Chapter 1 Introduction to Modeling and Simulation

System concept:

→ A system is a aggregation of objects where objects have regular interaction and interdependence to perform certain task.
→ A system exists and operates in time and space.
→ A system is collection of ~~entries~~ entities that act and interact together toward some end (specific task).

fig: A factory system

Consider a factory that makes and assembles parts into a product. Two major components of the system are the fabrication department making the parts and assembly department production the product. A purchasing department maintains a supply of raw materials and a shipping department dispatches the finished products. A production control department receives order and assign work to other department.

In looking at these system, we see that there are certain distinct objects each of which processes properties of interest. The term entity will be used to denote an object of interest in a system. The term attribute will denote a property of entity.

Date / /
Page

Any process that causes changes in the system will be called an activity. The term state of the system will be used to mean a description of all the entities, attributes and activities as they exist at one point in time.

System	Entities	Attributes	Activities
Traffic	cars	Speed, distance	Driving
Bank	customer	balance, credit status	Depositing
Communication	Messages	Length, priority	Transmitting
Super market	customers	Shopping list	checking out

System Environment:-

The changes occuring outside the system and effecting the system is called system environment. In case of factory system, for e.g. The factors controlling the arrival of order may considered to be outside influence of factory, so it's part of environment.

Date / /
Page

The term used in system environment are:
1. **Endogenous:**
It's used to describe the activities within the system.

2. **Exogenous:**
It's used to describe the activities in the environment that affects the system.
If the system has no exogenous activities then its called closed system and if it has exogenous activities then its called open system.

Simulation Modeling:-
Simulation modeling is the process of creating and analyzing a digital prototype of a physical model to predict its performance in the real world. Simulation modeling is used to help designers and engineers understand whether under what condition and in which ways a parts could fail and what loads it can withstand. Simulation model can also help to predict fluid flow and heat transfer patterns. It analyses the approximate working condition by applying the simulation software.

Uses of Simulation Modeling:-

→ Optimized geometry for weight and strength. Select materials that meet weight, strength and budget requirements.

Date / /
Page

→ Simulate part failure and identify the loading condition that causes them.
→ Access ~~exter~~ extreme environmental condition or loads not easily tested on physical prototype such as earthquake, shock load.
→ Verify hand calculations.
→ Validate the lightly safety & survival of a physical prototype before.

Deterministic and stochastic activities:

— Deterministic activity:
→ When the outcome of an activity can be described completely in terms of input of activity then it is called deterministic activity.
→ Output of model is fully determined by the initial condition and parameter values.
→ Contains no random elements
→ Output is deterministic for given set of inputs.
E.g: Among 52 cards we pick the card of no. 6 by seeing card, it is said to be deterministic.

— Stochastic activity:
→ The output of an activity varies randomly over the possible outcomes is called stochastic activity.
→ Some set of parameter values and initial condition lead to very output.
→ It contains random elements and output is random quantity.
E.g: If we pick the card without seeing card then we can get any card that is said to be stochastic.

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Continuous and Discrete System:

The system in which changes are predominantly smooth is called continuous system. Simply, the system in which the input signal and output signal both have continuous domain is said to be continuous system. Eg: Aircraft system

The system in which changes are predominantly discontinuous is called discrete system. Simply, the system in which input signal and output signals both have discrete domain is said to be discrete system. Eg: factory system

Types of Models:

- Models
  - Physical
    - Static
    - Dynamic
  - Mathematical
    - Static
      - Numerical
      - Analytical
    - Dynamic
      - Numerical
        - System Simulation

Date / / Page

① # Physical Model:-

Physical models are based on some analogy between such systems as mechanical and electrical and hydraulic. The system attributes are represented by measurements like voltage or the position of the shaft.

② Mathematical Model:-

This model uses symbolic notation and mathematical equation to represent a system. In this model, attributes are represented by variables and activities are represented by a mathematic function that interrelates variables.

③ Static Model:-

This model can be only show the values that system attributes takes when the system is in balance.

④ Dynamic Model:-

This model can show the values of the system attributes that changes over time due to the effect of the system attributes. Once a mathematical model is built, it must be then examined to see how it can be used to answer the question of interest in the system. The two method that are used to solve the mathematical model are:

i) Analytical ~~Models~~ Methods:

Analytical technique means using the deductive reasoning of mathematical theory to solve a model. E.g: Linear differential equation

Date / /
Page

let us consider the, if we know the distance to be travelled and velocity then we can work with the model to get as the time that will be required.

ii) Numerical Methods:-

Numerical methods involves applying computational procedures to solve equations. It produces a solution in steps, each step give a solution for once at a condition and calculation are repeated until a final solution is obtained.

iii) System Simulation:-

System simulation is a set of techniques that uses computers to imitate the operations of various real world task or processes through simulation. Computers are used to generate numeric models for the purpose of describing or displaying complex interaction among multiple variables within a system. It is also considered to be a dynamic mathematical model.

System Modeling:

A model is defined as a representation of a system to study the system. The model construct a conceptual framework that describes a system. A model of a system is a set of assumption and approximation about how the system works on the tasks drive a model of a system is called system model. Since, the purpose of the studies will determine

Date / /
Page

the nature of a information i.e. gather, there is no union model of a system.

Different model of the same system will be produced by different analyst interested in different expert of the system or by the same analyst as this understanding of the system changes. Studying a model instead of a real system is usually much easier, faster, cheaper and safer.

A model of a system can derived in a following two ways:

(1) Establishing a model structure: It determines the system boundary, entities, attributes, activities and state of a system.

(2) Supplying data: In this phase, the values are provides to the system attributes that can have relationship involve in the activities are defined.

These two jobs of creating a structure and providing the data are the parts of one tasks rather than as to separate tasks because they are so closely related neither can be done without others.

The assumptions about the system directs the gathering of the data and analysis of the data confirms the assumptions. To illustrate the process we consider the description of supermarket.

Principles used in modeling:

1) **Block Building:**

The description of the system should be organized in a series of blocks. The aim of constructing the block is to simplify the specification of the interactions within the system. Each block describes a part of the system that depends upon a few, preferably one, input variables and results in a few output variables. The system as a whole can then be described in terms of the interconnections between the blocks. Correspondingly, the system can be represented graphically as a simple block diagram.

2) **Relevance:**

The model should only include those aspects of the system that are relevant to the study objectives. As an example, if the factory system the study aims to compare the effects of different operating rules on efficiency, it is not relevant to consider the hiring of employees as an activity. While irrelevant information in the model may not do any harm, it should be excluded because it increases the complexity of the model and causes more work in solving the model.

Date / /
Page

3) **Accuracy:**
The accuracy of the information gathered for the model should be considered. In an aircraft system, the accuracy with which the movement of aircraft is described depends upon the representation of the airframe. If these are not accurate it gives a false result while testing a system for output.

4) **Aggregation:-**
A further factor to be considered is the extent to which several individual entities can be grouped together into larger entities.

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Verification, validation and calibration of model :-

After the development of a model is functionally complete, we should ask "does it work correctly"? There are two parts to this question

i) First, does it respond the way the analyst intended.
ii) Second, does it behave the way the real system works.

Verification:

Verification is the process of determining whether the simulation computer works as intended. It focuses on the internal consistency of a model. It checks that the implementation of the simulation model corresponds to the model. It concerns with the building the model right. It is utilized in the comparison of the conceptual model to the computer representation that implements the concept. It ask the question, Is the model implemented correctly in the computer? Are the input parameter and logical structure of the model correctly represented? It is the process of comparing the computer code with the model ensure that the code is correct implementation of the model.

Validation:-

Validation is concerned with the correspondence between the model and the reality. It is concerned with building the right model. It is utilized to determine that a model is an accurate representation of real system. Validation is usually achieved through the calibration of the model and iterative process of comparing the model to actual system behaviour and using discrepancies

Date / /
Page

between two and the inside gain to improve the model. This process is repated until model accuracy is a judge to be acceptable. It is the process of comparing the model output with the behaviour of the phenomenon.

Three step approach in validation process are,

a) Build a model that has a high phase validity.

b) Validate model assumptions.

c) Compare the model input and output transformation to correspond input / output transformation of the local system.

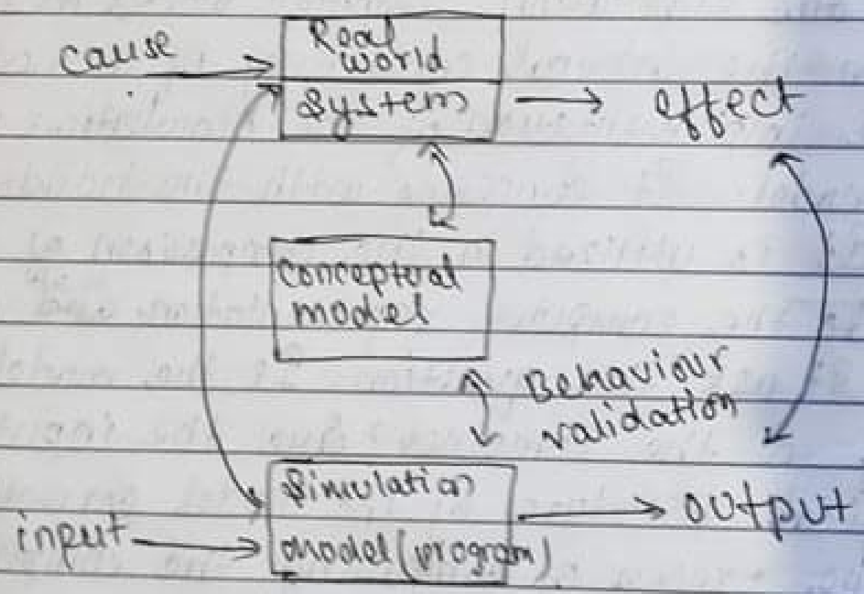

fig: Verification and validation activities

Calibration:-

It is an iterative process of comparing the model to the real system, making adjustments or major changes to the model, comparing the revised model to reality, making additional adjustments, comparing again and so on. It checks that the data generated by the simulation matches real data. The process is continued until the model is judged to be sufficiently accurate. In the context of optimization, calibration is an optimization procedure involved in system identification or during experimental design.

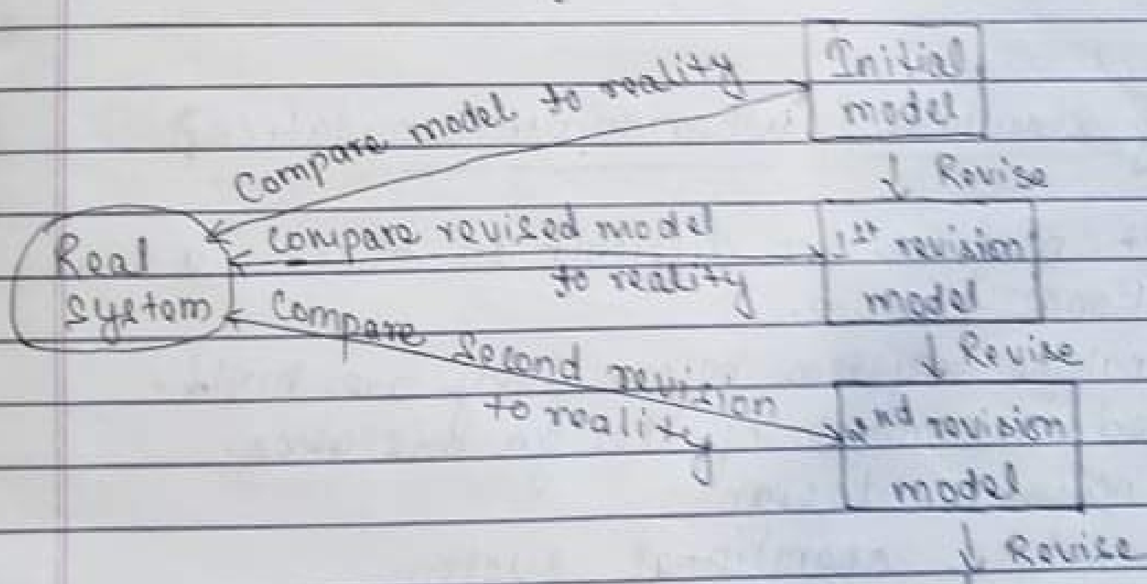

Fig: Calibration, verification and Validation Relationship

Chapter 2

System Simulation

Date / /

Page

System Simulation:-

Simulation is the representation of real life system by another system which inherits the important characteristics of the real system and allows experimentation on it. In other words, simulation is the process of defining a model of a real system and conducting an experiment with these models of system and evaluating various strategy for the operation of the system. Simulation has been used by researchers, analysts, designers and other professionals in the physical and non-physical experimentations and investigations.

Why simulate? / When to use simulation?

It may be too difficult to observe a real operational system.

→ To analyse system before they are built.
→ To reduce number of design mistakes.
→ To optimize design.
→ To analyse operational system.
→ To create virtual environments for training, entertainment.

Advantages of Simulation:-

→ Simulation helps to learn about real system.
→ Simulation models are comparatively flexible and can be modified.
→ Simulation technique is easier to use and can be

used for wide range of situation.

- → Simulation is a very good tool of training.
- → Simulation eliminates need of costly trial and error method of trying out new concepts.

