Content-Specific Broadcast Cellular Networks based on User Demand Prediction: A Revenue Perspective

Jihong Park and Seong-Lyun Kim
Dept. of Electrical & Electronic Engineering
Yonsei University
50 Yonsei-ro, Seodaemun-gu, Seoul, Korea
Email: {jhpark.james, slkim}@yonsei.ac.kr

***Abstract*—The Long Term Evolution (LTE) broadcast is a promising solution to cope with exponentially increasing user traffic by broadcasting common user requests over the same frequency channels. In this paper, we propose a novel network framework provisioning broadcast and unicast services simultaneously. For each serving file to users, a cellular base station determines either to broadcast or unicast the file based on user demand prediction examining the file's content specific characteristics such as: file size, delay tolerance, price sensitivity. In a network operator's revenue maximization perspective while not inflicting any user payoff degradation, we jointly optimize resource allocation, pricing, and file scheduling. In accordance with the state of the art LTE specifications, the proposed network demonstrates up to 32% increase in revenue for a single cell and more than a 7-fold increase for a 7 cell coordinated LTE broadcast network, compared to the conventional unicast cellular networks.**

***Index Terms*—LTE broadcast, eMBMS, unicast, resource allocation, delay, scheduling, pricing, revenue maximization**

I. Introduction

Explosive user traffic increase in spite of scarce wireless frequency-time resources is one of the most challenging issues for the future cellular system design [1]. LTE broadcast, also known as evolved Multimedia Multicast Broadcast Service (eMBMS) in the Third Generation Partnership Project (3GPP) standards [2], is one promising way to resolve the problem by broadcasting common requests among users so that it can save frequency-time resources [3]. The common user requests can be easily found in, for example, popular multimedia content or software updates in smart devices. By harnessing these overlapping requests of users, LTE broadcast enhances the total resource amount *per cell*. This plays a complementary role to the prominent small cell deployment approach providing more resource amount *per user* by means of reducing cell sizes [4].

To implement this technique in practice, it is important to validate the existence of sufficiently large number of common requests. According to the investigation in [5], discovering meaningful amount of common requests is viable even in YouTube despite its providing a huge amount of video files. That is because most users request popular files; for instance, 80% of user traffic may occur from the top 10 popular files. On the basis of this reason, AT&T and Verizon Wireless are planning to launch LTE broadcast in early 2014 to broadcast sports events to their subscribers [6].

The number of available common requests and its resultant saving amount of resources in cellular networks are investigated in [7], but it focuses on broadcast (BC) service while neglecting the effect of incumbent unicast (UC) service. Joint optimization of the resource allocations to BC and UC are covered in [8], [9] in the perspectives of average throughput and spectral efficiency. The authors however restrict their scenarios to streaming multimedia services where data are packetized, which cannot specify the content of data as well as the corresponding user demand of the files.

Leading from the preceding works, we propose a BC network framework being specifically aware of content and able to transmit generic files via either BC or UC service. The selection of the service depends on the following content characteristics: 1) file size, 2) delay tolerance, and 3) price discount on BC compared to UC. These characteristics are able to represent a content specified file in practice. For easier understanding, let us consider a movie file as an example. It is likely to be large file sized, delay tolerable (if initial playback buffer is saturated), and sensitive to the per-bit price of BC under usage-based pricing [10] owing to its large file size. An update file of a user's favorite application in smart devices can be a different example, being likely to be small file sized, delay sensitive, and less price sensitive.

Furthermore, this study devises a policy that a base station (BS) solely carry out BC/UC service selection based on user demand prediction. Corresponding to the policy, we maximize the network operator's revenue without user payoff degradation by jointly optimizing BC resource allocation, file scheduling, and pricing. To be more specific, the following summarizes the novelty of the proposed network framework.

- **BC/UC selection policy**: a novel BC/UC selection policy is proposed where a BS solely assigns one of the services for each user by comparing his expected payoffs of BC and UC if assigned, without degrading user payoff.
- **BC resource allocation**: optimal BC frequency allocation amount is derived in a closed form, showing the allocation is linearly increased with the number of users in a cell, and inversely proportional to UC price.
- **BC pricing**: optimal BC price is derived in a closed form, proving the price is determined proportionally to the number of users until BC frequency allocation uses up the entire resources.

