MA1011 - NUMERICAL METHODS

UNIT - II

INTERPOLATION AND APPROXIMATION

PART - A

1. **Interpolation:**

We are given a table values of X and Y. The process of finding the value of y (not given in the table) corresponding to the value of x is known as "interpolation".

Inverse Interpolation:

The process of finding the value of x (not given in the table) corresponding to the value of y is known as "inverse interpolation".

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2. Write Newton's forward and backward difference formula where they are used?

Newton's Forward Formula:

$$y(x) = y_0 + \frac{u\Delta y_0}{1!} + \frac{u(u-1)}{2!}\Delta^2 y_0 + \dots\dots + \frac{u(u-1)(u-2)\dots(u-\overline{n-1})}{n!}\Delta^n y_0.$$

Where $u = \dfrac{x - x_0}{h}$

It is used to find the value of y for the value of x near the beginning of the table value.

Newton's Backward Formula:

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \dots + \frac{v(v+1)\dots(v+n-1)}{n!}\nabla^n y_n.$$

Where $v = \frac{x-x_n}{h}$

It is used to find the value of y for the value of x at the end of the table value.

3. Obtain the interpolation Quadrate polynomial for the given data by using Newtons forward difference formula.

X :	0	2	4	6
Y :	-3	5	21	45

Table Calculation :

x	Y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-3			
		8		
			8	
2	5			
				0
		16		
4	21		8	
		24		
6	45			

$u = \frac{x - x_0}{h}$, $h = 2 - 0 = 2$.

$\therefore u = \frac{x-0}{2} = x/2$

Formula:

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0$$

$$\Rightarrow -3 + \frac{x}{2} \times 8 + \frac{x/2\,(x/2 - 1)}{2} \times 8$$

$$\Rightarrow -3 + 4x + x^2 - 2x$$

$$\Rightarrow x^2 + 2x - 3$$

4. Using Newtons backward difference formula. Write the formula for the first and the Second Order derivatives at the end value $x = x_n$ upto the fourth Order difference term.

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \ldots\ldots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \ldots\ldots\right]$$

5. State Newtons divided difference formula.

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$$
$$+ (x-x_0)(x-x_1)\ldots(x-x_{n-1}) f(x_0, x_1 \ldots x_n)$$

6. State the properties of divided differences.

The divided differences are Symmetrical in all their arguments.

1. $\Delta[f(x) \pm g(x)] = \Delta f(x) \pm \Delta g(x)$

($\because$ Linear Property)

2. $\Delta\, c f(x) = c\, \Delta f(x)$

3. The divided difference of n^{th} degree polynomial is Constant.

7. Show that $\underset{bcd}{\Delta^3}\left(\frac{1}{a}\right) = -\frac{1}{abcd}$

Given:

$\underset{bcd}{\Delta^3}\left(\frac{1}{a}\right) = -\frac{1}{abcd}$ Let us take $f(x) = \frac{1}{x}$; $f(a) = \frac{1}{a}$.

Solution

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
a	$\frac{1}{a}$	$\frac{\frac{1}{b}-\frac{1}{a}}{b-a} = -\frac{1}{ab}$	$\frac{-\frac{1}{cb}+\frac{1}{ab}}{c-a} = \frac{1}{abc}$	$\frac{\frac{1}{abc}-\frac{1}{bcd}}{a-d}$ $= -\frac{1}{abcd}$.
b	$\frac{1}{b}$	$\frac{\frac{1}{c}-\frac{1}{b}}{c-b} = -\frac{1}{cb}$	$\frac{-\frac{1}{cd}+\frac{1}{cb}}{d-b} = \frac{1}{bcd}$	
c	$\frac{1}{c}$	$\frac{\frac{1}{d}-\frac{1}{c}}{d-c} = -\frac{1}{cd}$		
d	$\frac{1}{d}$			

8. Find the Second divided differences arguments a, b, c if $f(x) = 1/x$.

$$\underset{b}{\Delta}\left(1/a\right) = f(a,b)$$

$$= \frac{f(b) - f(a)}{b-a}$$

$$= \frac{1/b - 1/a}{b-a} \Rightarrow \frac{a-b}{ab(b-a)}$$

$$f(a,b) = -1/ab$$

$$f(a,b,c) = \frac{f(b,c) - f(a,b)}{c-a}$$

$$= \frac{-1/bc + 1/ab}{c-a}$$

$$= \frac{(c-a)}{abc(c-a)} = 1/abc$$

9. If $f(x) = 1/x^2$ with arguments a, b find $f(a,b)$

$$f(a,b) = \frac{f(b) - f(a)}{b-a}$$

$$= \frac{1/b^2 - 1/a^2}{b-a}$$

$$= \frac{a^2 - b^2}{a^2b^2(b-a)} = \frac{(a+b)(a-b)}{a^2b^2(b-a)}$$

$$= -\frac{(a+b)}{a^2b^2}.$$

10. Find the 3rd divided differences of $f(x)$ given with the arguments 2, 4, 9, 10 where $f(x) = x^3 - 2x$.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	6	$\frac{56-6}{4-2} = 25$	$\frac{131-25}{9-2} = 15.14$	$\frac{189.66-15.14}{10-2} = 21.815$
4	56	$\frac{711-56}{9-4} = 131$	$\frac{1269-131}{10-4} = 189.66$	
9	711	$\frac{1980-711}{10-9} = 1269$		
10	1980			

11. Write Lagrange's Interpolation formula and inverse formula.

<u>Lagrange's Interpolation Formula:</u>

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots(x_0-x_n)} y_0$$

$$+ \frac{(x-x_0)(x-x_2)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots(x_1-x_n)} y_1$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots$$

$$+ \frac{(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})}{\ldots} y_n$$

Inverse Langrange's Interpolation formula:

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)}x_1 + \cdots + \frac{(y-y_0)(y-y_1)\cdots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\cdots(y_n-y_{n-1})}x_n$$

12. Compare Newtons formula with Lagrange's formula.

No	Newtons formula	Lagrange's formula
1.	Applicable only for the equal intervals	Applicable both for the equal and unequal intervals.
2.	The differences of the dependent Variable should be smaller.	The difference of this dependent Variable is smaller (or) not the formula can be Used.

