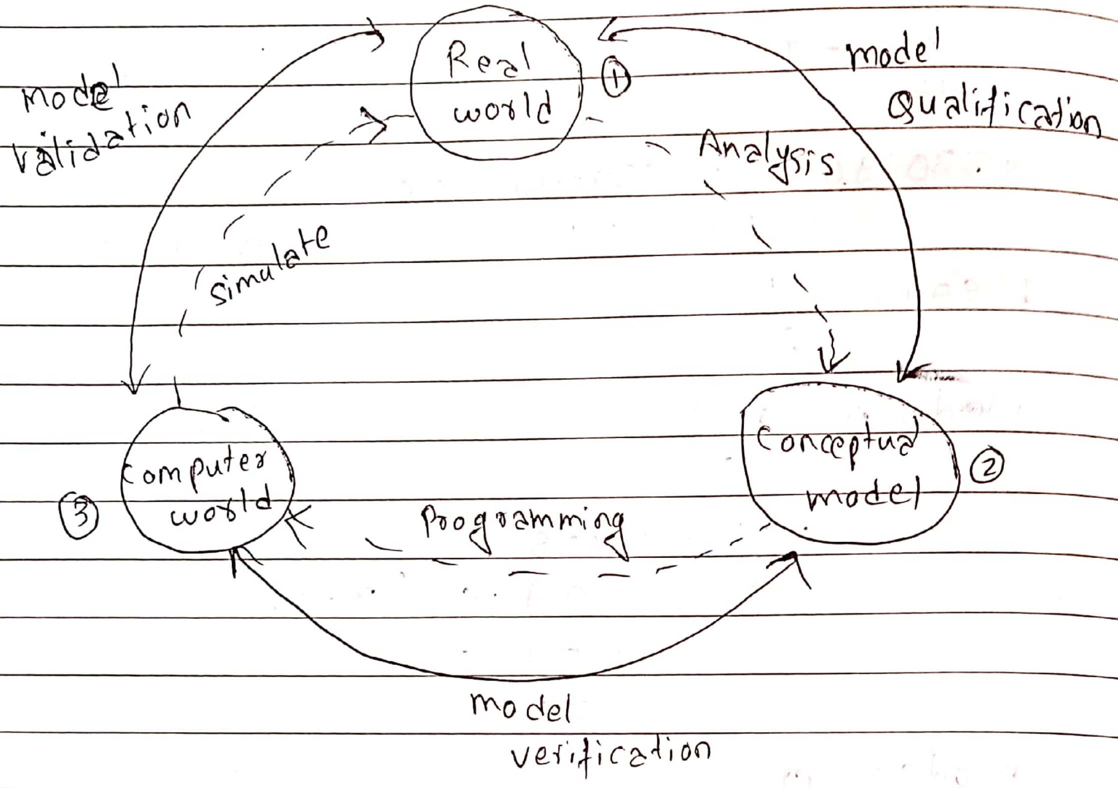
Unit-1 imitation

Introduction to Simulation



It is a process model of simulation to take the abstract (attribute) from the external world. The following valid steps are taken for simulation study

Step 1) Real world

The user or the person examine the equivalent abstract as to be static or dynamic from the external three dimensional world.

Step 2) The process are analyzed from the real world and the conceptual model is being drawn. Here, different problem solving techniques or modelling the parameter is being analyzed through flowcharts, algorithms, blue prints etc.

step 3: ~~Conceptual model~~ Conceptual model.

Whether the define model is qualified or not? If the valid model is to then it can be imitate (copy) and goes to the conceptual model

step 4: Computer Model

Here the different variant or the different flavour of programming tool kit such C, OOP, VB.net, JAVA, PHP, Abstract oriented. ALP is being implemented in this model.

step 5: The model verification goes through the static process. Hence, there is execution of simulation. but no validation is being occured.

step 6:

In this step, the external world & the programming simulation is being ~~mapp~~ matched Here, the execution from the real world is being validated & different testing (black box & sand box) is being done.

Step 7: Finally, the model is to be simulated with the real world parameters. Example:- Aro dynamic simulation, Factory, Conveyor belt simulation, construction engineering, Microprocessor IC, Wafer simulation.

Q What is simulation?

Simulation is the imitation of the real world process or system over time.

OR

Simulation is a set of technique that use computers to imitate the operation of various real world task or process through simulation.

* Areas or Application of Simulation

1 a) The contribution given in the simulation area by winter Sim - USA. More than 50 years of development.

i) Aeronautical Engineering

It is used to simulate or check the auto pilot conjection control.

ii) Biotech Manufacturing

It is used to check the nuclear projects, for the excitation & ~~immition~~ emmission of electron to nuclear fusion & fission

iii) Millitary & ~~Different~~ Defence Application

From the human ~~simulization~~ civilization to the second world war different simulation techniques were implemented ~~by~~ ~~a~~ Via Cryptography stenography, Holiography etc.

a) i) from modelling the leadership effect
(i) and recruite type in army and recruiting station

b) ii) Modelling the millitary requirement for non war fighting operation.

c) iii) Multi-tragetory performance for varying scenario sizes

iv) Constraction Engineering
In semi-conductor manufacturing the dispatching rules using the large facility models for the fabrication of semiconductor devices.

↳ For another use it is the ~~competition~~ comparison of 200 mm & 300 mm X-ray lithographye cells.

↳ (v) In the constraction Engineering for designing a dam embankment, it is used to analyzed tension (force), pressure etc is executed on the system.

↳ Investigation on the structural ~~skelt~~ steel erection process.

↳ Special purpose template for utility tunnel construction.

In the field of management it is used to investigate for the dynamics in a supply ~~service~~ oriented (production).

* ~~Field where simulation can be used~~

* History of Simulation

The historical prospective of simulation is ~~imulate~~ enumerated in a chronological order

1) 1940

A method name Montecarlo was developed by researcher John Vonneumann, Edward Teller etc where the physicist working on manhettern project to study neutron scattering.

2) 1960

The first special propose simulation were developed such as SIMSCRIPT by Harry Markowilz & RAND corporation

3) 1970

In this period, researcher was iniciated on mathematical foundation of simulation

4) 1980

During this period, PC based simulation software, GUI & ODP web development

5) 1990

In this period, webbased simulation, fancy animated graphics, simulation based optimization, markov - chain, Montecarlo methods were developed.

* When Simulation is appropriate tool - ?

1) Simulation enables the study of an experimentation with the internal interaction of a complex system, or sub system within in complex system. Eg:- Polynomial algorithm competition.

2) Informational, organization & environment change can be simulated & the effect of those alternation on the modala behaviour can be observed.

3) Simulation can be used as a device to reenforce analytic solution metharlogist.

4) By changing the simulations inputs & observing the resulting outputs, valuable insights may be obtained into which variables are more important & how variables interact.
Eg:- $ mkdir directory is different with
$ ...... mkdir diretory (output same but different in internal processing)

5) Simulation can be used to verified analytic solution.