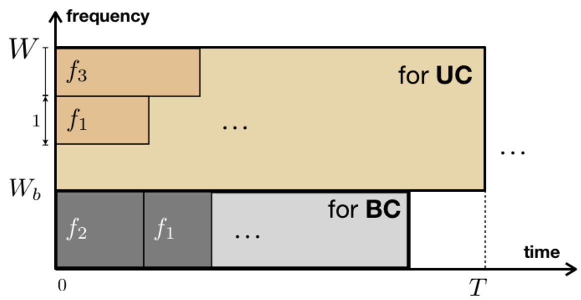


Fig. 1. Time-frequency resource allocation for unicast and broadcast services where W_b amount of frequency is allocated for broadcast while unity is allocated for unicast during T time slots

- **BC file scheduling**: optimal BC file order is derived in an operation-applicable form as well as a closed form for a suboptimal rule suggesting smaller sized and/or more delay tolerable files should be prioritized for BC.

As a consequence, we are able to not only estimate revenue in a closed form, but also verify the revenue from the proposed network keeps increasing along with the number of users unlike the conventional UC only network where the revenue is saturated after exhausting entire frequency resources. Considering 3GPP Release 11 standards, we foresee up to 32% increase in revenue for a single LTE broadcast scenario and more than a 7-fold increase for a multi-cell scenario.

II. System Model

A single cellular BS simultaneously supports downlink UC and BC services with W frequency bandwidth where BC files are slotted in a single queue. The BS serves N number of mobile users who are uniformly distributed over the cell region. Let the subscript k indicate the k-th user for $k \in \{1, 2, \cdots, N\}$, and define ϕ_k's as the locations of users. User locations are assumed to be fixed during T time slots, but change at interval of T independent of their previous locations. Let the subscripts u and b represent UC and BC hereafter, and P_u and P_b respectively denote UC and BC usage prices per bit. In order to promote BC use, the network offers price discount on BC so that it can compensate longer delay of BC.

A. User Request Pattern

Each user independently requests a single file at the same moment with a unit interval T time slots. Let the subscript i represent the i-th popular file for $i \in \{1, 2, \cdots, M\}$ where M denotes the number of all possible requests in a given region. Assume user request pattern follows Zipf's law (truncated discrete power law) as in YouTube traffic [5]. It implies the file i requesting probability p_i is given as $i^{-\gamma}/H$ where $H = \sum_{j=1}^{M} j^{-\gamma}$ for $\gamma > 0$. Note that larger γ indicates user requests are more concentrated around a set of popular files.

B. Network Operation

The following example sequentially describes the BS's operation to serve a typical user k requesting file i.

1) **Common request examination**: by inspecting user requests, BS becomes aware of the file i's size f_i as well as the number of file i requests n_i.

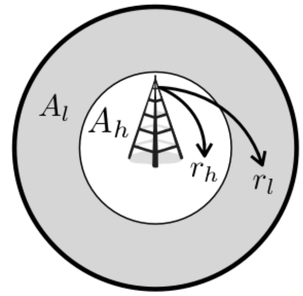


Fig. 2. Wireless channel model where a cellular base station provides average rate r_h for region A_h, and r_l for A_l

2) **Delay tolerance examination**: user k marks his requesting priority of the file i as in conventional peer-to-peer (P2P) services (e.g. high/low). Assuming BS has the full knowledge of users' quality-of-experience (QoE) patterns, this priority information corresponds to delay threshold θ_{ik}, allowable delay without degrading QoE.
3) **BC frequency allocation, pricing, and file scheduling**: by inspecting f_i, n_i, and θ_{ik}, BS allocates BC frequency amount W_b, and sets BC price P_b as well as optimizing BC file scheduling in a revenue maximizing order.
4) **BC/UC selection**: meanwhile in 3), BS assigns either BC or UC to user k in order to maximize revenue without inflicting the user's payoff loss.

Note that the pricing scheme we consider is similar to time-dependent pricing [10] in respect of its flattening user traffic effect by adjusting P_b over time. The target offloading traffic by the pricing is, however, novel since the conventional scheme aims at the entire user traffic but the proposed at *content-specific* traffic captured by n_i.

C. Resource Allocation

BS allocates W_b amount of BC frequency for handling the entire BC assigned requests. In compliance with the 3GPP Release 11 [2], the earmarked amount cannot be reallocated to UC requests during T as Fig. 1 visualizes. For each UC request, BS allocates a normalized unity frequency resource, to be addressed with a realistic unit in Section IV.

