* Rules of Interference:

To draw conclusion from the given premise we must be apply to some well defined steps that help reaching the conclusion. These steps of reaching conclusion are provided by the rules of interference. Here are some rules of interference given below:

(1) Rules 1: Modus pones (or law of Detachment):

when ever two propositions $p$ & $p \to q$ are both true then we confirm that $q$ is true. we write this rule as:

$$\frac{p,\ p \to q}{\therefore q} \qquad \frac{p \to q,\ p}{\therefore q}$$

This is valid rule of inference because the implication $p \wedge (p \to q) \to q$ is tautology.

Rule 2: Hypothetical Syllogism (Transitive rule):-
when ever two proposition $p \to q$ and $q \to r$ are both true, than we confirm that $p \to r$ is true, we write this as;

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array} \qquad \begin{array}{c} p \to q,\ q \to r \\ \hline \therefore p \to r \end{array}$$

This is valid rule of interference because the implication $((p \to q) \wedge (q \to r)) \to (p \to r)$ is tautology.
This rule can be extended to the larger no. implications as :-

$$\begin{array}{l} p \to q \\ q \to r \\ r \to s \\ \hline \therefore R \to S \end{array}$$

(3) Rule 3: Addition Rule: Due to tautology $p \to (p \vee q)$ the rule.

$$\begin{array}{c} \therefore p \\ \hline p \vee q \end{array}$$ is a valid rule of inference.

(4) Rule (4): Simplication Rule:
Due to tautology $(p \wedge q) \to p$.
The rule, $\dfrac{p \wedge q}{\therefore p}$ is a valid rule of inference.

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Rule 5: Conjunction Rule:

Due to the tautology $((p) \wedge (q)) \rightarrow (p \wedge q)$.

The rule, $\dfrac{p, q}{\therefore p \wedge q}$ is a valid rule of inference.

Rule 6: Modus Tollens:

Due to the tautology $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$.

The rule,

$$\begin{array}{l} \neg q \\ \underline{p \rightarrow q} \\ \therefore \neg p \end{array}$$

is a valid rule of inference.

Rule 7: Resolution Rule:

Due to tautology $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$,

The rule,

$$\begin{array}{l} p \vee q \\ \underline{\neg p \vee r} \\ \therefore q \vee r \end{array}$$

is a valid rule of inference.

Summary:

| S.N | Name | Rule of Inference | Tautology |
|---|---|---|---|
| Rule 1 | Modus pones | $\dfrac{p, p \rightarrow q}{\therefore q}$ | $p \wedge (p \rightarrow q) \rightarrow q$ |
| Rule 2 | Transitive Rule | $\dfrac{p \rightarrow q, q \rightarrow r}{\therefore p \rightarrow r}$ | $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ |

~~Exercise~~

| | | | |
|---|---|---|---|
| Rule 3 | Addition Rule | $\dfrac{p}{\therefore p \vee q}$ | $p \rightarrow (p \vee q)$ |
| Rule 4 | Simplification Rule | $\dfrac{p \wedge q}{\therefore p}$ | $(p \wedge q) \rightarrow q$ |
| Rule 5 | Conjunction Rule | $\dfrac{p, q}{\therefore p \wedge q}$ | $((p) \wedge (q)) \rightarrow (p \wedge q)$ |
| Rule 6 | Modus Tollens | $\dfrac{\neg q, p \rightarrow q}{\neg p}$ | $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ |
| Rule 7 | Resolution Rule | $\dfrac{(p \vee q) \wedge (\neg p \vee r)}{\therefore q \vee r}$ | $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ |
| Rule 8 | Disjunctive syllogism | $\dfrac{p \vee q \quad \neg p}{\therefore q}$ | $(p \vee q) \wedge (\neg p) \rightarrow q$ |

∴ Valid Argument:

An argument in propositional logic is valid if the truth of all its premises implies that conclusion is True.

i.e. the argument with premises $p_1, p_2, \ldots p_n$ and conclusion $q$ is valid when, $(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology.

Ex: Determine whether the following statement is valid or not.

Statement: If Ram is a human then Ram is mortal. Ram is human. Therefore Ram is mortal.

**Solution:** let the propositions be:

p: Ram is human

q: Ram is mortal

The given ~~statement are~~ hypothesis are:

$p \to q$ & $p$ and conclusion is $q$.

Now the statement is valid if $(p \wedge (p \to q)) \to q$ is tautology.

Truth Table:

| $p$ | $q$ | $p \to q$ | $p \wedge (p \to q)$ | $(p \wedge (p \to q)) \to q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

$\therefore$ so $(p \wedge (p \to q) \to q)$ is tautology, the given statement is valid.

$\therefore$ Fallacy:

The fallacies are the arguments that are convencing but are not true and produce faulty inferences.