6) Simulation model design for training, how learning without the cost & disruption of on the job learning. eg:- Working with GNS simulator.

7) Animation shows a system in simulated operation so that the plan can be visualized. Eg:- CAD or Autocat it can render the two dimensation object into 3D object.

8) The modern system like factory, hydropower water fabrication plans, service org etc is so complex that the interaction can be treated through simulation.

* When Simulation is not an appropriate tool?

It is based on the article by Banks and Gibson 1997 who give the rule for evaluating when the simulation is not appropriate

1) Simulation should not be used when the problem can be solved by common sense.
Eg:- i) 4-2=2 gives the resultant value 2
ii) an automobile tag facility serving the customer who arrive randomly at an average rate 100/hr & are it is solved at the mean rate at 12/hr.

iii) To determine the minimum number ~~nee~~ of server needed simulation is not necessary.

1)(a) It just compute $\frac{100}{12} = 8.33$ indicate that at least 9 server are needed.

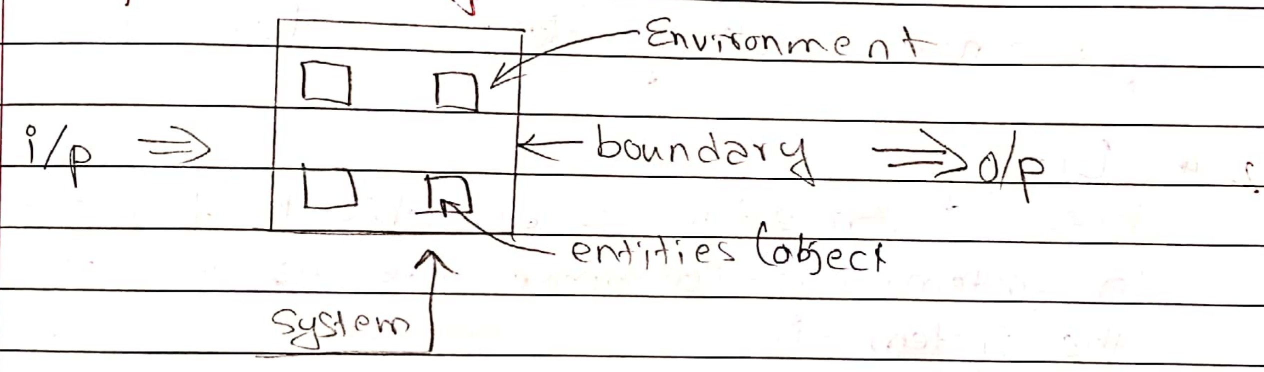
2) Simulation can be done, if the problem can be solved analytically.

3) Simulation should not be used if the ~~first~~ cost exceed the saving.

4) Simulation should not be performed if the resource or time are not available.

5) If no data is available, not even the estimation of simulation is not advised.

Component of System.



System	Entities	Attributes	Activities	Event	State variables
① Banking	Customers	Account Balance	Making deposites	Arrival Departure	No. of customer waiting
2) Production	Machine	speed capacity ~~break down~~ break	wealding stamping	break down	status ~~halt~~ of machine idle or down time
3 communication	(Packet) Message	length, destination	transmitting, ~~arrival of destination, no. of m~~	arrival at destination	no. of message waiting to be transmitted.

System → group of objectives joined together (for some purpose)

eg: automobile facility.

1 * System

It is defined as group of objects in sample space that are joined together in some regular interaction towards to accomplished of some purpose.

The system consists of input attributes and the desired level of output. The system is separated by its input & boundary. A system is often affected by changes occuring outside the system which is the system environment. For example:-

In the Banking system, the arrival of customer, making deposits etc.

2 * Entity

An entity is an object of interest in a system. Eg:- Customer are the entities of the system.

3 * Attributes

Attributes is the characteristics or properties of entities eg (Account Balance)

4 * Activities

Any process causing in the system is called an activities. eg: Making deposite in the bank.

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5 * State of system:

The collection of variables necessary to describe the system at any time, relative to the objective of the study. In another words, state of the system mean, the description of all entities, attributes & activities is they exist at one point in time.

6 * Event

An event is define as an instantaneous occurrance that may change the state of the system. (time) eg:- customer arrival, customer departure etc.

7 * Endogenous System (inside)

It is used to describe activities & events occurring within the system.

Eg:- cash withdraw from a cash counter of the bank.

8) Exogenous System (out)

The ~~teerm~~ term exogenous system is used to describe activities & events in the environment that affect the system

Eg:- cash withdraw from ATM.

9) Closed System

A system for which there is no exogenous activity and the event is set to closed system.

Eg:- Amount transfered from A person to B person ~~person to~~ from same bank

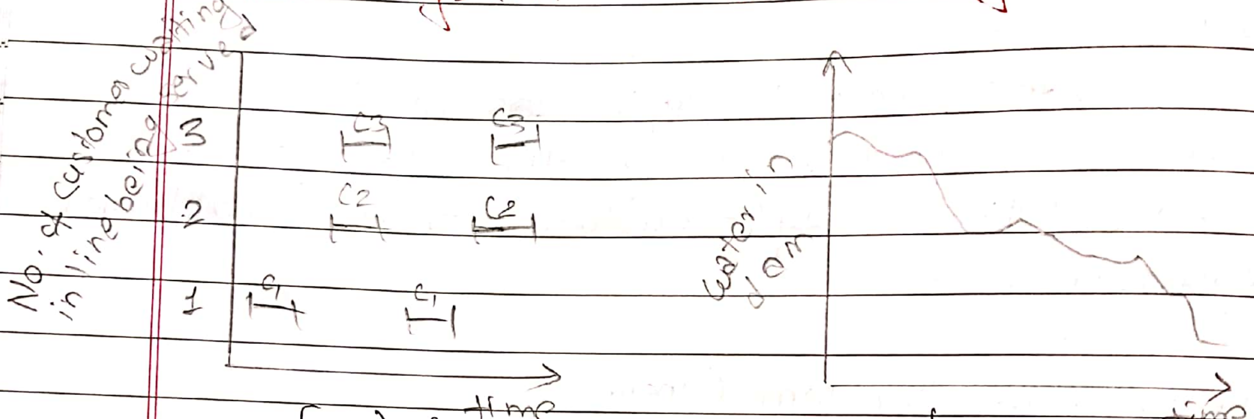
ii) Water in a tank.

10) Open system

A system for which there is exogenous activities & the events is said to be the open system :- Amount is

Eg :- Banking system transaction from person A to person B from the different bank

Discrete system & continuous system



(a) Discrete (b) Continuous

Discrete system (Fig a)

It is one in which the state variables change only at a discrete set of time.

Eg :- Banking system in which number of customer state variable changes only when a customer arrives or service provided to customer i.e customer depart from the system.

