Unit - 3
Continuos System

Q) What is continuous model

When the continuous system is modeled mathematically, the variables of models representing the attributes of system are control by continuous function.

The distributed lag model is an example of continuous model. Since the continuos system, the relationship the variables describe the rate at which the value of variable change. this system consists of differential equation.

→ Continuos system simulation use the notation of differential eqn to represent the change on the basic parameter of the system with respect to the time. Hence, the mathematical model for continuos system simulation is usually represented by differential & partial differential eqn $\left(\frac{dt}{dt}\right)$

where, dt = rate of change

eg of continuous system simulation are
- Predator Pray model (Parasitee model)
- Pure Persuit model
- Serial chase model
- Projectile simulation

1) Predator Pay model [Parasite - Host Model

Predator $\xrightarrow{eat}$	Prey
Lion	Zebra
Beer	fish
Fox	Rabbit
Rabbit	Lettuce
Grasshopper	leaf

- Italian mathematician (Lokta volterra)
→ Ecological Predator pray host model

let,
a → Rate of growth of preg (↑)
b → Rate at which predetors destroy prey
C → Rate at which predetors increase consuming prey

γ – death rate

2

- Let, prey population at time(t) be $y_1(t)$ predator pop^n at time (t) be $y_2(t)$

→ On absence of predators the prey grow exponentially

$$y_1'(t) = a \cdot y_1(t) \quad [\because a > 0]$$

→ Now assume death rate of the prey due to interaction of potential to $y_1(t)$, $y_2(t)$ with positive propotional constant.

$$y_1'(t) = a \cdot y_1(t) - b y_1(t) \cdot y_2(t)$$

→ Without prey predators will die exponentially

$$y_2'(t) = -\gamma y_2(t) \quad [\because \gamma > 0]$$

→ Since birth depends on both pop^n size $[c > 0]$

$$y_2'(t) = -\gamma y_2(t) + c \cdot y_1(t) \cdot y_2(t)$$

From above eq^n

$$y_1'(t) = a \cdot y_1(t) - b y_1(t)$$

$$y_2'(t) = -\gamma y_2(t) + c y_1(t) \cdot y_2(t)$$

$\therefore \gamma$ = death rate

For the given pop^n

$y_1(t)$, $y_2(t)$

we compute for

$$\underbrace{(e^{at}, 0)}_{\text{Birth}}, \quad \underbrace{(0, e^{-at})}_{\text{Death}}$$

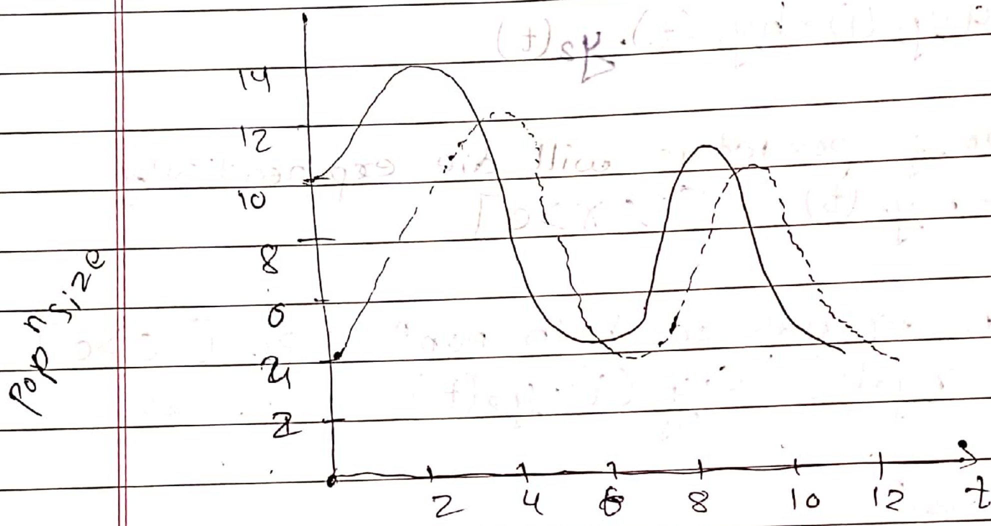
From the system we find the solution

$$y_1'\left(\frac{\gamma}{y_1} - c\right) + y_2'\left(\frac{a}{y_2} - b\right) = 0$$

Integrating both sides

$$r \log y_1(t) + a \log y_2(t) - b y_2(t) = \text{constant}$$

Lokta-Volterra
① fired order
② Non linear differential eq^n
③ Dynamics of biological system



-·-·· = Prey
—— = Predator

Differential equation 2 types
1) Linear Differential eq^n
2) Partial Differential eq^n

1) Linear Differential eq^n.