D. User Payoff

Let U_{ik} denote the payoff of user k when downloading file i via UC. Consider the payoff has the following characteristics: logarithmically increasing with f_i; logarithmically decreasing with its downloading completion delay after exceeding θ_{ik} [11]; and linearly decreasing with cost under usage-based pricing [10]. Define r_k^u as the spectral efficiency when user k is served by UC. Consider delay sensitive UC users such that UC downloading completion delays always make them experience QoE degrading delays. , i.e. $f_i/r_k^u > (\theta_{ik} + 1)$. Additionally, we neglect any queueing delays on UC. The payoff U_{ik} then can be represented as follows.

$$U_{ik} = \log\left(\frac{1 + f_i}{f_i/r_k^u - \theta_{ik}}\right) - P_u f_i \tag{1}$$

Note that $U_{ik} > 0$ as we are only interested in the users willing to pay for at least UC service.

In a similar manner, consider B_{ik} indicating the payoff of user k when downloading file i via BC. Let r_k^b denote the BC spectral efficiency of user k. We further define s_i as the size of the broadcasted files until the BC downloading of file i completes. This captures the effect of BC file scheduling. The payoff B_{ik} can be represented as below.

$$B_{ik} = \log\left(\frac{1+f_i}{s_i/\left(W_b r_k^b\right) - \theta_{ik}}\right) - P_b f_i \ . \tag{2}$$

To maximize revenue while guaranteeing at least UC payoff amount, BS compares U_{ik} and B_{ik}, and assigns either UC or BC service, to be further elaborated in Section III-A.

E. Wireless Channel

We consider distance attenuation from difference user locations ϕ_k, and adaptive modulation and coding (AMC) which changes modulation and coding schemes (MCS) depending on wireless channel quality [12]. While UC can adaptively adjust MCS based on its serving user's channel quality, the MCS for BC resorts to aim at the worst channel quality user because BC has to apply an identical MCS to all its users. BC average spectral efficiency is therefore not greater than the UC's.

To be more specific, as Fig. 2 illustrates, we consider a cell region A divided into A_h and A_l. BS can provide high spectral efficiency r_h to A_h, but low spectral efficiency r_l to A_l for $r_l \le r_h$. Let $|A|$ denote the area of a region A. The probability that user k is located within A_h, $\Pr\{\phi_k \in A_h\}$, is given as $|A_h|/|A|$, independent of k [13]. Define r_u as UC average spectral efficiency of user k, represented as:

$$r_u = r_l + (r_h - r_l)\Pr\{\phi_k \in A_h\} \ . \tag{3}$$

Similarly, average BC spectral efficiency r_b is given as:

$$\begin{aligned} r_b &= r_l + (r_h - r_l)\Pr\{\phi_k \in A_h\}^{N_b} & (4)\\ &\approx r_l \qquad \text{as } N \to \infty & (5)\end{aligned}$$

where N_b denotes the number of BC users. Note that (5) is because N_b is an increasing function of N.

III. Revenue Maximizing BC Network Management

In order to maximize revenue, we optimize BC frequency bandwidth W_b, price P_b, and file scheduling. For more brevity, assume sufficiently large N such that BC average spectral efficiency is approximated as r_l as in (5).

A. BC/UC Selection Policy and Problem Formulation

We firstly propose a BC/UC selection policy guaranteeing allowable user payoff, and then formulate the average revenue maximization problem under the policy. Assume that users predict to be served by UC as default, and hence BS should guarantee at least the amount of UC payoff for every service selection. For user k, revenue maximizing service selection policy is described in the following two different user payoff cases:

1) If $B_{ik} \ge U_{ik}$, BS firstly assigns UC as much as possible until UC resource allocation reaches $(W - W_b)T$ because $P_u \ge P_b$. After using up the entire UC resources, BS then assigns BC;
2) If $B_{ik} < U_{ik}$, BS resorts to assign UC in order to avoid payoff loss.

Note that this policy not only maximizes revenue, but also, albeit not maximizes, enhances user payoff.

