Unit II: System Simulation

Introduction to Simulation:

Simulation is one of the most powerful tools available to decision-makers responsible for the design and operation of complex processes and systems. It makes possible to study, analysis and evaluation of a situation that would not be otherwise possible. In an increasingly competitive world, simulation has become an essential problem-solving methodology for engineers, designers, and managers.

Simulation can be defined as the process of designing a model of a real system and conducting an experiment with these models of system and, or evaluating various strategies for the operation of the system. Thus, it is important that the model we design in such a way, model behaviour mimics the behaviour of a real system.

Simulation allows us to study the situation even though we are unable to experiment directly with the real system, either the system doesn't exist or it is too difficult or expensive to directly manipulate it.

We consider the simulation to include both constructions of the model and experimental use of the model for studying problems. Thus, we can think of simulation as an experiment and applied methodology which seeks to:

i. Describes the behaviour of the system.
ii. Use of model to predict future behaviour i.e. the effect that will be produced by changes in the system or in its method of operation.

When To Use Simulation?

Following are some of the purposes for which simulation may use:

a. Simulation is very useful for experiments with the internal interactions of a complex system, or of a subsystem within a complex system.

b. Simulation can be employed to experiment with new designs and policies, before implementing them.

c. Simulation can be used to verify the results obtained by analytical methods and to reinforce the analytical techniques.

d. Simulation is very useful in determining the influence of changes in input variable on the output of the system.

e. Simulation helps in suggestion modification in the system under investigation for its optimal performance.

Advantages:

Simulation has several advantages. First of all basic concept of simulation is easy to understand and hence often easier to justify to management or customer than some of the analytical methods. Also, a simulation model may be more. Because its behaviour has been compared to that of a real system or because it requires fewer simplifying assumptions and hence captures more of true characteristics of the system understudied.

Other Advantages Include:

a. We can taste new designs, layouts, etc. without assigning resources to their implementation.

b. It can be used to explore new staffing policy, operating procedure, design rules, organizational structure, information flows, etc. with out disturbing the ongoing operation.

c. It allows us to identify a bottleneck (jams) in information, matters, products, flows and test option for increasing the flow rates.

d. It allows us to test a hypothesis about how or why a certain phenomenon occurs in the system.

e. It allows us to control time. Thus we can operate the system several month or years of experience in a matter of seconds allowing us to quickly look at a long time or we can slow down the phenomenon for study.

f. It allows us to gain insights into how a model the system actually works and understanding of which variables are most important up to a performance.

g. Its great strength is its ability to let us experiment with new and unfamiliar situations.

Disadvantages:

Even though simulation has many strengths and advantages, it is not without drawbacks. And some are:

a. Simulation is an art that requires specialized trainers and therefore, skill labels of practice vary widely. The utility of the study depends upon the quality of the model and the skill of the models.

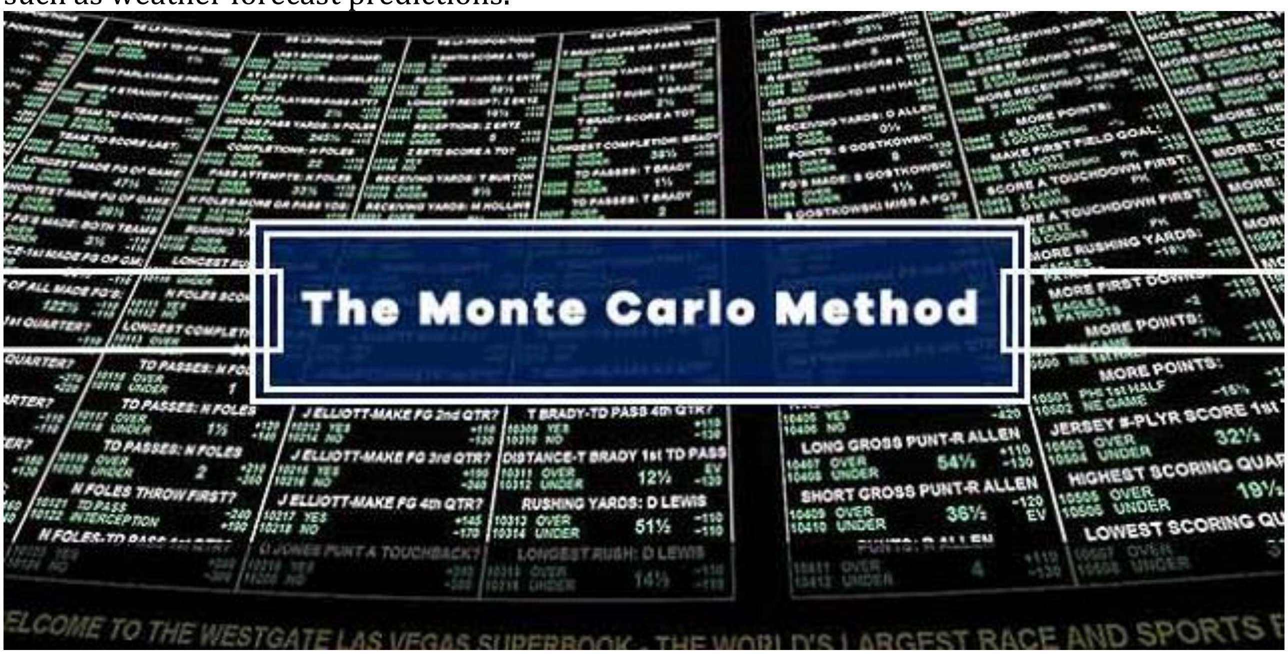
b. Gathering highly reliable input data can be time consuming and the resulting data is sometimes highly comprised of insufficient data or poor management decisions.

c. Simulation models are input and output models i.e. they yield the portable output of the system for a given input. These are, therefore, run rather than solved. They do not yield optional rather they serve as a tool for analysis of the behaviour of a system under conditions specified by the experiments.

The Technique of Simulation – Monte Carlo Method:

Monte Carlo simulation is a computerized mathematical technique to generate random sample data, based on some known distribution for numerical experiments. This method is applied to risk quantitative analysis and decision-making problems. This method is used by the professionals of various profiles such as finance, project management, energy, manufacturing, engineering, research & development, insurance, oil & gas, transportation, etc.