13. Find the divided difference for the following data.

X :	2	5	10
Y :	5	29	109

X	Y	$\Delta f(x)$	$\Delta^2 f(x)$
2	5		
		$\frac{29-5}{5-2} = 8$	
5	29		$\frac{16-8}{10-2} = 1$
		$\frac{109-29}{10-5} = 16$	
10	109		

14. Given $f(0) = -2$, $f(1) = 2$, $f(2) = 8$. Find the root of $f(x) = 0$ Using Newtons interpolation formula.

X	Y	ΔY	$\Delta^2 Y$
0	-2		
		4	
1	2		2
		6	
2	8		

$$u = \frac{x - x_0}{h} \; ; \; = \frac{x-0}{1} \quad \therefore \boxed{u = x}$$

Formula:

$$y(x) = y_0 + \frac{u}{1!}\Delta y + \frac{u(u-1)}{2!}\Delta^2 y$$

$$= -2 + x \cdot 4 + \frac{x(x-1)}{2} \cdot 2$$

$$= -2 + x \cdot 4 + \frac{x(x-1)}{2} 2$$

$$= -2 + 4x + x^2 - x$$

$$y = x^2 + 3x - 2$$

$$y(x) = 0 \Rightarrow x^2 + 3x - 2 = 0$$

$$x = \frac{-3 \pm \sqrt{17}}{2}$$

15. Write the formula to find Cubic Spline.

Let $S(x)$ is a cubic Polynomial. $S''(x)$ is linear in each interval (x_{i-1}, x_i).

$$\boxed{\begin{aligned} S(x) = \frac{1}{6h}&\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right] \\ &+ \frac{1}{h}\left[(x_i - x)\left(y_{i-1} - \frac{h^2}{6} M_{i-1}\right)\right] \\ &+ \frac{1}{h}\left[(x - x_{i-1})\left(y_i - \frac{h^2}{6} M_i\right)\right] \end{aligned}}$$

Where,

$$\boxed{M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]}$$

$$i = 1, 2, \ldots n-1.$$

PART- B

1. From the following data estimate the number of persons earning weekly wages between 60 and 70 rupees.

Wages in Rs : Below	40	40-60	60-80	80-100	100-120
No. of Persons	250	120	100	70	50.

Solution:

Table Calculation:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
60	370		-20		
		100		-10	
80	470		-30		20
		70		10	
100	540		-20		
		50			
120	590				

To find the no. of Persons Earning less than Rs.7[illegible]

$$u = \frac{x - x_0}{h} = \frac{70-40}{20} = 1.5$$

$$\boxed{u = 1.5}$$

$$y(70) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \ldots\ldots$$

$$= 250 + 1.5(120) + \frac{1.5(1.5-1)}{2}(-20) + \ldots\ldots$$

$$= 423.59.$$

$$\boxed{y(70) = 424}$$

No. of Persons Earning less than Rs. 60 is 370.

No. of Persons Earning between 70 and 60 is

$$\Rightarrow 424 - 370$$

$$= 54.$$

2. The population of a town is as follows

Year X : 1941	1951	1961	1971	1981	1991
Population Y : 20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

Solution:

Table Calculation:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	$\boxed{20}$					
		$\boxed{4}$				
1951	24		$\boxed{1}$			
		5		$\boxed{1}$		
1961	29		2		$\boxed{0}$	
		7		1		$\boxed{1}$
1971	36		3		$\boxed{1}$	
		10		$\boxed{2}$		
1981	46		$\boxed{5}$			
		$\boxed{5}$				
1991	$\boxed{51}$					

To estimate the population during 1946 use Newton's forward formula and for 1976 we use backward formula.

<u>Formula</u>:

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y$$
$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y.$$

$$u = \frac{x - x_0}{h} = \frac{1946-1941}{10} = \frac{5}{10} = 0.5$$

$$\boxed{u = 0.5}$$

$$\boxed{y(1976) = 40.809}$$

$\therefore$ Increase in population during the Period 1946 to 1976 is $\Rightarrow 40.809 - 21.69$

$\Rightarrow 19.119$ lakhs.

3. Obtain the root if $f(x)=0$ by Lagrange's inverse interpolation given that $f(30)=-30$; $f(34)=-13$, $f(38)=3$, $f(42)=18$.

x :	30	34	38	42
$y=f(x)$:	-30	-13	3	18

To find the root, find x when $y=0$.