For simplicity without loss of generality, assume the required resource amount for UC user demand exceeds the entire UC resources, $(W - W_b)T$. As there is no more available UC resource, P_u is set as a maximum value due to no price discount motivation on UC. It results in the revenue from UC is fixed as $P_u(W-W_b)T$. By contrast, the revenue from BC still can be increased if $B_{ik} \ge U_{ik}$ holds. As a consequence, the average revenue in a cell region A is represented as follows.

$$\mathcal{L}_0 := \mathsf{E}_k\left[P_b \sum_{i=1}^{M} f_i \sum_{k=1}^{n_i} \mathbf{1}\{B_{ik} \ge U_{ik}\}\right] + P_u(W - W_b)T$$

The left and right halves of $\mathcal{L}_0$ respectively indicate the average revenues from BC and UC, and $\mathbf{1}\{\cdot\}$ is an indicator function which becomes 1 if a condition inside the function is satisfied, otherwise 0. Unfortunately, $\mathcal{L}_0$ is an analytically intractable nonlinear function due to $\mathbf{1}\{B_{ik} \ge U_{ik}\}$. In order to detour the problem, consider the following Lemma.

Lemma 1. For $(P_u - P_b)f_i < 1$, the inequality $\mathcal{L}_0 \ge \mathcal{L}$ holds where $\mathcal{L}$ is defined as:

$$P_b N \sum_{i=1}^{M} f_i p_i \left[1 - \frac{s_i \theta_i r_u}{W_b r_b}\{1 - (P_u - P_b) f_i\}\right] + P_u(W - W_b)T$$

and $\theta_i := \mathsf{E}_k\left[1/\left(f_i - r_k^u \theta_{ik}\right)\right]$.

Proof: See Appendix. ■

Note that θ_i indicates the aggregate delay tolerance of file i among users for a given f_i and r_k^u. Additionally, the assumption $(P_u - P_b)f_i < 1$ does not imply small sized files since f_i is a normalized value. Applying $\mathcal{L}$ in the result of Lemma 1, the lower bound of $\mathcal{L}_0$, yields the corresponding problem formulation given as:

$$\begin{aligned} &\textbf{P1}. \max_{W_b, P_b, s_i} \ \mathcal{L} \\ &\quad \text{subject to} \\ &\qquad 0 \le P_b \le P_u, \\ &\qquad 0 \le W_b \le W, \\ &\qquad s_i > s_j \text{ or } s_i < s_j, \ \forall i,j \in \{1,2,\cdots,M\}. \end{aligned}$$

The last inequality condition means BC files are slotted in a single queue while BS transmits each file only once. In respect to $\mathcal{L}$ in **P1**, the following sections sequentially derive optimal BC network components, W_b^*, P_b^*, and s_i^*.

B. BC Frequency Allocation

Define F as $\sum_{i=1}^{M} f_i p_i$ implying the average requesting file size per user, which is a given value independent of our network design. Consider small f_i and sufficiently large N as assumed at the beginning of Section III, we can derive a closed form solution of the optimal BC frequency allocation in the following Proposition.

Proposition 1. Optimal BC frequency allocation W_b^* is given as follows.

$$W_b^* \approx \min\left(\frac{NF}{4P_uT}, W\right)$$

Proof: See Appendix. ■

The proposition shows the optimal BC frequency allocation is determined regardless of BC spectral efficiency r_b and price P_b. Moreover, it provides the network design principles that the BC frequency amount is proportional to N and inversely proportional to UC price P_u. The latter is because it becomes necessary to enhance BC downloading rate by allocating more amount of frequency to BC when BC service becomes less price competitive (smaller P_u).

C. BC Pricing

We can derive the optimal BC price in a closed form in the following Proposition.

Proposition 2. Optimal BC price is given as follows.

$$P_b^* \approx \min\left\{\frac{1}{2}\left(\frac{Nr_bF^2}{4P_uTr_uS^*} + P_u\right), P_u\right\}$$

where $S^* = \sum_{i=1}^{M} s_i^*\theta_i f_i p_i$

Proof: See Appendix. ■

The result shows that P_b^* is strictly increasing with N within the range from $P_u/2$ to P_u. It implies price increase is more effective to enhance revenue than price discount although the discount may promote more BC use. This result plays a key role to design a BC file scheduler for detouring a recursion problem in Section III-D. In addition, it is worth mentioning that BC file scheduler affects P_b^* by adjusting S^* since s_i^* therein varies along with the order of BC files, to be further elaborated in the following section.