An example of linear differential equation with constant of coefficent to describe the wheel suspension system of an automobile can be given as

$$M\ddot{x} + D\dot{x} + kx = k_1 f(x)$$

Here, the dependent variable x appears first & second derivative

$\dot{x}$ = 1st derivate

$\ddot{x}$ = 2nd derivative

The simple differention eqn can model the simplest continuous system & they have one or more linear differential eqn in a constant coefficient. It is an often possible to solve the problem without using system simulation technique. i.e. we can solve such eqn using analytical method.

However the non linear involves into the model, it may be impossible or at least very difficult to solve such model without simulation.

Partial Differential eqn

When more than one independent variables occurs in a differential eqn the eqn is said to be a partial differtial eqn

It can involves the derivate of same independent variable with respect to each of independent variable

Differential eqn both linear & non linear occur frequenty in an scientific & engineering study. The reason for this is that most physical & chemical process involves rate of change which required differential eqn to represent their mathematical discriptions.

Since the partial differential eqn also represent a growth rate & continuous model can also be applied to a problem of social or economic nature.

Date
Page

* Analog Computer

Before the invention of digital computer, there existing devices whose behaviour is equivalent of mathematical operation such as ADD, SUB or integration. Putting together this device in manner specification by a mathematical model or equational system allowed us to simulate the system. Some device may have predict for simulation of continuous system. & it is called a analog computer or differential analyzer.

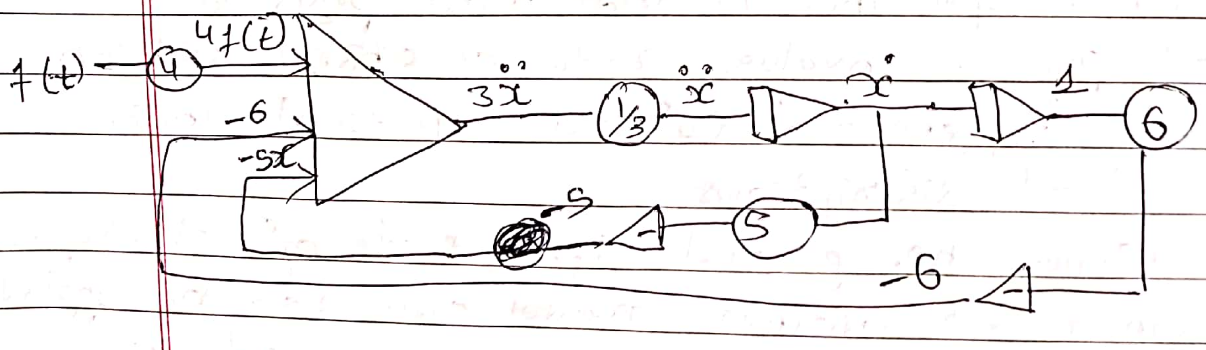
Analog Methode

The components used in analog Method representation are:-

1) ○
Scale factor applied to variable
2) ▷ : Addition
3) ▷ : Integration

③ Draw the analog method for the following system eqⁿ

① $4f(t) = 3x^2 + 5x + 6$
$3x^2 = 4f(t) - 5x - 6$



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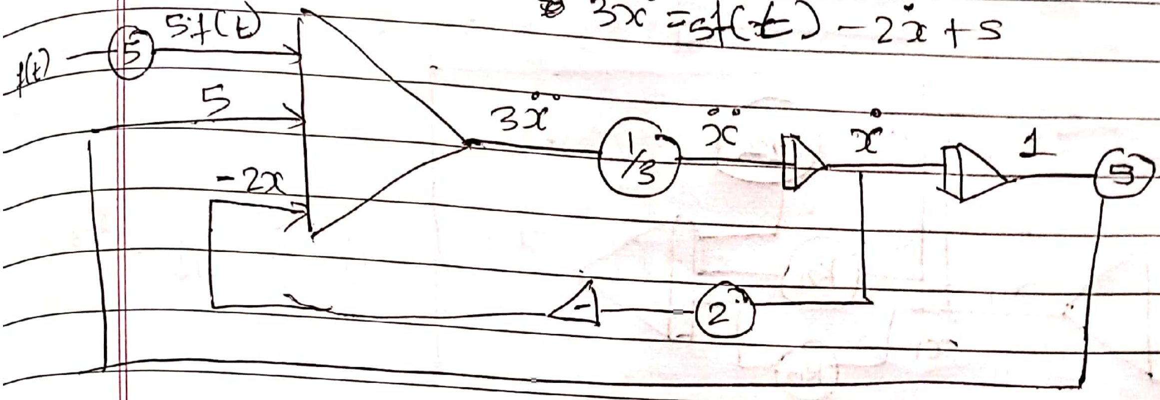
separate from highest order