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)}x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}x_1$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)}x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}x_3 .$$

$$y(1946) = 20 + 0.5(4) + \frac{0.5(0.5-1)}{2}1 + \frac{0.5(0.5-1)(0.5-2)}{6}1 + 0$$

$$+\frac{0.5(0.5-1)(0.5-2)(0.5-3)(0.5-4)}{5.4.3.2.1} \times 1$$

$$\Rightarrow 20 + 2 + \frac{0.5(-0.5)}{2} + \frac{0.5(-0.5)(-1.5)}{6}$$

$$+\frac{0.5(-0.5)(-1.5)(-2.5)(-3.5)}{120}.$$

$$\boxed{y(1946) = 21.69}$$

<u>To find y(1976):</u>

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n +$$

$$\frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \frac{v(v+1)(v+2)(v+3)(v+4)}{5!}\Delta^5 y_n$$

$$v = \frac{x - x_n}{h} = \frac{1976-1991}{10} = -\frac{15}{10} = -1.5$$

$$\boxed{v = -1.5}$$

$$y(1976) = 51 + (-1.5)5 + \frac{(-1.5)(-1.5+1)}{2}5 + \frac{(-1.5)(-1.5+1)(-1.5}{6}$$

$$+\frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)}{24}(1) + \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+}{120}.$$

Put $y=0$

$$x = \frac{(0+13)(0-3)(0-18)}{(-30+13)(-30-3)(-30-18)}(30) + \frac{(0+30)(0-3)(0-18)}{(-13-0)(-13-3)(-13-18)}(34)$$

$$+ \frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)}(38) + \frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)}(42)$$

$$\boxed{x = 37.23}$$

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4. Using Newtons forward interpolation formula find the Cubic polynomial which takes the following values.

x :	0	1	2	3
$f(x)$:	1	2	1	10

Evaluate $f(4)$ using Newtons backward formula.

Solution:

Here $x=4$

$$v = \frac{x - x_n}{h} = \frac{4-3}{1} = 1 \qquad \therefore \boxed{v=1}$$

Formula:

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \dots$$

Put $y=0$

$$x = \frac{(0+13)(0-3)(0-18)}{(-30+13)(-30-3)(-30-18)}(30) + \frac{(0+30)(0-3)(0-18)}{(-13-0)(-13-3)(-13-18)}(34)$$

$$+ \frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)}(38) + \frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)}(42)$$

$$\boxed{x = 37.23}$$

A. Using Newtons forward interpolation formula find the Cubic polynomial which takes the following values.

x :	0	1	2	3
f(x) :	1	2	1	10

Evaluate f(4) using Newtons backward formula.

Solution:

Here $x = 4$

$$v = \frac{x - x_n}{h} = \frac{4-3}{1} = 1 \qquad \therefore \boxed{v=1}$$

Formula:

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		$\boxed{12}$
2	1		$\boxed{10}$	
		$\boxed{9}$		
3	$\boxed{10}$			

$$y(4) = 10 + 1(9) + \frac{1(1+1)}{2}(10) + \frac{6}{6}\times 12.$$

$$= 10 + 9 + 10 + 12$$

$$= 41.$$

To find the polynomials using forward formula.

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \dots\dots$$

$$u = \frac{x - x_0}{h} = \frac{x-0}{1} = x \quad \therefore \boxed{u = x}$$

$$y = 1 + x.1 + \frac{x(x-1)}{2!}(-2) + \frac{x(x-1)(x-2)}{6}\times 12$$

$$= 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$$

$$= 2x^3 - 7x^2 + 6x + 1.$$

Put $y=0$

$$x = \frac{(0+13)(0-3)(0-18)}{(-30+13)(-30-3)(-30-18)}(30) + \frac{(0+30)(0-3)(0-18)}{(-13-0)(-13-3)(-13-18)}(34)$$

$$+ \frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)}(38) + \frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)}(42)$$

$$\boxed{x = 37.23}$$

A. Using Newtons forward interpolation formula find the Cubic polynomial which takes the following values.

x :	0	1	2	3
f(x) :	1	2	1	10

Evaluate $f(4)$ using Newtons backward formula.

Solution:

Here $x = 4$

$$v = \frac{x - x_n}{h} = \frac{4-3}{1} = 1 \qquad \therefore \boxed{v=1}$$

Formula:

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \dots$$

5. Find $f(x)$ on a polynomial in x for the following data by Newton's dividend differences formula.

x :	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

Table Calculation:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		$\frac{33-1245}{-1-(-4)}=-404$			
-1	33		$\frac{-28-(-404)}{0-(-4)}=94$		
		$\frac{5-33}{0-(-1)}=-28$		$\frac{10-94}{2-(-4)}=-14$	
0	5		$\frac{2-(-28)}{2-(-1)}=10$		$\frac{13+14}{5-(-4)}=3$
		$\frac{9-5}{2-0}=2$		$\frac{88-10}{5-(-1)}=13$	
2	9		$\frac{442-2}{5-0}=88$		
		$\frac{1335-9}{5-2}=442$			
5	1335				

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2)$$

$$+ (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$

$$= 1245 + (x+4)(-404) + (x+4)(x+1)(94)$$

$$+ (x+4)(x+1)(x)(-14)$$

$$+ (x+4)(x+1)(x-2)(3).$$

$$= 3x^4 + x^3 - 14x + 5$$

6. Find F(8) by Newton's divided difference formula

x :	4	5	7	10	11	13
f(x) :	48	100	294	900	1210	2028

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
4	48					
		52				
5	100		15			
		97		1		
7	294		21		0	
		202		1		0
10	900		27		0	
		310		1		
11	1210		33			
		409				
13	2028					

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2)$$

$$= 48 + (x-4)52 + (x-4)(x-5)15 + (x-4)(x-5)(x-7)$$

Put $x = 8$.

$$f(8) = 448.$$

7. Find the Cubic Spline for following data given below: 10

x :	0	1	2
y :	0	1	0

To Find $y(0.5)$ and $y'(1)$.