From the above fig(a) dotted line (-) represent the enterarrival of customer C1, C2, C3 at a discrete time units.

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Continuous system

It is one in which the state variable changes continuously over time.
Eg: During the winter season, the level of water decreases gradually & during the rainy season, the level of water increases gradually.

System Modelling

A model is define as the representation of a system for the purpose of studying the system. It is a mathematical representation of the system.

Types of Modelling (7 types)

1) ~~Mathematical~~ model

This model gives the symbolic & mathematical equations to represent the system. The system attributes are represented by variables & the activities of mathematical function.
Eg:- $y = mx + c$ ~~as~~ is the mathematical model for slope of line

2) Physical model

This model are based on some analogy between mechanical & electrical system.
Eg: ~~ext~~ analyzing the velocity, revolution per minute, its mass, acceleration, etc of the moving bus.

3) Static Model

Static Model can show the values that the system attribute value doesn't change over the time.

Eg 1) Monte Carlo simulation for finding simulation value of pie.

eg (2) Scientists has used this model in which sphere represent atoms, sheet of metal to connect the sphere to represent atomic bonds

eg (3) It is also use in for modelling the network graph.

4) Dynamic Model

It represent the system is the change over the time. example:- Simulation in bank.

5) Deterministic model

It contain no random variables There have no set of inputs which will result in a unique set of output.

Example: Arrival of patient to the dentist at the schedule of appoinment time.

6) Stochastic Model

It has more number of random variables. Random input leads to random output.

Example

(i) Simulation of a bank involve random interarrival & service time.

(ii) Monte Carlo simulation for computing the constant value for π [Area of circle $= \pi r^2$]

$$\left[F_{charge} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \right]$$

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iii) For finding the area of an irregular object like amoeba, paramecium'

7) Discrete & Continuous Model:

It is used for analogous manner. Simulation model may be mixed both with discrete & continuous event. Choice is based on the characteristics of the system & the objective of the study.

Steps in Simulation Study

1) Problem Formulation

Every study ~~beings~~ begins with a statement of a problem provided by policy makers. Analyst ensure is clearly ~~ensure~~ understood. If it is developed by the analyst policy maker should understand & agree with it.

2) Setting of objectives & overall project plan.

The objective indicate questions to be answered by ~~sum~~ simulation at this point a determination should be made concerning whether simulation is appropriate methodology. The overall project plans should include:-

- a statement of a alternative system.
- a method for evaluating the effectiveness of this alternatives.
- transferred for the study in terms of no. of people involve.
- cost of the study
- the no. of deals required to accomplish each

phase of the work with anticipated users.

3. Model Conceptualization :-

The construction of this model of a system is ~~collasping~~ probably as much are as science. The art of modelling is enhanced by an ability.

i) to abstract the ~~initial~~ essential feature of problem

ii) to select & modify basic assumption that characterized the system

iii) To ~~increase~~ increase and elavorate the model until the useful approximation result.

Thus, it is the best to start with a simple model & build toward greater complexity. Model conceptualization enhanced the quality of the resulting model & increase the confidence of modeluser in the application of the model.

4. Data Collection :

There is constant interplay between the construction of model & the collection of needed input data. It is done on the early stage. The objective kind of data are to be collected

5. Model Translation :- computer made better

The real world system result in models that require a great deal of information

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storage & computation. It can be programmed by using simulation language for the special purpose simulation software.

Simulation language are powerful & flexible. Simulation software ~~modeila~~ models development time can be reduced.

inside pogming would

6) **Verified :-** given input does gives desire output

It pretends to the computer program & checking the performace. If the input parameter & logical structure are correctly represent, verification is ~~va~~ completed.

To check equivalent

7) **Validated :-** final result outside the programming world

It is the determination that the model is an accurate representation of the real system. It is achieved through the calibration of model, & iterative process of comparing a model to actual system behaviour.

8) **Experimental Design :-**

The alternatives that are to be simulated must be determined. ~~With~~ Which alternatives to simulated may be a functional ~~branch~~ of runs. For each system design, discussion need to be made concern :-

a) Length of the initialization period.
b) Length of simulation runs.
c) The number of replication to be made of each run.

~~4) production runs & analysis:~~

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9. Production runs & analysis:-

They are used to estimate measures of performance for the system design that are being simulated.

10) More runs:-

Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed & what design those additional experiment should follows.

11) Documentation & reporting:

Two types of documentation

a) Program documentation:- can be used again by the same or different analysts to understand how the program operates. Further modification will be easier. Model users can be change the input parameters for better performance.

b) Process documentation;

Gives the history of a simulation project. The result of all analysis should be reported clearly & concisely in a final report. This enables to review the final formulation & alternatives, results of the experiment & the recommended solution of the problem. The final report provides a ~~vechicle~~ vehicle of certification.

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(11) Stepwise implementation of simulation study:-
Success depends on the previous steps. If the model user has been throroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.
The simulation model building can be broken into 4 phase.

Phase I

- Consists of Step 1 & 2
- It is period of discovery / orientation.
- The analyst may have to restart the process if it is not fine-tuned.
- Recalibrations & Clarification may occurs in this phase or another phase.

Phase II

- Consists of steps 3, 4, 5, 6, & 7
- A continuing interplay is required among the steps.
- Exclusion of model user result in implications during implementation.

Phase III

- Consist of steps 8, 9 & 10
- conceives ~~A thoroughly~~ plan for experimentation.

- Conceives a thorough plan for experimenting
- Discrete-event stochastic is a statistical experiment.