Disadvantages of Simulation:-

- → Model building requires special training.
- → Simulation results may be difficult to interprete due to randomness.
- → Simulation modeling and analysis can be time consuming and expensive.

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Steps of Simulation study:-

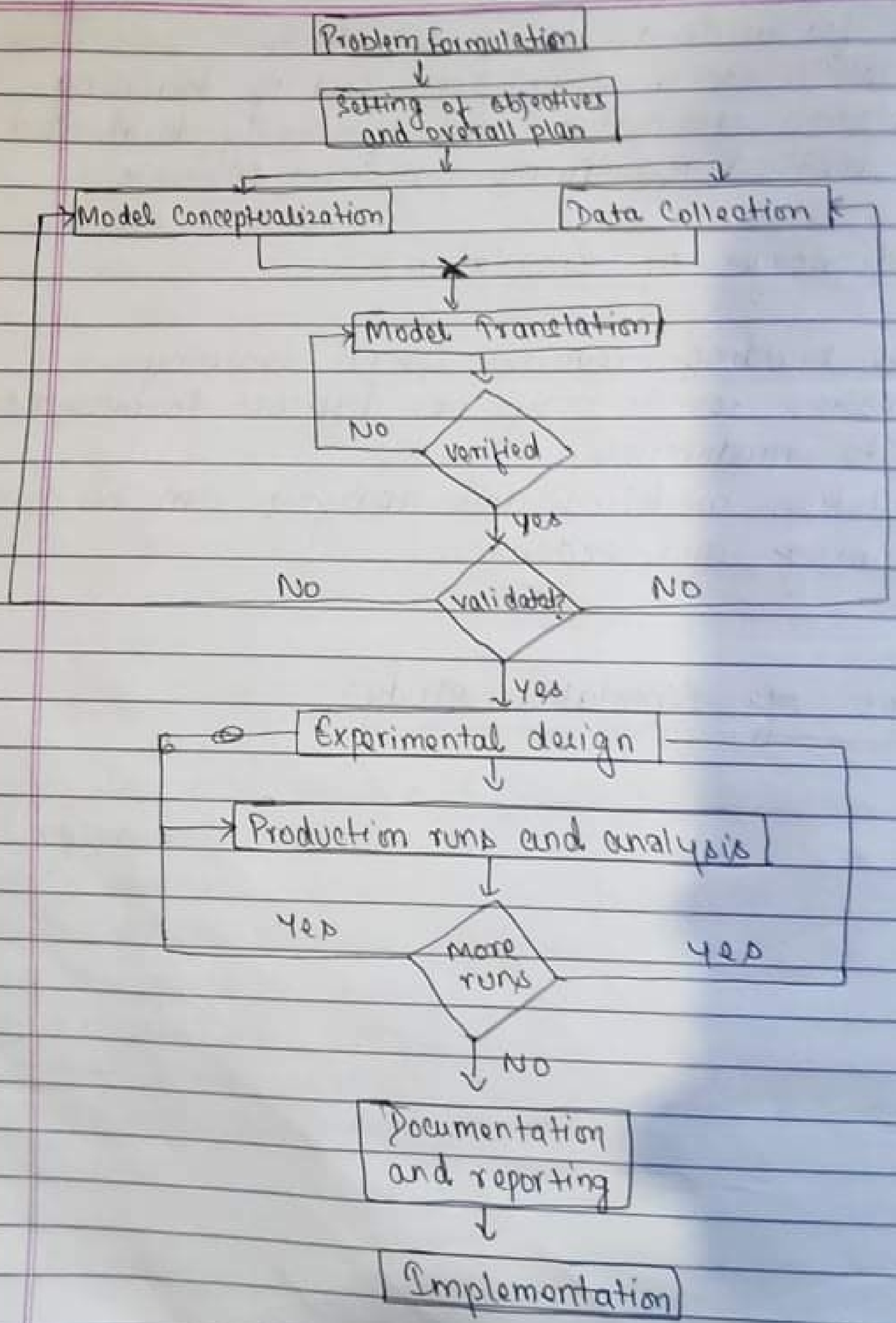

fig: Steps of Simulation study

i) Problem formulation:-
Clearly state the problem.

ii) Setting of objectives and overall plan:
How we should approach the problem.

iii) Model conceptualization:-
Establish a reasonable model.

iv) Data Collection:-
Collect the data necessary to run the simulation such as arrival rate, arrival process, service discipline, service rate, etc.

v) Model Translation:-
Convert the model into a programming language.

vi) Verification:
Verify the model by checking if the program works properly use common sense.

vii) Validation:
Check if the system accurately represent the real system.

viii) Experimental design:
How many runs? For how long? What kinds of input variations?

Date / /
Page

ix) **Production runs and analysis:**
Actual running the simulation, collect and analyze the output.

x) **Repeatition:-**
Repeat the experiments if necessary.

xi) **Documentation, ~~and~~ reporting and implementation:-**

It implements the results.

Comparison of simulation and analytical methods:-

→ Simulation gives specific solutions rather than general solutions whereas analytical method gives general solution.
→ The step by step nature of simulation technique means that the amount of computation increases very rapidly as the amount of detail increases.
→ Simulation provide a powerful extension of non-mathematical solutions.
→ Simulation provide a quicker or more convenient way of deriving results.

Difference between Simulation and Analytical Method:

Basis	Simulation	Analytic
Input Parameterization	Measured or invented	Measured or invented (with certain limitations)
Model components	Virtually anything	Composed of limited basic building blocks.
Model outputs	Anything that can be measured.	Equilibrium measures
Effort to construct model	Arbitrary	Modest
Computational cost	Typically large	Typically small
Underlying concepts	Probability or statistics	Algebra to stochastic process
Special properties	Credible	Insight, optimization

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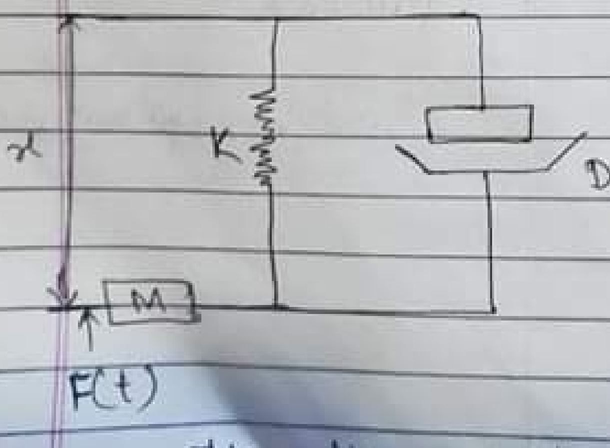
Experimental Nature of Simulation:-

Simulation is an experimental problem solving technique. Many simulation runs have to be made to understand the relationships involved in the system, so the use of simulation in a study must be planned as a series of experiments. A simulation study of the automobile wheel suspension system that was analyzed using analytical method would proceed by following the motion of wheel under different conditions. The relationship between D, K and M to prevent oscillation which was previously discovered analytically, would have to be discovered by observing the values that results in the motion being non-oscillatory.

Thus, simulation is based on observation and experiment.



D → Damping force depends on velocity

The differential equation of automobile wheel suspension system is: $M\ddot{x} + D\dot{x} + Kx = KF(t)$
where x is the distance moved, M is mass, K is stiffness of spring and D is damping factor of shock absorber.

Distributed Lag Model:-

If the regression model includes not only the current but also the lagged (past) values of the explanatory variables is called a distributed lag model. If the model includes one or more lagged values of the dependent variable among its explanatory variables is called an auto-regressive model. This is known as a dynamic model.

In other word, distributed lag model is ~~model~~ defined as a type of model that have the property of changing only at fixed interval of time and based on current values of variables on other current values of variables and values that occured in previous intervals.

In economic studies, some economic data are collected over uniform time intervals such as a month or year. This model consist of linear algebraic equations that represent continuous system but data are available at fixed point in time.

$$Sales_t = \alpha + \beta_1 price_t + \beta_2 price_{(t-1)} + \beta_3 \cdot price_{(t-2)} + u_t$$

E.g: $Sales_{(2019)} = \alpha + \beta_1 price_{(2019)} + \beta_2 price_{(2018)} + \beta_3 price(2017) + u_{2019}$

Date / /
Page

E.g.: Mathematical model of national economy:-

Let,

C - Consumption
I - Investment
T - Taxes
G - Government Expenditures
Y = National Income

then,

$$C = 20 + 0.7(Y - T)$$
$$I = 2 + 0.1Y$$
$$T = 0.2Y$$
$$Y = C + I + G$$

All equations are expressed in billions of rupees. This is static model and can be made dynamic by lagging and variables as follows.

$$C = 20 + 0.7(Y_{-1} - T_{-1})$$
$$I = 2 + 0.1Y_{-1}$$
$$T = 0.2Y_{-1}$$
$$Y = C_{-1} + I_{-1} G_{-1}$$

Any variable that can be expressed in the form of its current value and one or more previous value is called a lagging variable and hence this model is given the name distributed lag model. The variable in a previous interval is denoted by attaching -n suffix to the variables. Where 'n' indicates the n^{th} interval.

Q.N. What is distributed lag model? Find the growth in national consumption for five years using the model given. Assume the initial income $Y_{-1} = 80$ and take the government expenditure in the five years as follows:

Year	G
1	20
2	25
3	30
4	35
5	40

Soln

From the distributed lag model, we have the set of equations,

$$I = 2 + 0.1Y_{-1}$$
$$Y = 45.45 + 2.27(I + G)$$
$$T = 0.2Y$$
$$C = 20 + 0.7(Y - T)$$

where,

I = Investment
Y = National Income
T = Taxes
C = Consumption
G = Government Expenditure.

At first year:

$$I = 2 + 0.1 \times 80 = 10$$
$$Y = 45.45 + 2.27(10 + 20)$$
$$= 113.55$$
$$T = 0.2 \times 113.55 = 22.71$$
$$C = 20 + 0.7(113.55 - 22.71)$$
$$= 83.58$$

∴ The national consumption for first year = 83.58

96.3, 102.65 & ~~109.01~~

At second year:

$$I = 2 + 0.1 \times 80 = 10$$
$$Y = 45.45 + 2.27(10 + 25) = 124.9$$
$$T = 0.2 \times 124.9 = 24.98$$
$$C = 20 + 0.7(124.9 - 24.98) = 89.94$$

At third year:

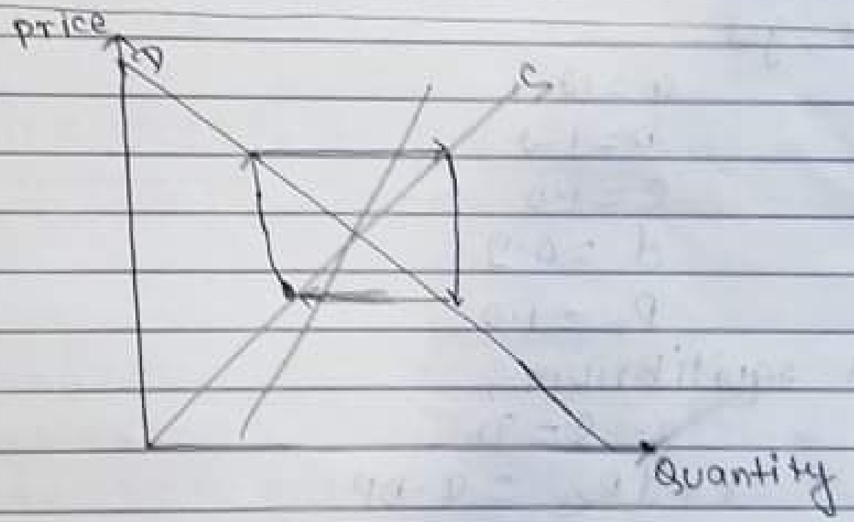
$$I = 2 + 0.1 \times 80 = 10$$
$$Y = 45.45 + 2.27(10 + 30) = 136.25$$
$$T = 0.2 \times 136.25 = 27.25$$
$$C = 20 + 0.7(136.25 - 27.25) = 96.3$$

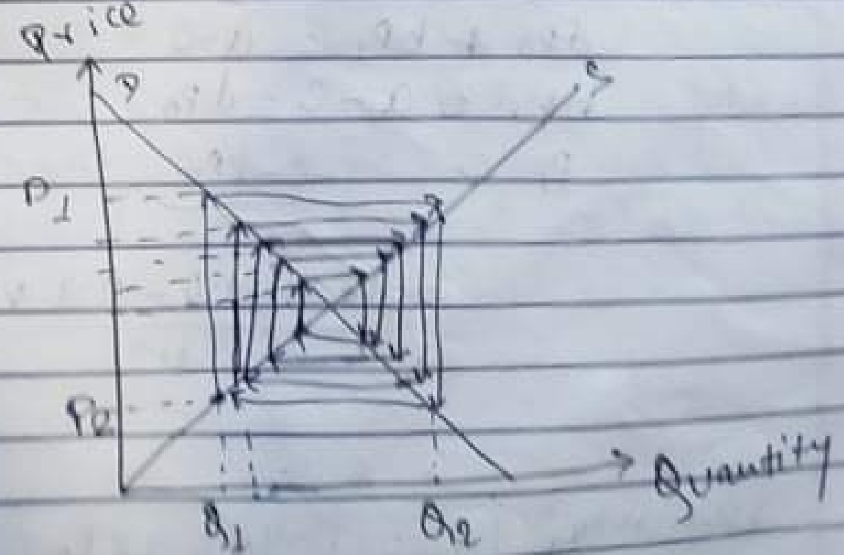
At fourth year:

Cobweb Models:

- It is an economic model.
- In the model of supply and demand, the price ~~exhaust~~ adjust so that the quantity supplied and quantity demand are equal.
- The equilibrium is not always clear so that some slopes are unstable.
- Nicholas Kaldor analyse the model in 1934 coined the term "Cobweb theorem" and applying it on agriculture markets.

