_r:

1. For every p_r, my algorithm starts with the counterfactual price level if $b = 37.5\%$ of firms were regulated, and computes thresholds ϕ_d, ϕ_o, ϕ_r with root finding algorithms.
2. With these thresholds, I can find new equilibrium aggregate price level, revenue, emissions, and output.
3. The loop is run again with the new price level, and the above steps are repeated until the difference converges to a certain level.

[50]Intuitively, free allocation makes being regulated more attractive such that the minimum emission threshold is higher.

4. Then, I interpolate all values for different p_r and save the approximated, continuous functions of p_r.

For varying $\underline{e}$ or f_r, the algorithm is adjusted accordingly.