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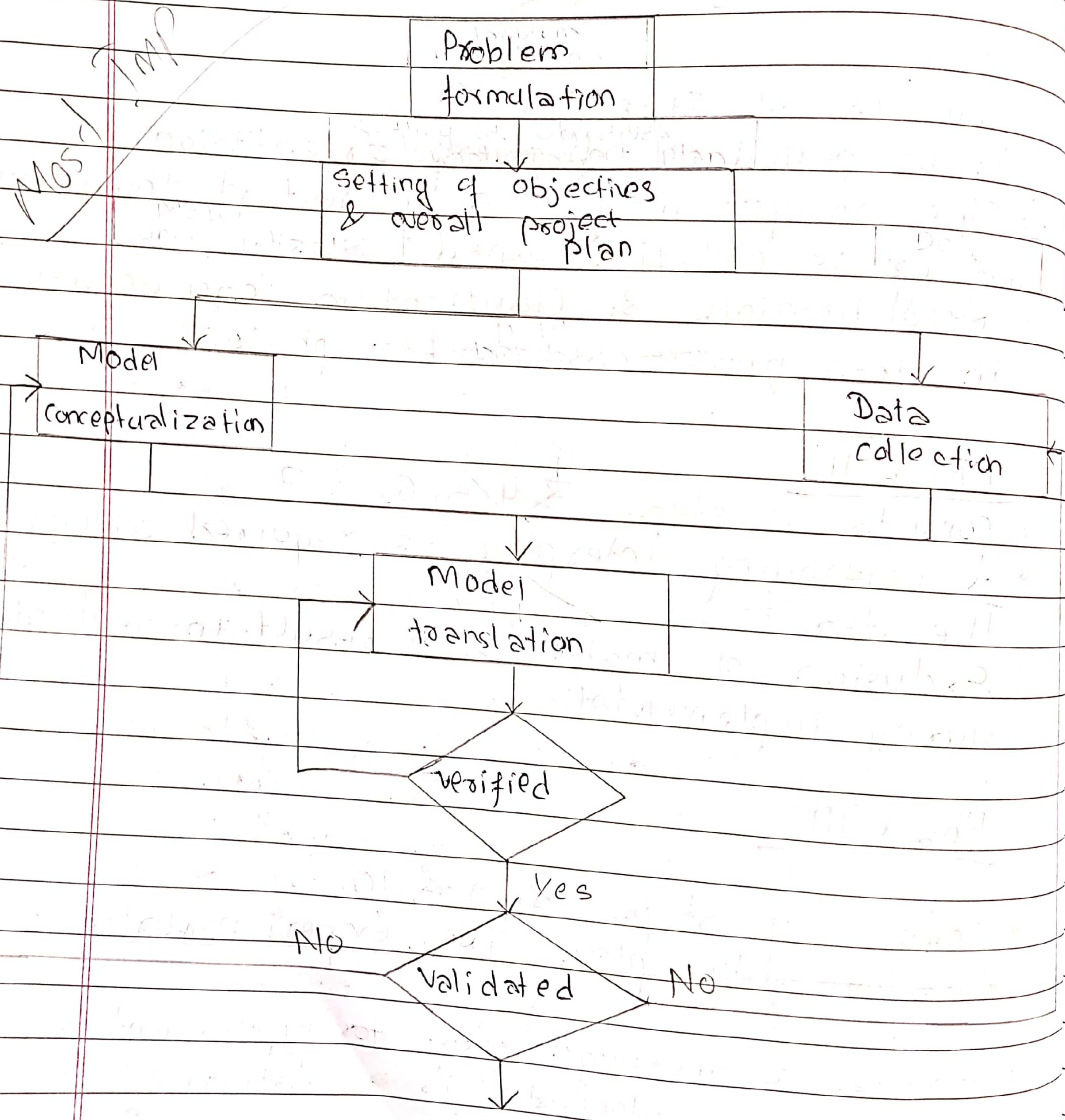
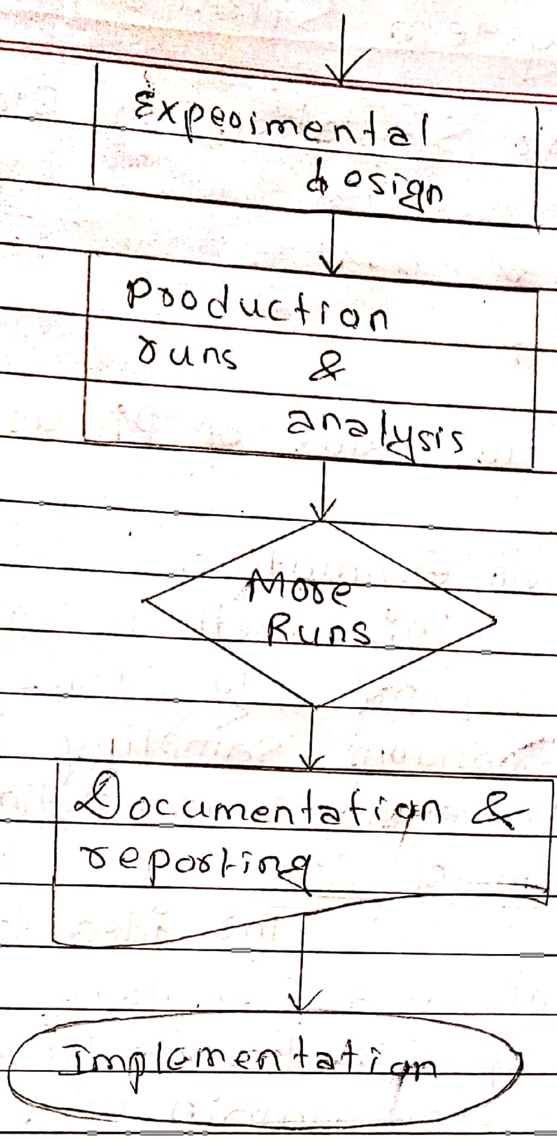
- The output variables are estimates that contain random error & therefore proper statistical analysis is required.

IV Phase

- Consists of Steps 11 & 12
- Successful implementation depends on the involvement of user & every steps successful completion.

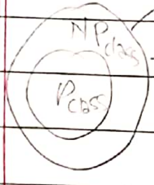



Experimental design
Production runs & analysis
More Runs
Documentation & reporting
Implementation

Unit - 2

~~Mont~~ System Simulation

Monte Carlo Simulation or Static simulation



Estimating the value of Pi using Monte Carlo

1) Monte Carlo estimation

This methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. It example is Estimation of Pie

2) Estimitation of Pi

The idea is to simulate random (x,y) points in a 2-D plane with domain as a square of side 1 unit. Imagine a circle inside the same domain with same diameter and inscribed into the square. We then calculate the ratio of number points that lied inside the circle & total number of generated points. Refer to the image below:

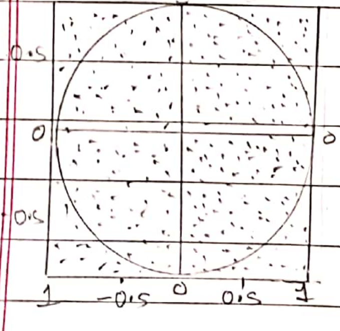


Fig:

We know that area of the square is 1 unit sq which that of circle is

$\pi * \left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$. Now for a very large number of generated points,

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$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\text{no. of points generated inside the circle}}{\text{total no. of points generated or no. of points generated inside the square.}}$$

that is,

$$\pi = 4 * \frac{\text{no. of points generated inside the circle}}{\text{no. of points generated inside the square}}$$

The beauty of this algorithm is that we don't need any graphics or simulation to display the generated points. We simply generate random (x,y) pairs & then check if $x^2+y^2 \leq 1$. If yes, we increment the number of points that appears inside the circle. In randomized & simulation algorithm like Monte Carlo the more the number of iterations, the more accurate the result is.

