hw6

November 19, 2022

[1]:
```
import numpy as np
import numpy.linalg as la
import matplotlib.pyplot as plt
```

1 1.

1.1 a)

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1/2 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1.2 b)

[2]:
```
A = np.array([[0,0,1,0], [.5, 0, 0, 0], [.5, 1, 0, 1], [0, 0, 0, 0]])
pi = np.array([0.25, 0.25, 0.25, 0.25])

converged = False
while not converged:
    newpi = A@pi

    if np.allclose(newpi, pi, rtol=1e-10):
        converged = True

    pi = newpi

print("Page Rank: ", pi)
```

```
Page Rank:  [0.4 0.2 0.4 0. ]
```

1.3 c)

[3]:
```python
G = 0.8 * A + ((1 - 0.8) * .25 * np.ones((4,1))@np.ones((4,1)).T)

print(G)
```

```
[[0.05 0.05 0.85 0.05]
 [0.45 0.05 0.05 0.05]
 [0.45 0.85 0.05 0.85]
 [0.05 0.05 0.05 0.05]]
```

[4]:
```python
converged = False
pi = np.array([0.25, 0.25, 0.25, 0.25])
while not converged:
    newpi = G@pi

    if np.allclose(newpi, pi, rtol=1e-10):
        converged = True

    pi = newpi

print("Page Rank: ", pi)
```

```
Page Rank:  [0.36320755 0.19528302 0.39150943 0.05       ]
```

1.4 d)

In this case we'd have the first row with all 1s and all other rows with all 0s ie:

$$M = \begin{bmatrix} 1 & 1 & 1 & ... \\ 0 & 0 & 0 & ... \\ 0 & 0 & 0 & ... \\ ... & ... & ... & ... \end{bmatrix}$$

And for G:

$$G = \alpha * \begin{bmatrix} 1 & 1 & 1 & ... \\ 0 & 0 & 0 & ... \\ 0 & 0 & 0 & ... \\ ... & ... & ... & ... \end{bmatrix} + (1-\alpha)\frac{1}{n}\mathbb{1}\mathbb{1}^T$$

$$G = \begin{bmatrix} \alpha & \alpha & \alpha & ... \\ 0 & 0 & 0 & ... \\ 0 & 0 & 0 & ... \\ ... & ... & ... & ... \end{bmatrix} + \begin{bmatrix} \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & ... \\ \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & ... \\ \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & ... \\ ... & ... & ... & ... \end{bmatrix}$$

$$G = \begin{bmatrix} \alpha + \frac{1-\alpha}{n} & \alpha + \frac{1-\alpha}{n} & \alpha + \frac{1-\alpha}{n} & ... \\ \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & ... \\ \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & ... \\ ... & ... & ... & ... \end{bmatrix}$$

$G\pi = \pi$

$$\begin{bmatrix} \alpha + \frac{1-\alpha}{n} & \alpha + \frac{1-\alpha}{n} & \alpha + \frac{1-\alpha}{n} & ... \\ \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & ... \\ \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & \frac{1-\alpha}{n} & ... \\ ... & ... & ... & ... \end{bmatrix} \begin{bmatrix} x \\ y \\ y \\ ... \end{bmatrix} = \begin{bmatrix} x \\ y \\ y \\ ... \end{bmatrix}$$

$x = x(\alpha + \frac{1-\alpha}{n}) + y(n-1)(\alpha + \frac{1-\alpha}{n})$

$y = x\frac{1-\alpha}{n} + y(n-1)\frac{1-\alpha}{n}$

We also know:

$x + y(n-1) = 1$

So:

$$x + y(n-1) = 1$$

$$y(n-1) = 1 - x$$

$$y = \frac{1-x}{n-1}$$

$$x = x\left(\alpha + \frac{1-\alpha}{n}\right) + \left(\frac{1-x}{n-1}\right) \cdot (n-1)\left(\alpha + \frac{1-\alpha}{n}\right)$$

$$x = x\left(\alpha + \frac{1-\alpha}{n}\right) + (1-x)\left(\alpha + \frac{1-\alpha}{n}\right)$$