<u>Solution:</u>

Here $h=1$; $n=2$; $i = 1,2,\ldots\ldots n-1$

$i = 1,2,\ldots\ldots 2-1$

$i = 1,2,\ldots\ldots 1$

Assume that $M_0 = M_2 = 0$.

Where

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}[y_{i-1} - 2y_i + y_{i+1}]$$

$$M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$$

$$(0) + 4M_1 + (0) = 6[0 - 2(1) + 0]$$

$$4M_1 = 6[-2+0]$$

$$4M_1 = -12$$

$$\boxed{M_1 = -3}$$

Let $x_{i-1} \le x \le x_i$

$x_0 \le x \le x_1$

$0 \le x \le 1$

By, the Cubic Spline Formula,

$$S(x) = \frac{1}{6h}[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i]$$

$$+ \frac{1}{h}[(x_i - x)(y_{i-1} - \frac{h^2}{6}(M_{i-1})]$$

$$+ \frac{1}{h}[(x - x_{i-1})(y_i - \frac{h^2}{6} M_i)]$$

Put $i=1$,

$$y(x) = S(x) = \frac{1}{6}\left[(x_1-x)^3 M_0 + (x-x_0)^3 M_1\right]$$
$$+ 1\left[(x_1-x)\left(y_0 - \frac{1}{6}(M_0)\right)\right]$$
$$+ 1\left[(x-x_0)\left(y_1 - \frac{1}{6}M_1\right)\right]$$

$$= \frac{1}{6}\left[(1-x)^3(0) + (x-0)^3(-3)\right]$$
$$+ \left[(1-x)\left(0-\frac{1}{6}(0)\right)\right]$$
$$+ \left[(x-0)\left(1-\frac{1}{6}(-3)\right)\right]$$

$$= \frac{1}{6}\left[0-3x^3\right] + [0] + [(x)(1.5)]$$

$$\boxed{S(x) = -0.5x^3 + 1.5x}$$

To find $y(0.5)$ and $y'(1)$:

Here $y(x) = S(x) = -0.5x^3 + 1.5x$

$$y(0.5) = -0.5(0.5)^3 + 1.5(0.5)$$
$$= -0.0625 + 0.75$$

$$\boxed{y(0.5) = 0.6875}$$

Here $y(x) = -0.5x^3 + 1.5x$

$$y'(x) = -(3)\times 0.5x^2 + 1.5$$
$$y'(1) = -3\times 0.5(1)^2 + 1.5$$
$$= -1.5 + 1.5$$

$$\boxed{y'(1) = 0}$$

By Using Cubic Spline formula,

$$S(x) = \frac{1}{6h}\left[(x_i - x)^3 (M_{i-1} + (x - x_{i-1})^3 M_i\right]$$

$$+ \frac{1}{h}\left[(x_i - x)(y_{i-1}) - \frac{h^2}{6}M_{i-1})\right]$$

$$+ \frac{1}{h}\left[(x - x_{i-1})(y_i - \frac{h^2}{6}M_i)\right]$$

Put $i = 1$ (i.e) $x_{i-1} \le x \le x_i$ $(-1 \le x \le 0)$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 + (x - x_0)^3 M_1\right] + \frac{1}{1}\left[(x_1 - x)(y_0 - \frac{1}{6}M_0)\right]$$

$$+ \left[(x - x_0)(y_1 - \frac{1}{6}M_0)\right]$$

$$= \frac{1}{6}\left[(0 - x)^3 10 + (x+1)^3 (-2)\right] + \left[(0 - x)(1 - \frac{1}{\cancel{6}_3}\cancel{10}^5)\right]$$

$$+ 1\left[(x+1)(0 - \frac{1}{6}(-2)\right]$$

$$= \frac{2}{6}\left[-5x^3 - (x+1)^3\right] + \left[(-x)(-\frac{2}{3})\right] + \left[(x+1)(\frac{1}{3})\right]$$

$$= \frac{1}{3}\left[-5x^3 - x^3 - 3x^2 - 3x - 1\right] + \frac{2}{3}x + \frac{1}{3}x + \frac{1}{3}$$

$$= \frac{1}{3}\left[-6x^3 - 3x^2 - 3x - 1\right] + \frac{2}{3}x + \frac{1}{3}x + \frac{1}{3}$$

$$= -2x^3 - x^2 - \cancel{x} - \frac{1}{3} + \cancel{x} + \frac{1}{3}$$

$$= -2x^3 - x^2$$

$$\therefore S(x) = -2x^3 - x^2 \quad \text{at } (-1 \le x \le 0)$$

Thus $\boxed{\begin{aligned} y(x) = S(x) &= -0.5x^3 + 1.5x \\ y(0.5) &= 0.6875 \\ y'(1) &= 0 \end{aligned}}$

8. Find the Cubic spline for the following data with $M_0 = 10$ $M_2 = 10$.

x :	-1	0	1
y :	1	0	1

<u>Solution</u> :

Here $h = 1$; $n = 2$ $i = 1, 2, \ldots n-1$

$i = 1, 2, \ldots\ldots 1$

Here $M_0 = M_2 = 10$.