D. BC File Scheduler

Each file i is tagged with a weighting factor w_i by BS. BS examines the scheduling file priorities by comparing w_i's. The file scheduling affects s_i defined in Section II-D, so we maximize $\mathcal{L}$ in terms of s_i as follows.

Proposition 3. *(Optimal Scheduler)* Broadcasting files in a descending order of w_i^* is the optimal scheduling rule maximizing $\mathcal{L}$ in **P1** where

$$w_i^* := \theta_i p_i \left\{1 - \frac{f_i}{2}\left(P_u - \frac{Nr_bF^2}{4P_uTr_uS^*}\right)\right\}.$$

Proof: For a given P_b^*, consider the subproblem of **P1**:

$$\textbf{P2.} \min_{s_i} \sum_{i=1}^{M} s_i\theta_i f_i p_i \left\{1 - (P_u - P_b^*)f_i\right\}$$
$$\text{subject to}$$
$$s_i > s_j \text{ or } s_i < s_j,\ \forall i, j \leq N.$$

Applying the Smith's indexing rule in [14] and Proposition 2 leads to yield the result of the statement in Proposition 3. ■

Note that w_i^* is recursive since S^* in w_i^* is a function of s_i^* which is also a function of w_i^*. This cannot be solved analytically, and therefore we resort to derive the value by simulation in Section IV. In order to provide more fundamentally intuitive understanding, we consider the following suboptimal but closed form solution.

Corollary 1. *(Suboptimal Scheduler)* Broadcasting files in a descending order of $\bar{w}_i^*$ is a suboptimal scheduling rule enhancing $\mathcal{L}$ in **P1** where

$$\bar{w}_i^* := \theta_i p_i \left(1 - \frac{P_u f_i}{2}\right).$$

Proof: Exploiting the boundary values of P_b^* in Proposition 2 at Proposition 3 enables to bypass the recursion problem, completing the proof. ■

Although the proposed scheduler is suboptimal, it still shows close-to-optimal behavior, to be verified by Fig. 3 in Section IV. The suboptimal scheduler provides the following network design principle: more delay tolerable (larger θ_i), more popular (larger p_i), and/or smaller files (smaller f_i) should be prioritized for BC if f_i is sufficiently small such that $P_u f_i/2 < 1$.

E. Revenue Gain

In a revenue perspective, we compare the proposed BC/UC network and conventional cellular networks where only UC operates. As a performance metric, we consider *revenue gain* R defined as the revenue of the proposed BC/UC network divided by that of the UC only network. By combining Propositions 1–3, our proposed network framework shows the following revenue gain.

Proposition 4. The revenue gain R is given as follows.

$$R \approx 1 + \frac{NF}{2WT}\left\{\min\left(\frac{Nr_bF^2}{4{P_u}^2Tr_uS^*}, 1\right) + 1 - \frac{G}{P_u}\right\}$$

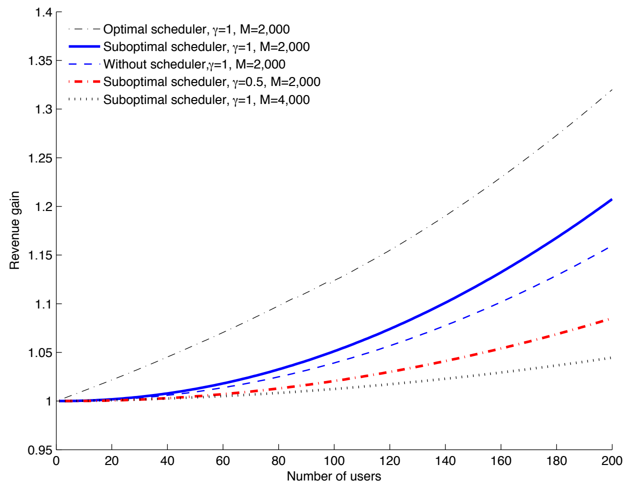
where $G := (0.5 + \sum_{i=1}^{M} s_i^*\theta_i p_i/S^*)$

Proof: Applying the results of Propositions 1–3 into $\mathcal{L}$ yields the following maximized revenue of the proposed network: $NF\left(P_b^* - G/2\right) + P_uWT$. Dividing it by the UC only network's revenue P_uWT while applying Proposition 2 concludes the proof. ■