Examples :-

Interval = 5

Output : Final Estimation of Pi = 2.56

Interval = 10

Output : Final Estimation of Pi = 3.24

Interval = 100

Output : Final Estimation of Pi = 3.0916

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MONTE - CARLO SIMULATION

Numerical Analysis on the Basis for random number

Question :

Kantipur Dental Clink who schedules all their patients for 30 mins appointments some of the patients take more or less than 30 mins depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities & the time actually needed to complete the work.

category	Time required (in mins)	No. of patients
Filling	45	40
Crown	60	15
Cleaning	15	15
Extracting	45	10
Checkup	15	20

Simulate the dentist clinic for 4 hours and find out the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the time clinic at exactly their scheduled, arrival time starting at 8:00 am

Use the following random number for handling the above problem. 40, 82, 11, 34, 25, 66, 17, 79

Soln

Steps in MONTE-CARLO Simulation

step 1: Establishing probability distribution

step 2: Calculating cumulative probability distribution.

step 3: Setting random number intervals.

step 4: Generating random numbers.

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Step 1: Establishing probability distribution.

Category	Time required (min)	No. of patients	Probability
Filling	45	40	40/100 = 0.4
Crown	60	15	0.15
Cleaning	15	15	0.15
Extracting	45	10	0.1
Checkup	15	20	0.2

Step 2 Calculate cumulative probability distribution

Step 3 Setting random number intervals.

Category	Probability	Cumulative probability	Random number
Filling	0.4	0.4	00-39
Crown	0.15	0.4+0.15=0.55	40-54
Cleaning	0.15	0.55+0.15=0.70	55-69
Extracting	0.1	0.70+0.1=0.8	70-79
Checkup	0.2	0.8+0.2=1.00	80-99

Random No: Make a particular interval or particular range.

Step 4: Generating random numbers.

Eight patients is in the system (denal), as determined in given random no's. Simulation goes for 4 hours. (i.e 240 mins)

1 hour = 60 min   4 hour = 240 min

Serving each person @30 min = 240 min/30 min = 8 patients (i.e doing the simulation for 8 patients for 4 hours)

Patient No	Scheduled Arrival Time	Random No	Category	Service Time
1	8:00	40	crown	60
2	8:30	82	Checkup	15
3	9:00	11	Filling	45
4	9:30	34	Filling	45
5	10:00	25	filling	45
6	10:30	66	Cleaning	15
7	11:00	17	filling	45
8	11:30	79	Extracting	45

Patient No	Arrival Time	Service Start	Service Duration	Service End	waiting min	Idle time
1	8:00	8:00	60	9:00	0	0
2	8:30	9:00	15	9:15	30	0
3	9:00	9:15	45	10:00	15	0
4	9:30	10:00	45	10:45	30	0
5	10:00	10:45	45	11:30	45	0
6	10:30	11:30	15	11:45	60	0
7	11:00	11:45	45	12:30	45	0
8	11:30	12:30	45	1:15	60	0
8						0
					$\Sigma = 285$	

Average waiting Time $= \frac{285}{8} = 35.625$ mints.

Doctor to ~~the~~ be free (was never) = Idle time
or average idle = 0 min

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Principle used in Modeling

1) Block Building:-

The description of system should be organized in a series of blocks. The aim is to specific the specification of the interaction within the system. The system as a whole can then be discussed in terms of inter connection between the blocks.

2) Relevance:

Only include those aspect that has relevant to study of objectives. Irrevelant information should be excluded as increases the complexity of model.

3) Accuracy:

The accuracy of the information gather for the model should be considered. For Ex Aircraft system, an engineer is responsible for ~~estal~~ estimating the fuel consumption may be satisfied with the simple representation. Another enginee responsible for comfort of the passenger needs to consider vibration and with one detail description of the year frame.

4) Aggregation:

It is the extend, a number of individuals, entities can be grouped together into large entities

Distributed lag model or (Dynamic Model)

- The model that have properties of changing only at fixed interval of time & of using the current values of the variables on other current values & values that are occured in previous interval is known as Distributed Lag model.

- There are types of dynamic models because time factor is involved in them.

- As a rule, this model consists of Linear Algebric Equation.

- As an example, consider the following dynamic model of National economy as following where C is the consumption, I is investment
T is tax
G is Govenrment Expenditure
Y is National Income.

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$$C = 20 + 0.7\,(Y - T)$$
$$I = 2 + 0.1\,Y$$
$$T = 0.2\,Y$$
$$Y = C + I + G$$
eqⁿ ①

Eqⁿ ① is the static model. But it can be make dynamic by picking a fixed interval time. Say a one year and expressing the current year values of the variable's in terms of values in the previous year.

Any variable that appear in the form of current values in one or more previous year values is called lagged variables. Value of the previous year is denoted by suffix with $t-1$ (log y_{-1}) The static model can be made dynamic by lag all variables as follows:

$$C = 20 + 0.7\,(y_{-1} - T_{-1})$$
$$I = 2 + 0.1\,y_{-1}$$
$$T = 0.2\,y_{-1}$$
$$y = C_{-1} + I_{-1} + G_{-1}$$
eqⁿ ②

$\frac{183}{500} \times 0.366$

For making accuracy

$\frac{m}{N} \times A = 39$

$\frac{39}{75} \times 75 = 39$

Monte Carlo Integration

(3) integrate $I = \int_2^5 x^2 \cdot dx$

Solve by
① Analytically model [i.e monte Carlo]
② Mathematical model [mathematical calculas]