$$x = (\cancel{x} + 1 - \cancel{x})\left(\alpha + \frac{1-\alpha}{n}\right)$$

$$x = \alpha + \frac{1-\alpha}{n}$$

and then

$$y = \frac{1-\alpha-\frac{1-\alpha}{n}}{n-1}$$

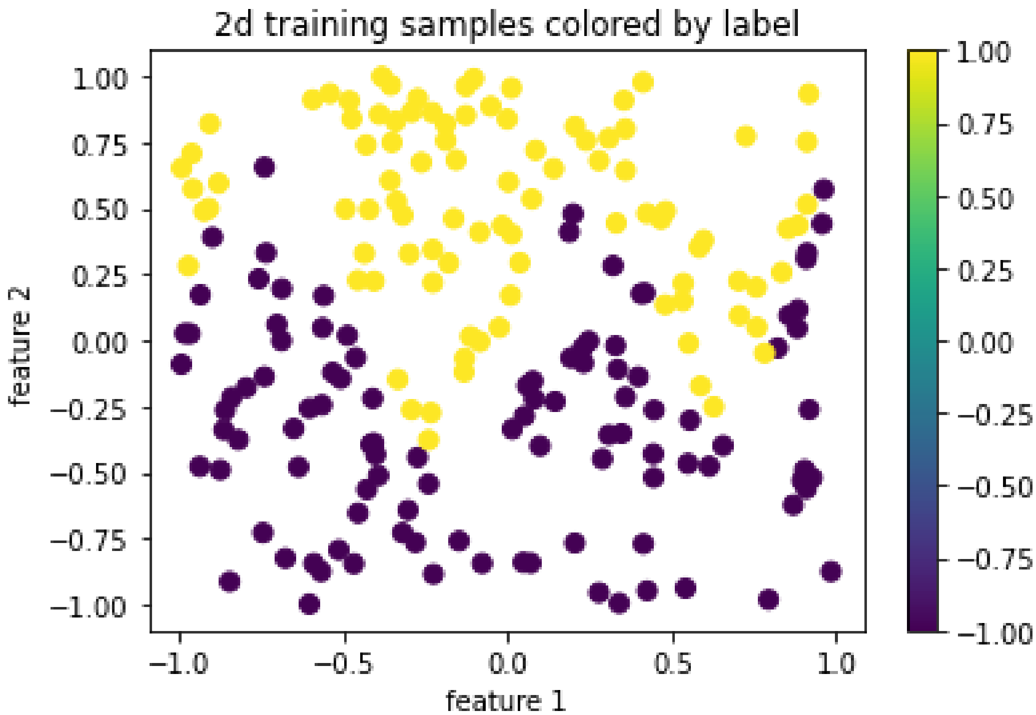
$$y = \frac{n-\alpha n-1-\alpha}{n(n-1)}$$

$$y = \frac{\cancel{(n-1)}(1-\alpha)}{n\cancel{(n-1)}} = \frac{(1-\alpha)}{n}$$

So we have $\pi = \begin{bmatrix} \alpha + \frac{1-\alpha}{n} \\ \frac{1-\alpha}{n} \\ \cdots \\ \frac{1-\alpha}{n} \end{bmatrix}$

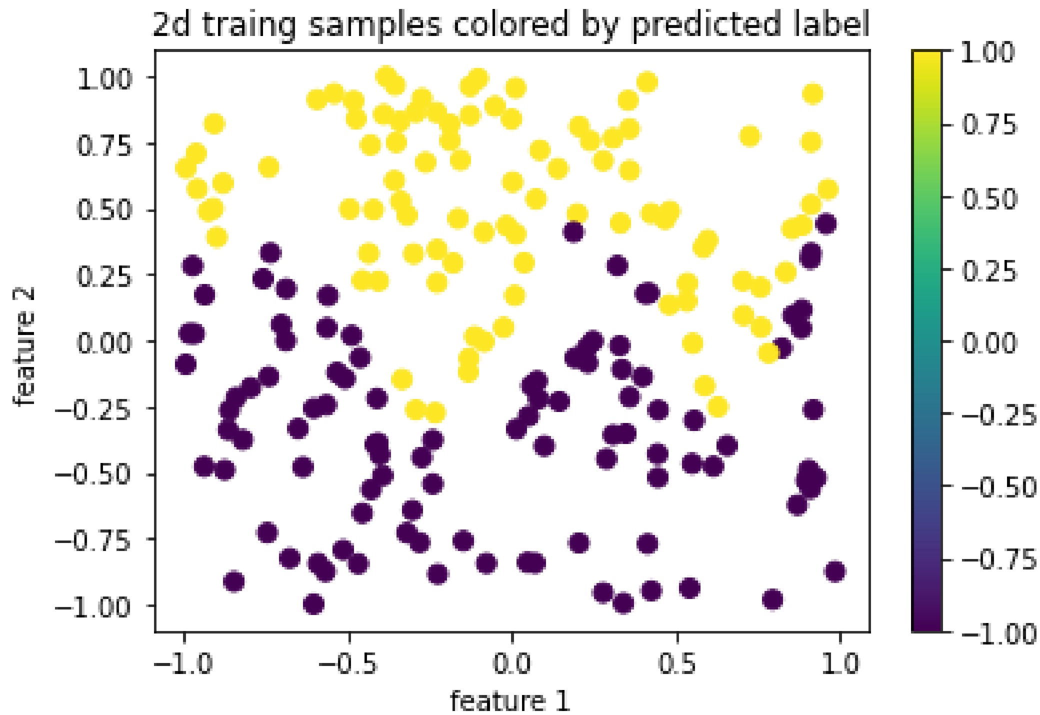
2 2.

