Study Topics for MAT 265 Final

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1 Functions and Limits

1.1 Functions and Their Representations

- Definition of a function (vertical line test).

- Domain and Range

- Piecewise-defined functions.

1.2 A Catalog of Essential Functions

Know what these functions are and some basic graphs.

- Linear Functions

- Polynomials

- Power Functions

- Rational Functions

- Trigonometric Functions

- Exponential Functions

- Vertical/Horizontal Shifts

- Vertical/Horizontal Stretching and Reflecting

- Composition of Functions

1.3   The Limit of a Function

- Know when a limit exists i.e. left and right hand limits.

1.4   Calculating Limits

- Limit Laws

- Squeeze Theorem

- Important limit to know: $\displaystyle\lim_{x \to 0} \frac{\sin(x)}{x}$

1.5   Continuity

- Know the definition of continuity.

- Continuity from the left and right.

- Know which functions are continuous everywhere on their domain.

- Intermediate Value Theorem

1.6   Limits Involving Infinity

- Definition of vertical asymptote.

- Definition of horizontal asymptote.

2   Derivatives

2.1   Derivatives and Rates of Change

- Definition of the derivative of a function at a point.

- Using the derivative to find the slope of the tangent line of a curve at a point.

- Average rate of change and instantaneous rate of change.

2.2 The Derivative as a Function

- Calculating $f'(x)$ using the definition of derivative.

- How can a function fail to be differentiable at a point?

- $f(x) = absx$ is not differentiable at $x = 0$.

2.3 Basic Differentiation Formulas

- Derivative of a constant.

- Power Rule

- Derivative of position is velocity. Derivative of velocity is acceleration.

2.4 Product and Quotient Rules

- Product and Quotient Rules.

- Derivative of Trigonometric functions.

2.5 Chain Rule

- Chain Rule (i.e. inside function, outside function, etc).

2.6 Implicit Differentiation

- Equations that are not solved for a single variable.

- y is "determined" by x.

- Application of the Chain Rule.

2.7 Related Rates

- Application of Implicit Differentiation.

- Know some basic volume and area formulas.

- Follow strategy guidelines on pg 129.

2.8 Linear Approximations and Differentials

- Linearization of f at $x = a$ can also be thought of as the tangent line to the curve at $x = a$.

- Differentials used to approximate error.

3 Inverse Functions

3.1 Exponential Functions

- Properties of exponents.

- The number e.

- Graphs of exponential functions.

3.2 Inverse Functions and Logarithms

- Definition of one-to-one function and the Horizontal Line Test.

- Definition of inverse of a function.

- Domain and Range of an inverse function.

- Graph of the inverse function compared to the graph of the original function.

- How do you find the inverse of a function algebraically?

- Derivative of the inverse function.

- Logarithmic functions are the inverse of which other functions?

- Properties of Logs

- The Natural Log, $\ln(x)$

- Graph of Logs

3.3 Derivatives of Logarithmic and Exponential Functions

- Know the formulas for the derivatives of both of these functions.

- Use logarithmic differentiation to find the derivative of functions like $f(x) = x^x$.

3.5 Inverse Trigonometric Functions

- Know the graph of $\arctan(x)$! You should know the graph of the other inverse trig functions, but that one is the most important.

- Calculate things like $\cos(\arctan(x))$ using triangles.

- <u>NOTE</u>: $\tan^{-1}(x) \neq \dfrac{1}{\tan(x)}$ and similarly for the other inverse trig functions.

- Derivatives of the inverse trigonometric functions.

3.7 Indeterminate Forms and L'Hospital's Rule

- Know which indeterminate forms L'Hospital's Rule works for.

- Know how to go from an indeterminate form which cannot be used with L'Hosptial's Rule to an indeterminate form which can.

4 Applications of Differentiation

4.1 Maximum and Minimum Values

- Know the difference between absolute max/min and local max/min.

- Extreme Value Theorem

- Know how to find critical points (numbers).

4.2   Mean Value Theorem

- Rolle's Theorem, which is a specific case of the Mean Value Theorem.

- Mean Value Theorem

- If $f'(x) = g'(x)$ on an interval (a, b), then $f(x) = g(x) + c$ for some constant c.

4.3   Derivatives and the Shapes of Graphs

- Know how to find intervals of increase/decrease.

- The First Derivative Test

- Know how to find intervals of concavity.

- Know the definition of inflection points.

- The Second Derivative Test.

4.4   Curve Sketching

- Follow the 8 steps for sketching a graph.

4.5   Optimization Problems

- Do plenty of examples in this section! There are a lot of word problems and it takes much practice to understand the setup of such problems.

4.7   Antiderivatives

- Know the definition of an antiderivative of a function.

- Know the general form of the antiderivative of a function.

- Know the basic antiderivative formulas.

5 Integrals

5.1 Areas and Distances

- Estimate the area under a curve using right-endpoint, left-endpoint, and midpoint approximations.

- Know how to setup of the area in terms of a limit of sums.

5.2 The Definite Integral

- Know the definition of the definite integral.

- Know how to go between definite integrals and a limit of sums.

- Properties of the Definite Integrals.

5.3 Evaluating Definite Integrals

- Evaluation Theorem.

- Know the difference between indefinite and definite integrals.

- The integral of a rate of change is the net change.

5.4 The Fundamental Theorem of Calculus

- Know how to use the Fundamental Theorem of Calculus, part 1 to find derivatives of certain functions very easily.

- Evaluation Theorem is the Fundamental Theorem of Calculus, part 2.

- Know have to use integrals to find the average value of a function, f_{ave}, on an interval $[a, b]$.

- Mean Value Theorem for Integrals