② for mathematical

$I = \int_2^5 x^2 dx$

$f(x) = y = x^2$

x	0	1	4	9	16	25
y	0	1	2	3	4	5

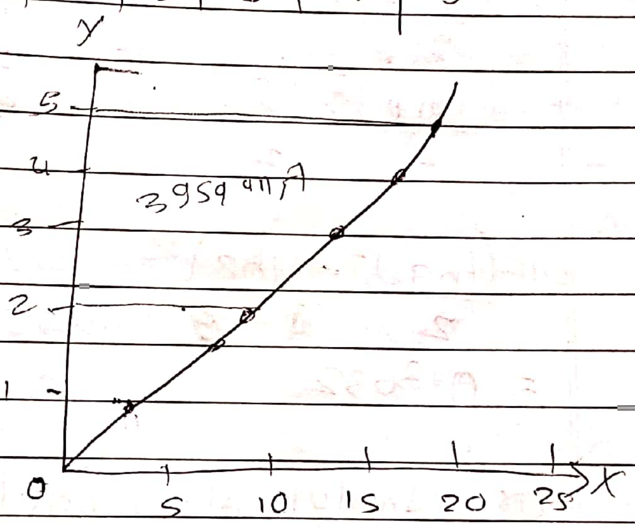
$I = \int_2^5 x^2 \cdot dx$

$= \frac{x^3}{3} \Big|_2^5$

$= \frac{5^3}{3} - \frac{2^3}{3}$

$= \frac{1}{3}(125-8)$

$= 39$ sq unit



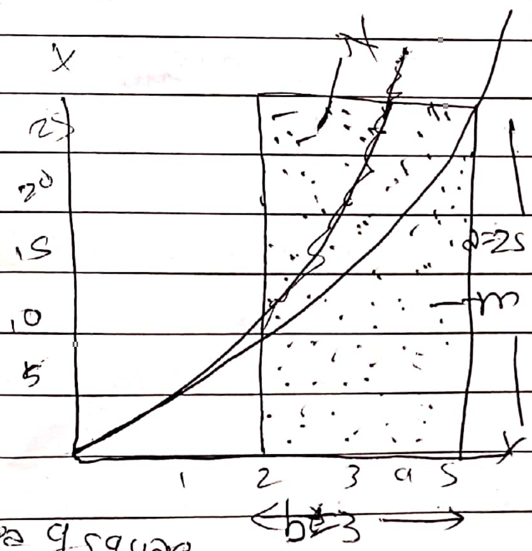
① Analytically model

Area of sq (A) $= b \times l = 3 \times 25 = 75\, cm^2$

Random dots on (m) = 25
Random dots on (N) = 50

Area of ~~the shaded region~~ $= \frac{m}{N} \times$ Area of square

$= \frac{25}{50} \times 75 = 37.5$



$\frac{m}{N} \times A = \frac{455}{1250} \times 1$ $\quad y = \frac{u^2}{2}$

$$\int_2^3 \frac{\ln x}{x}\,dx \quad \text{—(1)}$$

y	0	1	2	3	4	5
x	0	0	0.3465	0.366	0.3465	0.32

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$ —(2)

$du = \frac{1}{x}\,dx$

From (1) & (2)

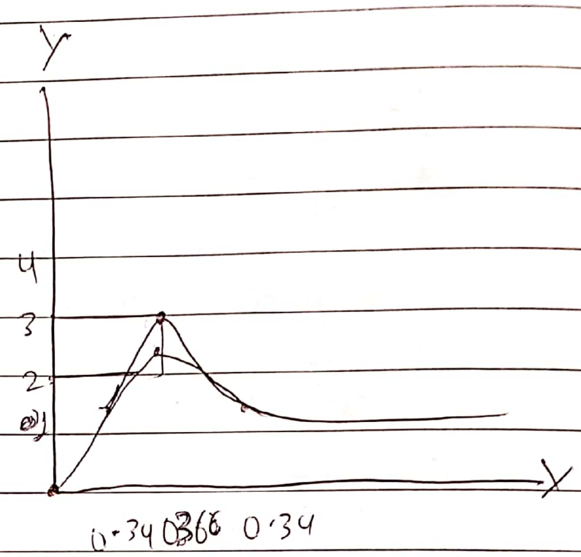
$\int u\,du$

$= \frac{u^2}{2} + C$

$= \left.\frac{(\ln x)^2}{2}\right|_2^3$

$= \frac{(\ln 3)^2}{2} - \frac{(\ln 2)^2}{2}$

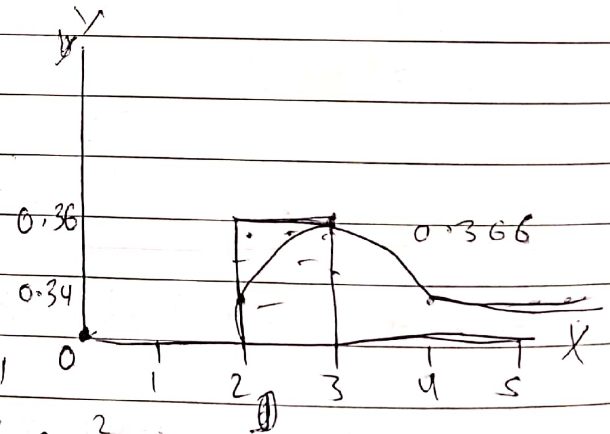
$= 0.3632$



→ For Analytical model

$l = 0.366$
$b = 1$

Area of square (A) = $l \times b$
$= 0.366 \times 1$
$= 0.366\ cm^2$



Random dots on m = 82
Random dots on

Q. Comparison of Simulation & Analytical method

Simulation gives the specific solution for given set of data where as analytical method gives solution for all of the condition:-

- To find the general solution, the simulation need to be repeated many time.

- For analytical study, the simulation should be model by using some specific formate.

- Simulation consists of lots of steps by step ~~breach~~ execution. If the simulation is run for several time than there will be increase in ~~deati~~ detail complexicity.

- Ideal way of simulation use extension of mathematical solution.

Experimental Nature of Simulation

Simulation observe the way in which all the variables of the model change with time.

- Instead of using analytical solution, a system might be ~~the~~ tested against lots of input samples & output ~~can be discovered~~ is observed.
- The relationship between input & output can be discovered
- It requires lots of simulation ~~run~~ learn.
- Popular in simulation study
- It must be planned as a series of experimentation.

Techniques used in Monte carlo Simulation

It is a computation algorithm, that delay on the ~~repetative~~ repeatate random sample to obtain numerical figure. (1) It solves the problem using statistical analysis on the output generated from the repeated random sample.

(2) It is used in physical & mathematical problems that are more useful when the problem is difficult

Example of Domain Distributed Lag Model or Dynamic Model or Cobweb model

- Cobweb model is the numerical & analytical approach to debicate the relation of demand & supply in the market
- Cobweb model explain the modularity recurring cycles

in the output price & prices of the farm products
- Cobweb theory was 1st suggested by professor N. Kaldor in 1934.
- Cobweb theory is based upon Lag concept (time series delay analysis).
- It is a dynamic model.
- The example domain can be the economic phenomenon such as the production of grains, wheat, national income etc.
- The supply of wheat is lagged function of price, so there is lagged in supply.
- There is no lagged in the demand side.
- It describe the cyclical supply & the demand of the market where the amount of produced must be choose before the price
- producer expectations ~~about~~ assume the price are assumed to be based on observation of previous price.
- The cobweb model is based on time lag between the supply & demand decision
- Agriculture market are context where the cobweb model migh applied since there is a lag between planting, harvesting.

Supply & Demand in Distributed lag model can be expressed as follows

D (Demand) $= a - bP_t$

S (Supply) $= c + d P_{t-1}$

where, constant a, b, c, d have their usual meaning
~~whe~~ b & d = slopes in the demand & supply function respectively

P = ~~present year~~ current market price P_{-1} = ~~first~~ previous year price.

$P_t = P_0$ Dynamic → Distributed → cobweb

$P_0 = 1$

Q ~~Use the cobweb to investigate~~

Q Draw a cobweb model of the following initial condition: $P_0 = 1$, $a = 12.4$, $b = 1.2$, $c = 1.0$, $d = 0.9$

Given,

D (Demand) $= a - bP$

S (supply) $= c + dP_{-1}$

We assume market is cleared, For equilibrium point

$S = D$

Using given values

$D = 12.4 - 1.2P$

& $S = 1 + 0.9P_{-1}$

initial $P_0 = 1$ Given

1st Iteration,

We assume, $S = D$

$1 + 0.9P_0 = 12.4 - 12P_1$

or, $P_1 = \dfrac{12.4 - 1 - 0.9P_0}{1.2}$

As we are given initial value of price.