Where $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6/1\left[y_0 - 2y_1 + y_2\right]$$

$$10 + 4M_1 + 10 = 6\left[1 - 0 + 1\right]$$

$$20 + 4M_1 = 12$$

$$4M_1 = 12 - 20 = -8$$

$$\boxed{M_1 = -2}$$

Put $i=2$ (ie) $x_{i-1} \le x \le x_i$ $(0 \le x \le 1)$.

$$S(x) = \frac{1}{6}\left[(x_2-x)^3 M_1 + (x-x_1)^3 M_2\right]$$

$$+ \left[(x_2-x)(y_1-\frac{1}{6}M_1)\right] + \left[(x-x_1)(y_2-\frac{1}{6}M_2)\right]$$

$$= \frac{1}{6}\left[(1-x)^3(-2) + (x-0)^3 10\right] + \left[(1-x)(0-\frac{1}{6}(-2))\right]$$

$$+ \left[(x-0)(1-\frac{1}{6}(10))\right]$$

$$= \frac{2}{6}\left[-(1-x)^3 + 5x^3\right] + (1-x)\left(\frac{1}{3}\right) + x\left(1-\frac{5}{3}\right)$$

$$= \frac{1}{3}\left[5x^3 + x^3 - 3x^2 + 3x - 1\right] + \frac{1}{3} - \frac{x}{3} - \frac{2}{3}x$$

$$= \frac{1}{3}\left[6x^3 - 3x^2 + 3x - 1\right] + \frac{1}{3} - x$$

$$= 2x^3 - x^2 + \cancel{x} - \cancel{\frac{1}{3}} + \frac{1}{3} - \cancel{x}$$

$$= 2x^3 - x^2$$

$\therefore S(x) = 2x^3 - x^2$ at $0 \le x \le 1$

Thus

$$\boxed{\begin{aligned} S_1(x) &= -2x^3 - x^2 \quad (-1 \le x \le 0) \\ S_2(x) &= 2x^3 - x^2 \quad (0 \le x \le 1) \end{aligned}}$$

Question Bank

Numerical Methods - MA1251.

SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS

1. Find the positive root of $x^3 - 2x - 5 = 0$ by the Regula falsi method.

2. Using method of false position find a root of the equation $x^3 - 3x - 5 = 0$.

3. Find an approximate root of $x \log_{10} x - 1.2 = 0$ by Regula Falsi method.

4. Solve the eq $x \tan x = -1$ by Regula falsi method starting with $a = 2.5$ and $b = 3$ correct to 3 decimal places.

5. Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton-Raphson method.

6. Using Newton's iterative method find the root b/w 0 and 1 of $x^3 = 6x - 4$ correct to 2 decimal places.

7. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's correct to 6 decimal places.

8. Find by NR method, the root of $\log_{10} x = 1.2$.

9. Find the root of $\cos x = xe^x$ by Newton-Raphson method. Take $x_0 = .5$

10. Solve by Newton's method, a root of $e^x - 4x = 0$.

11. Find the least +ve root of ~~xe~~ $xe^{-2x} = .5\sin x$ Correct to 3 decimal places using Newton-Raphson method.

12. By Newton Raphson method find a non-zero of ~~2x~~ $x^2 + 4\sin x = 0$.

13. Find a root of $x \log_x x - 1.2 = 0$ by N.R method Correct to 3 decimal places.

14. Using N.R method, Solve $x \log_x x = 12.34$ Start with $x_0 = 10$.

15. Find the real root of $x^3 - 2x - 5 = 0$ using N.R method.

16. Find the root of $x = 2\sin x$, near 1.9 correct to 3 decimal places by applying N.R method twice.

17. Write down N.R formula for finding $\sqrt{a}$ Where $\sqrt{a}$ where a is a +ve no. & hence find $\sqrt{5}$.

18. Obtain Newton's Iterative formula for finding $\sqrt{N}$, where N is a +ve Real no. Hence evaluate $\sqrt{142}$.

19. Find the Iterative formula for finding the Value of $1/N$, where N is a real number, using Newton-Raphson method, Hence evaluate $1/26$ Correct to 4 decimal places.

Numerical Methods

Unit 1

Solutions of Equations and Eigen value problems

2 mark Questions :-

1. State and iterative formula for regular falsi method to solve $f(x) = 0$.

solution:

The iteration formula to find a root of the equation $f(x) = 0$ which lies between $x = a$ and $x = b$ is

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}.$$

2. Explain Regular Falsi method of getting a root.

solution:

Let $f(x) = 0$ be the given equation.

(i) Find two numbers 'a' and 'b' such that $f(a)$ and $f(b)$ are of opposite signs. Then a root lies between 'a' and 'b'.

(ii) The first approximation to the root is given by,

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

(iii) If $f(x_1)$ and $f(a)$ are opposite signs, then the actual root lies between 'x_1' and 'a'. Now replacing 'b' by 'x_1' and keeping 'a' as it is we get the next closer approximation 'x_2' to the actual root.

(iv) This procedure is repeated till the root is found to the desired degree of accuracy.

3. How to reduce the number of iterations while finding the root of an equation by Regular falsi method?

Solution:-

The number of iterations to get a good approximation to the real root can be reduced, if we start with a smaller interval for the root.

4. What is the order of convergence of Newton Raphson method if the multiplicity of the root is one (or) What is the rate of convergence in Newton Raphson method?

Solution:-

Order of convergence of Newton Raphson method is 2.

5. When should we not use Newton Raphson method?

Solution :-

If ~~f(x)~~ x_1 is the exact root and x_0 is its approximate value of the equation $f(x) = 0$, we know that $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$.

If $f'(x_0)$ is small, the error $\frac{f(x_0)}{f'(x_0)}$ will be large and the computation of the root by this, method will be a slow process or may even be impossible.