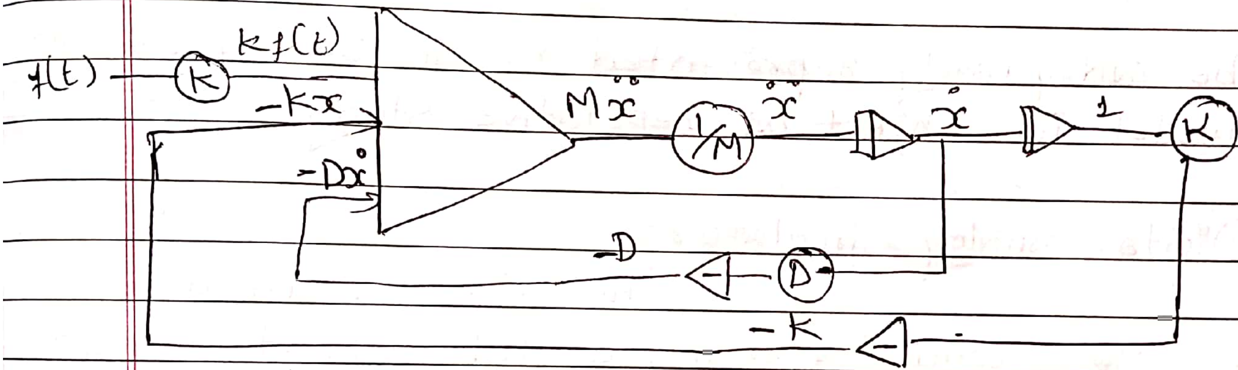
(2) $3\ddot{x}+2\dot{x}-5=5f(t)$

$3\ddot{x}=5f(t)-2\dot{x}+5$



(3) Q: $M\ddot{x}+D\dot{x}+Kx=Kf(t)$

$= M\ddot{x}=Kf(t)-D\dot{x}-Kx$



Hybrid Computer

The term hybrid computer has come to describe the combination of traditional analog computer elements, giving smooth continuous output & elements carrying out such non linear digital operation. Such as storing values switching & performing logical operation. The term had the meaning of extending analog computer capability due to the addition of special purpose & often specially constructed devices.

Hybrid computer may be used to simulate the system that are mainly continuous. Hybrid computer are also useful when a system that can

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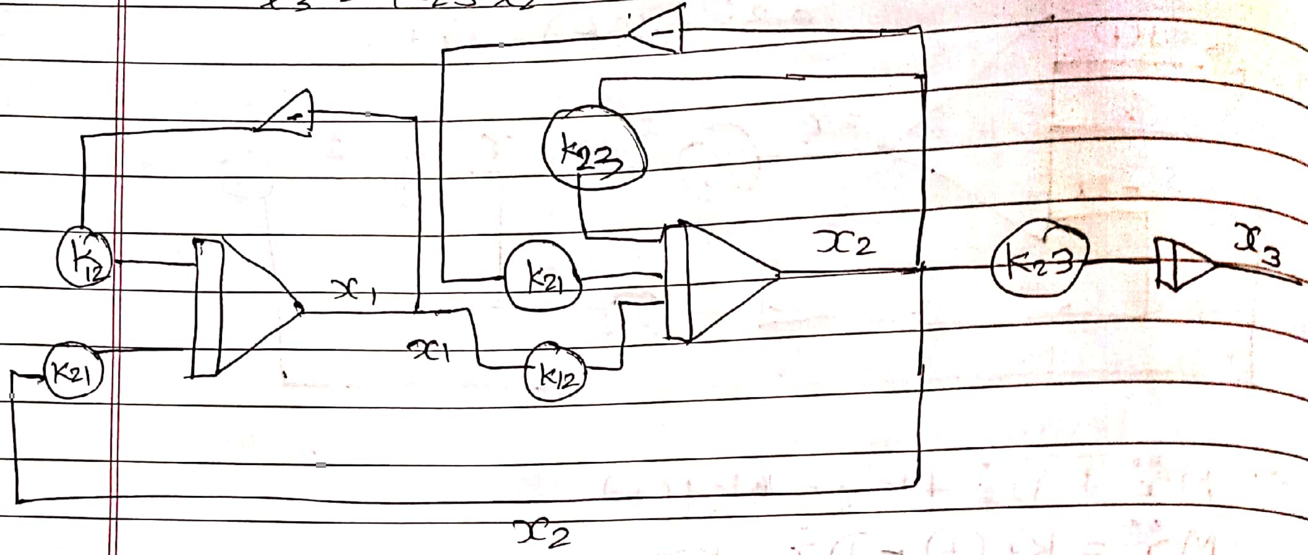
Imp $\dot{x}_1 = -K_{12}x_1 + K_{21} \cdot x_2$