→ The equilibrium price is at the intersection of supply and demand curves.





Date / /
Page

Q.N. Draw the cobweb model of a market economy for the following conditions:

$$D = 12.4 - 1.2P$$
$$S = 1.0 + 0.9P_{-1}$$
$$P_0 = 1.0$$

Sol^n

Comparing above equation with the standard equation of market model (cobweb model) is,

$$D = a - bP$$
$$S = c + dP_{-1}$$
$$D = S$$

we get,

$$a = 12.4$$
$$b = 1.2$$
$$c = 1.0$$
$$d = 0.9$$
$$P_0 = 1.0$$

At equilibrium,

$$S = D$$
$$c + dP_{-1} = a - bP$$
$$\text{or, } c + dP_0 = a - bP_1 \quad (\because P_{-1} = P_0)$$
$$dP_0 + bP_1 = a - c$$
$$bP_1 = a - c - dP_0$$
$$P_1 = \frac{a - c - dP_0}{b}$$
$$= \frac{12.4 - 1.0 - 0.9 \times 1.0}{1.2}$$
$$= 8.75$$

Similarly, $\boxed{P_1 = \frac{11.4 - 0.9 \times P_0}{1.2}}$ — (1)

from eq^n (1)

$$P_2 = \frac{11.4 - 0.9 \times P_1}{1.2} = \frac{11.4 - 0.9 \times 8.75}{1.2} = 2.93$$

$$P_3 = \frac{11.4 - 0.9 \times P_2}{1.2} = \frac{11.4 - 0.9 \times 2.93}{1.2} = 7.29$$

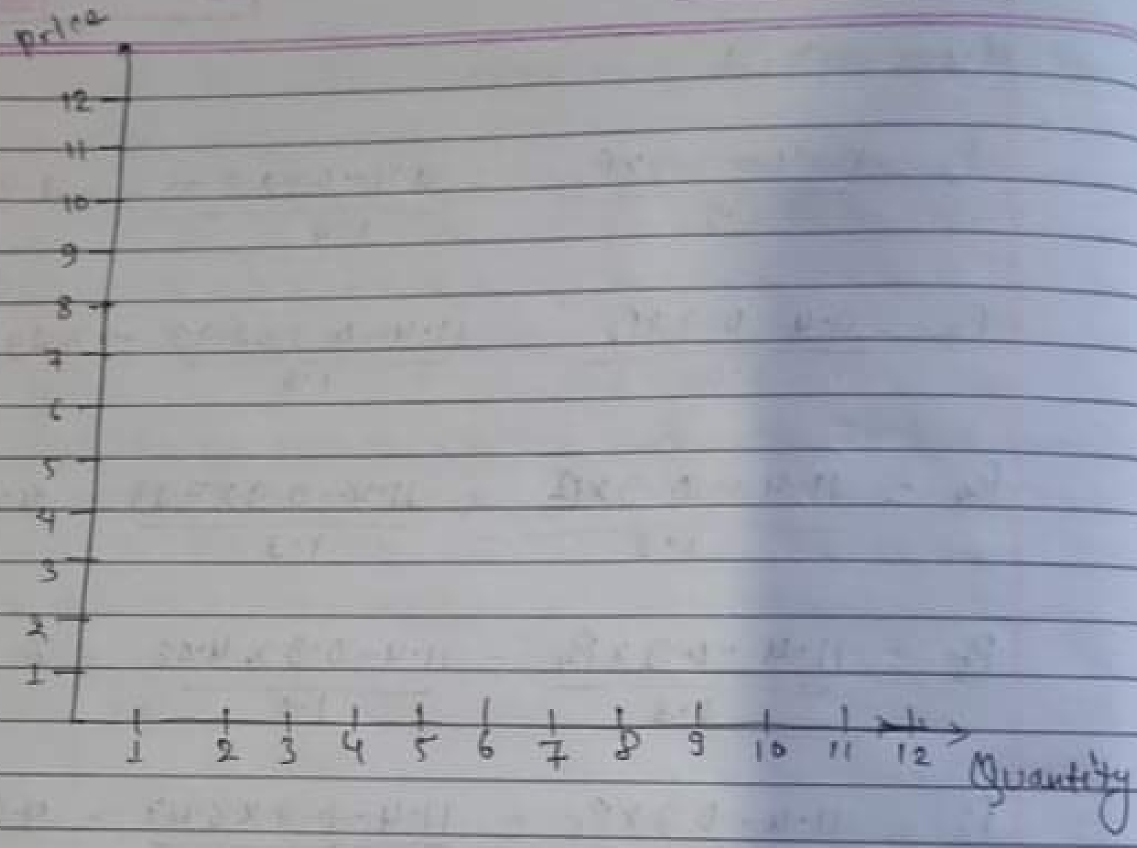
$$P_4 = \frac{11.4 - 0.9 \times P_3}{1.2} = \frac{11.4 - 0.9 \times 7.29}{1.2} = 4.02$$

$$P_5 = \frac{11.4 - 0.9 \times P_4}{1.2} = \frac{11.4 - 0.9 \times 4.02}{1.2} = 6.47$$

$$P_6 = \frac{11.4 - 0.9 \times P_5}{1.2} = \frac{11.4 - 0.9 \times 6.47}{1.2} = 4.64$$

Now,

Price	Supply S, $S = 1.0 + 0.9P_{-1}$	Demand $D = 12.4 - 1.2P$
$P_0 = 1.6$	$S_0 = 1.0 + 0.9 \times 1 = 1.9$	$D_0 = 12.4 - 1.2 \times 1 = 11.2$
$P_1 = 8.75$	$S_1 = 1.0 + 0.9 \times 8.75 = 8.87$	$D_1 = 12.4 - 1.2 \times 8.75 = 1.9$
$P_2 = 2.93$	$S_2 = 1.0 + 0.9 \times 2.93 = 3.65$	$D_2 = 12.4 - 1.2 \times 2.93 = 8.83$
$P_3 = 7.29$	$S_3 = 1.0 + 0.9 \times 7.29 = 7.56$	$D_3 = 12.4 - 1.2 \times 7.29 = 3.64$
$P_4 = 4.02$	$S_4 = 1.0 + 0.9 \times 4.02 = 5.15$	$D_4 = 12.4 - 1.2 \times 4.02 = 7.6$
$P_5 = 6.47$	$S_5 = 1.0 + 0.9 \times 6.47 = 6.85$	$D_5 = 12.4 - 1.2 \times 6.47 = 4.6$
$P_6 = 4.64$	$S_6 = 1.0 + 0.9 \times 4.64 = 5.17$	$D_5 = 12.4 - 1.2 \times 4.64 = 5.19$


Price
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12
Quantity

Time Advance Mechanism:-

- → Discrete event simulation (DES) models are dynamic in nature.
- → Time advance mechanism advances simulated time from one value to another.
- → Simulation clock gives the current value of simulated time.
- → We must keep track of the current value of the simulated time as the simulation proceeds.

There are two main approaches for advancing the simulation clock time and they are;

i> Fixed Increment Time Advance Mechanism (FITAM)

- → In this mechanism, there is equal time between consecutive two events.
- → Events occur at a fixed increment.
- → Events occuring between time increments must be moved to an increment boundary.
- → Simple to implement, but not an accurate realization of occurence of events.
- → In figure, time between e_0 and e_1 is equal to time between e_1 and e_2.

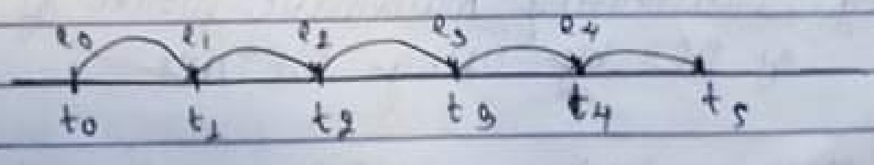


Fig: SSQM using FITAM

Date / /
Page

ii) Next Event Time Advanced Mechanism :- (NETAM).

→ This approach is generally used for DES.
Eg: SSQM (Single Server Queuing Model). In this mechanism, simulation clock changing occur next event not in fixed time.

→ The most common used approach:

1) The simulation clock is initialized to zero.
2) Time of occurrence of future events are determined.
3) The simulation clock is then advanced to the time of the occurrence of the next event.
4) The system is updated taking in account that the event has occured.
5) Update the time of the occurrence of the next event.
6) Go to step 3.
7) Repeat until a stopping condition is satisfied.

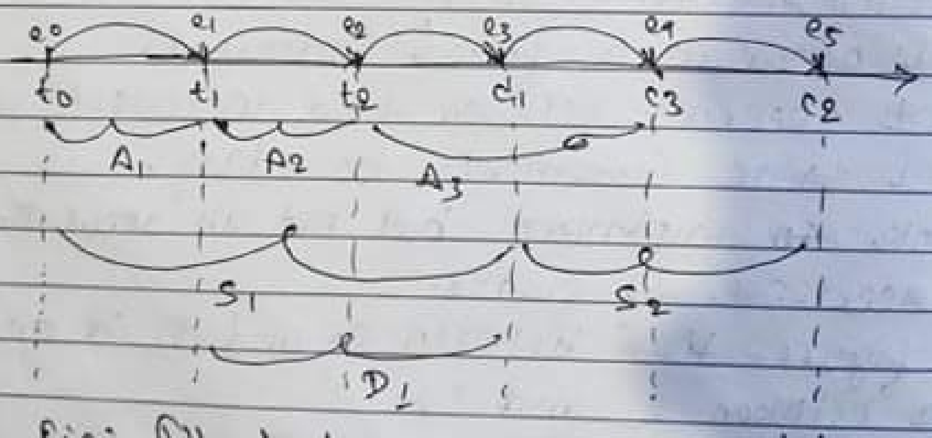


Fig: Illustrating mathematic model of SSQM of DES by using NETAM.

- The state variables of SSQM are defined as follows:

t_i → time of arrival of i^{th} customer.

$A_i \rightarrow (t_i - t_{i-1})$
→ inter-arrival time between $(i-1)^{th}$ and i^{th} customer arrival.

$S_i \rightarrow$ time that server actually spend serving i^{th} customer

$D_i \rightarrow$ Delay in queue of i^{th} customer

$C_i \rightarrow (t_i + D_i + S_i)$

$\rightarrow$ time for i^{th} customer to get served and departure time.

Queuing Model and its characteristics:-

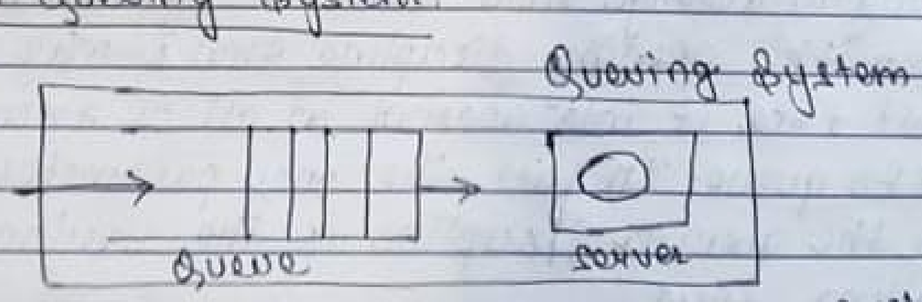
→ Queuing systems are the waiting line queues, one of the most important areas where the technique of simulation has been extensively employed. The waiting lines or queues are a common site in real life.

Eg: People at railway ticket window, vehicles at a petrol pump, products at machining center, etc.

→ The objective in the analysis of queuing situations is to balance the waiting and idle time so as to keep the total cost at minimum.

→ By engineer A.K. Earlang (1920) who studied the waiting line queues of telephone calls in Copenhagen, Denmark.

Model Queuing System:-



Queuing system   Server System

→ Using queuing models to:

- describe behaviour of queuing systems.
- Evaluate system performance.

Characteristics of Queuing System:-

- Arrival Process:
  - the distribution that determines how the tasks arrives in the system.
- Service Process:
  - the distribution that determines the task processing time.
- Number of servers:
  - total no. of servers available to process the tasks.

Queuing Discipline:

It represents the way the queue is organized (rules of inserting and removing customers to or from the queue).

a) FIFO b) LIFO c) SIRO (Service in Random Order)
d) SPT (Shortest Processing Time)
e) SPR (Service According to Pride)

Most quantitative parameters like average queue length, average time spend in system do not depend upon the queuing discipline that's why most models don't take it into account at all or assume the number FIFO queue. In fact, the only parameter that depends on the queuing discipline is the variance of the waiting time.

