* Introduction to Proofs:

A proof is a valid argument that establishes the truth of a mathematical statement. A proof can use the hypothesis of the theorem if any, axioms assume to be true, and previously proven theorem. Using these ingradients and rule of inference. The final step of the proof establishes the truth of the statement being proved.

In practice, the final step of the proof is usually just the conclusion of the theorem.

A less important theorem i.e. helpful in the proof of other results is called a lemma. Complicated proof are usually easier to understand when they are proved using a series of lemmas, when each lemma is proved individually.

A corollary is a theorem that can be establish from a theorem that has been proved.

A conjucture is a statement i.e. being proposed to be a true statement usually on the basis of some partial evidence, a heuristic argument, on the intution of an expert. When a proof of conjucture is found a conjucture becomes a theorem.

* Method of Proof:

Problem Solving or proving is not the science maile banako question paper bo pdf share garera hai

so there is no hard and fast rule i.e. applied in the problem solving. However, there are some guiding methods that helps us to solve different kind of problem.

1. Direct proof:

A direct proof of a conditional statement $p \to q$ is constructed when first step is assumption that p is true. subsequent step are constructed using rule of inference, with the final step showing that Q must be true.

The implication $p \to q$ can be proved by using that if p is true then q must also be true.

~~Doe~~ To carry out such a proof, we assume that hypothesis p is true and using information already available if conclusion q becomes true then the argument becomes valid.

Ex: If a and b are odd integer, then a+b is an even integer.

proof: $p \to q$ represents, we know that the fact that is a number is an odd number, we can represent as $2m+1$ where m is an integer.

Similarly, if a number is even it can be represented as $2n$ where n is an integer.

let,

$a = 2m+1$, n is an integer (i.e. a is odd)

$b = 2n+1$, n is an integer (i.e. b is odd)

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Now,

$a+b = 2m+1+2n+1$

$= 2m+2n+2$

$= 2(m+n+1)$

Since $(m+n+1)$ is an integer.
Hence, $(a+b)$ is even. proved

Q. Prove that if n is an odd integer then n^2 is an odd integer.

proof: we know that the fact, that is a number is an odd integer, we can represent as: integer

let: $p \to q$ represent: If n is odd integer then n^2 is odd.
We assume that the hypothesis of this statement is true, i.e. n is odd integer.

so,
by using the fact. we can represent n as; $n = 2a+1$
where a is an integer.

now,

$n^2 = (2a+1)^2$

$= 4a^2+4a+1$

$= 2(2a^2+2a)+1$

since, $2a^2+2a$ is an integer.
we can conclude that n^2 is odd integer. proved

Q. using direct proof, prove that for every positive integer n, n^3+n is even.

soln:

From the fact, a positive integer n can be either even or odd. ~~For~~ any positive even integer n can be represented as 2K where k is an integer. Similarly any positive odd integer n can be represented as $2k+1$ where k is an integer.

The given statement is $p \to q$: If n is positive integer then n^3+n is even.

using direct proof we can prove the statement by assuming the hypothesis true.

Case I:

Let n is even integer. i.e $n = 2k$

Then,

$$n^3+n = (2k)^3 + 2k$$
$$= 8k^3+2k$$
$$= 2(4k^3+k)$$

$\therefore n^3+n$ is even. ($\because 4k^3+k$) is an Integer.

Case II:

let n is odd integer:

i.e. $n = 2k+1$

Then, $n^3+n = (2k+1)^3 + (2k+1)$
$$= 8k^3 + 6k^2 + 6k + 1 + 2k + 1$$
$$= 8k^3 + 6k^2 + 8k + 2$$
$$= 2(k^3 + 3k^2 + 4k + 1)$$

$x = p/q \neq 0$

Since, $(4k^3+3k^2+4k+1)$ is integer, n^3+n is even.
Hence, for any positive integer n, n^3+n is even.
proved

Q. Prove that the sum of two rational number is rational number

proof:
The given statement can be written as:
$p \to q$: If ~~p is~~ x & y are rational numbers than sum of x & y is rational number.