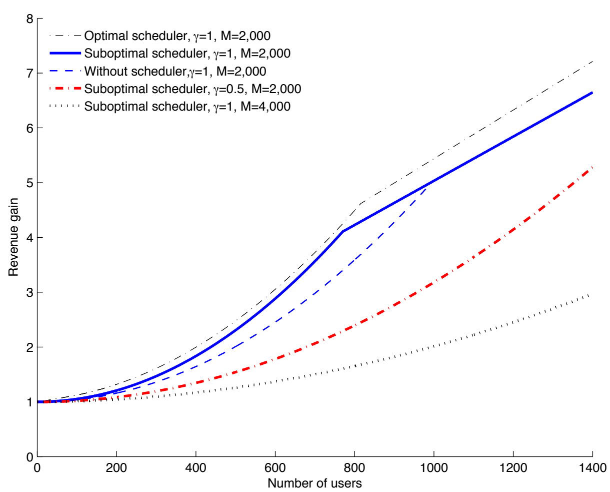
Interestingly, the proposed network always achieves positive revenue gain for sufficiently large files such that $P_u > G$ where G defined in Proposition 4 is a decreasing function of f_i (recall S^* in G and s^* therein is an increasing function of f_i by definition in Section II-D). For those files, the revenue gain R increases with the order of N^2, converging to the order of N for large N when $P_b^* = P_u$ as the effect of N diminishes. It is worth mentioning that R grows even when frequency-time resources become scarce (smaller WT) thanks to the thrifty nature of BC in frequency. In addition, the result captures the design of BC file scheduler affects revenue by adjusting S^* (and G, a function of S^*).

IV. Numerical Results

We consider two different LTE broadcast network scenarios in accordance with 3GPP Release 11 standards [2].



(a) Scenario 1: Single cell LTE broadcast



(b) Scenario 2: 7 cell coordinated LTE broadcast

Fig. 3. Revenue gains of (a) a single cell and (b) 7 cell coordinated LTE broadcast networks under the following environmtnets: with the optimal/suboptimal scheduler, without scheduler, lower popular file concentration γ of user requests, and larger number of possible requesting files M

A. Single Cell LTE Broadcast

The first scenario is a typical single cell operates LTE BC, having the number of users N up to 200 with the entire frequency amount W given as 10 MHz. For BC, BS is able to allocate up to 60% of W. For UC, BS allocates average 2.5 MHz to a single UC user until the downloading completes. At A_h, the average spectral efficiency r_h is given as 2.4 bps/Hz whereas r_l at A_l is 45 % degraded from r_h where $|A_l| = 9|A_h|$. These correspond to MCS index 19 with 64QAM and the index 12 with 16QAM respectively [12]. The number of possible requesting files M in the cell is fixed as 2,000, and the Zipf's law exponent γ is set as 1 as default. File sizes are uniformly distributed from 160 to 634 MBytes, which may correspond with 4.8 to 19 minute long 1080p resolution video content. User delay threshold θ_{ik} is uniformly

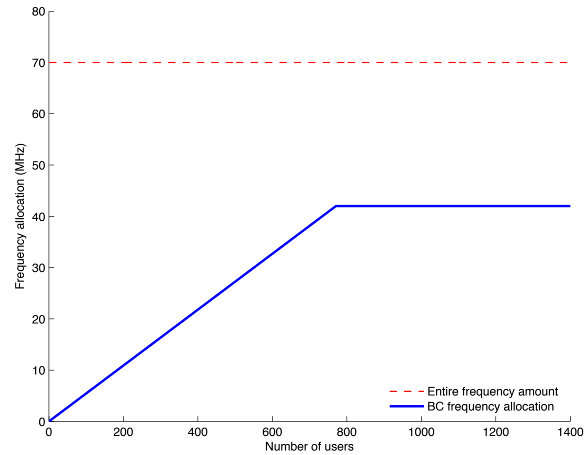


Fig. 4. Optimal broadcast frequency allocation of the 7 cell coordinated LTE broadcast network with the proposed suboptimal scheduler for increasing the number of users when $\gamma = 1$ and $M = 2{,}000$