So the fallacies are contengencies rather than tautologies.

Ex

$q$

$p \to q$

---

$\therefore p$ is a fallacy.

Ex: If the economy of the Nepal is poor, then the education system in Nepal will be poor. The education system in Nepal is poor. Therefore, economy of Nepal

Ex:

If you do every problem of MFCS then you are perfect in MFCS. You are perfect in MFCS.
Therefore, you did every problem of MFCS.
let,

p: You do every problem of MFCS.
q: You are perfect.

Hypothesis: $p \to q$, & $q$ Conclusion: $p$

i.e. $p \to q$

$q$

$\therefore p$

Using Truth table for $((p \to q) \wedge q) \to p$ we get,

| $p$ | $q$ | $p \to q$ | $(p \to q) \wedge q$ | $((p \to q) \wedge q) \to p$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | F |
| F | F | T | F | T |

# Rules of Inference for Quantified Statement :

(1) Universe Instatiation:

If the proposition of the form $\forall x\, p(x)$ is supposed to be true then the universal quantifier canbe dropped out to get $p(c)$ is true for every arbitary universe of discourse.

i.e.
$$\frac{\forall x\, p(x)}{\therefore p(c)}$$

Ex: The universe of discourse of all man is mortal implies that prakash is mortal where prakash is a man.

(2) Universal generalization:

If all the instances of c makes $p(c)$ true, then $\forall x\, p(x)$ is true.

i.e.
$$\frac{p(c) \text{ for an arbitiary } c}{\therefore \forall x\, p(x)}$$

(3) Existential Instatiation:

If the proposition of the form $\exists x\, p(x)$ is supposed to be true then there is an element c in the universe of discourse such that $p(c)$ is true.

i.e.
$$\frac{\exists x\, p(x)}{\therefore p(c) \text{ for such element } c}$$

Here, c is not arbitary, it must be specified.

4) Existential generalization:

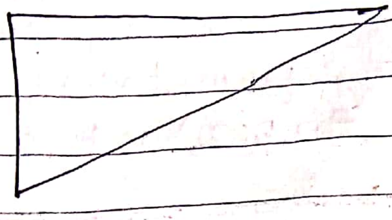
If at least a element $c$ from the universe of discourse make $p(c)$ true, then $\exists x\, p(x)$ is true.

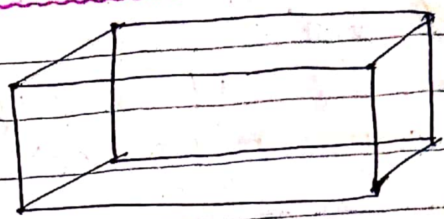
$$\therefore \frac{p(c) \text{ for some element } c}{\therefore \exists x\, p(x)}$$

| S.N | Rules of Inference for Quantified Statement | Name of Rules |
| --- | --- | --- |
| 1. | $\dfrac{\forall x\, p(x)}{\therefore p(c)}$ | Universe Instantiation |
| 2. | $\dfrac{p(c) \text{ for an arbitrary } c}{\therefore \forall x\, p(x)}$ | universal generalization |
| 3. | $\dfrac{\exists x\, p(x)}{\therefore p(c) \text{ for such element } c}$ | Existential Instantiation |
| 4. | $\dfrac{p(c) \text{ for some element } c}{\therefore \exists x\, p(x)}$ | ~~Exit~~ Existential Generalization |

* Invarient:

A property preserved by isomorphism of graph is called a graph invarient. For instance isormorphic simple graph most have the same number of vertices, same no. of edges, same degree sequence and there is a one to one correspondence between set of vertices of the graph.





Q. Show that the hypothesis $(p \wedge q) \vee r$ and $r \to s$ imply the conclusion $p \vee s$.

Solution:

Given that hypothesis are:
$(p \wedge q) \vee r$ & $r \to s$ & the conclusion is: $p \vee s$

Now, using the rule of inferences we get as follows:

| Step | Reason |
|---|---|
| ① $(p \wedge q) \vee r$ | Given hypothesis |
| ② $p \vee r$ | Using simplification rule no. ① |
| ③ $r \to s$ | Given hypothesis |
| ④ $\neg r \vee s$ | Conditional equivalence law |
| ⑤ $r \vee p$ | Cummutative law of equivalence of ② |
| ⑥ $\therefore s \vee p$ | using Resolution law on ④ & ⑤. |
| ⑦ $p \vee s$ | Cummutative law ⑥ |

Hence, the given hypothesis imply the conclusion $p \vee s$.

Q. use inference Rules to show i.e. the hypothesis "
"Jasmine is reading or it is not snowing" and "it is Snowing or Bart is playing Hockey" implies that "Jasmine is reading or Bart is playing Hockey".

Solution:

let, the proposition be:

$p$: Jasmine is reading

$q$: It is snowing

$r$: Bart is playing Hockey.

∴ The given premises can be written as:

$(p \vee \neg q)$, $q \vee r$

and the conclusion is, $p \vee r$

now, using the rule of Inference we get as follows:

| Step | Reason |
|---|---|
| (1) $p \vee \neg q$ | Given Hypothesis |
| (2) $\neg q \vee p$ | Cummulative law of equivalence of (1) |
| (3) $q \vee r$ | Given hypothesis |
| (4) $p \vee r$ | Using resolution law on (2) & (3) |

∴ Hence, the given hypothesis imply the conclusion. $p \vee r$.

* Show that the hypotheses "If you send me an email message, then i will finish writing the program", "If you donot send me an email message, then i will go to sleep early", and "If I go to sleep early, then I will wakeup feeling refreshed", lead to the conclusion "If I donot finish writing the program, then I will wake up feeling refreshed".

Soln:

let the proposition be:

p: you send me an email message.

q: I will finish writing the program.

r: I will go to sleep early.

s: I will wake up feeling refreshed.

The given hypotheses can be written as:

$p \to q$, $\neg p \to r$, and $r \to s$

& the conclusion is: $\neg q \to s$

Using rule of inference we can derived the conclusion as follows:

| Steps | Reason. |
|---|---|
| ① $p \to q$ | Given Hypothesis |
| ② $\neg p \to r$ | Given hypothesis |
| ~~③ $q \to r$~~ | |
| ③ $\neg q \to \neg p$ | Contrapositive of ① |
| ④ | |

| Steps | Reasons |
|---|---|
| ① $p \to q$ | Given hypothesis |
| ② $\neg q \to \neg p$ | Contrapositive of ①. |

③ $\neg p \rightarrow r$ Given hypothesis

④ $\neg q \rightarrow r$ Hypothetical syllogism of ② & ③

⑤ $r \rightarrow s$ Given hypothesis

⑥ $\neg q \rightarrow s$ Hypothetical syllogism of ④ & ⑤.

Q. Construct an argument using rules of inference to show that the hypothesis. "If doesnot rain or it is not foggy, then the raining."

Q. Show that the hypotheses "It is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny". "If we donot go swimming, then we will take a pokhara trip", and "If we take a pokhara trip, then we will be home by sunset" lead to conclusion "we will be home by sunset".

Sol: let the propositions be:

p: It is sunny this afternoon

q: it is colder than yesterday

r: we will go for swimming

s: we will take a pokhara trip

t: we will be home by sunset.

The given hypotheses can be written as:

$\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, $s \rightarrow t$

and conclusion is $t$:

Q. Use Rule of inference to show that the hypotheses "Randy works Hard". "If randy works hards, then he is a dull boy," and "if Randy is a dull boy, then he will not get the job" implies the conclusion "Randy will not get the job".

Soln:

let, the proposition be:

p : Randy works hard

q : He is dull boy

r : He will get the job

Given hypotheses can be written as :

$p, \; p \to q, \; q \to \neg r$,

& the conclusion is $\neg r$.

Rules

| Rules | | Reasons |
|---|---|---|
| 1) | $p$ | Given hypothesis |
| 2) | $p \to q$ | Given hypothesis |
| 3) | $q$ | Modus ponens on 1 & 2 |
| 4) | $q \to \neg r$ | Given hypothesis |
| 5) | $\neg r$ | Modus ponens on 3 & 4 |

∴ Hence, the given hypothesis imply the conclusion; $\neg r$.

Q. Prove or disprove the validity of argument: "Every living thing is a plant or Animal". "Hari's dog is alive and is not a plant", "All Animals have hearts". Hence, Hari's dog has heart.

Soln:

$L(x)$ = x denotes living things

$P(x)$ = x denotes plants

$A(x)$ = x denotes Animal

$H(x)$ : x has heart.

d : represents Hari's dog.

Then the given hypotheses are:

$\forall x\,(P(x) \vee A(x))$, $L(d) \wedge \neg P(d)$, $\forall x\, A(x) \to H(x)$

and the conclusion is : $H(d)$.

Now, using Rule of inference can derive the conclusion as follows:

| Step | | Reasons |
|---|---|---|
| 1 | $\forall x\,(P(x) \vee A(x))$ | Given hypotheses |
| 2 | $L(d) \wedge \neg P(d)$ | Given hypotheses |

③ $\forall x\ A(x) \rightarrow H(x)$ — Given hypothesis

④ $\neg p(d) \vee A(d)$ — using Universal instantiation on ①

⑤ $\neg p(d)$ — Using simplification instantiation on ②.

⑥ $\neg p(d) \vee H(d)$ — Addition Rule

⑦ $A(d) \vee H(d)$ — Resolution rule in ④ & ⑧

⑧ $A(d) \rightarrow H(d)$ — ~~Res~~ universal instantiation on ③.

⑨ $\neg A(d) \vee H(d)$ — conditional equivalence law on ⑧

⑩ $H(d) \vee H(d)$ — Resolution on ⑦ & ⑨

11) $H(d)$ — Idempotent law:

∴ Hence, The conclusion, Hari's dog has heart is "valid".

Q. Show that the premises "A student in this class has not read the book", and "Everyone in the class passed the first exam" imply the conclusion "someone who passed the first exam has not read the book".

Soln: let, the proposition be:

$C(x)$ = $x$ denotes a student of the class.

$B(x)$ = $x$ has read the book.

$P(x)$ = $x$ has passed the first exam.

Then the given hypothesis are:

$\exists x\ C(x) \wedge \neg B(x)$, $\forall x\ C(x) \rightarrow P(x)$

& the conclusion is: $\exists x\ P(x) \wedge \neg B(x)$

Using Rule of inference we can derive the conclusion as:

| Step | Reason. |
|---|---|
| 1. $\exists x\ C(x) \wedge \neg B(x)$ | Given hypothesis |
| 2. $C(a) \wedge \neg B(a)$ | Using Existential Instantiation in 1 |
| 3. $\forall x\ C(x) \rightarrow P(x)$ | Given hypothesis |
| 4. $C(a) \rightarrow P(a)$ | Universal Instantiation in 3 |
| 5. $C(a)$ | Simplification on 2 |
| 6. $P(a)$ | Modus pones on 4 |
| 7. $\neg B(a)$ | using simplification on 1 |
| 8. $P(a) \wedge \neg B(a)$ | conjuction on 6 & 7 |
| 9. $\exists x\ P(x) \wedge \neg B(x)$ | Existential generalization on 8. |

Hence, The conclusion "someone who passed the first exam has not read the book" is true.

Q. Find the contrapositive, converse, inverse & negation of the statement "If it is sunny today, then we will go to the beach".

Sol:

let the proposition be:

p: It is sunny today

q: we will go to the beech.

The Given hypotnesis

$p \rightarrow q$

(1) Contrapositive:

According to contrapositive hypotneis be:

$\neg q \rightarrow \neg p$

And the Contrapositive statement is:

"If we will not goto beech, then it is not sunny today.

(2) Converse:

$q \rightarrow p$

And the converse statement is:

" If it is ~~not~~ sunny today then we will ~~not~~ goto beech"

(3) Inverse:

$\neg p \rightarrow \neg q$

And the ~~Converse~~ Inverse statement is:

" If it is not sunny toda

(2) Converse:

$q \rightarrow p$

And the converse statement is:

" If we will go to the beech, then it is sunny today".

(3) Inverse:

$\neg p \rightarrow \neg q$

And the ~~Stateme~~ Inverse statement is:

"~~If the~~ it is not sunny today, then we will not go to the beech".

(4) Negation.

$$\neg(p \to q)$$

equivalent with,

$$= \neg p(\neg p \vee q)$$

$$= \neg\neg p \wedge \neg q$$

$$= p \wedge \neg q$$

~~The~~ The Negation statement is,

"If it is sunny today, then we will not go to the beech".

Q. use the rule of inference to show that "~~If is~~ It is not raining or Ram has his umbrella", "Ram doesnot have ~~this~~ his umbrella or he doesnot get wet" and "It is raining or Ram doesnot get well" and "Ram doesnot get wet".

Soln:

let, the proposition be:

p: It is raining

q: Ram has his umbrella.

r: he get wet

Then,

$$(\neg p \vee q), (\neg q \vee \neg r) \ \& (p \vee \neg r)$$

the conclusion is : $\neg r$

| Rules | Reason |
|---|---|
| (1) $\neg p \vee q$ | Given Hypothesis |
| (2) $q \vee \neg p$ | commulative law of (1) |
| (3) $\neg q \vee \neg r$ | Given hypothesis |
| (4) $\neg p \vee \neg r$ | using Resolution (2) & (3) |
| (5) $p \vee \neg r$ | Given hypotheses |
| 6) $\neg r \vee \neg r$ | using resolution law from (4) & (5). |
| 7) $\neg r$ | using idempotent from (6). |

Q. Verify that the following argument is valid by using the rules of inferences.

If clinton doesnot live in France, then he doesnot speak french.
Clinton doesnot drive a Datson.
If clinton lives in France, then he rides a bicycle.
Either clinton speaks French or he rides a Datsun
Hence, clinton rides a Bicycle.

Soln:

let,

p : clinton live in France.

q : He speaks french.

r : clinton drive a Datsun.

s : He rides a bicycle.

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Given hypotheses be:

$(\neg p \to \neg q)$, $\neg r$, $p \to s$, $q \vee r$

& the conclusion is : s.

| Rules | Reason |
|---|---|
| (1) $\neg p \to \neg q$ | Given Hypothesis |
| (2) $q \to p$ | contrapositive of (1) |
| (3) $\neg r$ | Given hypothesis |
| ~~(4) Hypo.~~ | a. |
| (4) $p \to s$ | Given Hypothesis |
| (5) $q \to s$ | Hypothetical syllogism (2) & (4) |
| (6) $q \vee r$ | Given hypothesis |
| (7) $q$ | Disjunctive syllogism of (3) & (6) |
| (8) $s$ | Modes pones using (4) & (7). |

Q) Show that the hypothesis:

$\neg s \wedge c$, $w \to s$, $\neg w \to t$. & $t \to h$ leads to the conclusion h.

| Rules | Reason. |
|---|---|
| (1) $\neg s \wedge c$ | Given hypothesis |
| (2) $\neg s$ | Simplification Rule of (1) |
| (3) $w \to s$ | Given hypothesis |
| (4) $\neg w$ | Modus pones |
| (5) $\neg w \to t$ | Given hypothesis |
| (6) $t$. | Modus pones |

⑦ $t \to h$ Given hypothesis

⑧ $h$ Modus pones.

## * Mathematical Induction:

It is an extremely important proof technique that can be used to prove mathematical theorem & statement to analyze the complexity of algorithm and correctness of statement or computer program.

Here, induction means the method infering a general statement from the validity of particular cases.

let, $p(n)$ be a statement on positive integer $n$ to prove $p(n)$ is valid. Using induction we need to follow following steps.

① Basic steps:

In this step we need to verify that $p(n)$ is true for $n=0$ or $1$.

② Induction hypothesis:

In this step we need to verify that $p(n)$ is true for $n=k$ i.e. $p(k)$ is true.

(3) Inductive step:

In this step by using Induction hypothesis we prove that $p(n)$ is true for $n = k+1$.

i.e. $p[p(n_0) \wedge \forall(p, k) \rightarrow p(k+1)] \rightarrow \forall n\, p(n)$

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Q. use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.

Soln:

let, $p(n)$ denotes the proposition : $n^3 - n$ is divisible by 3.

(1) Basic step:

let, $p(n)$ is true for $n = 1$

i.e. $p(1) = 1^3 - 1 = 0$ (i.e. true because 0 is divisible by 3).

(2) Induction Hypothesis:

let $p(n)$ is true for $n = k$,

$p(k) = k^3 - k$ is divisible by 3.

(3) Inductive step:

we have to show that $p(n)$ is true for $n = k+1$ using induction hypothesis.

Here,

$$p(k+1) = (k+1)^3 - (k+1)$$
$$= k^3 + 3k^3 + 3k + 1 - k - 1$$
$$= (k^3 - k) + 3(k^3 + k)$$

Since, ~~both~~ the term divisible by 3 'using induction

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hypothesis. we conclude that $n^3-n$ is divisible by 3 for any positive integer n.

Q. Show that $2n+1$ is odd for any positive integer n using M.I.

soln:

let, p(n) be the proposition $2n+1$ is odd.

(1) Basic:

let p(n) is true for $n=1$,

i.e. $p(1) = 2\times1+1 = 3$ (i.e. true because 3 is odd)

(2) Induction hypothesis:

let, p(n) is true for $n=k$,

$p(k) = 2k+1$ is odd number.

(3) Inductive step:

we have to show that p(n) is true for $n=k+1$ using induction hypothesis.

Here,

$p(k+1) = 2(k+1)+1$

$= 2k+2+1$

$=(2k+1)+2$

Since, $(2k+1)$ is odd and by adding odd & even we get odd result. So we can conclude $2n+1$ is odd for any positive integer n.

Q) Show that 2n is even for any positive integer n using M.I.

Soln:

let, p(n) be the proposition 2n is even.

(1) Basic:

let, p(n) is true for n=1.

i.e. $p(1) = 2 \times 1 = 2$ (i.e. true because 2 is even)

(2) Induction hypothesis:

let, p(n) is true for n=k

p(k) = 2k is even number.

~~(3) Induction hypothesis:~~

~~let, p(n) is true for~~

(3) Inductive step:

we have to show that p(n) is true for n=k+1 using induction hypothesis.

Here, $p(k+1) = 2(k+1)$

$= 2k+2$

∴ Hence, (2k) is even and by adding by even we got even result. So we can conclude 2n is even for any positive integer n.

Q. ~~Show that~~ prove that $1.1! + 2.2! + 3.3! + .... + n.n! = (n+1)! - 1$

~~Solution~~: whenever n is a positive integer.

Solution:

let the proposition p(n) is $1.1! + 2.2! + 3.3! ... n.n! = (n+1)! - 1$

① **Basic step**

Taking $n=1$

$1.1! = (1+1)! - 1$

i.e. $1 = 2-1 = 1$ (True)

(Because 1 is positive integer)

② **Induction hypothesis:**

let, $p(n)$ is true for $n=k$.

i.e. $1.1! + 2.2! + 3.3! + \ldots + k.k! = (k+1)! - 1$

③ **Inductive step:**

we have to show that the $p(n)$ is true for $n=k+1$

i.e. $1.1! + 2.2! + 3.3! + \ldots + k.k! + (k+1).(k+1)! = (k+1+1)! - 1$

$= (k+2)! - 1$

Here,

L.H.S =

$1.1! + 2.2! + \ldots + k.k! + (k+1)(k+1)!$

$= (k+1)! - 1 + [(k+1)(k+1)!]$ (using induction hypothesis)

$= (k+1)[k! + (k+1)!] - 1$

$= (k+1)k! [1 + k+1] - 1$

$= (k+1) k! (k+2) - 1$

$= (k+2)(k+1)k! - 1$

$= (k+2)! - 1 = $ R.H.S.

5!

5×4!

6×5!

Show that if $n$ is a positive integer then
$1+2+3+4+\dots+n = \frac{n(n+1)}{2}$

Solution:-

let the proposition $p(n)$ is.
$1+2+3+4+\dots+n = \frac{n(n+1)}{2}$

Basic step:

taking $n=1$
$1 = \frac{n(n+1)}{2}$
$= \frac{1(1+1)}{2} = 1$ (true)

let $P(n) = P(k)$
$1+2+3+4+\dots+k = \frac{k(k+1)}{2}$

Induction Hypothesis:

we have to show that the $p(n)$ is true for $n=k+1$
i.e. $1+2+3+4+\dots k + (k+1) = \frac{(k+1)(k+1+1)}{2}$
$= \frac{(k+1)(k+2)}{2}$

Hence, L.H.S.
$1+2+3+4+\dots k+(k+1)$
$\Rightarrow \frac{k(k+1)}{2}+(k+1)$
$= \frac{(k^2+k)+(2k+2)}{2}$
$= \frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$

R.H.S

Q. Use mathematical induction to show that $1 + 2 + 2^2 + 2^4 + \dots + 2^n = 2^{n+1} - 1$ for all non negative integer n.

Soln: Let the proposition ~~p~~ p(n) be:

$1 + 2 + 2^2 + 2^4 + \dots + 2^n = 2^{n+1} - 1$

(1) Basic step:

let, p(n) be true for n = 1.

$2^1 = 2^{1+1} - 1$

$2 = 2^2 - 1$

$2 = 3$ (False)

Again, Taking n = 0

$2^0 = 2^{1+0} - 1$

$1 = 2 - 1$

$1 = 1$ (True)

(2) Induction step:

let, p(n) be true for n = k

$1 + 2 + 2^2 + 2^4 + \dots + 2^k = 2^{k+1} - 1$ (True)

(3) Inductive hypothesis:

let, p(n) be true for n = k + 1.

$1 + 2 + 2^2 + 2^4 + \dots + 2^k + 2^{k+1} = 2^{(k+1)+1} - 1$

$= 2^{k+2} - 1$

L.H.S.

$= 2^{k+1} - 1 + 2^{k+1}$

$= 2^k . 2 - 1 + 2^k . 2$

$= \cancel{2.2^k}\ 2 . 2^{k+1} - 1$

$= 2^1 . 2^{k+1} - 1$

$= 2^{k+2} - 1$

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Q. Conjuncture a formula for the sum of first n positive odd integers. Then prove your conjuctor using mathematical induction.

Soln: The sum of first n-odd integer for n = 1, 2, 3, 4, 5 ... is as:

for n = 1, sum = 1
for n = 2 sum = 1 + 3 = 4
for n = 3 sum = 1 + 3 + 5 = 9
for n = 4 sum = 1 + 3 + 5 + 7 = 16

Thesis from the above sequence we can get the conjucture formula;
The sum of first n odd integer is,
$1 + 3 + 5 + 7 + \dots + (2n-1) = n^2$

Let, the proposition p(n) is:
$1 + 3 + 5 + 7 + \dots + (2n-1) = n^2$

(1) Basic Step:
Taking n = 1
$1 + 3 + 5 + 7 + \dots + (2 \times 1 - 1) = 1^2$
$1 = 1$ (True)

(2) Induction hypothesis:
let, p(n) is true for n = k
i.e. $1 + 3 + 5 + 7 + \dots + (2k-1) = k^2$ (True)

(3) Induction Step:
we have to show that the p(n) is true for n = k+1
i.e. $1 + 3 + 5 + 7 + (2k-1) + (2(k+1)-1) = (k+1)^2$

L.H.S

$= k^2 + (2(k+1) - 1)$

$= k^2 + (2k+2-1)$

$= k^2 + (2k+1)$

$= k^2 + 2k + 1$

$= (k+1)^2$ R.H.S. proved

Q. Conjucture a formula for the sum of first n positive even integers. Then prove your conjuctor using mathematical induction.

Soln:

The sum of first n-positive even integer for $n = 1, 2, 3, 4, 5 \ldots$ ~~2, 4, 6, 8~~ is as:

for $n = 1$ sum $= 2$

$n = 2$ Sum $= 2 + 4 = 6$

$n = 3$ Sum $= 2 + 4 + 6 = 12$

$n = 4$ Sum $= 2 + 4 + 6 + 8 = 20$

Thus from above sequence we can get conjunction formula, The sum of first n even integer is:

$2 + 4 + 6 + 8 + \ldots + 2n = n^2 + n$

let, the proposition p(n) be:

$2 + 4 + 6 + 8 \ldots + 2n = n^2 + n = n(n+1)$

① Basic step:

Taking $n = 1$

$2 \times 1 = 1^2 + 1$

$2 = 2$ (True)

(2) Induction hypothesis:

let $p(n)$ is true for $n = k$.

$2 + 4 + 6 + 8 \ldots\ldots + \cancel{2k} = n^2 + n + 2k = k^2 + k$

(3) Inductive step:

$2 + 4 + 6 + 8 + \ldots\ldots + 2k + 2(k+1) = \cancel{(k+1)^2 + (k+1)}$

LHS.

$= (k+1)(k+2)$

$k^2 + k + 2(k+1)$

$k^2 + k + 2k + 2$

$k(k+1) + 2(k+1)$

$(k+1)(k+2)$

2012 | spring Use mathematical induction to prove the sum.

$4 + 9 + 14 + 19 + \ldots\ldots + 5n - 1 = \dfrac{n(3+5n)}{2}$

soln:

(i) ~~Ba~~ let, proposition be:

$4 + 9 + 14 + 19 + \ldots\ldots + 5n - 1 = \dfrac{n(3+5n)}{2}$

(1) Basic:

Taking $n = 1$.

$5 \times 1 - 1 = \dfrac{1(3 + 5 \times 1)}{2}$ or, $4 = 8/2$ or, $4 = 4$ (True)

(2) Induction hypothesis.

let, p(n) be true for n=k

$4+9+14+19+ \ldots + 5k-1 = \frac{k(3+5k)}{2}$ (True)

(3) Inductive step:

let, p(n) be true for n=k+1

i.e. $4+9+14+19+ \ldots + (5k-1) + 5(k+1) - 1 = \frac{(k+1)\{3+5(k+1)\}}{2}$

$= \frac{(k+1)(5k+8)}{2}$

$= \frac{(k+1)(3+5k+5)}{2}$

$= \frac{(k+1)(8+5k)}{2}$

L.H.S.

$= \frac{k(3+5k)}{2} + 5(k+1) - 1$

$= \frac{k(3+5k)+10(k+1)+2}{2}$

$= \frac{(k+1)[k(5+3)+10]+2}{2}$

$= \frac{3k+5k^2+10k+10+2}{2}$

$= \frac{5k^2+13k+12}{2}$

$= \frac{5k^2+12k+1k+12}{2}$

$= \frac{5k(k^2+12)+1(k+12)}{2}$

L.H.S.

$\frac{k(3+5k)}{2} + 5(k+1) - 1$

$= \frac{k(3+5k)+10(k+1)-2}{2}$

$= \frac{3k+5k^2+10k+10-2}{2}$

$= \frac{5k^2+13k+8}{2}$

$= \frac{5k^2+8k+5k+8}{2}$

$= \frac{5k(k+1)+8(k+1)}{2}$

$= \frac{(k+1)(8+5k)}{2}$ R.H.S. proved.

Q. Use mathematical Induction to prove that $2^n < n!$ for every positive integer n with $n \geq 4$.

let the proposition be:

$2^n < n!$

(1) Basic step:

Taking n=4

$2^4 < 4!$

$16 < 24$ (True)

(2) Induction hypothesis.

$2^k < k!$

(3) Inductive step:

$2^{k+1} < (k+1)!$

L.H.S = $2^{k+1} = 2^k . 2$
$< k! . 2$ (using I.H)
$< k! . (k+1)$ $(k \geq 4)$
$< (k+1)!$

Q. State the principle of mathematical induction prove by mathematical induction : $3^n > n^3$,
for all positive integer $n \geq 4$.

Soln: let, the proposition:
$3^n > n^3$

① Basic step:
Taking $n = 4$,
$3^4 > 4^3$
$81 > 64$ (True)

② Induction Hypothesis:
$3^k > k^3$ (True)

③ Induction step:

$3^{k+1} > (k+1)^3$

LHS.
$3^{k+1} = 3^k . 3^1$
$= 3 . 3^k$
$< (k+1) 3^k$
$> (k+1) k^3$
$> (k+1)^3$
R.H.S.

Q. use mathematical induction to prove:

① $1^2+2^2+3^2+4^2+ \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integer n.

② $1^3+2^3+3^3+ \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

③ $1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

① $1^2+2^2+3^2+4^2+ \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integer n.

soln:

let, the proposition be:

$1^2+2^2+3^2+4^2+ \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

① Basic step:

let p(n) be true for n=1, $n^2 = \frac{1(1+1)(2\times1+1)}{6}$

$1^2 = \frac{2\times3}{6}$

$1=1$ (True)

② Induction hypothesis:

let p(n) be true for n=k.

$1^2+2^2+3^2+4^2+ \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$

③ Inductive step:

let p(n) be true for n=k+1.

$1^2+2^2+3^2+4^2+ \dots + k^2+(k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$

$= \frac{(k+1)(k+2)(2k+2+1)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

Taking LHS:

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

or, $\frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$

or, ~~$k(k+1)(2k^2+k)+6(k^2+2k+1)$~~

or, $\frac{(k+1)[k(2k+1)+6(k+1)]}{6}$

or, $\frac{k+1[2k^2+k+6k+6]}{6}$

or, $\frac{(k+1)(2k^2+7k+6)}{6}$

or, $\frac{(k+1)[2k^2+4k+3k+6]}{6}$

or, $\frac{(k+1)[k(2k+3)+2(2k+3)]}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$ R.H.S proved

(2) $1^3+2^3+3^3+ \ldots +n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Sol^n:

(1) Basic step: Taking $n=1$,

$$1^3 = \left(\frac{1(1+1)}{2}\right)^2$$

$$1 = 1^2$$

$\therefore (1=1)$ (True)

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(2) Induction hypothesis: Taking $n=k$

$1^3+2^3+3^3+\dots+k^3=\left(\frac{k(k+1)}{2}\right)^2$

(3) Inductive step:

Taking $n=k+1$

$1^3+2^3+3^3+\dots+k^3+(k+1)^3=\left(\frac{(k+1)(k+1+1)}{2}\right)^2$

$=\left(\frac{(k+1)(k+2)}{2}\right)^2$

Taking LHS.

$\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3$

$=\left(\frac{k^2(k+1)^2+4(k+1)^3}{4}\right)$

$=\frac{(k+1)^2(k^2+4(k+1))}{4}$

$=\frac{(k+1)^2(k+2)^2}{4}=\left(\frac{(k+1)(k+2)}{2}\right)^2$ R.H.S

(3) $1.2+2.3+3.4+\dots\dots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

soln:

(1) Basic step: Taking $n=1$

$1(1+1)=\frac{1(1+1)(1+2)}{3}$

$2=6/3$

$2=2$ (true)

(2) Induction hypothesis:

$1.2+2.3+3.4+\dots+k(k+1)=\frac{k(k+1)(k+1)}{3}$

③ Inductive step:

$1.2 + 2.3 + 3.4 + \ldots + \cancel{n(n+1)}\ k(k+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$

L.H.S.

$\frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$

$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$

$= \frac{(k+1)(k+2)[k+3]}{3}$

R.H.S

Assignment:

④ $1.2.3 + 2.3.4 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

Sol:

① Basic step:

$1(1+1)(1+2) = \frac{1(1+1)(1+2)(1+3)}{4}$

$6 = 24/4$

$6 = 6$ (true)

② Induction hypothesis:

$1.2.3 + 2.3.4 + \ldots \cancel{+ n(n+1)(n+2)}$

$k(k+1)(k+2) = \frac{k(k+1)(k+2)(k+3)}{4}$

(3) Inductive step:

$\cancel{1.2+2.}$ $1.2.3+2.3.4+ \ldots + k(k+1)(k+2)+(k+1)(k+2)(k+3)$

$= \frac{(k+1)(k+2)(k+2)(k+3)(k+4)}{4}$

L.H.S =

$\frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$

$= \frac{k(k+1)(k+2)(k+3)+4(k+1)(k+2)(k+3)}{4}$

$= \frac{(k+1)(k+2)(k+3)[k+4]}{4}$

$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$ . R.H.S proved

(5) prove that $n^3+2n$ is divisible by 3.

(6) prove that $n^5-n$ is divisible by 5.

(7) prove that $n^3-n$ is divisible by 6.

(8) $1^2+3^2+5^2+ \ldots +(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

(9) $1+2+2^2+2^3+ \ldots +2^n = 2^{n+1}-1$

(10) $1+a+a^2+ \ldots + a^k = \frac{a^{k+1}-1}{a-1}$