$\dot{x}_2 = K_{12}x_1 - (K_{21} + K_{23})x_2$

$\dot{x}_3 = K_{23}x_2$



be adequately represented by an analog computer model in subject of repetative study.

Digital analog simulators

To avoid the disadvantage of analog computer many digital computer programming language have been return to produce digital analog simulator. They allow or facilate a continuos model to be programmed on a digital computer is essentially the same way as it solved on analog computer.

The language contain microinstructions that carry the action of additions, integration, & ~~science~~ sign changer. A program is written to link together with micro instruction in the same way as operational amplifier are connected together in analog computer.

Since more powerful digital computer & programming language have been developed for the purpose of simulating continuous system on digital computer, The digital analog simulator are now in expensive use.

Q Write a CSMP program of an automobile suspension system

$M\ddot{x} + D\dot{x} + Kx = KF(t)$

Ans

$M\ddot{x} = KF(t) - D\dot{x} - Kx$

$\ddot{x} = \frac{1}{M} \{KF(t) - D\dot{x} - Kx\}$

```
TITLE AUTOMOBILE SUSPENSION SXSTEM
**
PARAM D = (5.656, 16.968, 39.582, 56.56)
*
CONST M = 20 , F = 1.0 , K = 400.0
X2DOT = (1.0/M) * (K * F - D * XDOT - K * X)
XDOT = INTGRL (0.0, XDOT)
TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.05,
OUTDEL = 0.05
PRINT X, XDOT, X2DOT
PRTPLT X
LABLB DISPLACEMENT VERSUS TIME
END
STOP
```

Unit-V

Probability Concept & Random Number Generation

discrete should have finite value

Linear Congruential Method (LCM)

- Generating random number.
- Proposed by Lehmer 1951
- Method produces a sequence of integers $X_1, X_2 \ldots$ between 0 & $m-1$ by following a recursive relationship.

$$X_{i+1} = (aX_i + c) \bmod m \qquad i = 0, 1, 2$$

The multiplier, The increment, The modulus

- The initial value X_0 is called seed. The selection of the values for a, c, m & X_0 drastically affects the statistical properties & the cycle length.

a) If $c \neq 0$ in the above eq^n, then it is called as Mixed Congruential method.

b) If $c = 0$ the form is known as the Multiplicative Congruential method.

The random number (R_i) between 0 & 1 can be genereted by

$$R_i = \frac{X_i}{m}, \quad i = 1, 2 \ldots$$

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If $a=17$ $X_0=27$ $C=43$ $M=100$

Q:

Range = 0 - 99

Now,

Formula,

$$X_{i+1} = (aX_i + c) \bmod M$$

$X_0 = 27$ $i=0$

1) Random value:

$$X_1 = [17 \times 27 + 43] \bmod 100$$
$$= 2$$

$$R_1 = \frac{X_1}{m} = \frac{2}{100} = 0.02$$

2nd Random value

$$X_2 = [17 \times 2 + 43] \bmod 100$$
$$= 77$$

$$R_2 = \frac{X_2}{m} = \frac{77}{100} = 0.77$$

3rd Random value

$$X_3 = [17 \times 77 + 43] \bmod 100$$
$$= 52$$

$$R_3 = \frac{X_3}{m} = \frac{52}{100} = 0.52$$

4th Random value

$$X_4 = (17 \times 52 + 43] \bmod 100$$
$$= 27$$

$$R_4 = \frac{X_4}{m} = \frac{27}{100} = 0.27$$

5th Random value

$$X_5 = [17 \times 27 + 43] \bmod 100$$
$$= 2$$

$$R_5 = \frac{X_5}{m} = \frac{2}{100} = 0.02$$

$10000\overline{)721}(0$

6^{th} Random value

$X_6 = [17 \times 2 + 43] \bmod 100$
$= 77$

$R_6 = \frac{77}{100} = 0.77$

Q. Design a 4 digit random no. using linear Congruential method with value $X_0 = 21$, $a = 34$, $c = 7$
Find r_1, r_2, r_3, r_4, r_5.