This method was first used by scientists working on the atom bomb in 1940. This method can be used in those situations where we need to make an estimate and uncertain decisions such as weather forecast predictions.



Important Characteristics:

Following are the three important characteristics of Monte-Carlo method.

a. Its output must generate random samples.

b. Its input distribution must be known.

c. Its result must be known while performing an experiment.

Advantages:

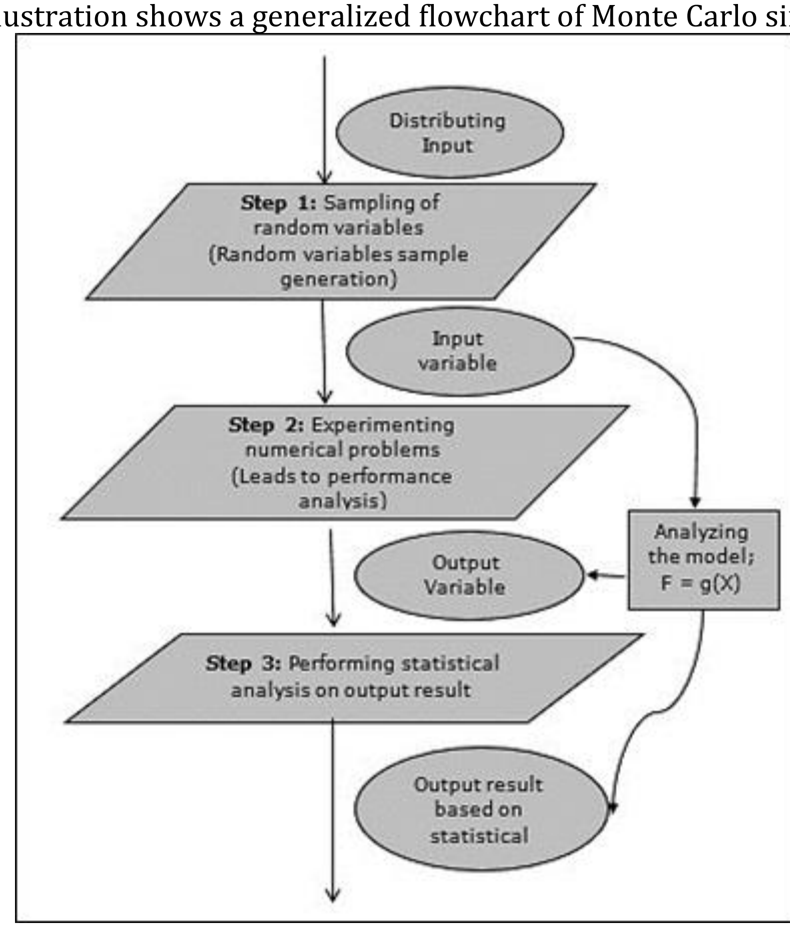
a. Easy to implement.
b. Provides statistical sampling for numerical experiments using the computer.
c. Provides an approximate solution to mathematical problems.
d. Can be used for both stochastic and deterministic problems.

Disadvantages:

a. Time consuming as there is a need to generate a large number of sampling to get the desired output.
b. The results of this method are only the approximation of true values, not the exact.

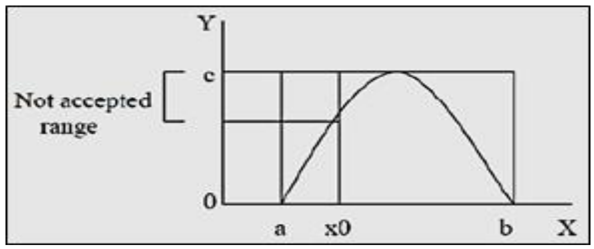
Flow Diagram:

The following illustration shows a generalized flowchart of Monte Carlo simulation.



Problem Depicting Monte Carlo Method:

This method is applied to solve both deterministic as well as stochastic problems. There are many deterministic problems also which are solved by using random numbers and interactive procedure of calculations. In such a case, we convert the deterministic model into a stochastic model, and the results obtained are not exact values, but only estimates.



For example, we shall consider the problem taking integral of a single variable over a range which corresponds to finding the area under graph representing the function f(x). Let us suppose that f(x) is positive and has **a** and **b** as bounds above by **c**. Then as shown in the figure, the function f(x) will be contained within the rectangle of sides c and (b-a). Now, we can pick up points at random within the rectangle and determine whether they lie beneath the curve or not. The points selected are assumed to be obtained from a uniformly distributed random number generator. Two successive samplings are made to get X and Y coordinates so that X is in the range **a** to **b** and Y is in the range **0** to **c**. The fraction of points that fall or below the curve will be approximately the ratio of the area under the curve to the area of the rectangle. If **N** points are drawn and **n** of them fall under curve then;

$$\frac{\textit{number of points inside the curve}}{\textit{number of points falling inside rectangle}} = \frac{\textit{area of curve}}{\textit{area of rectangle}}$$

Here,
Curve = f(x),
n=randomly selected points laying inside the curve,
N=total numbers of points selected,
Area of rectangle=c*(b-a)
Now we have;

$$\frac{n}{N} = \frac{\int_a^b f(x)dx}{c * (b - a)}$$

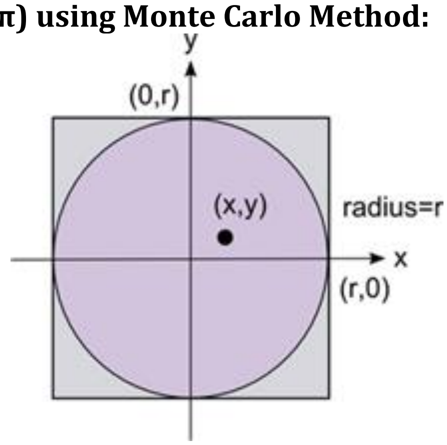
Which is the mathematical statement of the Monte Carlo method?

The accuracy increase as **N** increases. After enough points have taken, the value of the integral (i.e. the area under the curve represented by the function f(x)) is obtained by **n/N * c * (b - a).**

The computational technique is shown in the figure. At each trial, the value of x is selected at random between ‘a’ and ‘b’, say X0. Similarly, the second random number is selected between 0 and C to give y. If y <= f(x0) then point is inside the curve and count ‘n’ otherwise point will not lie in the curve and the next point will be picked.