[5]:
```
n = 200
p = 2
X = 2 * (np.random.rand(n,p)-.5)
y = np.sign(X[:,1]-(X[:,0]**2/2+np.sin(X[:,0]*7)/2))

plt.figure(1)
plt.scatter(X[:,0], X[:,1], 50, c=y)
plt.colorbar()
plt.xlabel('feature 1')
plt.ylabel('feature 2')
plt.title('2d training samples colored by label')
plt.show()
```

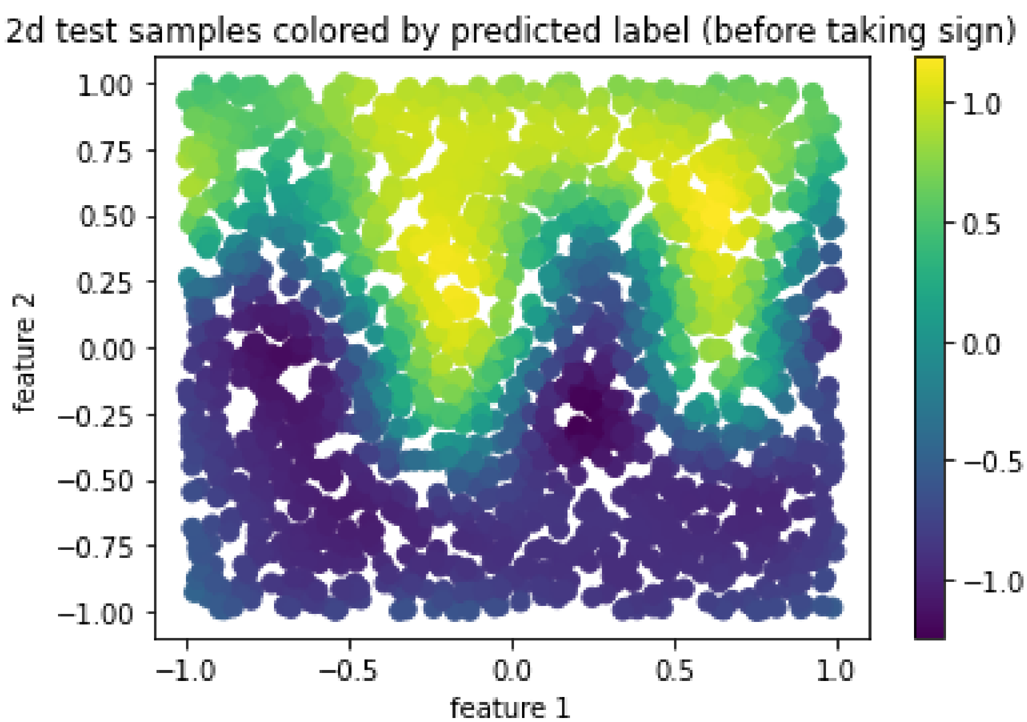


[7]:
```python
def train_set(sigma, show = False):
    lam = 1
    norms2 = (np.array(la.norm(X, axis=1)).T)**2
    innerProds = X@X.T
    dist2 = np.matrix(norms2).T@np.ones([1, n])\
    + np.ones([n, 1])@np.matrix(norms2) -2*innerProds

    K = np.exp((dist2/(2 * sigma)) * -1)
    alpha = la.inv(K + lam*np.identity(K.shape[0]))@y
    yhat = K@alpha.T
    y2 = np.array(np.sign(yhat)).reshape(1,-1)
    accuracy = (y == y2).sum()/n
    if show:
        plt.figure(2)
        plt.scatter(X[:,0], X[:, 1], 50, c=y2)
        plt.colorbar()
        plt.xlabel('feature 1')
        plt.ylabel('feature 2')
        plt.title('2d traing samples colored by predicted label')
        plt.show()

    return accuracy, alpha, norms2, innerProds
```

```python
def test_set(sigma, alpha, norms2, innerProds, show = False):
    lam = 1
    ntest = 2000
    Xtest = 2*(np.random.rand(ntest,p) - .5)
    norms2_test = (np.array(la.norm(Xtest, axis=1)).T)**2
    innerProds_test = Xtest@X.T
    dist2_test = np.matrix(norms2_test).T@np.ones([1, n])\
    + np.ones([ntest, 1])@np.matrix(norms2) - 2*innerProds_test

    K_test = np.exp((dist2_test/(2 * sigma)) * -1)
    ytest = K_test@alpha.T
    ytest_sign = np.sign(ytest).reshape(1,-1)
    ycorrect = np.sign(Xtest[:,1]-(Xtest[:,0]**2/2+np.sin(Xtest[:,0]*7)/2))
    accuracy = (ycorrect == ytest_sign).sum()/2000
    if show:
        plt.figure(3)
        plt.scatter(Xtest[:,0], Xtest[:, 1], 50, c=np.array(ytest))
        plt.colorbar()
        plt.xlabel('feature 1')
        plt.ylabel('feature 2')
        plt.title('2d test samples colored by predicted label (before taking␣
 ↪sign)')
        plt.show()

    return accuracy

sigma = .05
lam = 1

train_acc, alpha, norms2, innerprods = train_set(sigma, True)
print("Training Accuracy: ", train_acc)

test_acc =  test_set(sigma, alpha, norms2, innerprods, True)
print("Test Accuracy ", test_acc)
```

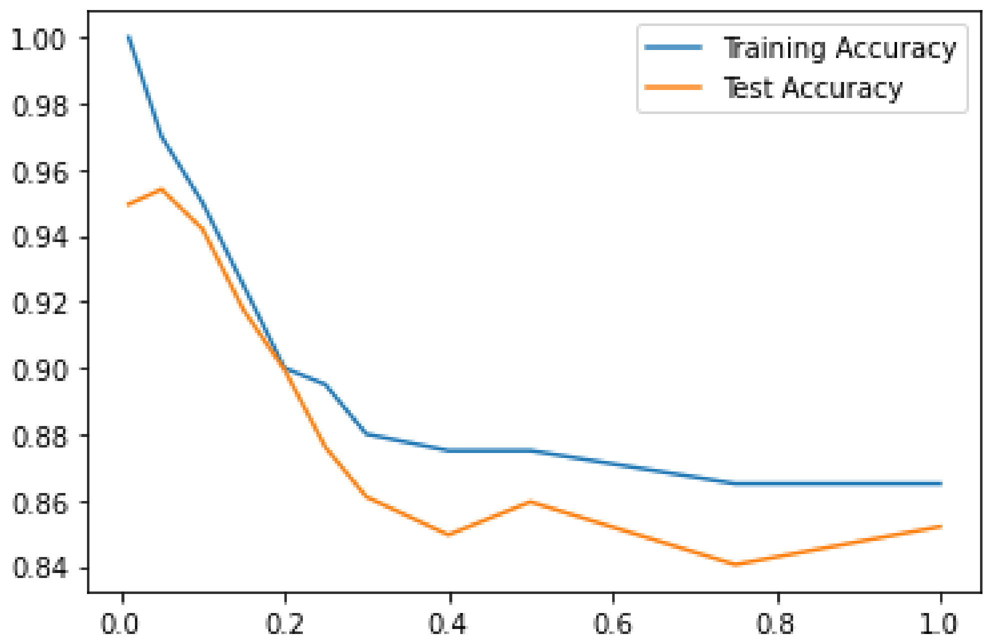


`Training Accuracy:  0.97`



```
Test Accuracy  0.96
```

[8]:
```python
train_accs = []
test_accs = []
sigmas = [0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.75, 1]
for sigma in sigmas:
    train_acc, alpha, norms, innerprods = train_set(sigma)
    test_acc =  test_set(sigma, alpha, norms, innerprods)
    train_accs.append(train_acc)
    test_accs.append(test_acc)

plt.plot(sigmas, train_accs, label="Training Accuracy")
plt.plot(sigmas, test_accs, label="Test Accuracy")
plt.legend()
plt.show()
```



As sigma increases the accuracy for the training set goes down. The test set seems to increase for a moment, but then also decreases as sigma increases.

[ ]:
```

```