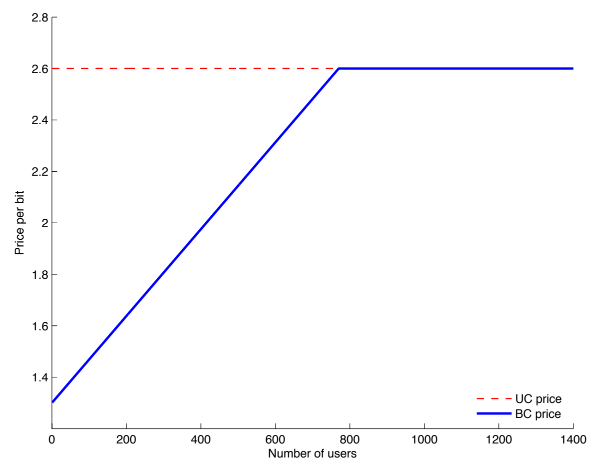


Fig. 5. Optimal broadcast price of the 7 cell coordinated LTE broadcast network with the proposed suboptimal scheduler for increasing the number of users when $\gamma = 1$ and $M = 2{,}000$

distributed from 0.6 to 6 seconds. Furthermore, T is set as 2 minutes and P_u as 2.6 a normalized value having no unit.

Fig. 3(a) shows up to 32% gain in revenue for a single cell LTE broadcast network, including the effect of the 4.7% increment from the suboptimal scheduler proposed in Section III-D. Moreover, scheduler design becomes more important when N increases due to its increasing effect on revenue gain. In addition, the result captures the revenue gain is highly depending on user request concentration γ (Zipf's law exponent) as well as the number of possible requesting file M in a cell. Specifically, doubling γ from 0.5 decreases revenue gain by up to 12.7%, and M from 2,000 does by 16.3%.

B. 7 Cell Coordinated LTE Broadcast

The second scenario we consider is a Multicast Broadcast Single Frequency Network (MBSFN) [12] where 7 neighboring cells are synchronized and operate LTE broadcast like a single cell. Assuming we neglect inter-cell interference, all the simulation settings are the same as in the single cell case except for the increased entire frequency amount W by 70 MHz and the number of users N by up to 1,400. As a result, Fig. 3 shows the proposed network with the suboptimal scheduler achieves up to 720% revenue. The result also verifies that the revenue gain increasing rate with respect to N converges to a linear scaling law when $P_b^* = P_u$ (see Fig. 5 at $N \geq 770$) as expected in Section III-E The effect of gain increment by the scheduler increases as anticipated in the single cell case for small N. This tendency, however, is no longer valid after exceeding $N = 770$, where having the maximum 70.6% revenue increment by means of the suboptimal scheduler, and the effect of scheduler diminishes along with increasing N. The reason is there is no more available BC frequency since then, and thus revenue cannot be increased by any operations of BS other than the increasing number of common requests due to N. This behavior can be further justified by Fig. 4 and 5 respectively representing the linear growing rates of W_b^* and P_b^* with increasing N, as well as the convergence to the maximum values for $N \geq 770$.

V. Conclusion

In this paper, we propose a BC network framework adaptively assigning BC or UC based on user demand prediction by examining content specific information such as file size, delay tolerance, and price sensitivity. For the purpose of the network operator's revenue maximization, the proposed framework jointly optimizes resource allocation, pricing, and file scheduling under a novel BC/UC selection policy.

Although a BS solely assigns BC or UC service without informing users of the possible selections, the proposed policy does not degrade but even enhance user payoff. In addition, this study provides closed form solutions that enables to understand the fundamental behavior of the proposed framework and give meaningful network design insights; for instance, revenue gain scaling order becomes N from N^2 as N increases. We consequently observe up to 32% increase in revenue for a single cell and more than 7 times for 7 cell coordinated LTE broadcast networks compared to the conventional networks.

The future work we are heading in is to extend the proposed framework into more general multi-cell scenarios which may rigorously incorporate inter-cell interference modeling.

Acknowledgement

This research was supported by the Ministry of Science, ICT and Future Planning, Korea, under the Communications Policy Research Center support program supervised by the Korea Communications Agency (KCA-2013-001).