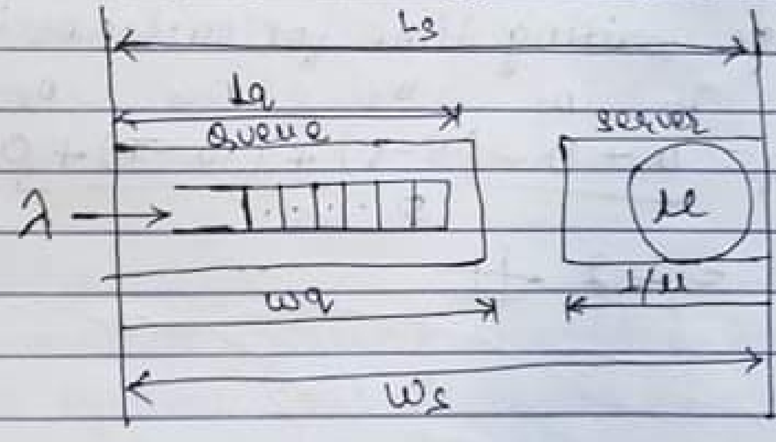
Date / /
Page

Measures of queue:-

- Typical measure of system performance include server utilization, length of waiting line, delays of customers, etc.
- System performance is predicted as a function of input parameters.
- Input parameters are arrival rate of customers, service demand of customer, rate at which server works, arrangement of servers, etc.
- for relatively simple system compute mathematically.
- for realistic models of complex system, simulation is usually required.

Single Server Queuing System:- (SSQM):-

We have M/M/1 queuing system i.e. poisson distributed arrival and service and one server.



where, λ = arrival rate of jobs
μ = service rate of server
L_s = average no. of customer in a queuing system.
L_q = average no. of customer in queue.
W_s = average waiting time in whole system.
W_q = average waiting time in the queue
$1/\mu$ = average service time.

Date / /
Page

Q.N. Assume a system with a single server and a FIFO queue discipline. let arrival occurs at 0, 3, 5, 7 and 16 and departures occur at 2, 8, 10, 14 and 20 and calculate average waiting time by the customers in the whole system and average waiting time by the customer in the queue.

Soln

0	2	3	5	7	8	10	14	16	20
$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
A_1	D_1	A_2	A_3	A_4	D_2	D_3	D_4	A_5	D_5

$\therefore$ Average waiting time for customer in system,

$$w_s = \frac{(2-0)+(8-3)+(10-5)+(14-7)+(20-16)}{5}$$

$= 4.6$ time units.

$\therefore$ Average waiting time for customer in queue,

$$w_q = \frac{\overset{A_1}{0} + \overset{A_2}{0} + \overset{A_3}{(8-5)} + \overset{A_4}{(10-7)} + \overset{A_5}{0}}{5}$$

$= 1.2$ ##

11 Types of System Simulation:

A Simulator is a device, computer program, or system that performs a simulation. A simulation is a method for implementing a model over time. There are three types of commonly use simulations:

1) Live:

Simulation involving real people operating real systems.
- Involve individuals or groups.
- May use actual equipment
- Should provide a similar area of operations
- Should be close to replicating the actual activity.

2) Virtual:

Simulation involving real people operating simulated systems. Virtual simulations inject Human-In-the-Loop in a central role by exercising.
- Motor control skills (eg. flying an aeroplane)
- Decision skills (e.g. committing fire control resources to action)
- Communication skills (e.g. members of a C4I team)

3) Constructive:

Simulation involving simulated people operating simulated systems. Real people can stimulate (make inputs) but are not involved in determining outcomes. Constructive simulations offer the ability to:
- Analyze concepts
- Predict possible outcomes
- Stress large organizations
- Make measurements
- Generate statistics
- Perform analysis

POCO
SHOT ON POCO F1

Date / /
Page

Q.N In a health clinic, the average rate of arrival of patient is 12 patients per hour. On an average, a doctor can save patients at the rate of 1 patient every 4 minutes.

i) Assume the arrival at patients follows a poisson distribution and service to patient follow an exponential distribution and calculate server utilization.

ii) Find average no. of patients waiting line in clinic.

iii) Find average waiting time in waiting line or in queue and also the average waiting time in clinic.

Soln

Given,

λ = patients rate = 12 /hrs

μ = doctor's service rate

$= 1$ in 4 min

$= \frac{60}{4}$ hrs

$= 15$ / hr

We have,

i) Server utilization, $\rho = \frac{\lambda}{\mu}$

$= \frac{12}{15}$

$= 0.8$

ii) The average no. of patients in clinic (l_s),

$$l_s = \frac{\rho}{1-\rho} = \frac{0.8}{1-0.8}$$

$$= 4$$

Average no. of patients waiting line in clinic (l_q),

$$l_q = \rho \times l_s = 0.8 \times 4$$

$$= 3.2$$

iii) Average waiting time in clinic (w_s),

$$w_s = \frac{l_s}{\lambda} = \frac{4}{12} \text{ hr} = \frac{1}{3} \text{ hr}$$

$$= 0.33 \text{ hr}$$
$$= 0.33 \times 60 \text{ min}$$
$$= 20 \text{ min}$$

Average waiting time in waiting line or in queue (w_q),

$$w_q = \frac{l_q}{\lambda} = \frac{3.12}{12}$$

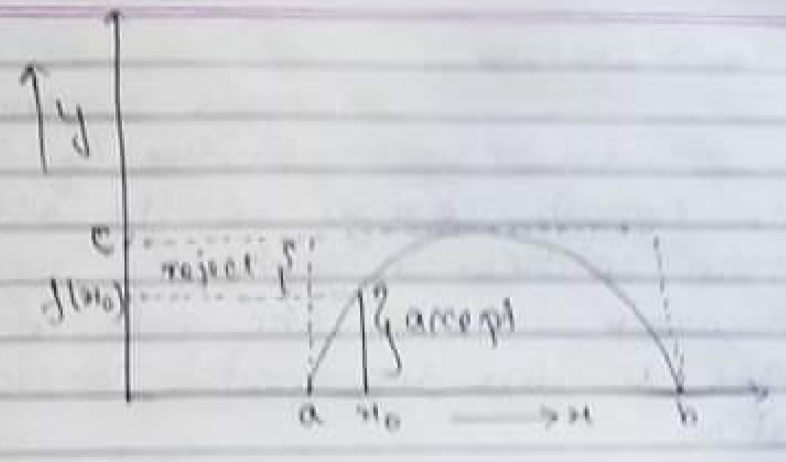
$$\simeq 0.266 \text{ hr}$$
$$= 0.266 \times 60 \text{ min}$$
$$= 16 \text{ min}$$

Date / /
Page

The Techniques of Simulation- Monte Carlo Method:-

- Originally used for variable reduction.
- Uses random numbers as the input sample.
- Computational technique applied to static model.
- Define a function and select random numbers.
- Test the function against the random numbers.
- The good accuracy can be achieved by increasing the no. of random samples.
- let $f(x)$ be a function define and has lower and upper bounds a, b and c.
- The function will be bounded by the area $c*(b-a)$.
- let us consider 'N' random points are taken of which only 'n' numbers of points lie within the curve.

$\Rightarrow$



- The ratio will lead to

$$\frac{n}{N} = \frac{\int_a^b f(x)\cdot dx}{c * (b-a)}$$

- It seems like when the value of 'n' is increased, the accuracy will be increased but since the value of 'n' again depends on 'N'. The accuracy is increased for greater value of 'N'.
- Only the points satisfy $y \leq f(x_0)$ are accepted otherwise rejected.
- If rejected then another point is selected randomly.

Chapter 3 Continuous Systems

Date / / Page

Continuous system:

Continuous system is one in which the pre-dominant activities of the system cause smooth changes in the attributes of the system entities. In continuous system, the relationship describe the rate at which attributes changes so that the model consist of differential equation.

Differential Equation:

It is the equation which simulate the continuous system.

E.g:

$f(x) = Ax + B$ is linear equation

$z = Bx^2 + C$ is non-linear equation

$g = a\frac{dx}{dt} + b\frac{dx}{dr}$ is partial differential equation.

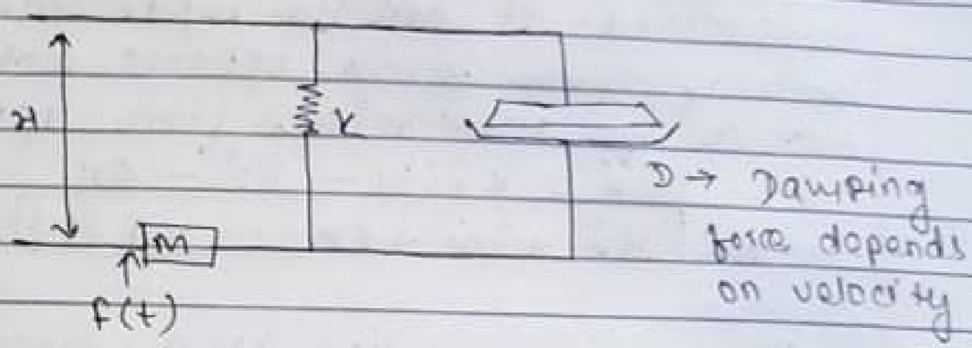
A differential equation is an equation which involves the derivates of dependent variable along with its coefficient.

$$M\ddot{x} + D\dot{x} + Kx = K\,F(t)$$

OR,

$$M\frac{dx^2}{dt} + D\frac{dx}{dx} + Kx = KF(t)$$

To illustrate differential equation we take example of equation that describe the automobile wheel suspension system that is derived from mathematical principles



Here, In this system we take a reference point of a wheel to measure its vertical displacement variable. 'x' gives the displacement. The velocity of wheel is $\dot{x}$. The acceleration of wheel is $\ddot{x}$.

The mechanical law determine the relationship between applying force and movement of the body i.e. acceleration of the body is proportional to applied force.

i.e. $\ddot{x} \propto F$

If there is no force then there is no acceleration. The coefficient of proportionality between force and acceleration, acceleration is the mass of the body $F = ma$. In case of suspension wheel 'M' is a mass and F(t) is applied force.

i.e. $M\ddot{x} \propto F(t)$

$M\ddot{x} = K F(t)$

where 'K' is proportionality constant.

The shock absorber exerts a resisting force that depends on the velocity of the wheel. The force is directly proportional to velocity and can be represented by $D\dot{x}$.

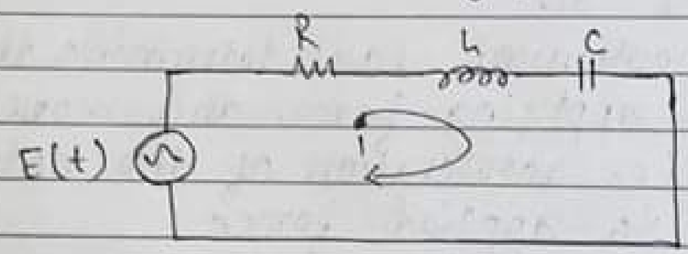
Similarly, spring exerts resisting force which depends upon the extend to which it has been compressed. It can be represented by 'Kx'. Both of these forces opposes motion. Thus, they should be subtracted from applied force.

i.e. $M\ddot{x} = KF(t) - D\dot{x} - Kx$

or, $\boxed{M\ddot{x} + D\dot{x} + Kx = KF(t)}$

Now this equation is a linear differential equation of automobile wheel suspension system.

Differential equation for R-L-C circuit:-



from KVL;

$$E(t) = V_R + V_L + V_C \quad \text{--- (i)}$$

Here,

Voltage drop across resistor (R) is:

$$V_R = i \cdot R$$

Since, $i = \frac{dq}{dt}$, $V_R = R \cdot \frac{dq}{dt}$ — (ii)

Voltage drop across inductor (L) is,

$$V_L = L\frac{di}{dt}$$

Date / /
Page

$$V_L = L\frac{d}{dt}\left(\frac{dq}{dt}\right) \qquad \left[\because i = \frac{dq}{dt}\right]$$

$$\boxed{\therefore V_L = L\frac{d^2q}{dt^2}}$$

Voltage drop across ~~conductor~~ capacitor (C) ~~is~~, with capacitance (C) is,

$$V_C = \frac{q}{C} \quad \text{—— (iv)}$$

from eqn (i), (ii), (iii) and (iv),

$$\boxed{E(t) = R\cdot\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}}$$

which is the required linear differential equation for R-L-C circuit.

Analog Computer:-

Before general availability of digital computers, there existed devices whose behaviour is equivalent to a mathematical operation putting together combination of such devices in a manner specified by a mathematical model of a system allowed the system to be simulated. Specific devices have been created for particular system but ~~which show~~ with so a general technique, it is customary to refer them as analog computer or when they are primarily used to solve differential equation models as differential analyzer.

Date / /
Page

Advantages of Analog Computer:

- Representation of analog computer is more natural in the sense that they represents the structure of the system.
- Under certain condition analog computer is more faster than digital computer which can solve equations simultaneously.

Disadvantages of analog computer:-

- Limited accuracy.
- They are designed for specific system.
- The analog computer are based on assumptions.
- Expensive to make as it included more hardware components.

Standard symbols representing analog computers:-

i) 

$\rightarrow$ scale factor

ii) 

$\rightarrow$ Adder

iii) 

$\longrightarrow$ Integrator

iv) 

$\longrightarrow$ Sign Changer

Q.N. Design an analog computer for automobile suspension system.
Imp

Soln

We know that,

Automobile suspension system is represented by following differential equation:

$$M\ddot{x} + D\dot{x} + Kx = Kf(t)$$

Solving equation for the highest order derivative gives:

$$M\ddot{x} = Kf(t) - D\dot{x} - Kx$$

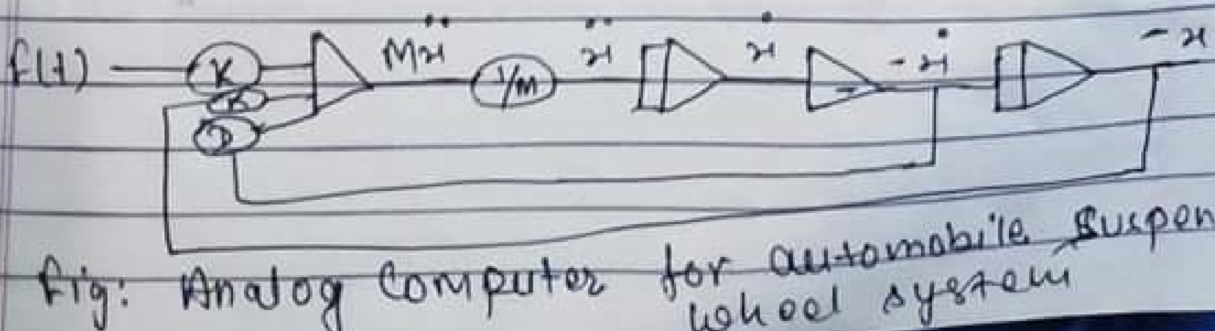


Fig: Analog Computer for automobile suspension wheel system

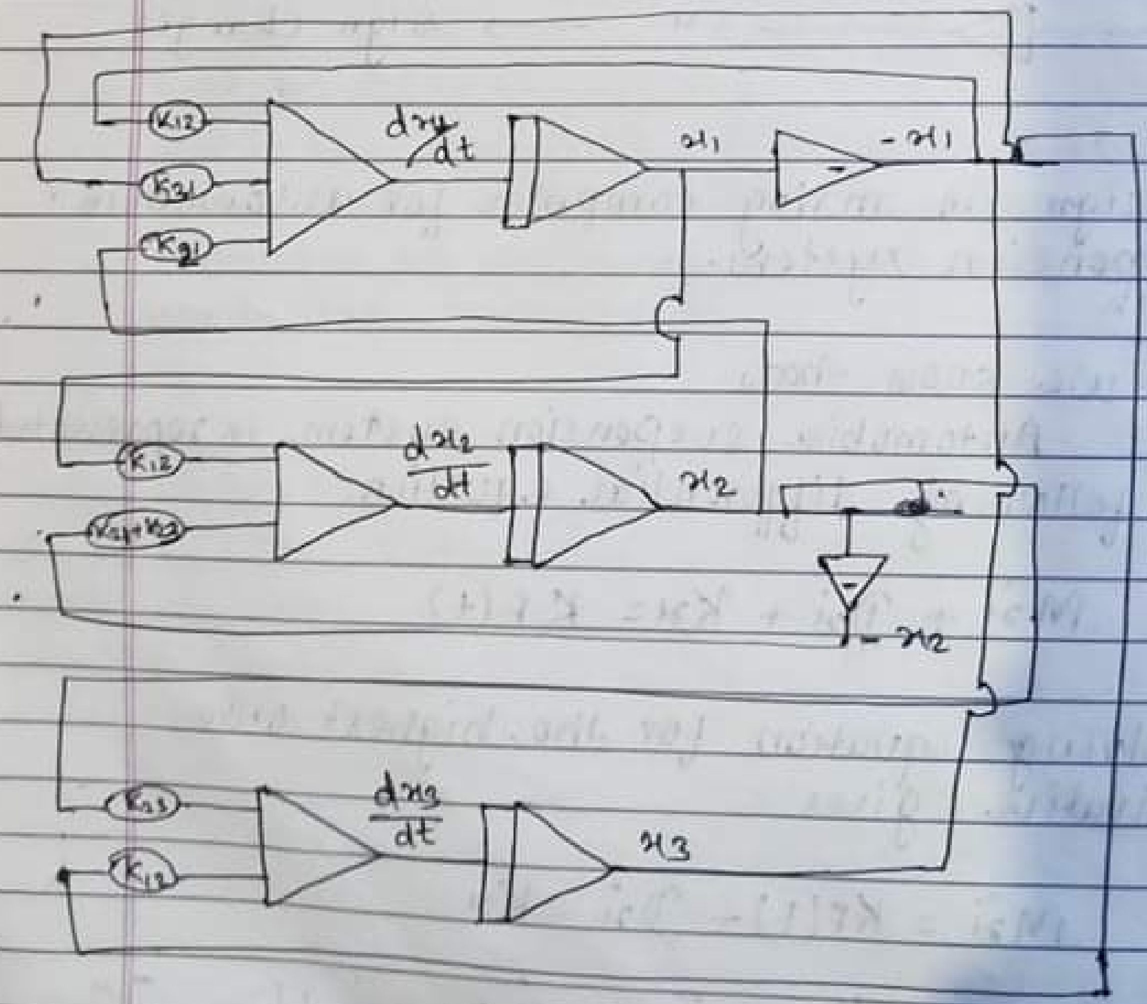
Imp from (2) (1) Ques in

Design Analog Computer for the following equation:

$$\frac{dx_1}{dt} = -K_{12}x_1 + K_{21}x_2 + K_{31}x_3$$

$$\frac{dx_2}{dt} = \cancel{K_{22}x_1}\ K_{12}x_1 - (K_{21} + K_{23})x_2$$

$$\frac{dx_3}{dt} = K_{23}x_2 - K_{12}x_1$$



Assignment

(1) Hybrid Computer:

(2) Digital Analog Simulator

(3) Real Time Simulation

(4) Feedback System

(5) Interactive System

Continuous System Simulation Language (CSSLs):

CSSL are developed design continuous model digital analog simulator, has became system specific as if was designed for particular system. To remove this, many CSSL, have been developed. These language uses familiar statements to input in digital computers. The problems were programmed from the equation. They also contains macros, subroutines, to perform functions of analog elements. It also allows to use special purpose elements that corresponds to operations that are particularly important in specific types of application. These languages allow variety of algebraic and logical expressions to describe the relationship of variables.

11 Continuous System Model Program III (CSMP III):

CSMP III is CSSLs which is used to program continuous model which was solved by analog method. The program is constructed in 3 general steps:

i) Structural statement
ii) Data statement
iii) Control statement

- Structural statement:

Structural statement define the model that use FORTRAN FORTAN statement. All the mathematical operations are carried out as in FORTAN.

E.g. $x = \frac{6y}{w} + (z-2)^2$

FORTAN statement:

```
x = 6.0 * y/w + (z - 2.0)**2.0
```

- Data statement:

In data statement, numerical values are assigned to parameters, constant and initial condition (INCON). INCON can be used for initial values of integration function. CONST can be used to set values for constant. PARAM is used to set parameters for individual. It can be used to set series of numeric values for one parameters.

E.g.

```
CONST   A = 0.5 , XDOT = 1.2
PARAM   D = (0.25, 0.5, 0.75 ...
```

Date / /
Page

- **Control statement:-**
Control statement specify the options in assembly and in execution of program and choice of output. TIMER is used to specify time intervals.

E.g.
TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.1, OUTDEL = 0.1

where,
DELT → Integral Interval
FINTIM → Finish Time
PRDEL → Interval at which to plot result.

~~Ddier~~ CSMP-III of
Automobile Suspension Wheel for
$M = 2.0$, $f = 1$ and $K = 400$

$$\frac{M d^2x}{dt^2} = K f(t) - D\frac{dx}{dt} - Kx$$

Soln

⇒ We have,
$$\frac{M d^2x}{dt^2} = K f(t) - D\frac{dx}{dt} - Kx$$

or, $$\frac{d^2x}{dt^2} = \frac{1}{M}\left(K f(t) - D\frac{dx}{dt} - Kx\right)$$

Date / /
Page

```
TITLE Automobile Suspension Wheel PARAM
D = (5.1, 16.96, 31.42, 39.66)

X2DOT = (1.0/M)*(K*F - K*X - D*XDOT)

XDOT = INTGRL (0.0, X2DOT)
  x = INTGRL (0.0, XDOT)
CONST M = 2.0, F = 1.0, K = 400 ← Data part
TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.05
                                  OUTDEL = 0.05

PRINT x, XDOT, X2DOT
PRTPLT x
LABEL DISPLACEMENT VERSUS TIME
END
STOP
```

⟹

Assiggment

Date / /
Page

Hybrid Computers:

The term hybrid computers have emerged to describe the combination of traditional analog computer elements (that gives the smooth continuous output and carry out non-linear operations) as well as the circuit components that have the capacity of storing values, switching operations and performing a logical operation. The scope of analog computers has been considerably extended by developments of solid-logic electronic device. Hybrid computer may be used to simulate a system that is mainly continuous and also have some digital elements
Eg: An artificial satellite for which both the continuous equation of motion and the digital controls signal must be simulated. A hybrid computer is useful when a system that can be adequately represented by an analog computer model is subject of a repetitive study.

Digital Analog Simulators:

To avoid the disadvantage of analog computers, many digital computer programming languages have written to produce digital-analogue simulator. These allow a continuous model to the programmed on a digital computer in the same way as it is solved on an analog computer. These languages contain macro instructions that carry out the action of address, integrators and sign chargers.

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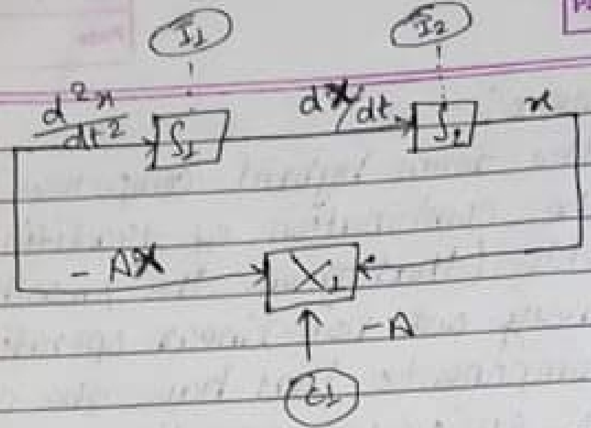


A program uses these macro-instructions to link them together in essentially the same way as operational amplifiers are connected in analog computers. Later more powerful techniques of applying digital computers to the simulation of the continuous system have been developed. Due to those, digital-analogue simulators are not now in common uses

Real Time Simulation:-

It refers to a computer model of a physical system that can execute at the same rate as the actual "wall clock" time. In other words, the computer model runs at the same rate as the actual physical system.

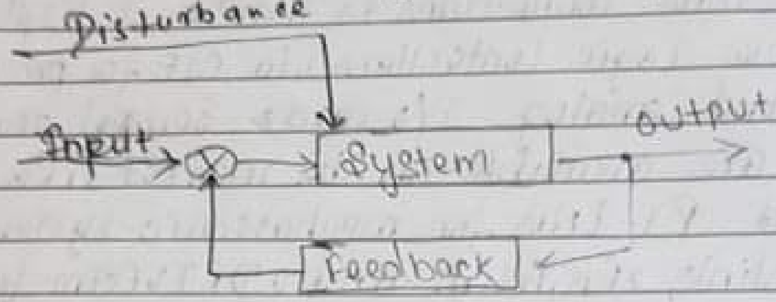
Eg: If a tank takes 10 minutes to fill in the real word, the simulation would take 10 minutes as well.

Real time simulation occurs commonly in computer gaming, but also is important in the industrial market for operator training and off-line controller tuning. Computer languages like LabVIEW, VisSim, and Simulink allow quick creation of such real-time simulations and have connections to industrial displays and Programmable Logic Controllers via OLE for process control or digital and analog I/O cards. Several real time simulators are available on the market like xPC Target and RT-LAB for mechatronic systems and using Simulink, eFPGAsim and eDRIVEsim for power electronic simulation and eMEGAsim, HYPERSIM and RTDS for power grid real time (RTS) simulation.

Feedback System:-

Feedback systems have a closed-loop structure that brings results from past action of the system back to control future action, so feedback systems are influenced by their own past behaviour. Extending the blind control example, a feedback system would be a system that not only opens the blinds when the sun rises but also adjusts the blinds during the day to ensure the room is not subjected to direct sunlight.

Even though the open system can consist of many parts and thus become very complex (these systems have high detail complexity), experience shows that the behaviour of even small feedback systems consisting of only a few parts (and thus low detail complexity) can be very difficult to predict in practice: despite low detail complexity, these systems have high dynamic complexity. In business prototyping, we deal with both kinds of systems. System dynamics is particularly good at capturing the dynamics of feedback systems.



Interactive System:

Interactive systems are those computer systems characterized by significant amounts of interaction between humans and the computer. Most users have grown up using Macintosh or Windows computer operating systems, which are prime examples of graphical interactive systems. Editors, CAD-~~CA~~ CAM (Computer Aided Design-Computer Aided Manufacture) systems and data entry systems are all computer systems involving a high degree of human-computer interaction. Games and simulations are interactive systems. Web browsers and Integrated Development Environments (IDEs) are also examples of very complex interactive systems.