→ Given,
$x_0 = 21$ $a = 34$ $c = 7$
Here, 4 digit random number so $M = 10000$
Now, Using formula

→ $x_{i+1} = (a \times x_i + c) \bmod M$ $x_0 = 21$ $i = 0$
$x_{0+1} = (34 \times 21 + 7) \bmod 10000$
$x_1 = 721$

$R_1 = \frac{x_1}{10000} = \frac{721}{10000} = 0.0721$

→ $x_2 = (34 \times x_1 + 7) \bmod 10000$
$= (34 \times 721 + 7) \bmod 10000$
$= 4521$

$R_2 = \frac{x_2}{M} = \frac{4521}{10000} = 0.4521$

→ $x_3 = (34 \times 4521 + 7) \bmod 10000 = 3721$
$R_3 = \frac{X_3}{M} = \frac{3721}{10000} = 0.3721$

→ $x_4 = (34 \times 3721 + 7) \bmod 10000 = 6521$
$R_4 = \frac{X_4}{M} = \frac{6521}{10000} = 0.6521$

→ $x_5 = (34 \times 6521 + 7) \bmod 10000 = 1721$
$R_5 = \frac{X_5}{M} = \frac{1721}{10000} = 0.1721$

$\frac{3/1}{64)12(}$ $\frac{\cancel{12}}{64}$ $\cancel{6}$ 3 $\cancel{32}$16

classmate

Date

Page

$a \neq 1$ (always)

Generate 5 digit number using additive ~~aditive~~ congeruent no. with value $x_0 = 5$ $c=7$, $m=64$

→ Given

$a = 1$ $\quad C = 7$ $\quad x_0 = 5$ $\quad m = 64$

Now,

$x_{i+1} = (a x_0 + c) \bmod M$

→ $x_1 = (1 \times 5 + 7) \bmod 64 = \cancel{1875}\ \cancel{1875}\ 12$

$R_1 = \frac{x_1}{M} = \frac{\cancel{1875}\,12}{64} = \cancel{29}\ \cancel{2968}\ 0.1875$

→ $x_2 = (1 \times \cancel{1875}^{12} + 7) \bmod 64 = 19$

$R_2 = \frac{x_2}{M} = \frac{19}{64} = 0.2968$

→ $x_3 = (1 \times 19 + 7) \bmod 64 = 26$

$R_3 = \frac{x_3}{M} = \frac{26}{64} = 0.4062$

→ $x_4 = (1 \times 26 + 7) \bmod 64 = 33$

$R_4 = \frac{x_4}{M} = 0.5156$

→ $x_5 = (1 \times 33 + 7) \bmod 64 = 40$

$R_5 = \frac{x_5}{M} = \frac{40}{64} = 0.625$

Linear congruence Generation:

It produces the sequence of integer $x_1, x_2 \ldots$ between 0 & m-1 may following by recursive relationship.

$x_{i+1} = (a x_i + c) \bmod m \quad i = 0, 1, 2 \ldots m-1$

If $c \neq 0$ then the form is called mixed congruential generator.
If $c = 0$, then the form is called multiplicative " "
If $a = 1$, then the form is called additive congruential generator.

S1 Define hypothesis for testing uniformity

$H_0 \sim R_i [0,1]$
$H_1 \nsim R_i [0,1]$

S2 Divide total number of observation, N into mutually exc equal no of close 'n'

$n \Rightarrow E_i \geq 5$
Expected frequency

$\frac{N}{n} \geq 5$

$E_i \geq \frac{N}{n}$ Total number

S3 Test, Stastics

$$\chi_0^2 = \sum_{i=1}^{n} \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

$\frac{100}{n} \geq 5$
$n \leq 20$

Interval, O_i E_i N/n $(O_i - E_i)^2/E_i$

S4 Determine critical value for given LOS with $(n-1)$ dof $\rightarrow$ degree of freedom
(α)
Level of significance

S5 $\chi_0^2 > \chi_{\alpha, n-1}^2 \Rightarrow H_0$ rejected
not uniform

Null Alter

H_0 accepted when