Using direct proof, we can prove ~~that~~ the statement by assuming the hypothesis (p) true.
Suppose, that x & y are two rational numbers then by definition of rational number, it follows that there are two integer p and q where $q \neq 0$ such that $x = \frac{p}{q}$ and integer m and n with $n \neq 0$, such that $y = m/n$.

Now, $x+y = p/q + m/n = \frac{pn+qm}{qn}$

Since, ~~$q \neq 0$ &~~ $q \neq 0$ & $n \neq 0$ it follows that $qn \neq 0$.
Hence, we have expressed $A+B$ as ~~ration~~ ratio of two integers $(pn+qn)$ and qn with $qn \neq 0$.

Hence, $x+y$ is rational number. Thus, sum of two rational number is rational. proved

* Indirect Proof:

we have $p \to q \equiv \neg q \to \neg p$ i.e. contrapositive of implication is equivalent to implication. This is the basic for indirect proof. we proof the implication $p \to q$ by assuming that the conclusion is false and using known facts we show that the hypothesis is also false. And hence, the given arguement is true.

Ex. If the product of two integers a and b is even then either a is even or b is even.

Solution:

The statement is:

$p \to q$: If the product of two integers a and b is even, then either a is even or b is even.

Using, indirect proof we proof the statement by showing $\neg q \to \neg p$ is true.

let, a and b are both odd.

by using the fact that we can represent a and b as;

$a = 2n+1$ where n is ~~an~~ integer.

$b = 2y+1$ where y is an integer.

Now,

The product of a & b is

$a * b = (2n+1)(2y+1)$

$= 2n(2y+1) + 1(2y+1)$

$= 4ny + 2n + 2y + 1$

$= 2(ny + n + y) + 1$

Since, $2ny + n + y$ is an integer $a * b$ is odd.

Hence, the statement is true.

Q. Show that if $3n+2$ is odd then n is odd using indirect proof::

Solution:

The statement is.

$p \to q$: $3n+2$ is odd then n is odd.

using indirect proof we proof the statement by showing $\neg q \to \neg p$ is true.

let, n is even.

by using fact that we can represent n is,

$n = 2m$ where m is integer.

Now,

$n = 2m.$

$= 6m+2$

$= 2(3m+1)$ is even.

* Proof By Contradiction:

The Steps in proof of implication $p \to q$ by contradiction are:

step ①: Assume $p \wedge \neg q$ is true.

step ②: Try to show that the assumption is false.

when the assumption is found to be false then the implication $p \to q$ is true.

Since, $p \to q \equiv \neg p \vee q$

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$\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv \neg\neg p \wedge \neg q \equiv p \wedge \neg q$

~~# If n is even then n is divisible by 2.~~

Ex: If a^2 is an even number, then a is an even number

solution:-

The given statement $p \to q \Rightarrow$ If a^2 is even number then a ~~a is using proof by contradiction method~~ is even.

Using proof by contradiction method we prove the statement by assuming the $p \wedge \neg q$ true and showing the assumption is false.

So let, $p \wedge \neg q$ is true i.e.: a^2 is an even number and a is an odd number".

By using the fact that, any odd integer a can be represented as:

$a = 2k+1$ where k is an integer.

Now,

$a^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$

i.e. a^2 is odd & our assumption is false.

Hence, The given statement "........" is true. proved

* So that if n is an integer and n^3+5 is odd, then n is even. using contradiction and contraposition.

① By using contradiction:

The given statement $p \to q$ = If n^3+5 is odd, then n is even.

Now, using ~~contradiction:~~ contraposition:

Using indirect proof we proof the statement by showing

$\neg q \to \neg p$ is true.

let, n is odd.

by using the fact that we can represent n is;

$n = 2m+1$ where m is integer.

Now,

n^3+5

$= (2m+1)^3+5$

$= 8m^3 + \cancel{6} 3.4m^2 + 3\times 2.m.1 + 1 + 5$

$= 8m^3 + 12m^2 + 6m + 6$

$= 2(4m^3 + 6m^2 + 3m + 3)$ is even.

ii) using contradiction:

let $p \to n^3+5$ is odd.

q: n is even integer

we assume,

$p \wedge \neg q$ is ~~true~~ false.

i.e. p: n^3+5 is odd integer

$\neg q \to n$ is odd number.