$P_{-1} = P_0 - 1$

$P_1 = \dfrac{12.4 - 1 - 0.9 \times 1}{1.2}$

$\boxed{\therefore P_1 = 8.75}$

2nd Iteration:

$1 + 0.9P_1 = 12.4 - 1.2P_2$

or, $P_2 = \dfrac{12.4 - 1 - 0.9P_1}{1.2}$

$\boxed{\therefore P_2 = 2.9375}$

3rd iteration:

$1 + 0.9P_2 = 12.4 - 1.2P_3$

or, $P_3 = \dfrac{12.4 - 1 - 0.9 \times 2.9375}{1.2}$

$\boxed{P_3 = 7.2968}$

4th iteration:

$1 + 0.9P_3 = 12.4 - 1.2P_4$

or, $P_4 = \dfrac{12.4 - 1 - 0.9 \times 7.2968}{1.2}$

$\boxed{\therefore P_4 = 4.0273}$

5th iteration:

$1 + 0.9P_4 = 12.4 - 1.2P_5$

$P_5 = \dfrac{12.4 - 1 - 0.9 \times 4.0273}{1.2}$

$\boxed{P_5 = 6.4795}$

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6th iteration

$1+0.9P_5 = 12.4 - 1.2P_6$

$P_6 = \frac{12.4 - 1 - 0.9 \times 6.4795}{1.2}$

$\boxed{P_6 = 4.63}$

7th iteration

$1+0.9P_6 = 12.4 - 1.2P_7$

$P_7 = \frac{12.4 - 1 - 0.9 \times 4.63}{1.2}$

$= 6.0275$

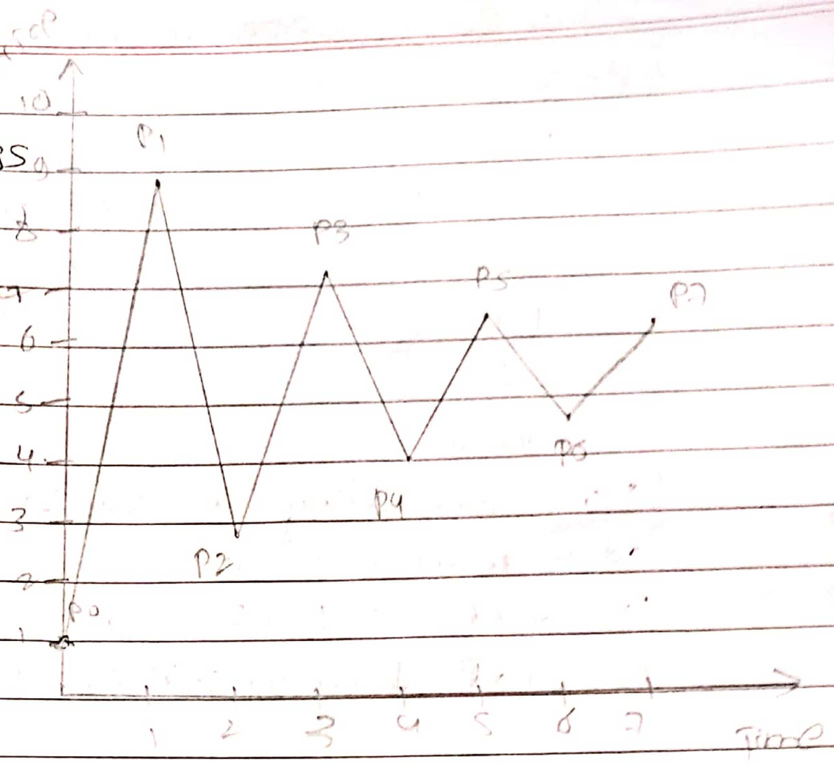

Fig: Fluctuation of Market Price

Now Evaluating Demand & Supply in every interval of time basis on the price

1st interval $P_0 = 1$

$D_0 = a - bP_0 = 12.4 - 1.2 \times 1 = 11.2$

$S_0 = c + dP_0 = 1 + 0.9 \times 1 = 1.9$

2nd iteration

$P_1 = 8.75$

$D_1 = 12.4 - 1.2 \times 8.75 = 1.9$

$S_1 = 1 + 0.9 \times 8.75 = 8.875$

3rd iteration

$P_2 = 2.93$