The application of the Monte Carlo Method for evaluation of Pi (π) is converting a deterministic model into a stochastic model. Some examples that use random sampling in problem-solving are as follows:
a. To find the area of irregular surface figure
b. Numerical Integration of single-variable function
c. A Gambling Game.
d. Random Walk Problem

Determine the value of Pi (π) using Monte Carlo Method:



$$\frac{Area\ of\ quadrant\ of\ circle}{Area\ of\ Rectangle} = \frac{Number\ of\ points\ inside\ the\ curve}{Number\ of\ points\ inside\ the\ rectangle}$$

$$or, \frac{{}^{1}/_{4}\,\pi r^2}{r^2} = \frac{n}{N}$$

$$\therefore\ \pi = \frac{4n}{N}$$

We use random number generation method to determine the sample points that lie inside or outside the curve. Let (x_0, y_0) be an initial guess for the sample point than from a linear congruential method of random number generation:

$X_{i+1} = (a_{xi} + c) \bmod m$

$Y_{i+1} = (a_{yi} + c) \bmod m$

Where a & c are constants, m is the upper limit of generated random number. If $y \le y_i$ then increment n.

Example:

We have circle equation = $x^2 + y^2 = 1$ or $y = \sqrt{(1 - x^2)}$ Now, generate the random numbers x and y within the interval 0 and 1.

For x: $x_0 = 27$, a = 17, c = 0, m = 100

For y: $y_0 = 47$, a = 17, c = 0, m = 100

X'	Y'	$\sqrt{(1 - x^2)}$	In/Out
0.59	0.99	0.962	In
0.03	0.83	0.99	In
0.51	0.11	0.86	In
0.67	0.87	0.74	Out

Now,

Points inside the curve (n) = 3

Points inside the rectangle (N) = 4

Value of pi = (n/N)*4

= 3

Numerical Integration using Monte Carlo Method:

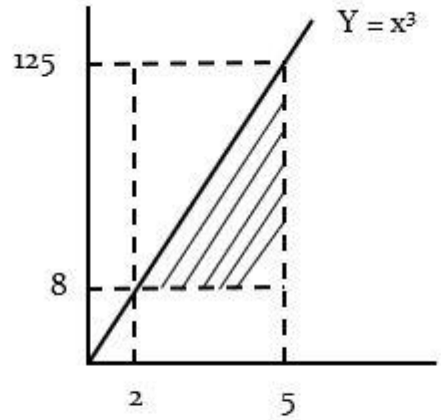
$$I = \int_a^b f(x)dx$$

From Monte Carlo Method

$$I = \frac{n}{N} * (b - a) * c$$

Example:

$$I = \int_2^5 x^3 dx \text{ using Monte Carlo Method}$$



We have to generate a random number for x and y. For x (random number be in range 2 to 5) & for y (random number be in range 8 to 125).

Here, the area of the rectangle under the given condition = (5 - 2) * (125 - 8) = 351, also we know, ***I = n/N * area of rectangle***

Now, we can select the random points inside the curve (using the random number generation method).

For x: x0 = 23, a = 17, c = 0, m = 50

Fir y: y0 = 61, a = 59, c = 0, m = 125

X	X' = X*0.1	Y	X'3	In/Out
23	2.3	59	12.167	Out
41	4.1	99	68.921	Out
47	4.7	91	103.823	In
49	4.9	119	117.649	Out
33	3.3	21	35.937	In
11	1.1	114	1.331	Out
37	3.7	101	50.653	Out

We get,

Points inside the curve (n) = 2

Points inside the rectangle (N) = 6

*I = n/N * area of rectangle*

*I = 2/6 * 351*

I = 117

Comparison of Simulation and Analytical Method:

Once we have built a mathematical model, it must then be examined to see how it can be used to answer the questions of interest in the system is supposed to represent. If the model is simple enough, it may be possible to work with its relationships and quantities to get an exact, analytical solution.

In the d = v*t example, if we know the distance to be travelled and the velocity, then we can work with the model to get t = d/v as the time that will be required. This is a very simple, closed-form solution obtainable with just paper and pencil, but some analytical solutions can become extraordinarily complex, requiring vast computing resources; inverting a large non-sparse matrix is a well-known example of a situation in which there is an analytical formula known in principle, but obtaining it numerically in a given instance is far from trivial.

If an analytical solution to a mathematical model is available and is computationally efficient, it is usually desirable to study the model in this way rather than via a simulation. However, many systems are highly complex, so that valid mathematical models of them are themselves complex, precluding any possibility of an analytical solution. In this case, the model must be studied using simulation, i.e., numerically exercising the model for the inputs in question to see how they affect the output measures of performance.

While there may be an element of truth to pejorative old saws such as “method of last resort” sometimes used to describe simulation, the fact is that we are very quickly led to simulation in many situations, due to the sheer complexity of the systems of interest and of the models necessary to validly represent them.

Given, then, that we have a mathematical model to be studied using simulation (henceforth referred to as a simulation model), we must then look for particular tools to execute this model (i.e. actual simulation).

Difference Between Simulation and Analytic:

Basis	Simulation	Analytic
Input Parameterization	Measured or Invented	Measured or invented (with certain limitations)
Model Components	Virtually anything	Composed of limited basic building blocks
Model Outputs	Anything that can be measured	Equilibrium measures
Effort To Construct Model	Arbitrary	Modest
Computational Cost	Typically large	Typically small
Underlying Concepts	Probability or Statistics	Algebra to stochastic processes
Special Properties	Credible	Insight, optimization

Experimental Nature of Simulation:

The simulation technique makes no specific attempt to isolate (separate) the relationship between any particular variables; instead, it observes how all variables of the model change with time. The relationship between the variables must be derived from these observations. Simulation is, therefore, essentially an experimental problem-solving technique. Many simulation runs have to be made to understand the relationships involved in the system, so the use of simulation in a study must be planned as a series of experiments.

