CHAPTER 2

Logic and Inductions:

The term logic term came from Greek word "logos" which is sometime translated also reason.

In artificant intelligence (AI) logic is defined as representation language of knowledge. There are lots of application of logic in the field of computer science such as designing circuit, programming, program verification etc.

The logic on the basic of it's representation capability are of two types.

① propositional logic
② predicate logic

propositional logic:

proposition is a declarative sentences that is either true or false, but not both.

Propositional logic is also known as sentential logic is the branch of logic that studies way of joining or modifying proposition to form more complicated proposition as well as logical relationship.

In propositions that are two logic of sentences Simple Sentences (atomic) and compound.

Compound Sentences express logical relationship between automic sentences.

Example:- (1) 2+2 = 5 (false) (proposition)
(2) 6-2 = 5 (false) "
(3) 2+2 = 4 (True) "
(4) Kathmandu is capital city of Nepal (True) "
(5) x+y = 10 (Not proposition)
(6) x>y (Not proposition)
(7) open the door (Not proposition)
(8) what time it is? (Not ")

* propositional variable:
Any statement whose truth value doesnot depend ~~one~~ on another proposition is called simple proposition.
In propositional logic these are often called propositional constant or sometime logical constant. Proposition are denoted by smaller letters P, Q, R, S, which are called propositional variable.

The truth value of Propositional is denoted by T for True and F for false proposition. The table which consist of all possible truth value of any proposition is called truth table.

* Logical operators or connectives: *

Logical operator are used to form compound propositions from existing simple (atomic) proposition.

① Negative ($\neg$) : NOT :

let p be any proposition. The negative of given proposition p denoted by $\neg p$ (read as not p) is called negation of p.

Ex- ↳ I love poets

↳ Negation of the statement is : I donot love poets

↳ If I love poets is denoted by p. It is negation I donot love poets is denoted by $\neg p$.

p	$\neg p$
T	F
F	T

Truth table of Negation.

② Conjunction (AND) : $\wedge$

The Proposition that is obtain by the AND operator is also called conjuction of propositions. Given that two proposition p & q the conjunction of p & q is represented as $p \wedge q$.

Ex: p : Ram is a boy

q : Ram is smart

$p \wedge q$: Ram is a boy and he is smart.

p	q	$p \wedge q$
T	T	T
F	T	F
T	F	F
F	F	F

Truth table of conjunction

③ Disjunction : OR : V

The proposition obtain by the user of OR ~~Operator~~ operator is also called Disjunction proposition. Given two proposition p & q the Disjunction of p & q is represented p∨q.

Ex:

p: It is cold.

q: It is raining

p∨q: It is cold or it is raining

p	q	p∨q
T	T	T
F	T	T
T	F	T
F	F	F

Truth Table of Disjunction

④ Exclusive OR : XOR : ⊕

Given two proposition p and q, the proposition exclusive OR of p & q is denoted by p⊕q is the proposition ~~to~~ i.e. True whenever only one of the proposition p or q is true, false otherwise.

Ex:

p: prakash drinks tea in the morning.

q: prakash drink coffee in the morning.

p⊕q: prakash drink either coffee or tea in the morning.

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p	q	p ⊕ q
T	T	F
F	T	T
T	F	T
F	F	F

Truth table of Exclusive OR

(5) Implication (→) (conditional implies):

Given two proposition p & q, the proposition: implication p→q is the proposition ~~to~~ i.e. false when p is true and q is false, otherwise true.

Here p is called Hypothesis or premise and q is called Conclusion or consequences.

There are different terminology to express p→q like.

1. If p then q
2. q is consequence of p
3. p is Suffecient for q.
4. q whenever p
5. q provides p
6. p only if q

Ex.:

p: Bina is pregnant.

q: She can give birth

p→q: If bina is pregnant then she can give birth

p	q	$p \to q$
T	T	T
F	T	T
T	F	F
F	F	T

Truth table of Implication

(6) Double Implication or Biconditional ($\leftrightarrow$):
let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q". $p \leftrightarrow q$ is true when both p and q have same truth values and is false otherwise. It is also called Bi-implementation. Other common ways to express $p \leftrightarrow q$ ~~are if p is necessary~~

→ p is necessary and sufficient for q.

→ p iff q.

→ if p then q and conversely.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Fig. Truth table

* Construct the truth table of compound proposition.
$(p \vee \neg q) \to (p \wedge q)$

p	q	$\neg q$	$(p \vee \neg q)$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

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* Translate the given sentences into logical expressions.

(1) Sita is sick and it is raining implies that Ram stayed up late night.

Sol^n:

let, the proposition:

p: sita is sick

q: It is raining

r: Ram stayed up late high.