Hence the method should not be used in cases where the graph of the function when it crosses the x axis is nearly horizontal.

6. What is the iterative formula of Newton Raphson method?

Solution :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

7. State the order of convergence and convergence condition for Newton Raphson method.

Solution :-

condition for convergence is $|f(x) f''(x)| < |f'(x)|^2$

The order of convergence is 2.

8. If $g(x)$ is continuous in $[a,b]$, then under what condition the iterative method $x = g(x)$ has a unique solution in $[a,b]$?

Solution:-

Let $x = r$ be a root of $x = g(x)$. Let I be an interval combining the point $x = r$. If $|g'(x)| < 1$ for all x in I, the sequence of approximation $x_0, x_1, \ldots x_n$ will converge to the root r, provided that the initial approximation x_0 is chosen in I.

9. Explain briefly Gauss - Jordan iteration to solve simultaneous equations.

solution:-

consider the system of equations $AX = B$

If A is a diagonal matrix the given system reduces to,

$$\begin{bmatrix} a_{11} & 0 & \ldots & \ldots & 0 \\ 0 & a_{22} & \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

This system reduces to the following n equations.

$$a_{11}x_1 = b_1, \; a_{22}x_2 = b_2, \ldots a_{nn}x_n = b_n$$

Hence, we get the solution directly as

$$x_1 = \frac{b_1}{a_{11}}, \quad x_2 = \frac{b_2}{a_{22}}, \ldots \quad x_n = \frac{b_n}{a_{nn}}.$$

The method of obtaining the solution of the system of equations by reducing the matrix A to a diagonal matrix is known as Gauss Jordan Elimination method.

10. For solving a linear system, compare Gaussian elimination method and Gauss Jordan method.

solution :.

	Gauss elimination method	Gauss Jordan method.
1.	Coefficient matrix is transformed into upper triangular matrix	coefficient matrix is transformed into diagonal matrix
2.	Direct method	Direct method
3.	We obtain the solutions by back substitution method	No need of back substitution method

11. State the principle of Gauss Jordan method?

solution :-

Coefficient matrix is transformed into diagonal matrix.

12. Write a sufficient condition of Gauss seidal method to converge.

Solution :-

The process of iteration by Gauss seidal method will converge if in each equation of the system, the absolute values of the largest coefficient is greater than the sum of the absolute values of the remaining co-efficients.

[The coefficient of a matrix should be diagonally dominant].

13. Gauss elimination and Gauss Jordan are direct methods while ——— and ——— are iterative methods.

Solution :-

(i) Gauss seidal method.

(ii) Gauss Jacobi method.

14. True (or) false 'iteration method will always converge'. ?

solution :

False.

15. Give two indirect methods to solve a system of linear equations?

solution:

(i) Gauss Jacobi method

(ii) Gauss seidal method.

16. solve the system of equations

$2x - 3y + 20z = 25$

$20x + y - 2z = 17$ by Gauss Jacobi iteration method.

$3x + 20y - z = -18$

solution :-

As the coefficient matrix is not diagonally dominant as it is we rewrite the equation

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

Now the diagonal elements are dominant in the coefficient matrix, we write x, y, z as

$$x = \frac{1}{20}[17 - y + 2z]$$

$$y = \frac{1}{20}[-18 - 3x + z]$$

$$z = \frac{1}{20}[25 - 2x + 3y]$$

Let the initial conditions be $x=0,\ y=0,\ z=0$.

First iteration:-

$$x^{(1)} = \frac{1}{20}\left[17-0+0\right] = 0.85$$

$$y^{(1)} = \frac{1}{20}\left[-18\right] = -0.9.$$

$$z^{(1)} = \frac{1}{20}\left[25\right] = 1.25$$

Second iteration:-

$$x^{(2)} = \frac{1}{20}\left[17-y^{(1)}+2z^{(1)}\right]$$

$$= \frac{1}{20}\left[17-(-0.9)+2(1.25)\right] = 1.02$$

$$y^{(2)} = \frac{1}{20}\left[-18-3x^{(1)}+z^{(1)}\right]$$

$$= \frac{1}{20}\left[-18-3(0.85)+1.25\right] = -0.965$$

$$z^{(2)} = \frac{1}{20}\left[25-2x^{(1)}+3y^{(1)}\right]$$

$$= \frac{1}{20}\left[25-2(0.85)+3(-0.9)\right] = 1.03$$

17. Solve by Gauss seidal method $x-2y=-3$, $2x+25y=15$ correct to 4 decimal places.

solution:-

$$x-2y=-3$$

$$2x+25y=15$$

$$x=-3+2y$$

$$y=\frac{1}{25}\left[15-2x\right]$$

Let the initial value $y=0$.

First iteration:-

$$x^{(1)} = -3+0 = -3$$

$$y^{(1)} = \frac{1}{25}[15+6] = 0.84.$$

Second iteration:

$$x^{(2)} = -3+2y^{(1)} = -3+2(0.84) = -1.32$$

$$y^{(2)} = \frac{1}{25}[15-2x^{(2)}] = \frac{1}{25}[15-2(-1.32)] = 0.7056$$

Third iteration:

$$x^{(3)} = -3+2y^{(2)} = -3+2(0.7056) = -1.5888$$

$$y^{(3)} = \frac{1}{25}[15-2x^{(3)}] = \frac{1}{25}[15-2(-1.5888)] = 0.7271$$

Fourth iteration:

$$x^{(4)} = -3+2y^{(3)} = -3+2(0.7271) = -1.5458$$

$$y^{(4)} = \frac{1}{25}[15-2x^{(4)}] = \frac{1}{25}[15-2[-1.5458)] = 0.7237$$

Fifth iteration:

$$x^{(5)} = -3+2y^{(4)} = -3+2(0.7237) = -1.5526$$

$$y^{(5)} = \frac{1}{25}[15-2x^{(5)}] = \frac{1}{25}[15-2(-1.5526)] = 0.7242$$

Sixth iteration:

$$x^{(6)} = -3+2y^{(5)} = -3+2(0.7242) = -1.5516.$$

$$y^{(6)} = \frac{1}{25}[15-2x^{(6)}] = \frac{1}{25}[15-2(-1.5516)] = 0.7241$$

Seventh iteration:

$$x^{(7)} = -3+2y^{(6)} = -3+2(0.7241) = -1.5518$$

$$y^{(7)} = \frac{1}{25}[15-2x^{(7)}] = \frac{1}{25}[15-2(-1.5518)] = 0.7241$$

20. As soon as a new value for a variable is found by iteration, it is used immediately in the following equations. This method is called ______

solution :-

Gauss seidal method of iteration.

21. state True or False :-

The convergence in the Gauss seidal method is thrice as fast as in Jacobi's method.

solution :

The statement is false. In fact, the rate of convergence of Gauss seidal method is roughly twice that of Gauss - Jacobi.

22. Find the inverse of the coefficient matrix by Gauss Jordan elimination method.

solution :

coefficient matrix $A = \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$

Given :-

$5x - 2y = 10$

$3x + 4y = 17$

$[A, I] = \left[\begin{array}{cc|cc} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$

$$\sim \left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \quad R_1 \leftrightarrow \frac{R_1}{5}$$

$$\sim \left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 0 & 26/5 & -3/5 & 1 \end{array}\right] \quad R_2 \leftrightarrow R_2 - 3R_1$$

$$\sim \left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 0 & 1 & -3/26 & 5/26 \end{array}\right] \quad R_2 \leftrightarrow R_2 \times \frac{5}{26}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 2/13 & 1/13 \\ 0 & 1 & -3/26 & 5/26 \end{array}\right] \quad R_1 \leftrightarrow R_1 + \frac{2}{5} R_2$$

$$A^{-1} \sim \begin{bmatrix} 2/13 & 1/13 \\ -3/26 & 5/26 \end{bmatrix} = \frac{1}{26} \begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}.$$

23. Find the power method, the largest eigen value of $\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ correct to two decimal places, chose $[1, 1]^T$ as the initial eigen vector.

<u>solution</u>:-

Let $X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$AX_1 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0.8 \end{bmatrix} = 5X_2.$$

$$AX_2 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 4.8 \\ 3.4 \end{bmatrix} = 4.8 \begin{bmatrix} 1 \\ 0.71 \end{bmatrix} = 4.8X_3$$

$$AX_3 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.71 \end{bmatrix} = \begin{bmatrix} 4.71 \\ 3.13 \end{bmatrix} = 4.71 \begin{bmatrix} 1 \\ 0.67 \end{bmatrix} = 4.71X_4$$

$$AX_4 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 4.67 \\ 3.01 \end{bmatrix} = 4.67\begin{bmatrix} 1 \\ 0.65 \end{bmatrix} = 4.67 X_5$$

$$AX_5 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 4.65 \\ 2.95 \end{bmatrix} = 4.65\begin{bmatrix} 1 \\ 0.63 \end{bmatrix} = 4.65 X_6$$

$$AX_6 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.63 \end{bmatrix} = \begin{bmatrix} 4.63 \\ 2.89 \end{bmatrix} = 4.63\begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = 4.63 X_7$$

$$AX_7 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = \begin{bmatrix} 4.62 \\ 2.86 \end{bmatrix} = 4.62\begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = 4.62 X_8$$

The eigen value = 4.62 and the corresponding eigen vector $= \begin{bmatrix} 1 \\ 0.62 \end{bmatrix}$

24. Determine the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ correct to two decimal places using power method.

<u>solution</u> :-

$$AX_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2X_2$$

$$AX_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2X_3$$

This shows that the largest eigenvalue = 2

The corresponding eigen vector $= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

I. Numerical Methods

Unit - 2 Interpolation and Approximation

2 Mark Questions

1. State Lagrange's interpolation formula

Solution :-

Let $y = f(x)$ be a function which takes the values $y_0, y_1, \dots y_n$ and corresponding to $x_0, x_1, \dots x_n$

The lagranges interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} \cdot y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} \cdot y_1 + \dots\dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} \cdot y_n$$

2. What is the lagranges formula to find y, if three sets of values $(x_0, y_0), (x_1, y_1)$ and (x_2, y_2) are given

Solution :-

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \cdot y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \cdot y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \cdot y_2.$$

3. What is the assumption we make when Lagrange's formula is used?

Solution :-

Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not whether the difference of y become smaller or not.