Types of System Simulation:

A simulator is a device, computer program, or system that performs a simulation. A simulation is a method for implementing a model over time. There are three types of commonly uses simulations:

1. Live:

Simulation involving real people operating real systems.
a. Involve individuals or groups
b. May use actual equipment
c. Should provide a similar area of operations

d. Should be close to replicating the actual activity

2. Virtual:

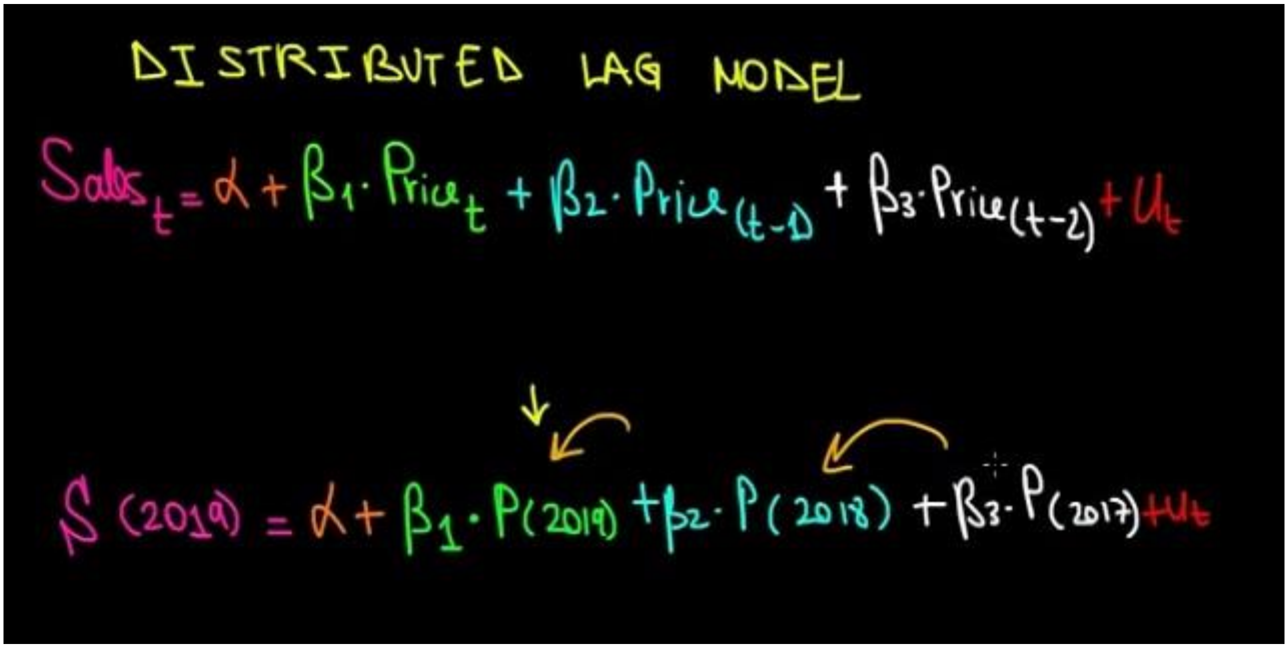
Simulation involving real people operating simulated systems. Virtual simulations inject Human-In-The-Loop in a central role by exercising:
a. Motor control skills (e.g. flying an aeroplane)
b. Decision skills (e.g. committing fire control resources to action)
c. Communication skills (e.g. members of a C4I team)

3. Constructive:

Simulation involving simulated people operating simulated systems. Real people can stimulate (make inputs) but are not involved in determining outcomes. Constructive simulations offer the ability to:
a. Analyze concepts
b. Predict possible outcomes
c. Stress large organizations
d. Make measurements
e. Generate statistics
f. Perform analysis

Distributed Lag Model:

If the regression model includes not only the current but also the lagged (past) values of the explanatory variables (the x's) it is called a distributed lag model. If the model includes one or more lagged values of the dependent variable among its explanatory variables, it is called an autoregressive model. This is known as a dynamic model.



In other word, Distributed Lag Model is defined as a type of model that have the property of changing only at fixed interval of time and based on current values of variables on other current values of variables and values that occurred in previous intervals.

In economic studies, some economic data are collected over uniform time intervals such as a month or year. This model consists of linear algebraic equations that represent continuous system but data are available at fixed points in time.

For Example: Mathematical Model of National Economy:

Let,
C = Consumption
I = Investment
T = Taxes
G = Government Expenditures
Y = National Income

Then
C = 20 + 0.7 (Y – T)
I = 2 + 0.1 Y

T = 0.2 Y

Y = C + I + G

All the equation are expressed in billions of rupees. This is static model and can be made dynamic by lagging all the variables as follows:

C = $20 + 0.7(Y_{-1} - T_{-1})$

I = $2 + 0.1Y_{-1}$

T = $0.2Y_{-1}$

Y = $C_{-1} + I_{-1} + G_{-1}$

Any variable that can be expressed in the form of its current value and one or more previous value is called a lagging variable. And hence this model is given the name distributed lag model. The variable in a previous interval is denoted by attaching –n suffix to the variable. Where -n indicates the n^{th} interval.

Advantages of Distributed Lag Model:

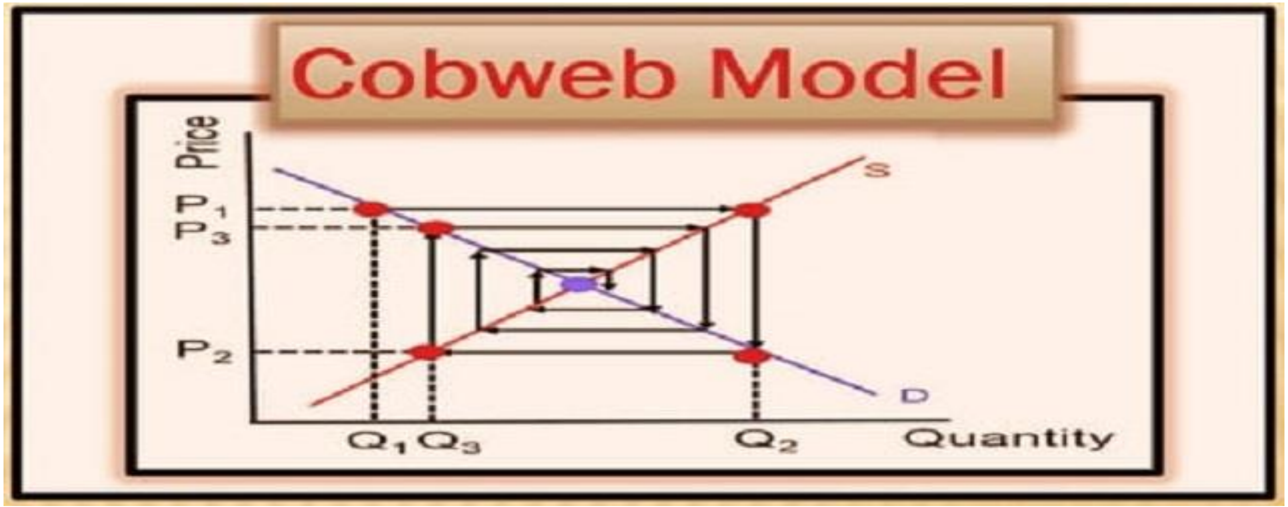
a. Simple to understand and can be computed by hand, computers are extensively used to run them.

b. There is no need for special programming language to organize the simulation task.