Then the given sentences can be expressed as $(p \wedge q) \to r$

(2) Sita is sick and it is not the case that it is raining

Sol^n:

let the proposition

p: Sita is Sick

q: It is raining

Then the given sentences can be expressed as $p \wedge \neg q$

(3) You cannot ride the roller couster if you are under 4 feet tall, unless you are older than 16 year old.

Sol^n: let the proposition,

p: You can ride the roller couster

q: You are under 4 feet tall

r: you are older than 16 years old

$$(p \wedge \neg r) \to \neg p$$

④ You can access the internet from the campus if you are BCA graduated or staff of the campus.
Given Soln:
let the proposition :
p: You can access the internet
q: You are BCA graduated
r: You are staff of the campus
$(q \vee r) \to p$

⑤ It is raining if and only if the road is muddy.
Given: let proposition
p: It is raining
q: Road is muddy
$(p \leftrightarrow q)$

⑥ It is not the case that it is raining and Sita is sick.
let the proposition
p: It is raining
q: Sita is sick
$\neg(p \wedge q)$

⑦ Ram get 'A' on the final exam but ram do not do every exercise in this book.
p: Ram get 'A' on the final exam.
q: Ram does all exercise in this book.
The expression is
$p \to \neg q$

* Translate the paragraphs into the logical expressions.
"whenever the system is being upgraded, users cannot access file system. If the user can access the file system then they can save new files. If users cannot save new file then the system software is not being upgraded.

→ Let the proposition be

p: system is being upgraded
q: User can access the file
r: They can save new files

(a) $p \rightarrow \neg q$
(b) $q \rightarrow r$
(c) $\neg r \rightarrow \neg p$

The given paragraph can be written as:
$(p \rightarrow \neg q) \wedge (q \rightarrow r) \wedge (\neg r \rightarrow \neg p)$

(c) Translate the logical expression into english sentences.
(a) $\neg p$ (b) $r \wedge \neg p$ (c) $\neg r \vee p \vee q$

when,

p = "it rained last night"
q = "The sprinkes came on last night"
r = "The lawn was wet this morning".

(a) $\neg p \Rightarrow$ "It didnot rain last night".
(b) $r \wedge \neg p \Rightarrow$ "The lawn was wet this morning and it didnot rain last night".

(c) $\neg r \vee p \vee q \Rightarrow$ "The lawn was not wet this ~~moring~~ morning or it rained last night or the sprinkles came on last night."

(8) Let p, q, r be the proposition:

p: You have the flu
q: you missed the final examination
r: you pass the course.

Express each of these proposition as an English sentences.

(a) $p \to q$ (b) $q \to \neg r$ (c) $(p \wedge q) \vee (\neg q \wedge r)$
(d) $\neg q \leftrightarrow r$

(a) $p \to q$ if you have the flu then you missed the final examination.

(b) $q \to \neg r$: you missed the final examination, you couldnot pass the Course.

(c) $(p \wedge q) \vee (\neg q \vee r)$: you have the flu and you missed the final examination and you pass the course.

(d) $\neg q \leftrightarrow r$: you didnot missed the final examination.

Let p and q be the propositions

p: It is below freezing

q: It is snowing

write these propositions using p and q and logical connective.

(a) It is not below freezing and it is not snowing.

(b) If it is below freezing, it is also snowing.

(c) It is either below freezing or it is snowing but it is not snowing if it is below freezing.

$$(p \vee q) \wedge (p \rightarrow \neg q)$$

(d) That it is below freezing is necessary and sufficient for it to be snowing $(q \leftrightarrow p)$

* Inverse, converse and contrapositive of Implication.

(1) Inverse:

The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$.

(2) Converse:

The converse of $p \rightarrow q$ is $q \rightarrow p$

(3) Contrapositive:

The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$

* Write inverse, converse, and contrapositive of the sentence.

If two angle are congruent, then they have same measure.

Let, the proposition be

p: Two angle are congruent

q: They have same measure

① Inverse : $p \to q$: $\neg p \to \neg q$ (If two angle are not congruent, they have not same measure)

② Converse : $p \to q$: $q \to p$ (They have same measure then two angle are congruent).

③ Contrapositive: $p \to q$: $\neg q \to \neg p$

Statement : They have not same measure the two angle are not congruent.

* Tautology, contradiction and contingency:

→ Tautology:

A compound proposition i.e. is always true no matter the truth value of atomic proposition that in it called tautology.

Ex (i) $p \vee \neg p$ (ii) $\neg(p \to q) \to \neg q$.