Here,

$\therefore n^3+5 = (2m+1)^3+5$

$= 8m^3 + 12m^2 + 6m + 1 + 5$

$= 8m^3 + 12m^2 + 6m + 6$

$= 2(4m^3 + 6m^2 + 3m + 3)$ is even.

Q: proofs by cases:

The imprication of the form $(p_1 \vee p_2 \vee p_3 \vee \ldots \vee p_n) \rightarrow q$ can be prove by using the tautology $(p_1 \vee p_2 \vee p_3 \vee \ldots \vee p_n) \leftrightarrow [(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge \ldots \wedge (p_n \rightarrow q)]$ i.e. we can show every implication $(p_i \rightarrow q)$ true for $i = 1, 2, 3, \ldots, n$.

Ex: If $|x| > 3$ then $x^2 > 9$ where x is a realnumber.
Since, $|x|$ is an absolute value of x.
Now,
(i) if $|x| > 3$, $x^2 > 9$ which is true.
(ii) If $|-x| > 3$, $x^2 > 9$ is also true.
Hence, If $|x| > 3$ then $x^2 > 9$. proved

Trivial and vacus proof:

It is possible to show that q is correct regardless of truth values of p then we can say that implication $p \rightarrow q$ is true, This is trivial proof. If we can show that p is false then the impiication $p \rightarrow q$ is true then this is called vacus proof.

Ex:

i) If x is an integer, then 3 is an odd integer. (Trivial proof)
ii) If a black is white then pink ~~blue~~ is blue. (This is Vacus proof)

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* proof By Equivalence:

we can proof the equivalence i.e. $p \leftrightarrow q$ by showing $p \to q$ & $q \to p$ is both true.

* Formal proof:

1) Provides ~~gives~~ an arguments supporting the validity of the statement.

2) Such as the proof of theorems.

3) Shows that the conclusion follows ~~flows~~ from ~~the~~ premises. (hypothesis)

4) May use premises axioms and results of other theorem.

5) It shows that the proof ~~flow~~ follow logically the set of hypothesis and axioms.

Ex:

proof by rule of inferences, mathematical induction etc.

* Informal proof:

→ The steps of the proof are not expressed in any formal language like propositional logic.

→ steps argued less formally using english language, mathematical formulas, believes and so on.

* One must always watch the consistency of the argument made, logics and its rules can often help us to decide the surrounding of the argument, if it ~~is~~ is in question.

→ we use informal proof to illustrate different methods of theorem proving

Q. prove that "if n is a perfect square, then n+2 is not a perfect square. using
a) direct proof
b) indirect proof
c) proof by contradiction:

a) using ~~direct proof~~:

Sol: The given statement is,

$p \to q$: "If n is a perfect square, then n+2 is ~~square~~, then n+2 not a perfect square".

a) Using direct proof:

Using the direct proof we prove the statement by assuming p true and showing q is also true:

From the fact that, if n is a perfect square it can be represented as $n = k^2$, where k is an integer.

then,

$n+2 = k^2+2$ (This is not a perfect square.).

$\therefore$ Hence, the statement is proved.

b) Indirect proof:

$\neg p$: n is not a perfect square

$\neg q$: n+2 is ~~not~~ perfect square

Using indirect proof we prove the statement by showing $\neg q \to \neg p$ ~~is true~~, assuming $\neg q$ is true

let, (n+2) is ~~not~~ a perfect square.

by using the ~~fact~~ given fact we can represent (n+2) as:

$n+2 = k^2$ where k is an integer.

$n = k^2 - 2$ i.e. not perfect square.

∴ Hence, the statement is true. proved

(iii) By using contradiction:

By Assuming $p \land \neg q$,

$p = n$ is a perfect square

$\neg q = n+2$ is perfect square.

let,

$n = k^2$ where k is an integer

$n+2 = k^2+2$ is not perfect square.

Thus the assumption is false, then the given statement is true. proved

Q. show that the square of an even number is even number. using direct proof.

Q. show that at least 10 of any 64 days choosen must fall on same day of the week.