4. Using Newton's divided difference formula find the missing value from the table.

x :	1	2	4	5	6
y :	14	15	5	-	9

Solution :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	14	$\frac{15-14}{2-1} = 1$		
2	15	$\frac{5-15}{4-2} = -5$	$\frac{-5-1}{4-1} = -2$	
4	5	$\frac{9-5}{6-4} = 2$	$\frac{2+5}{6-2} = \frac{7}{4}$	$\frac{7/4+2}{6-1} = \frac{3}{4}$
6	9			

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$$

$$+ (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + \cdots$$

$$= 14 + (x-1)(1) + (x-1)(x-2)(-2) + (x-1)(x-2)(x-4)\left(\frac{3}{4}\right)$$

$$= 14 + x - 1 - 2(x-1)(x-2) + \frac{3}{4}(x-1)(x-2)(x-4)$$

$$f(5) = 13 + 5 - 2(4)(3) + \frac{3}{4}(4)(3)(1)$$

$$= 18 - 24 + 9$$

$$= 3$$

5. Using Newtons divided difference formula determine $f(3)$ from the data

x :	0	1	2	4	5
$f(x)$:	1	14	15	5	6

Solution:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	1				
		$\frac{14-1}{1-0} = 13$			
1	14		$\frac{1-13}{2-0} = -6$		
		$\frac{15-14}{2-1} = 1$		$\frac{-2+6}{4-0} = 1$	
2	15		$\frac{-5-1}{4-1} = -2$		$\frac{1-1}{5-0} = 0$
		$\frac{5-15}{4-2} = -5$		$\frac{2+2}{5-1} = 1$	
4	5		$\frac{1+5}{5-2} = 2$		
		$\frac{6-5}{5-4} = 1$			
5	6				

By Newtons divided difference interpolation formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1)$$

$$f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + \cdots$$

$$= 1 + (x-0)13 + (x-0)(x-1)(-6) + (x-0)(x-1)(x-2)(1) + 0$$

$$= 1 + 13x - 6x(x-1) + x(x-1)(x-2)$$

$$f(3) = 1 + 39 - 6(3)(2) + 3(3-1)(3-2)$$

$$= 40 - 36 + 3(2)(1)$$

$$= 40 - 36 + 6$$

$$= 10.$$

6. Derive Newtons backward difference formula by using operator method

Solution:

$$P_n(x) = P_n(x_n + vh)$$

$$= E^v P_n(x_n)$$

$$= (1-\nabla)^{-v} y_n \quad \text{since } E = (1-\nabla)^{-1}$$

$$= \left[1 + v\nabla + \frac{v(v+1)}{2!}\nabla^2 + \frac{v(v+1)(v+2)}{3!}\nabla^3 + \cdots\right] y_n$$

$$= y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

$$\text{where } v = \frac{x - x_n}{h}.$$

7. When Newton's backward interpolation formula is used

Solution:

The formula is used mainly to interpolate the values of y near the end of a set of tabular values and also for extrapolating the values of y a short distance ahead (to the right) of y_0

8. Newtons forward interpolation formula used only for ___ intervals.

Solution:

Equidistant (or) equal intervals.

9. Write the newton's forward and backward formula:

Solution:

Newtons forward formula

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$\text{Where } p = \frac{x - x_0}{h}$$

Newtons Backward formula

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

Where $p = \frac{x - x_n}{h}$.

10. Write the Inverse Lagragian Formula :-

Solution:

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)} \cdot x_0 + \frac{(y-y_0)(y-y_2)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)} \cdot x_1 + \cdots$$

11. Write the cubic spline interpolation formula

Solution:

$$S_{i-1} + 4S_i + S_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right].$$

For $i = 1, 2, 3, \ldots n-1$.

For natural ~~natur~~ cubic spline,

$$S(x) = \frac{1}{6h}\left[(x_i - x)^3 S_{i-1} + (x - x_{i-1})^3 S_i)\right] + \frac{1}{h}\left[(x_i - x)(y_{i-1} - \frac{h^2}{6} S_{i-1})\right] + \frac{1}{h}\left[(x - x_{i-1})(y_i - \frac{h^2}{6} S_i)\right].$$

(20) Solve the System of Equations by
(i) Gauss Elimination method
ii) Gauss - Jordan method

$10x - 2y + 3z = 23$
$2x + 10y - 5z = -33$
$3x - 4y + 10z = 41$

(21) Solve the following System of equation by Gauss - Jordan method $5x + 4y = 15$ $3x + 7y = 12$.

(22) Using Gauss - Elimination method, Solve the System,
$3.15x - 1.96y + 3.85z = 12.95$
$2.13x + 5.12y - 2.89z = -8.61$.
$5.92x + 3.05y + 2.15z = 6.88$.

(23) Using the Gauss - Jordan method Solve the following Eq.
$10x + y + z = 12$
$2x + 10y + z = 13$
$x + y + 5z = 7$.

(24) Solve $x + 3y + 3z = 16$, $x + 4y + 3z = 18$, $x + 3y + 4z = 19$ by Gauss - Jordan.

(25) Solve by Gauss Jacobi & Gauss Seidel.
$27x + 6y - z = 85$
$x + y + 54z = 110$
$6x + 15y + 2z = 72$

(26) Solve by Gauss - Seidel.

$$4x + 2y + z = 14$$
$$x + 5y - z = 10$$
$$x + y + 8z = 20$$

(27) Solve by Gauss Jacobi & Gauss - Seidal

$$3x_1 - x_2 + x_3 = 1$$
$$3x_1 + 6x_2 + 2x_3 = 0$$
$$3x_1 + 3x_2 + 7x_3 = 4$$

(28) Using Gauss - seidal method Start with

$x = 1,\ y = -2,\ z = 3,$

$$x + 3y + 52z = 173.61$$
$$x - 27y + 2z = 71.31$$
$$41x - 2y + 3z = 65.46$$

29. Using Gauss - Jordan find Inverse.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

ii) $$A = \begin{bmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{bmatrix}$$

30 Find the numerically largest Eigen value

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & [illegible] \end{bmatrix}$$ by power metho