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¬ (p→q) → ¬q

p	¬p	p∨¬p
T	F	T
F	T	T

¬q	p	q	p→q	¬(p→q)	
F	T	T	T	F	T
T	T	F	F	T	T
F	F	T	T	F	T
T	F	F	T	F	T.

* Contradiction:

No matter truth value of atomic proposition that in it is called contradiction.

Ex

(1) p∧¬p

p	¬p	p∧¬p
T	F	F
F	T	F

(2) (p∨q) ∧ (¬p∧¬q)

p	q	¬p	¬q	p∨q	¬p∧¬q	
T	T	F	F	T	F T	F
T.	F	F	T	T	T.	
F	T	T	F	T	T	
F	F	T	T	F	T	

* Contengency:

The compound proposition i.e. neither tautology nor a contradiction is called contengency.

Ex (1) ¬p∧¬q

p	q	¬p	¬q	¬p∧¬q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

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* Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.

p	$\neg p$	q	$\neg q$	$p \to q$	$\neg q \wedge (p \to q)$	$(\neg q \wedge (p \to q)) \to \neg p$
T	F	T	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	T	F	T	T	T	T

Propositional equivalences:

* If two proposition are symantically identical then they are called propositionally equivalent or logically equivalent.

To test whether two propositions p and q are logically equivalent, the following steps are followed.

→ Construct the truth table of p

- Construct the truth table of q

- If the truth value set of p and q are same, then they are called equivalent.

* Assignment 1

1. Show that $(p \to q) \wedge (q \to r)$ and $p \vee q) \to r$ are logical equivalent.
2. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

Construct the truth table for the following

Logical equivalence:

The compound proposition p & q are called logically equivalent if $p \to q$ is a tautology. The notation $p \equiv q$ denotes that p & q are logically equivalent. The symbol $\Leftrightarrow$ is sometimes used instead of $\equiv$ to denotes equivalence.

(1) Identical laws:

$p \wedge T \equiv p$

$p \vee T \equiv p$

(2) Domination laws:

$p \vee T \equiv T$,

$p \wedge F \equiv F$

(3) Idempotent law:

$p \vee p \equiv p$, $p \wedge p \equiv p$

(4) Double Negation laws:

$\neg(\neg p) \equiv p$

(5) Commulative laws:

$p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$

(6) Association laws:

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(7) Distributive laws:

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

(8) Demorgan law:

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

(9) Absorption law:

$$p \wedge (p \vee q) \equiv p$$
$$p \vee (p \wedge q) \equiv p$$

(10) Negation laws:

$$p \vee \neg p \equiv T$$
$$p \wedge \neg p \equiv F$$

(11) Conditional laws:

(a) $p \to q \equiv \neg p \vee q$

(b) $p \to q \equiv \neg q \to \neg p$

(c) $p \vee q \equiv \neg p \to q$

(d) $p \leftrightarrow q \equiv \neg p \leftrightarrow q$

(e) $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

(f) $p \wedge q \equiv \neg(p \leftrightarrow \neg q)$

(g) $\neg(p \to q) \equiv p \wedge \neg q$

* Predicate logic:

Any declaritive statements involving variables often found in mathematical expression and in computer programs, which are nither true or false where the values are not specified is called predicate. logic involving predicate is called calculas. Similar to logic involving proposition is called propositional logic.

let, us take a statement $5 > 9$, this is a propositional statement because it is false. Now let us take $n > 5$. This statement is neither true nor false depending upon the value of x. We can say that any statement involving a variable is not proposition.

The predicate $x>5$ has two parts the variable x is subject of statement and another part '>5' called predicate. We can denote $x>5$ by $p(x)$ where p is predicate '>5' and x is the function. We can tell whether $p(x)$ gives values of P at x.

"Ex: let $p(x)$: $x+3<12$, find truth value of $p(5)$ and $p(10)$.

Solution: Given $p(x) \Rightarrow x+3<12$

when $x=5$, $p(5) \Rightarrow 5+3<12$ i.e. $8<12$ (True)

when $x=10$, $p(10) \Rightarrow 10+3<12$ i.e. $13<12$ (False.

Quantifier:

Quantifier are the tools that change the propositional function into a proposition.

Construction of proposition from the predicate using quantifiers is called quantification. The variables that appears in the statement can take different possible values and all the possible values that the variable can take forms the domain, called universal ~~discourse~~ discourse or universal set.

There are two types of quantifiers:

1. Universal Quantifier
2. Existential quantifier.

① Universal quantifier:

The phrase 'for all' denotes by $\forall$, is called universal quantifier. The process of conversing predicate into proposition using universal quantifier is called universal quantifier.

So, the universal quantification of $p(x)$ denoted by $\forall x\, p(x)$ is a proposition where $p(x)$ is true for all the values of x in universal set.

$\forall x\, p(x)$ is read as for all x, $p(x)$.

Ex: Take the universal set of all student of this class & $p(x)$: represents x takes MFCS class Then, $\forall x\, p(x)$ means all students of this class takes MFCS class.

② Existential quantifier:

The phrase "There exist", denoted by $\exists$ is called Existential quantifier.

The process of converting the predicate logic into proposition using existential quantifier. The existential quantification of $p(x)$ is denoted by $\exists x\, p(x)$.

$\exists x\, p(x)$ is read as; there exist x, such that $p(x)$ is true for atleast one x.

Ex: Take the universal set of all students of this College.

$p(x)$ represents, x takes MFCS class.

Then,

$\exists x\, p(x)$ means, at least one student in this college takes MFCS class.

or There exist at least one student in this college who takes MFCS class.

* Nested Quantifier:

When we use more than one quantifier in a sequence, then it is known as nested quantifier.

For example:

$\forall x \exists y\, p(x, y)$

Here, $\exists$ and $\forall$ are nested in a sequence.

let, $L(x, y)$: x loves y

(1) Everybody loves somebody.

$\forall x \exists y (x, y)$

(2) Someone love somebody.

$\exists x \exists y\, L(x, y)$

(3) Everybody loves Everybody

$\forall x \forall y\, L(x, y)$

Q. Translate the given sentences into logical expression.

(1) All man are mortal.

let, $p(x) = x$ is a man

$m(x) = x$ is mortal

$\therefore$ This statement can be written as:

$\forall x\, p(x) \rightarrow m(x)$

(ii) Any integer is either even or odd.

let,
$i(x)$ = x is an integer
$e(x)$ = x is even integer
$o(x)$ = x is odd integer

∴ This statement can be written as:

$\forall x\, I(x) \rightarrow o(x) \vee e(x)$

(iii) Some student of the CCT college passed the IOE entrance examination.

let,
$s(x)$ = x is student of CCT college.
$p(IOE)$ $p(x)$ = x passed the IOE entrance examination.

Then the statement can be written as

$\exists x\, s(x) \rightarrow p(x)$

$\exists x\, p(x, IOE)$

(iv) Everybody has claim buddha was born in Nepal.

let,
$c(x)$: x has claimed buddha was born in Nepal.

$\forall x\, c(x)$

(v) Nobody has claimed Buddha was Born in Nepal

$\neg \forall\, c(x)$

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(vi) Everybody has claimed Buddha was not born in Nepal.
↳ $\forall \neg c(x)$

(vii) Nobody has claimed Buddha was not born in Nepal.
$\neg \forall \neg c(x)$

<u>Note</u>: let $\forall x p(x)$ is a quantified statement, its negation is $\neg \forall x p(x)$, which is logically equivalent to $\exists x \neg p(x)$.
similarly,
$$\neg \exists x p(x) \equiv \forall x \neg p(x)$$
$$\forall x \neg p(x) \equiv \neg (\exists x p(x)]$$
$$\exists x \neg p(x) \equiv \neg (\forall x p(x)]$$

(viii) No girls in KTM is lovely.
let,
$L(x)$ = x is lovely girl in ktm
$\forall x \neg L(x)$

(ix) Not all girls are lovely.
$\exists x \neg L(x)$

Q. Let $L(x,y)$ be the statement "x loves y", where the domain for both x & y consists of all people in the world.
Use quantifier to express each of these statement.

(a) Everybody loves Jerry.

$L(x)$ = x loves Jerry

$\forall x\ L(x, \text{Jerry})$

(b) There is somebody whom everybody loves.

$\exists y\ \forall x\ L(x, y)$

(c) There is somebody whom Jerry doesnot love.

$\exists x\ \neg L(\text{Jerry}, x)$

(d) Everybody loves herself or himself.

~~let~~ let, $L(x)$: x loves herself or himself

$\forall x\ L(x, x)$

* Mathematical Induction :-

Any of the mathematical statements must be supported by arguments that make it correct. For this we need to know different technique, and rules that can be applied in the mathematical statement such that we can prove the correctness of the given mathematical statement.

Q. What are the free and bound variable? let, $L(x,y)$ denotes x loves y where the universe of discourse for x and y ~~is~~ is set of all people in the world. Translate the following expression into english sentences.

(a) $\forall x \exists y\ L(x, y)$

- Everybody loves somebody.

(b) $\exists y \forall x\ L(x, y)$

→ There is somebody whom everybody loves.

(c) $\exists x \forall y\ L(x, y)$

→ ~~Somebody~~ somebody loves Everybody.

(d) $\exists x \exists y\ L(x, y)$

→ somebody loves somebody.