Some estimates ~~sy~~ suggest that as much as 90% of computer technology development effort is now devoted to enhancements and innovation

Date / /
Page

interface and interaction. To improve efficiency and effectiveness of computer software, programmers, and designers not only need a good knowledge of programming languages, but a better understanding of human information processing capabilities as well. They need to know how people perceive screen colors, why and how to construct unambiguous icons, what common patterns or errors occur on the part of users and how user effectiveness is related to the various mental models of systems people possess.

10/22

CSMP-III of Electrical System (R-L-C circuit):-

⇒ We have,

The equations of electrical system (R-L-C circuit) is

$$L\frac{d^2q}{dt} + R\frac{dq}{dt} + \frac{q}{C} = E(t)$$

$$\text{or, } \frac{d^2q}{dt} = \frac{1}{L}\left(E(t) - R\frac{dq}{dt} - \frac{q}{C}\right)$$

where, L, R, C → Constant

```
TITLE Electrical System
CONST L=35.0, R=5.0, C=2.0          } data part
q2dot = (1.0/L) *(E(t) - R*q1dot - 1.0/C *q)  }
q1dot = INTGRL (0.0, q2dot)                     } Structural part
q = INTGRL (0.0, q1dot)                         }
TIMER DELT = 0.05, FINTIM=1.5,
      PRDEL = 0.5, OUTDEL = 0.5
PRINT q, q1dot, q2dot   }
PRTPLT q                } Control
LABEL Charge Versus Time } Statement
END                     }
STOP                    }
```

Feedback System:

desired heading (θ_i) → Gyroscope → Control Surface → Air frame

Actual Heading (θ_o)

Fig: Feedback System

Feedback system is used to define a system where there is coupling between output and input. An example of feedback system is aircraft system which has continuous control. As shown in figure, input is desired heading (θ_i) and output is actual heading (θ_o). The gyroscope of auto-pilot is able to detect the difference between two headings. A feedback is established by using the differences to operate control surfaces as change of the heading will affect the signal being used to control the heading. The differences between $\theta_i - \theta_o$ is the error signal. It is denoted by E. i.e. $\theta_i - \theta_o = E$

If the control surface is directly proportional to error, then force changing the heading is also proportional to error signal. The feedback in the auto-pilot system is negative feedback. The more the system output diverse from the desired value, stronger is the force to drive it back. For the positive feedback, the force tends to increase the deviation and the system becomes unstable.

Date / /
Page

Simulation of Autopilot System:-

The error signal 'E' has been defined as the difference between desired heading (θ_i) and actual heading (θ_o) i.e. $E = \theta_i - \theta_o$ —— (1)

We assume control system is proportional to error signal so that force changing the system is also proportional to error signal. The force applies torque to aircraft which turns the aircraft. Turning of aircraft produces resisting which is proportional to angular velocity of aircraft.

i.e. $\text{Torque} = KE - D\dfrac{d\theta_o}{dt}$ —— (11)

where K and D are constant.

In mechanical system, the acceleration of the body is proportional to applied force. Similarly, for turning motion angular acceleration of a body is directly proportional to applied torque. 'I' denotes inertia of body. Angular acceleration is second derivate of its heading.

i.e. $\text{Torque} = I\dfrac{d^2\theta_o}{dt^2}$ —— (111)

From eqn (1), (11) and (111),

$$I\frac{d^2\theta_o}{dt^2} = K(\theta_i - \theta_o) - D\frac{d\theta_o}{dt}$$

$$I\frac{d^2\theta_o}{dt^2} = K\theta_i - K\theta_o - D\frac{d\theta_o}{dt}$$

Date / /
Page

Dividing both sides by I and then putting $\frac{K}{I} = \omega^2$ and $\frac{D}{I} = 2\xi\omega$

$$\frac{d^2\theta_0}{dt^2} = \omega^2\theta_i - \omega^2\theta_0 - 2\xi\omega\frac{d\theta_0}{dt}$$

$$\boxed{\therefore \frac{d^2\theta_0}{dt^2} + 2\xi\omega\frac{d\theta_0}{dt} + \omega^2\theta_0 = \omega^2\theta_i}$$

Predator- Prey Model:-

→ It is a biological model of two population.

→ We consider the growth of two independent population. Given, two species of animals, interdependence might arise because one species (the prey) serves as food source for other species (predator).

→ Model of this type is called predator-prey model.

→ There is a deep mathematical connection between the predator and prey model.

→ This model shows that if predator population increases then prey population decreases and vice-versa.

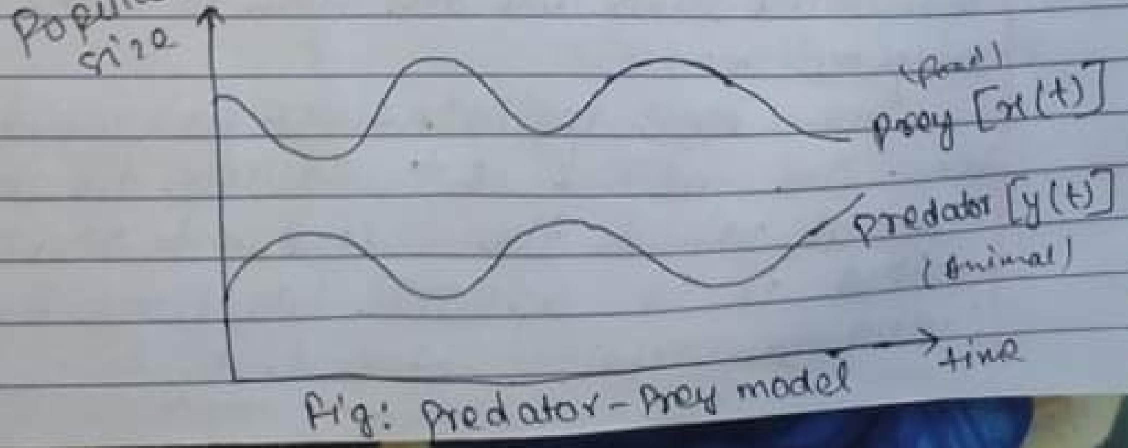


Fig: Predator-Prey model

Chapter 4

Discret System Simulation:-

- Discrete event simulation concerns the modeling of a system as it evolves over time by a representation in which state variable changes instantenously at separate point in time.
- Events defined as instantenous occurance that may change system. It can be done by hand calculation. But in order to solve real world simulation, data must be stored to simulate it on digital computer.

Eg: Single server Queuing Model (SSQM)

Representation of Time:-

The passage of time is recorded by a number referred to as clock time. It is usually said to zero at the begining of a simulation and subsequently indicates how many units of simulated time had passed since the beginning of a simulation. Unless, specifically stated otherwise, the term simulation time means the indicated clock time and not the time that a computer has taken to carry out the simulation. As a rule, there is no direct connection between simulated time and the time to carry out the computation.

Two basic methods exist for uploading clock time. One method is to advance the clock to the time at which the next event occur. The next method is to advance the clock by small (usually uniform) intervals of time and determine

Date / /
Page

at each interval whether an event is due to occur at that time. The first method is referred to as event oriented and the second method is said to be interval oriented.

Discrete system simulation is usually carry out by using the event oriented method while continuous system simulation normally uses the interval oriented method.

Generation of Arrival Pattern:-

Arrival pattern for particular system is specified for simulation. The exogeneous arrival can be designed for simulation. The sequence of inputs can be generated from the observation on particular system. The components of the system are tested. from records gathered from a running system i.e. representative of sequence of operations, the computer system will have to execute. This method is trace driven simulation.

Here, program monitors can be attached to the running system to extract data with no or little disturbance to the running system. The arrival time of an entity is recorded as one of the event time, the event of system is executed and the arrival time of next entity is calculated. This method is called bootstraping method. Here, one entity creates its successor.

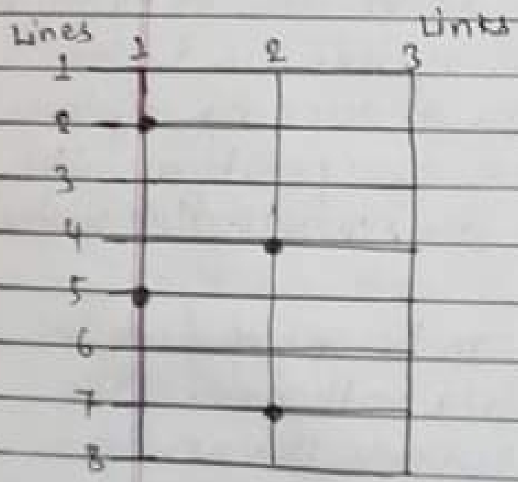
V. Imp

Simulation of Telephone System:-

The simulation of a discrete system can be explained by simulating a telephone system.

The telephone system has several telephones (here only 8 are shown) connected to a switchboard by time. The switchboard has several lines provided the condition that only one connection at a time can be made to each line. Any call that cannot be connected at the time, it arrives is immediately abandoned and then the system is called a lost call system. A call may be lost due to the following reasons:

a) If the called party is engaged (busy).
b) If no link is available (a blocked call).



$\boxed{\begin{array}{l} \text{Link} = n = 3 \\ \text{Lines} = 2^n = 2^3 = 8 \end{array}}$

Horizontal → Telephone lines
Vertical → Links

Fig: Simple telephone system

i) <u>Object of Simulation</u> (<u>Lost call System</u>):

To process a given number of calls and determine what portion is successfully completed, blocked or found to be busy calls. Let's consider the current state of the system as below:-

Lines

1	0
2	1
3	0
4	1
5	1
6	0
7	1
8	0

Links

Max no.	3
In use	2

Clock

1027

Next call

From	To	length

Arrival time

1057

Call in progress

From	To	End
4	7	1075
2	5	1053

Call counters

Processed	Completed	Blocked	Busy
131	98	5	28

Fig: System State 1

- In above table, 0 means the line is free. 1 means line is busy. To keep track of events clock time is included. Currently, the clock time is 1027. Here, unit of time is 1 sec. The clock updates at the next most event as simulation proceeds.
- Each call is entity. It's attributes are origin, destination, length at time at which call finishes.
- There is a list of call in progress showing which line is connected call finish time.
- To general arrival of calls, bootstrapping is used.
- Call can come from any line i.e. not busy at the time of arrival.
- The origin, destination and call length is generated at the time of arrival. The time of next arrival is 1057.
- The records are set for clock time 1027.
- There are two activities causing the events, new call can arrive and existing call can finish.
- In figure, there are three future events, the lines between 2 and 5 with finish time 1053, the call between 4 and 7 with finish time 1075 and a new call arrival at 1057, this call will call from line 1 to 7 for 20 sec.
- Simulation proceeds by executing a cycle of steps to simulate each events. In first step, the next potential event is scanned. In this case, the event is at 1053. Thus, clock is updated to clock time 1053. In 2nd step, the activity is selected that cause the event. In 3rd step, the event execution is tested. In 4th step, the records are changed to reflect the effect of events.

Date / /
Page

Lines

1	0
2	0
3	0
4	1
5	0
6	0
7	1
8	0

Links

Max no	3
In use	1

Clock

1053

Next arrival

from	To	length
1	7	20

Arrival time

1057

Call in progress

from	To	End
4	7	1075

Call ~~conductor~~ counter

Processed	Completed	Block	Busy
132	99	5	28

Fig: System state 2

Call is disconnected by setting 0 to line 2 and 5. After that necessary statics are gathered for simulation output and then simulation is continued.

Lines

1	0
2	0
3	0
4	1
5	0
6	0
7	1
8	0

Links

Max no.	3
In use	1

Clock

1057

Next Arrival

from	To	Length
3	6	98

Arrival Time

1063

Call in progress

from	To	End
4	7	1075

Call counter

Processed	Completed	Block	Busy
133	99	5	29

[1 ले 7 लाई call गर्दा Busy call हुन्छ due to 7 in busy state]

Fig: System state 3

For clock 1057, the next arrives. The clock is updated to arrival time and attributes of new arrival are generated. Here, test is carried out to check the availability of link and lines since 7 is busy, the call is lost.

ii) Delayed Call System:-

Now, let us assume the telephone system is modified that the calls that cannot be connected are not lost. They wait until they cannot be connected. This system is like a message passing switching system with store and forward capability. To keep the record of delay calls, it is necessary to build another list like call in progress. The first two steps are same as before lost call system. The system state at clock 1057 when call from line 1 arrives the system state changes as.

Lines

1	0
2	0
3	0
4	1
5	0
6	0
7	1
8	0

Links

Max no.	3
In use	1

Next arrival

From	To	length
3	6	98

Arrival time

1063

Clock

1057

Delayed Calls

From	To	End
1	7	[illegible]

Call in Progress

From	To	End
4	7	[illegible]

Call counter

Processed	Completed	Block	Busy
133	99	5	29

Fig: System State 3 (System state 1 and 2 same as lost call

Gathering Statistics

Commonly required statistics in discrete simulation system are.

i- Count:-

- To give the number of times an entity of a certain event occur.

ii- Summary measure:-

To give extreme values like mean values and standard deviation.

iii- Utilization:-

It is defined as the fraction or percentage of time of some entity is engaged.

iv- Occupancy:-

It is defined as the fraction of group of entities used at average.

v- Distribution:-

It is used for quantities such as queue length or waiting time.

vi- Transit Time:-

It is defined as time taken for an entity to move from one part of the system to other part.

Counters and Summary Statistics:-

Counters are the basis for most statistics. Some accumulates ~~partat~~ total, some records the current value in the system. The telephone system simulation use counters to record the total number of lost and busy calls. The mean of a set of N observations for x_i where $i = 1, 2, 3, \ldots N$.

$\therefore$ Mean(m) $= \frac{1}{N} \sum_{i=1}^{N} x_i$

$\therefore$ Standard deviation(σ) $= \left[\frac{1}{N-1} \sum_{i=1}^{N} (m - x_i)^2 \right]^{1/2}$

Measuring utilization and occupancy:-

Measuring the load on same entity is most common requirement of a simulation. What ~~(?)~~ fraction of the ~~type~~ time the item was engaged during the simulation run?

Measuring by utilization factor. Let time 't_b' be the time the item becomes busy and 't_f' be the time when the item becomes free.

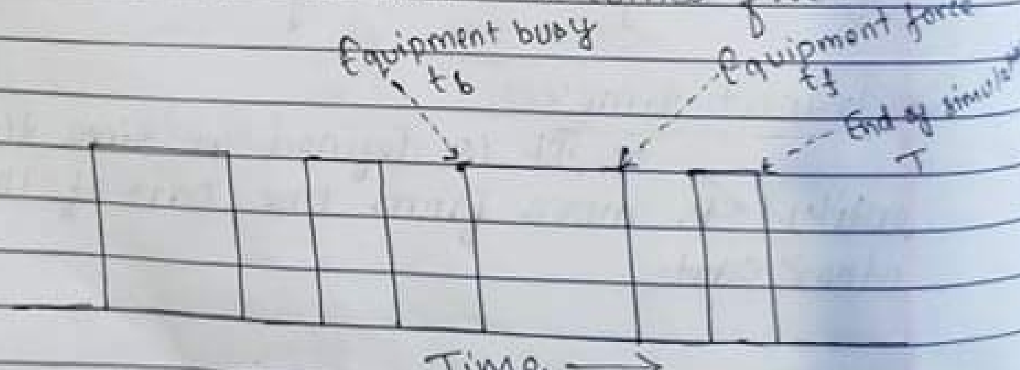


Fig: Utilization of equipments

Date / /
Page

The interval $t_f - t_b$ is added to a counter. At the end, the utilization is derived as:

$$U = \frac{1}{T} \sum_{r=1}^{N} (t_f - t_b)_r$$

Recording Distribution and Transit Time:-

To determine the distribution of a variable, we need to count the number of times the value of the variable falls within specific intervals. For each new value, it is compared with predefined intervals and for that interval counted up once. All information is kept in a table that requires:

i) The lower limit of tabulation (L)
ii) The interval size (dx)/(Δ)
iii) The number of intervals (N)

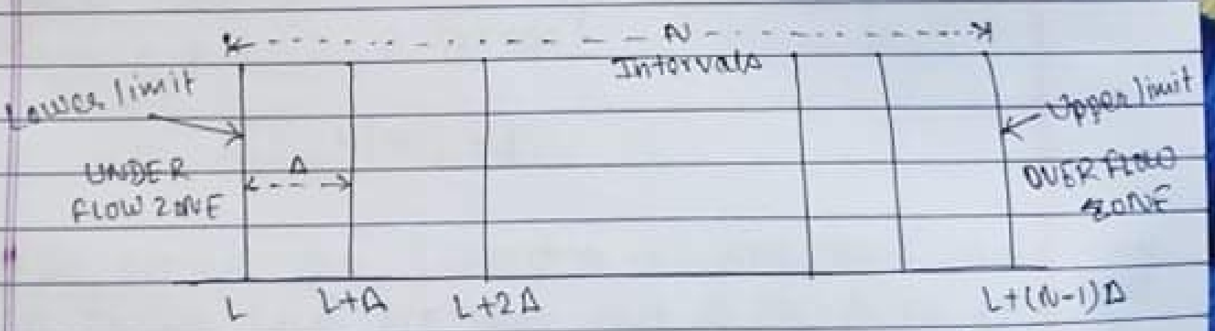


fig: Definition of a distribution table.

To measure transit time, the clock is used in the manner of a time stamp. Each entity holds time stamp. When entity reaches a point from which transit time is started, the arrival time is noted. later when the entity reaches to end point, the difference is computed.

Chapter 5 — Probability Concepts And Random Number Generation

Stochastic Variable:

- → Stochastic process give raise to stochastic variable
- → It can be discrete or continuous.
- → A stochastic process is described by a probability law called probability density function.

Discrete Probability functions:-

i) Probability Mass function (PMF):-

- → Distribution of probabilities.
- → If a variable can take n different values x_n ($n=1,2,3 \dots N$) and the probability of x_n being taken is $P(x_n)$ then the set of number comprise of $P(x_n)$ is probability mass function of a random variable x.
- → The number of $P(x_i)$ must satisfy the following two condition:
  - $P(x_i) >= 0$ for all i
  - $\sum_{i=1}^{n} P(x_i) = 1$

→ In certain case like dice roll, the PMF values may be known (e.g: 1/6). But most of the time PMF is counted from input sample.

Date / /
Page

ii) Cumulative Distribution Function (CDF) :-

→ It is a function which gives the probability of a random variable being less or equal to random variable being less or equal to a given value.

→ In discrete part, the CDF is denoted by $P(x_i)$. This function implies that x takes values less than or equal to x_i.

iii) Continuous Probability Function (CPF) :-

→ If the random variable is continuous and not limited to discrete values, it will have an infinite number of values ~~it~~ in an interval. Such a variable is defined by a function $f(x)$ called a probability density function (PDF). The probability that a variable x falls between x and $x+dx$ is expressed as $f(x)dx$ and the probability that x falls in the range x_1 to x_2 is given as:

$$P(x) = \int_{x_1}^{x_2} f(x)\, dx$$

Date / /
Page

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Random Number:-

A random number is a number generated by a process, whose outcome is unpredictable and which cannot be subsequently reliable reproduced. Random numbers are the basic building blocks for all simulation algorithm.

The properties of random numbers are given below:

→ The two important statistical properties are:
a) Uniformity b) Independence

→ Each random number R_i is an independent sample drawn from a continuous uniform distribution between 0 and 1. The probability density function is given by:

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

→ The expected value of each R_i is given below:

$$E(R) = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

→ The variance is given by,

$$V(R) = \int_0^1 x^2 dx - [E(R)]^2$$

$$= \left[\frac{x^3}{3}\right]_0^1 - \left[\frac{1}{2}\right]^2$$

$$= \left(\frac{1}{3} - 0\right) - \frac{1}{4}$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

POCO
SHOT ON POCO F1

→ The consequences of uniformity and independence properties are:

i) If the interval (0,1) is divided into n classes or subintervals of equal length, then the expected number of observation in each interval is N/n, where N is the total number of observations.

ii) The probability of observing a value in a particular interval is independent of previous values drawn.

Techniques for generation of random number:-

→ In computer simulation, where a very large number of random numbers is generally required, the random numbers can be obtained by the following methods:

i) Random number may be drawn from the random number tables stored in the memory of the computer.

ii) Using electronic devices.

iii) Using arithmetic operations.

The main techniques of generating random numbers are:

1) Congruence or Residue Method or Linear Congruential Method:-

→ In 1951 by Lehmer, this algorithm is described by the expression,

$$r_{i+1} = (ar_i + b) \text{ modulo } m \quad \text{---} \quad (1)$$

where,

a, b, m are constants

r_i and r_{i+1} are i^{th} and $(i+1)^{th}$ random number

r_0 is also required called seed value.

→ If $a > 1$, $b > 0$; it is of fixed type.

→ If $a = 1$, the expression reduces to the additive type.

i.e. $r_{i+1} = r_i + b$

→ If $b = 0$, the expression reduces to multiplicative type.

i.e. $r_{i+1} = a \cdot r_i$

Eg: Mixed multiplicative congruential (MMC) generator

$\{ r_{i+1} = (ar_i + b) \bmod m \}$

let $a = 13$, $b = 1$, $m = 19$ and $r_0 = 1$

Soln

when $i = 0$,

$r_1 = (13 \times 1 + 1) \bmod 19$
$= 14 \bmod 19$
$= 14$

$r_2 = (13 \times 14 + 1) \bmod 19$
$= 183 \bmod 19$
$= 12$

$r_3 = (13 \times 12 + 1) \bmod 19$
$= 157 \bmod 19$
$= 5$

$r_4 = (13 \times 5 + 1) \bmod 19$
$= 66 \bmod 19$
$= 9$

$r_5 = (13 \times 9 + 1) \bmod 19$
$= 118 \bmod 19$
$= 4$

∴ The random numbers are 14, 12, 5, 9, 4 and so on---

Date / /
Page

2) <u>Arithmetic Congruential Generator</u> :-

→ The random numbers are given by the equation,

$$r_{i+1} = (r_{i-1} + r_i) \bmod m$$

where,

$m \rightarrow$ constant

r_{i+1}, r_i, r_{i-1} are random numbers $(i+1)^{th}$, i^{th} and $(i-1)^{th}$

E.g.: Let $r_1 = 9$, $r_2 = 13$, $m = 17$

Soln

when $i = 2$,

$$r_3 = (r_1 + r_2) \bmod m = (9+13) \bmod 17$$
$$= 22 \bmod 17$$
$$= 5$$

when $i = 3$,

$$r_4 = (r_2 + r_3) \bmod m = (13+5) \bmod 17$$
$$= 18 \bmod 17$$
$$= 1$$

when $i = 4$,

$$r_5 = (r_3 + r_4) \bmod m = (5+1) \bmod 17$$
$$= 6 \bmod 17$$
$$= 6$$

when $i = 5$,

$$r_6 = (r_4 + r_5) \bmod m = (1+6) \bmod 17$$
$$= 7 \bmod 17$$
$$= 7$$

∴ The random numbers are, 9, 13, 5, 1, 6, 7 and so on.

Testing numbers for Randomness:-

A sequence of random numbers is considered to be random if,

i) The numbers are uniformly distributed i.e. every number has an equal chance of occurence.

ii) The numbers are not serially autocorrelated. This means that there is no correlation between adjacent pairs or numbers or that appears of the number does not influence the appearance of next number.

Test for random number:-

i) Uniformity test:-

- Compares the distribution of the set of numbers generated to a uniform distribution.
- Kolmogrov - Smirnov (K-S) test or Chi-square test

v.Imp

(a) Kolmogrov - Smirnov (K-S) test:-

This test compares the continuous CDF, $F(x)$ of the uniform distribution to the empirical CDF, $S_N(x)$ of the sample N observations.

By definition,

$$F(x) = x \; ; \; 0 \le x \le 1$$

$$S_N(x) = \text{no. of } R_1, R_2, R_3 \ldots R_N \text{ which}$$

$$S_N(x) = \frac{\text{are } \le x}{N}$$

Date / /
Page

Algorithm:-

1) Rank the data from smallest to greatest. Let $R(i)$ denote the i^{th} smallest observation among the N observation.

$$R(1) \leq R(2) \leq R(N)$$

2) Compute

$$D^+ = \max_{1 \leq i \leq N} \left\{ \frac{i}{N} - R(i) \right\}$$

$$D^- = \max_{1 \leq i \leq N} \left\{ R(i) - \frac{(i-1)}{N} \right\}$$

3) Compute.

$$D = \max \{ D^+, D^- \}$$

4) Determine the critical value D_α from the table for specific α and sample size N.

5) If $D > D_\alpha$, Null hypothesis is rejected. If $D < D_\alpha$, no difference has been detected; Null hypothesis is accepted (not rejected) and is acceptance criteria.

Date / /
Page

Q.N. Perform the uniformity test with a level of significance of $\alpha = 0.05$ on the following five generated numbers:
0.44, 0.81, 0.14, 0.05, 0.93 by using K-S test.

Soln

Here,

$N = 5$

$i = 1, 2, 3, 4, 5$

By using K-S test,

i	1	2	3	4	5
$R(i)$	0.05	0.14	0.44	0.81	0.93
i/N	0.2	0.4	0.6	0.8	1
$D^+ = \frac{i}{N} - R(i)$	0.15	$\boxed{0.26}$	0.16	-0.01	0.07
$\frac{(i-1)}{N}$	0	0.2	0.4	0.6	0.8
$D^- = R(i) - \frac{(i-1)}{N}$	0.05	-0.06	0.04	$\boxed{0.21}$	0.13

$\therefore D = \max(D^+, D^-)$

$D = \max(0.26, 0.21)$

From table,

Since, $D_{0.05} = 0.565 > 0.26$, the null hypothesis is accepted. and so, uniformity is also accepted.

SHOT ON POCO F1

Chi-Square Test (χ^2 test) :-

→ This test is used to check whether the sample of size, n from the generated sequence may be regarded as coming from a uniform (0,1) distribution.

→ Using the sample statistics, χ_0^2.

$$\boxed{\chi_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}}$$

where,

O_i = observed number in the i^{th} class.

E_i = Expected number in the i^{th} class

$$\boxed{E_i = \frac{N}{n}}$$

where, N = Total no. of observation

n = Expected no. of observation in a class

∴ $\boxed{\text{Degree of freedom} = n-1}$

Q.N.

Two digit random numbers generated by a multiplicative congruential method are given below. Test these data for uniform distribution using Chi-square. Is it acceptance at 95% confidence level? (Use $\chi^2_{0.005, 9} = 16.9$)

36, 91, 51, 02, 54, 06, 58, 06, 58, 02, 54, 01, 48,
97, 43, 22, 83, 25, 79, 95, 42, 87, 73, 17, 02,
42, 95, 38, 79, 29, 65, 09, 55, 97, 39, 83, 31,
77, 17, 62, 03, 49, 90, 37, 13, 17, 58, 11, 51,
92, 39, 78, 21, 66, 09, 54, 49, 90, 35, 84,

26, 74, 22, 62, 12, 90, 36, 83, 32, 75, 31, 94, 34, 87, 40, 07, 58, 05, 56, 22, 58, 77, 71, 10, 73, 23, 57, 13, 36, 89, 22, 68, 02, 44, 99, 27, 81, 26, 85, 22.

Soln

Class	Count	Frequency (O_i)	Difference $(O_i - E_i)$	$(Diff)^2$
$0 \le x < 10$		12	2	4
$10 \le x < 20$		8	-2	4
$20 \le x < 30$		12	-2	4
$30 \le x < 40$		12	2	4
$40 \le x < 50$		8	-2	4
$50 \le x < 60$		13	3	9
$60 \le x < 70$		5	-5	25
$70 \le x < 80$		10	0	0
$80 \le x < 90$		9	-1	1
$90 \le x < 10$		11	1	1

$N = 100$

$\Sigma(O_i - E_i)^2 = 56$

$$E_i = \frac{N}{n} = \frac{100}{10} = 10$$

where 'n' is the no. of class.

$$\chi_0^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= \frac{56}{10} = 5.6$$

Degree of freedom $= n-1$
$= 10-1$
$k = 9$

From table,

$\chi_0^2 = 16.92$ at $\alpha = 0.05 > 5.6$

Hence, the given set of random numbers is acceptable so far as its uniformity in distribution is concerned.

Pseudo Random Number:-

- Pseudo means 'fake'.
- Pseudo implies that the random numbers are generated by using some ~~[illegible]~~ known arithmetic operation.
- Since the arithmetic operation is known and the sequence of random numbers can be repeated obtained, the numbers cannot be called truly random.
- However, the pseudo random numbers generated by many computers routines, very closely fulfill the requirement of desired randomness.
- If the method of random numbers generation defective, the generated pseudo random numb...

Date / /
Page

may ~~be~~ have following departures from ideal randomness:

i) The generated numbers may not be uniformly distributed.

ii) The generated numbers may not be continuous.

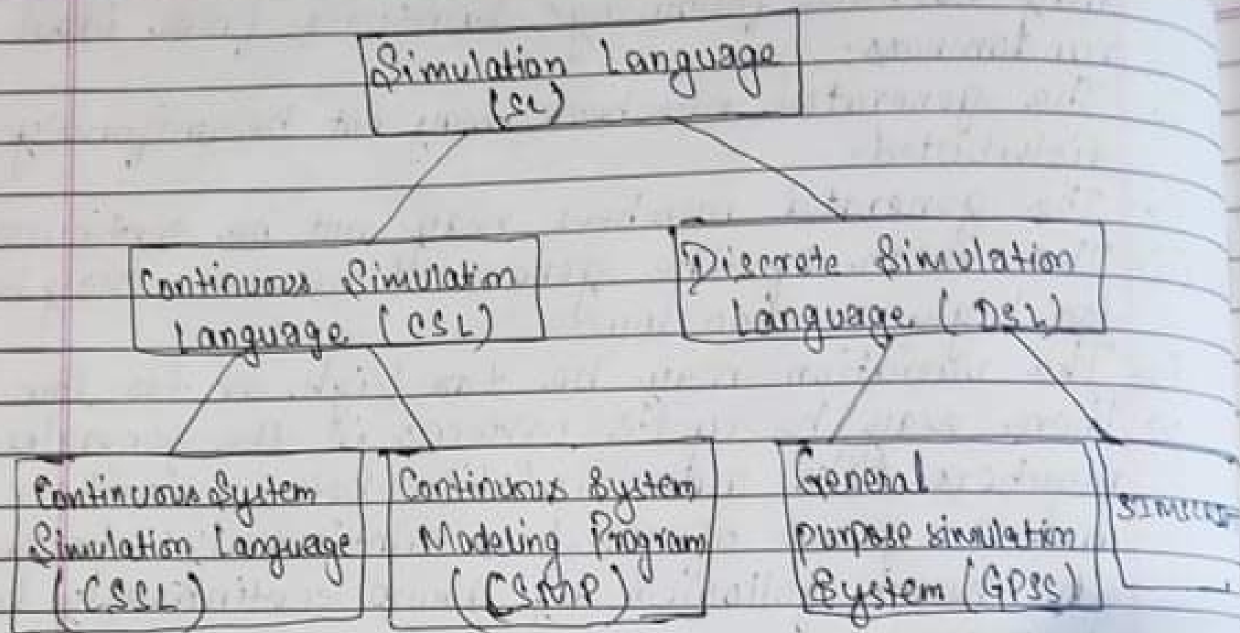
iii) The mean of the generated numbers may be too high or too low.

iv) The variation may be too high or too low.

v) There may be cyclic patterns in the generalized numbers like auto-correlation between numbers and a group of numbers continuously above the mean, followed by group continuously below of mean.

— Thus, before employing a pseudo random number generator, it should be properly validated, by testing the generated random number for randomness.

Chapter 6 Simulation Language



Simulation Language:-

- Simulation language provide conceptual framework in which the system analyst can work while building the model of system.
- There are varieties of simulation language.
- Simulation language can divided into two types Continuous Simulation Language and Discrete Simulation Language.
- If we are simulating military system, we have to use MILITRIAL but for simulating a dynamic model, we would use DYNAMO.

Date / /
Page

1) Continuous Simulation Language (CSL):-

→ They are designed for continuous model.
→ Before the digital computer came into use, analog computers were used for simulating continuous dynamic system.
→ It has two types: CSSL and CSMP

Describe in previous chapter (3)

2) Discrete Simulation language (DSL):-

→ They are designed for discrete model.
→ To represent the state of the system in the model, a set of numbers is used i.e. used in the discrete time system.
→ Discrete Simulation language is used to produce different programs, which helps to solve different complex work.
→ There are two types: ~~GS~~ GPSS and SIMSCRIPT

V.Imp i) General Purpose Simulation System (GPSS):-

→ The system to be simulated in GPSS is described as a block diagram in which block represents the activities and lines joining the blocks indicated the sequence in which the activities can be executed.
→ Where there is a choice of activities more than one line leaves a block and the condition for the choice is stated at the block.

→ Each block must give precise meaning.
→ There are 48 specific blocks, each of which represents a characteristics action of system.
→ The program should be written using these block diagrams.
→ Entities of the system depend upon the nature of the system.
→ E.g: A communication system is concerned with the movement of messages, a road transportation system with motor vehicles, a data processing system with records and so on.
→ In the simulation, these entities are called transactions.
→ The sequence of events in real time is reflected in the movement of transaction from block to block in simulated time.
→ Transaction start from GENERATE block and stops at TERMINATE block.
→ Transaction is hold in block and blocks can hold many transaction simultaneously.
→ Transfer of transaction from one block to another occurs ~~inst~~ instanteously at a specific time or when some change of system condition occurs.

Block System of GPSS:-

1) Action Time:-

i) ADVANCE

$\boxed{A, B}$

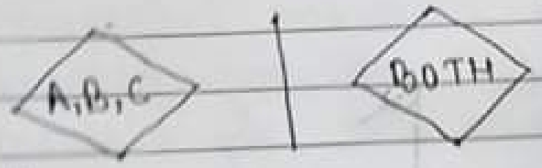
ADVANCE

ii) GENERATE

where, A → Mean
B → Modifier
C → Offset
D → Count
E → Priority
F → Parameter

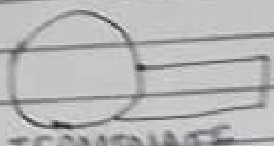
2) Succession of events:-

i) TRANSFER BLOCK



where,
A → Selection factor
B → Next Block B
C → Next Block C

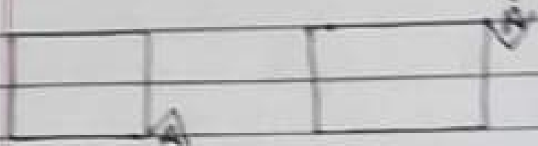
3) TERMINATE BLOCK



TERMINATE

4) Facilities

no. of release



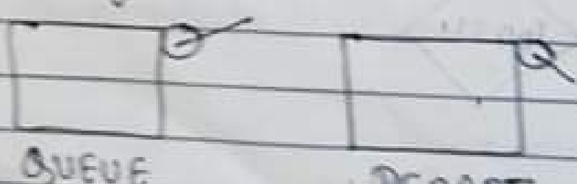
SEIZE RELEASE

5) Storage :

how many entities can enter?

ENTER LEAVE

6) Gathering Statistics.



QUEUE DEPART

MARK TABULATE

Facilities and Storage:-

→ A facility is defined as an entity that can be occupied by engaged by a single transaction at a time.
A storage is defined as an entity that can be occupied by many transaction at a time.

→ In addition, both facilities and storage can be made unavailable, as could occur if the equipment they represent breakdown and can be made available again, as occur when a repair has been made.

→ [Symbols]

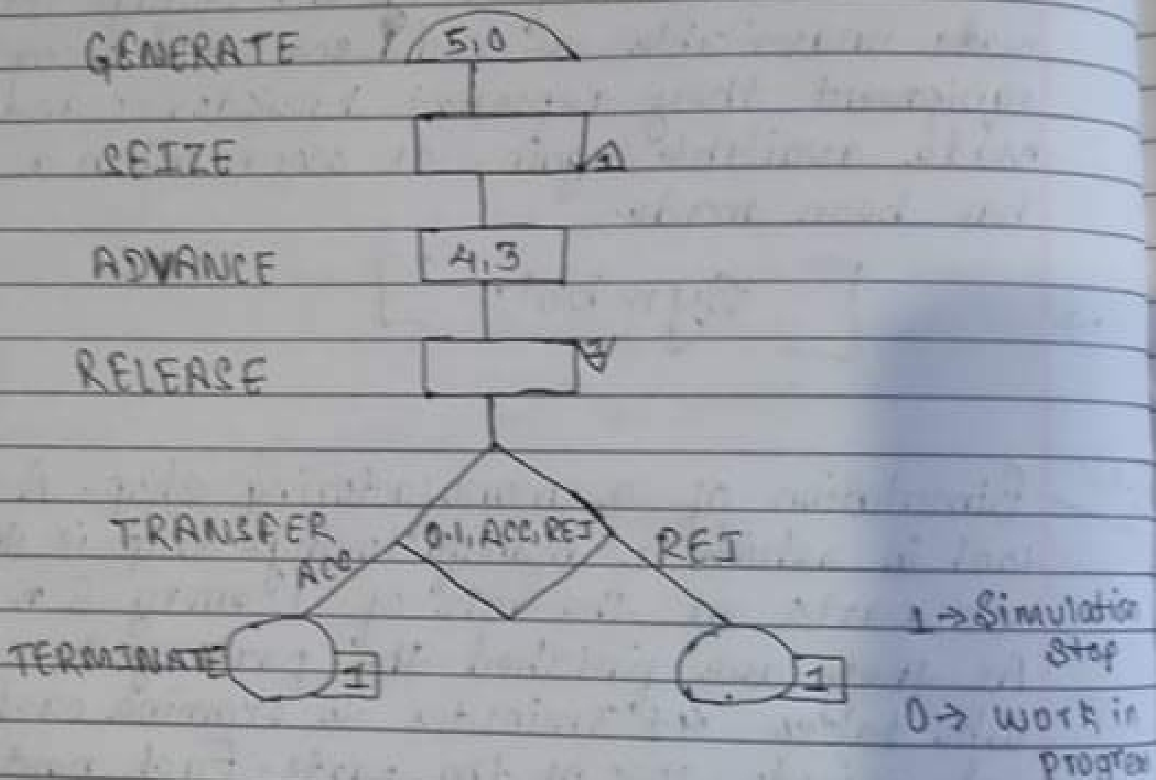
Q.N. Simulation of a manufacturing shop. A machine tool in which a manufacturing shop is turning out parts at the rate of 1 every 5 minutes. As they are finished the parts go to and inspector who takes 4 ± 3 minutes to examine each one and rejects 10% of the parts. Each part will be represented one transaction and the time unit selected for the problem will be one minute simulate for 1000 parts.

(i) Assuming for only one inspector.
(ii) Assuming for more than one inspector.

Sol^n

(i) When assuming for only one inspector:

The block diagram for simulation of a manufacturing shop:



The coding of above block diagram model is.

Date / /
Page

* MANUFACTURING SHOP MODEL

1. GENERATE	5	Create parts
2. SEIZE	1	Get inspector
3. ADVANCE	4,3	Inspect
4. RELEASE	1	Free inspector
5. TRANSFER	0.1,ACC,REJ	Select Rejects
6. ACC TERMINATE	1	Accepted parts
7. REJ TERMINATE	1	Rejected parts

*

START	1000	Run 1000 parts