$D_2 = 12.4 - 1.2 \times 2.93 = 8.884$

$S_2 = 1 + 0.9 \times 2.93 = 3.637$

4th iteration $P_3 = 7.29$

$D_3 = 12.4 - 1.2 \times 7.29 = 3.652$

$S_3 = 1 + 0.9 \times 7.29 = 7.561$

5th iteration P_4

$D_4 = 12.4 - 1.2 \times 4.02 = 7.576$

$S_4 = 1 + 0.9 \times 4.02 = 4.618$

6th iteration $P_5 = 6.47$

$D_5 = 12.4 - 1.2 \times 6.47 = 4.636$

$S_5 = 1 + 0.9 \times 6.47 = 6.823$

7th iteration $P_6 = 4.63$

$D_6 = 12.4 - 1.2 \times 4.63 = 6.844$

$S_6 = 1 + 0.9 \times 4.63 = 5.167$

Now, Price when market is cleared

$P = \frac{a-c}{b+d}$

$\because S = D$

$a - bP = c + dP$

or, $bP + dP = a - c$

$P = \frac{a-c}{b+d}$

$= \frac{12.4 - 1}{1.2 + 0.9}$

$= \frac{11.4}{2.1}$

$= 5.4286$

Now, evaluating the supply & demand using price when market is cleared

$D = a - bP = 12.4 - 1.2 \times 5.4286 = 5.8856$

$S = c + dP = 1 + 0.9 \times 5.4286 = 5.8857$

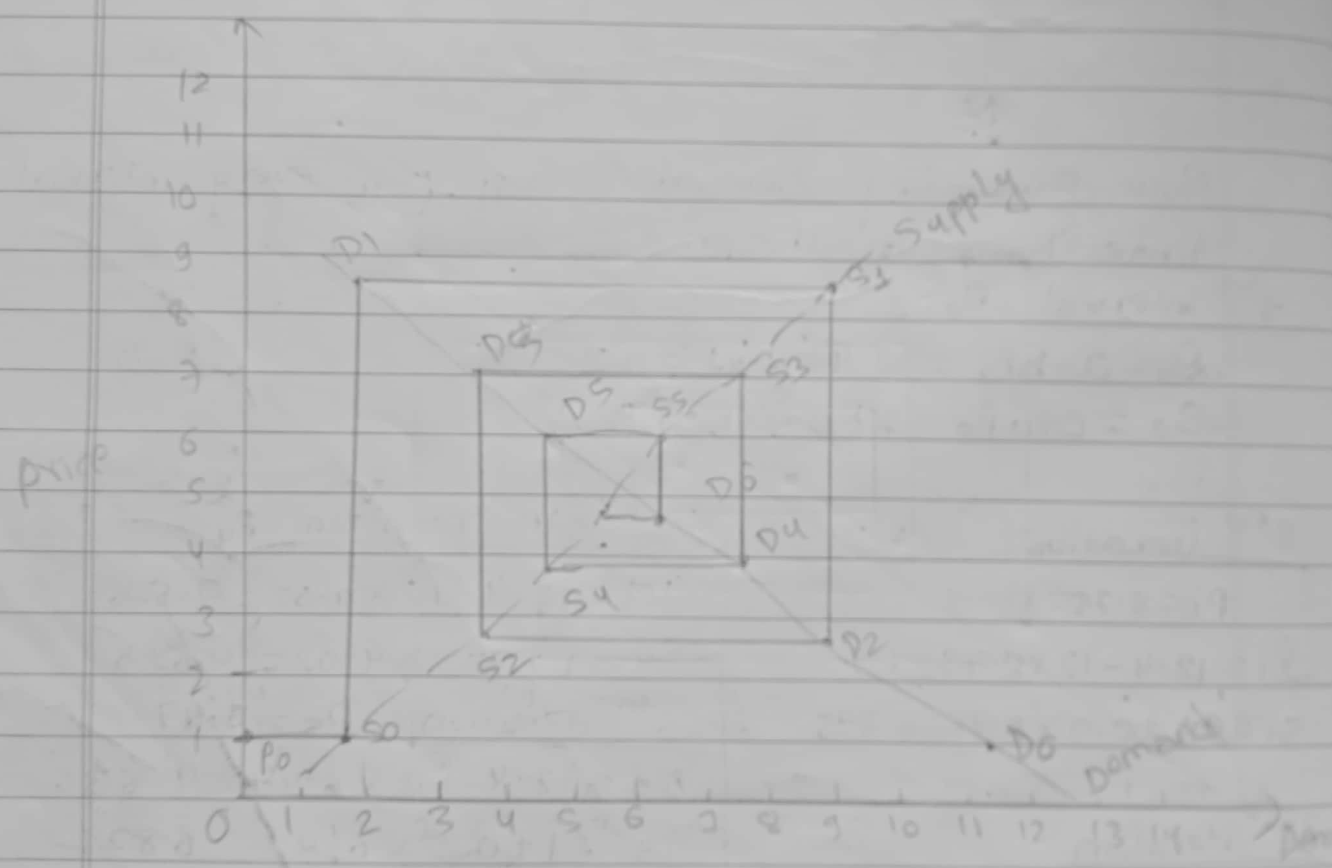

Fig- Supply Demand Cobweb Model

Now, Price when market is cleared

$$P = \frac{a-c}{b+d}$$

$\because S = D$

$a - bP = c + dP$

or, $bP + dP = a - c$

$$P = \frac{a-c}{b+d}$$

$$= \frac{12.4 - 1}{1.2 + 0.9}$$

$$= \frac{11.4}{2.1}$$

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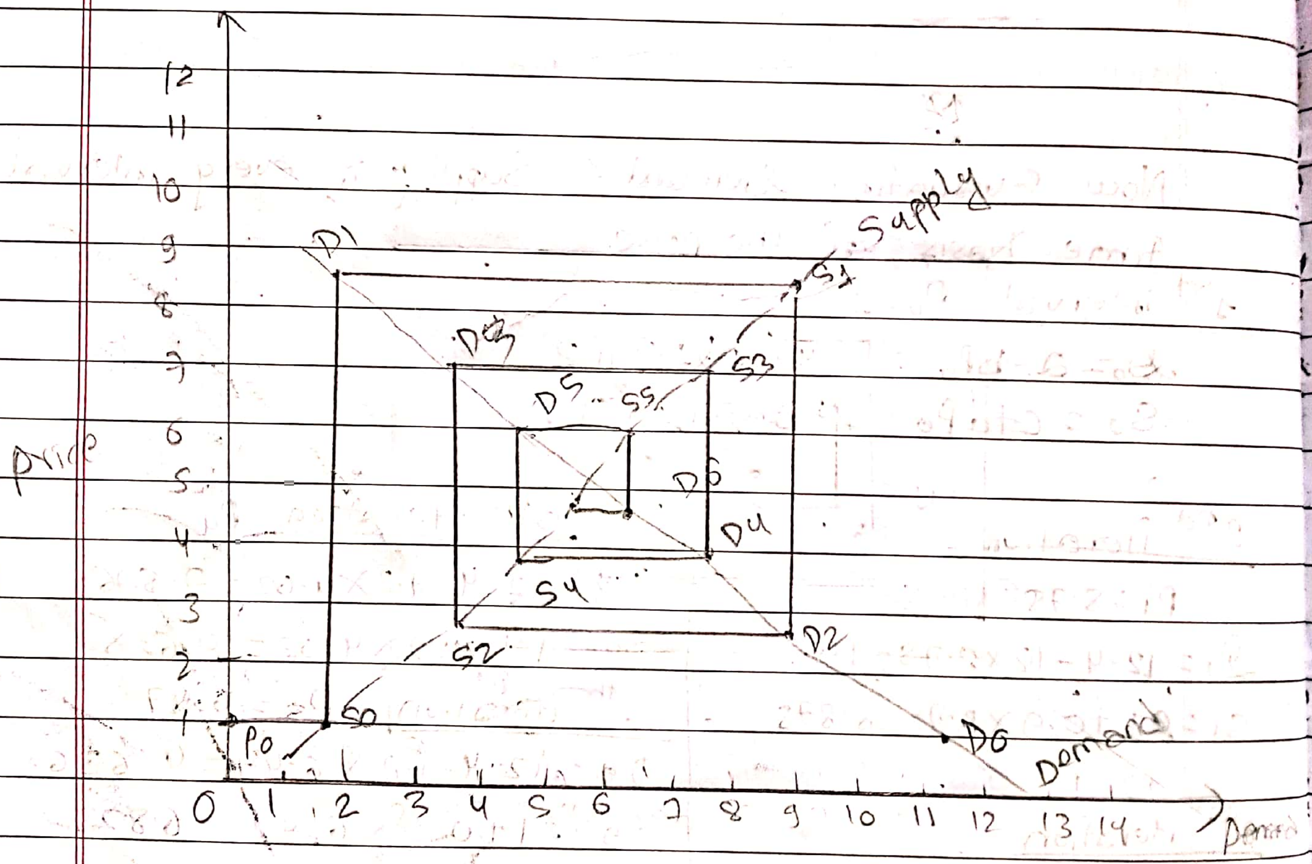

Fig: Supply Demand Cobweb Model

Q. