MATH 157: Mathematics in the world
Homework 8 (Due April 17th, 2017 at 4PM)

PLEASE SOLVE COMPLETELY BOTH OF THE FOLLOWING 2 PROBLEMS

Problem 1

1. Write a function `intersect_sorted(A, B)` which intersects two given sorted arrays. For example, if

$$A = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10],$$
$$B = [5, 7, 9, 11, 13],$$

   then the output should be

$$[5, 7, 9].$$

2. Let m and n be the lengths of A and B. What is the time-complexity of your solution (as a function of m and n)?

Problem 2

Recall the *sorted matrix* search problem from class (see Notes 18 on Canvas).

1. Implement `search_sorted_matrix(A, k)` which searches the sorted matrix A for a value k. It should return the position (i, j) of the first occurrence of k, or `None` if no matches exist. [1]

2. Let the size of A be $m \times n$. What is the time-complexity of `search_sorted_matrix` (as a function of m and n).

3. Use `search_sorted_matrix` to implement a function `find_sum_sq(n)` which finds a pair of integers $x, y > 0$ such that $x^2 + y^2 = n$. [2] The return value should either be a tuple (x, y) if one exists, or `None` otherwise. [3]

[1] If `A` is a Numpy matrix (2-dimensional array), the number of rows is `A.shape[0]`. Similarly, the number of columns is `A.shape[1]`.

[2] Such pairs may not be unique. If you are looking for a fun problem, try to find the smallest n for which two distinct pairs exist. What about three pairs?

[3] Hint: Use the matrix `np.fromfunction(lambda x, y: (1+x)**2 + (1+y)**2, (nrows, ncols))`.

4. What are the time- and space-complexities of `find_sum_sq` as a function of the input n?

5. Produce an alternative implementation `find_sum_sq_alternative` which does not use matrices. [4]

6. What are the time- and space-complexities of `find_sum_sq_alternative` as a function of the input n?

7. (Extra credit) Generalize your function `search_sorted_matrix` to sorted arrays of arbitrary dimension. What is the time-complexity of your solution? [5]

[4] Hint: Given a value n, we can check if x can take part of a suitable pair (x, y) by inspecting $n - x^2$. What are the possible values of x?

[5] Higher dimensional arrays are called *tensors* in mathematics and physics.