Cobweb Model:

Cobweb theory is the idea that price fluctuations can lead to fluctuations in supply which cause a cycle of rising and falling prices.

In a simple cobweb model, we assume there is an agricultural market where supply can vary due to variable factors, such as the weather.



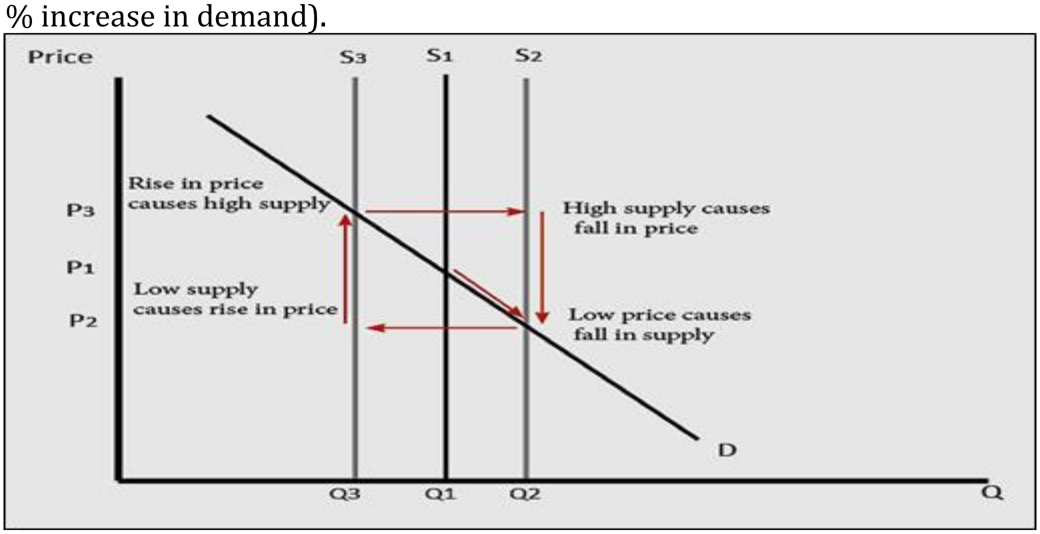
Assumptions of Cobweb theory:

a. In an agricultural market, farmers have to decide how much to produce a year in advance, before they know what the market price will be. (supply is price inelastic in short-term)

b. A key determinant of supply will be the price from the previous year.

c. A low price will mean some farmers go out of business. Also, a low price will discourage farmers from growing that crop in the next year.

d. Demand for agricultural goods is usually price inelastic (a fall in price only causes a smaller % increase in demand).



1. If there is a very good harvest, then supply will be greater than expected and this will cause a fall in price.

2. However, this fall in price may cause some farmers to go out of business. Next year farmers may be put off by the low price and produce something else. The consequence is that if we have one year of low prices, next year farmers reduce the supply.

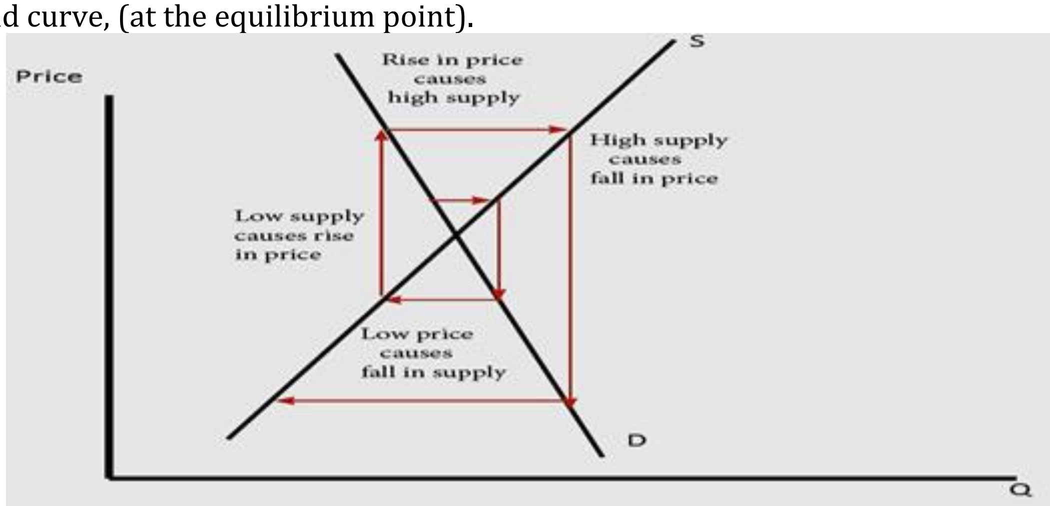
3. If supply is reduced, then this will cause the price to rise.

4. If farmers see high prices (and high profits), then next year they are inclined to increase supply because that product is more profitable.

In theory, the market could fluctuate between high price and low price as suppliers responds to past prices.

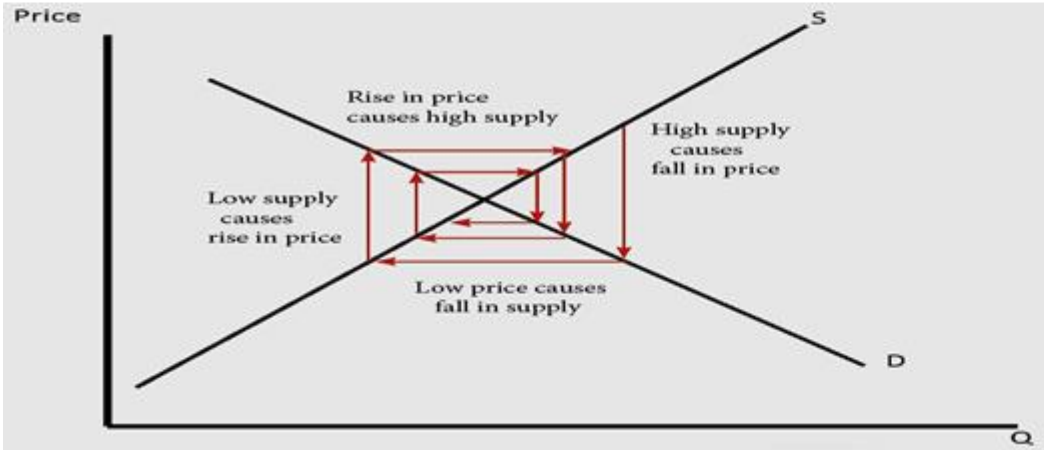
Cobweb Theory and Price Divergence:

The price will diverge from the equilibrium when the supply curve is more elastic than the demand curve, (at the equilibrium point).



If the slope of the supply curve is less than the demand curve, then the price changes could become magnified and the market more unstable.

Cobweb Theory and Price Convergence:



At the equilibrium point, if the demand curve is more elastic than the supply curve, we get the price volatility falling, and the price will converge on the equilibrium

Limitations of Cobweb theory:

1. Rational Expectations:

The model assumes farmers base next year's supply purely on the previous price and assume that next year's price will be the same as last year (adaptive expectations). However, that rarely applies in the real world. Farmers are more likely to see it as a 'good' year or 'bad year and learn from price volatility.

2. Price Divergence Is Unrealistic And Not Empirically Seen:

The idea that farmers only base supply on last year's price means, in theory, prices could increasingly diverge, but farmers would learn from this and pre-empt changes in price.

3. It May Not Be Easy Or Desirable To Switch:

A potato grower may concentrate on potatoes because that is his speciality. It is not easy to give up potatoes and take to aborigines.

4. Other Factors Affecting Price:

There are many other factors affecting price than a farmers decision to supply. In global markets, supply fluctuations will be minimized by the role of importing from abroad. Also, demand may vary. Also, supply can vary due to weather factors.

5. Buffer Stock Schemes:

Governments or producers could band together to limit price volatility by buying surplus.

Steps of Simulation Study:

1. Problem Formulation:

The study begins with defining the problem statement. It can be developed either by the analyst or client. If the statement is provided by the client, then the analyst must take extreme care to ensure that the problem is clearly understood. If a problem statement is prepared by the simulation analyst, the client must understand and agree with the formulation. Even with all of these precautions, the problem may need to be reformulated as the simulation study progresses.

2. Setting of Objectives and Overall Project Plan:

Another way to state this step is to "prepare a proposal." The objectives indicate the questions to be answered by the simulation study. Whether the simulation is appropriate or not is to be decided at this stage. The overall project plan should include a statement of the alternative systems and a method for evaluating the effectiveness of these alternatives. The plan includes several personnel, number of days to complete the task, stages in the investigation, output at each stage, cost of the study and billing procedures if any.

3. Model Conceptualization:

Model is a simplification of reality. The real-world system under investigation is abstracted by a conceptual model. It is recommended that modelling begins with a simple model and grows until a model of appropriate complexity has been achieved. For example, consider the model of a manufacturing and material handling system. The basic model with the arrivals, queues, and servers is constructed. Then, add the failures and shift schedules. Next, add the material-handling capabilities. Finally, add special features. Constructing an excessive complex model will add to the cost of the study and the time for its completion, without increasing the quality of the output.

Maintaining client involvement will enhance the quality of the resulting model and increase the client's confidence in its use.

4. Data Collection:

This step involves gathering the desired input data. The data changes over the complexity of the model. Data collection takes a huge amount of total time required to perform a simulation. It should be started at early stages together with model building. The collection of data should be relevant to the objectives of the study.

5. Model Translation:

The conceptual model constructed in Step 3 is coded into a computer recognizable form, an operational model. The suitable simulation language is used.

6. Verified:

Verification is concerning the operational model. Is it performing properly? If the input parameters and logical structure of the model are correctly represented in the computer, then verification is completed.

7. Validated:

Validation is the determination, that the model is an accurate representation of the real system. This is done by calibration of the model; an iterative process of comparing the model to the actual system behaviour. This process is repeated until model accuracy is acceptable.

8. Experimental Design:

The alternatives to be simulated must be determined. For each the scenario that is to be simulated, decisions need to be made concerning the length of the simulation run, the number of runs (also called replications), and the length of the initialization period.

9. Production Runs And Analysis:

The production runs, and their subsequent analysis is used to estimate measures of performance for the system design that is being simulated.

10. More Runs:

After the completion of the analysis of runs, the simulation the analyst determines if additional runs are needed and any additional experiments should follow.

11. Documentation And Reporting:

There are two types of Documentation: Program and Progress. Program documentation is necessary for numerous reasons. If the program is going to be used again by the same or different analysts, it may be necessary to understand how the program operates. This will enable confidence in the program so that the client can make decisions based on the analysis. Also, if the model is to be modified, this can be greatly facilitated by adequate documentation. Progress reports provide a chronology of work done and decisions made. It is the written history of a simulation project. The result of all the analyses should be reported clearly and concisely. This will enable the client to review the final formulation.

12. Implementation:

If the client has been involved throughout the study period, and the simulation analyst has followed all of the steps rigorously, then the likelihood of a successful implementation is increased.