Appendix

Proof of Lemma 1: Let X_k denote $\mathbf{1}\{B_{ik} > U_{ik}\}$. Since X_k's are independent of n_i, we can apply Wald's identity [15], yielding $\mathsf{E}_k\left[\sum_{k=1}^{n_i} X_k\right] = Np_i\mathsf{E}_k\left[X_k\right]$. The lower bound of X_k is derived as follows.

$$
\begin{aligned}
X_k &\geq 1 - e^{-(B_{ik}-U_{ik})} && (6)\\
&\approx 1 - \left(\frac{s_i/(W_b r_k^b) - t_{ik}}{f_i/r_k^u - t_{ik}}\right)\{1-(P_u-P_b)f_i\} && (7)\\
&\geq 1 - \frac{s_i}{W_b r_k^b\left(f_i/r_k^u - t_{ik}\right)}\{1-(P_u-P_b)f_i\} && (8)
\end{aligned}
$$

Combining these results completes the proof. ■

Proof of Proposition 1 and 2: The lower bound of average revenue gain $\mathcal{L}$ is a concave function with respect to P_b as well as W_b. We therefore can find the unique optimal point (P_b^*, W_b^*) via convex programming. Let P_b be fixed, and consider $\mathcal{L}$ in terms of W_b, yielding the solution given as:

$$
W_b^* = \sqrt{\frac{P_b N r_u \sum_{i=1}^{M} s_i \theta_i {f_i}^2 p_i}{P_u T r_b}}. \quad (9)
$$

Similarly, for a fixed W_b, the optimal BC price is given as follows.

$$
P_b^* = \frac{P_u}{2} + \left(4\sum_{i=1}^{M} s_i\theta_i {f_i}^2 p_i\right)^{-1} \sum_{i=1}^{M} p_i\left(\frac{W_b}{r_u} f_i - s_i\theta_i\right) \quad (10)
$$

Combining (9) and (10) proves Proposition 1. For Proposition 2, N/S^* increases with N since $s_i^* < N$ due to $f_i < 1$ where s_i^* is only a function of N in S^*. This proves P_b^* is an increasing function of N, completing the proof. ■

References

[1] "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2012–2017," *Cisco*, May 2013.
[2] D. Lecompte and F. Gabin, "Evolved Multimedia Broadcast/Multicast Service (eMBMS) in LTE-Advanced: Overview and Rel-11 Enhancements," *IEEE Comm. Magazine*, vol. 50, pp. 68–74, Nov. 2012.
[3] "LTE broadcasat: A revenue enabler in the mobile media era," *Ericsson*, Feb. 2013.
[4] J. Andrews, "Seven Ways that HetNets are a Cellular Paradigm Shift," *IEEE Comm. Magazine*, vol. 51, no. 3, 2013.
[5] M. Cha, H. Kwak, P. Rodriguez, Y.-Y. Ahn, and S. Moon, "I Tube, You Tube, Everybody Tubes: Analyzing the World's Largest User Generated Content Video System," *Proc., ACM IMC*, pp. 1–14, 2007.
[6] "AT&T to Use Lower 700 MHz D and E Block Spectrum for LTE Broadcast," *Ericsson*, Sep. 24 2013.
[7] S. M. Yu and S.-L. Kim, "Cost-effective broadcast in cellular networks," *availalbe at: http://arxiv.org/pdf/1304.4285.pdf.*
[8] C. Lou and L. Qiu, "QoS-aware Scheduling and Resource Allocation for Video Streams in E-MBMS towards LTE-A System," *Proc., IEEE VTC Fall*, pp. 1–5, Sep. 2011.
[9] J. F. Monserrat, J. Calabuig, A. Fernandez-Aguilella, and D. Gomez-Barquero, "Joint Delivery of Unicast and E-MBMS Services in LTE Networks," *IEEE Trans. on Broadcasting*, vol. 58, no. 2, pp. 157–167, 2010.
[10] S. Sen, C. Joe-Wong, S. Ha, and M. Chiang, "Incentivizing Time-shifting of Data: A Survey of Time-dependent Pricing for internet access," *IEEE Comm. Magazine*, vol. 50, no. 11, 2012.
[11] J. Shaikh, M. Fiedler, and D. Collange, "Quality of Experience from User and Network Perspectives," *Annals of Telecommunications*, vol. 65, pp. 47–57, Dec. 2010.
[12] R. Radhakrishnan, B. Tirouvengadam, and A. Nayak, "Channel Quality-Based AMC and Smart Scheduling Scheme for SVC Video Transmission in LTE MBSFN Networks," *Proc., IEEE Intl. Conf. on Comm.*, pp. 6514–6518, Jun. 2012.
[13] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Gemoetry and its Applications*. Wiley, 2nd ed., 1995.
[14] D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Optimization*. Athena Scientific, 1997.
[15] R. Gallager, *Discrete Stochastic Processes*. Springer, 1st ed., 1995.