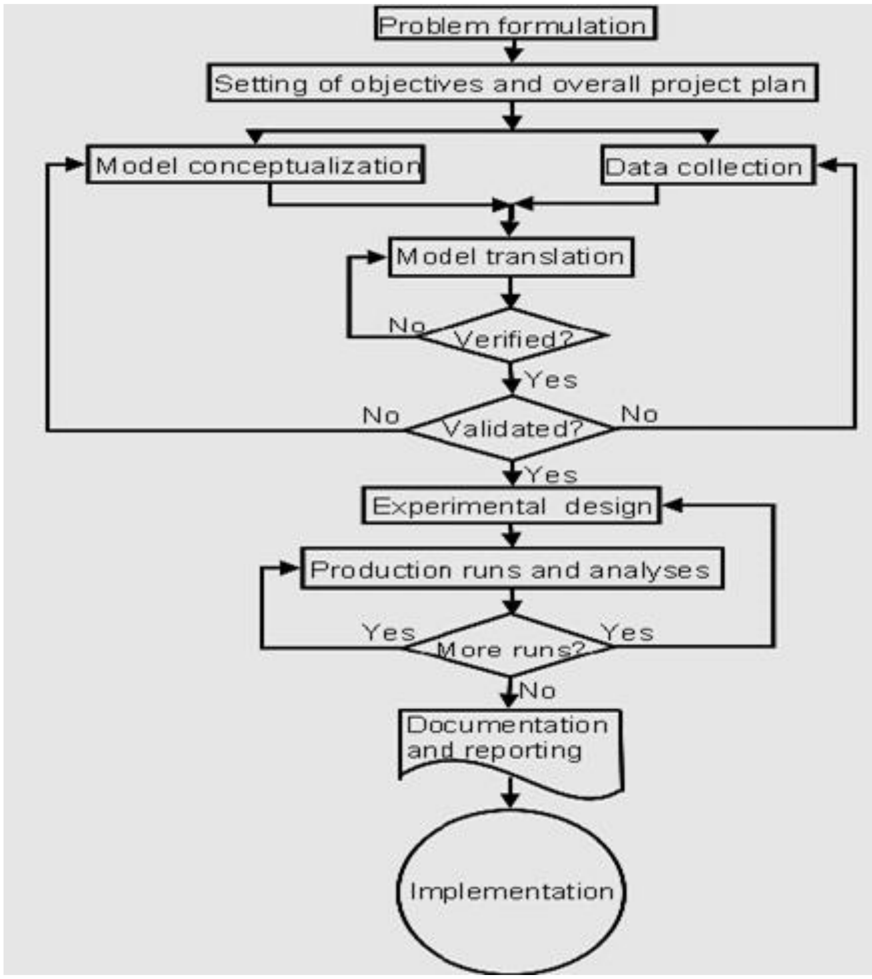


Fig: Steps of Simulation Study

Time Advancement Mechanism:

The simulation models we consider will be discrete, dynamic, and stochastic. Discrete event simulation concerns the modelling of a system as it evolves over time by a representation in which the state variables change instantaneously at separate points in time. These points in time are the one which an event occurs, where an event is defined as an instantaneous occurrence that may change the state of a system.

Because of the dynamic nature of discrete-event simulation model, we must keep track of the current value of simulation time as the simulation proceeds and we also need a mechanism to advance simulation time from one variable to another. We call the variable in a simulation model that gives the current value of simulation time the simulation clock or simulation time.

Simulation time means the integral clock time and not the time a computer was taken to carry out the simulation. Two principal approaches for advancing the simulation clock are:
a. Next Event Time Advance
b. Fixed Increment Time Advance

Queuing Models and its Characteristics:

The Queuing theory provides predictions about waiting times, the average number of waiting customers, the length of a busy period and so forth.

These predictions help us to anticipate situations and to take appropriate measures to shorten the queues.

A further attractive feature of the theory is quite an astonishing range of its applications. Some of the more prominent of these are telephone conversation, machine repair, toll booths, taxi stands, inventory control, the loading, and unloading of ships scheduling patients in the hospital clinics, production flow and applications in the computer field concerning program scheduling, etc.

A queuing system may be described as one having a service facility, at which units of some kind (called customers) arrive for service and whenever there are more units in the system than the service facility can handle simultaneously, a queue or waiting line develops. The waiting units take their turn for service according to a pre-assigned rule and after service, they leave the system. Thus, the input to the system consists of the customers demanding service and the output is the serviced customers.

Characteristics of the Queuing System:

1. The Arrival Pattern:

The arrival pattern describes how a customer may become a part of the queuing system. The arrival time for any customer is unpredictable. Therefore, the arrival time and the number of customers arriving at any specified time intervals are usually random variables. A Poisson distribution of arrivals corresponds to arrivals at random. In Poisson distribution, successive customers arrive after intervals which independently are and exponentially distributed. The Poisson distribution is important, as it is a suitable mathematical model of many practical queuing systems as described by the parameter "the average arrival rate".

2. The Service Mechanism:

The service mechanism is a description of the resources required for service. If there are an infinite number of servers, then there will be no queue. If the number of servers is finite, then the customers are served according to a specific order. The time taken to serve a particular customer is called the service time. The service time is a statistical variable and can be studied either as the number of services completed in a given time or the completion period of service.

3. The Queue Discipline:

The most common queue discipline is the "First Come First Served" (FCFS) or "First-in, First-out" (FIFO). Situations like waiting for a haircut, ticket-booking counters follow FCFS discipline. Other disciplines include "Last In First Out" (LIFO) where the last customer is serviced first, "Service In Random Order" (SIRO) in which the customers are serviced randomly irrespective of their arrivals. "Priority service" is when the customers are grouped in priority classes based on urgency. "Preemptive Priority" is the highest priority given to the customer who enters into the service, immediately, even if a customer with lower priority is in service. "Non-preemptive priority" is where the customer goes ahead in the queue but will be served only after the completion of the current service.

The Number of Customers allowed in the System:

In certain cases, a service system is unable to accommodate more than the required number of customers at a time. No further customers are allowed to enter until space becomes available to accommodate new customers. Such types of situations are referred to as finite (or limited) source queue. Examples of finite source queues are cinema halls, restaurants, etc.

On the other hand, if a service system can accommodate any number of customers at a time, then it is referred to as an infinite (or unlimited) source queue. For example, in a sales department, where the customer orders are received; there is no restriction on the number of orders that can come in so that a queue of any size can form.

The Number of Service Channels:

The more the number of service channels in the service facility, the greater the overall service rate of the facility. The combination of arrival rate and service rate is critical for determining the number of service channels. When there are several service channels available for service, then the arrangement of service depends upon the design of the system's service mechanism.
Parallel channels mean, several channels providing identical service facilities so that several customers may be served simultaneously. Series channel means a customer goes through successive ordered channels before service is completed. A queuing system is called a **one-server model,** i.e., when the system has only one server, and a **multi-server model** i.e., when the system has several parallel channels, each with one server.

a. Arrangement Of Service Facilities In Series:

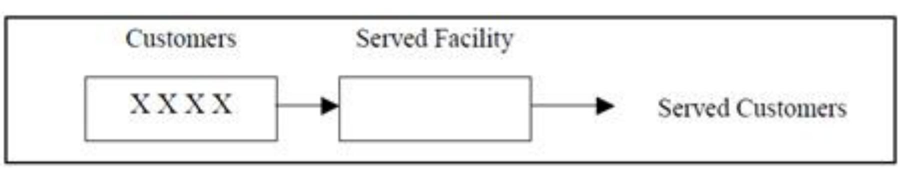


Fig: Single Queue Single Server

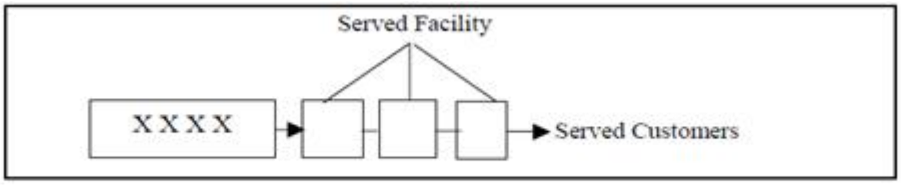
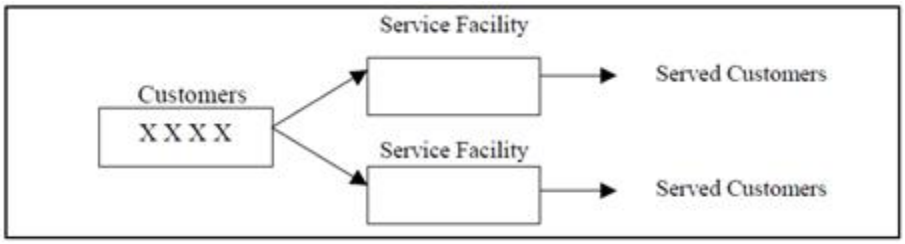
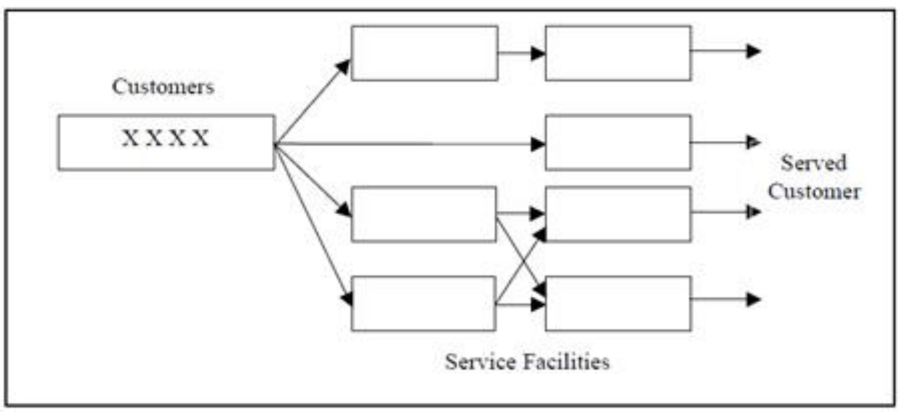


Fig: Single Queue, Multiple Server

b. Arrangement Of Service Facilities In Parallel:



c. Arrangement of Mixed Service facilities:



Measuring the Performance Of The System:

To measure the performance of the system, we estimate the following three qualities:

1. Estimate the expected average delay in a *d(n)* queue of *(n)* customers. The actual average delay for n customers depends on the inter-arrival and service time. From a single sum of simulation with customers delays D1, D2… Dn, the estimate of *d(n)* is given by:

$$\hat{d}(n) = \frac{\sum_{i=1}^{n} Di}{n} --------(1)$$

Note that a customer could have a delay of zero in case of an arrival finding the system empty or idle. If many delays were 0 then this could represent the system providing very good service.

This estimate of *d(n)* gives information about the system performance form the customer point of view.

2. Estimate the expected average number of customers in the queue (but not being served), denoted by *q(n).* It is different from the average delay in the queue because it takes over continuous time rather than over the customer (being discrete).

Let Q(t) = number of customers in queue at time (t), (t≥0).
T(n) = time required for 'n' delays in queue.
Pj = expected proportion of time that Q(t) = 1 (value of Pi will be between 0-1)
Then, the average number of customer queue;

$$q(n) = \sum_{i=0}^{\infty} iPi$$

Expected average number of customer in queue:

$$\hat{q}(n) = \sum_{i=0}^{\infty} i\hat{P}i --------(2)$$

Where $\hat{P}i$ is observed (rather than expected) portion of the time during the simulation that there were *i* customers in queue. Also, let *Ti* be the total time during the simulation that the queue is of length *i,* then *T(n) =* $T_1+T_2+T_3+$…… and $\hat{P}i = Ti/T(n)$ and so that we can rewrite the above equation as:

$$\hat{q}(n) = \frac{\sum_{i=0}^{\infty} iTi}{T(n)}$$

3. Measure How Busy The Server Is:

The expected utilization of the server is the expected proportion of time during the simulation that the server is busy and is thus, the number between 0 & 1; denote by *u(n).* From a single simulation, then, our estimate of *u(n)* is = $\hat{u}(n)$ the observed proportion of time during the simulation that the server is busy. Let us define the busy function as:

B(t) = 1 if the server is busy at time t

0 if the server is idle at time t

Then, $\hat{u}(n)$ could be expressed the proportion of time when *B(t)* equals to 1 and is given by:

$$\hat{u}(n) = \frac{\int_0^{T(n)} B(t)\,dt}{T(n)}$$

Single Server Queuing System:

Consider a single server queuing system, where the inter arrival time A1, A2… are independent and identically distributed (IID) random variables. A customer who arrives and finds the server idle enters the service immediately and the service time S1, S2… of the successive customers are IID random variables that are independent of inter-arrival times.

a. A customer who arrives and finds the server busy, joins the end of the single queue.

b. After completing service for a customer1, the server chooses the next customer from the queue (if any) in a FIFO manner.

c. The simulation will begin in the empty and idle state i.e. no customers are present and the service is idle.

d. At time 0, the system will begin waiting for the arrival of the 1st customer will occur, after the 1st inter-arrival time A1 rather than at time 0.

e. The simulation will continue until N numbers of customers have completed their delays in a queue.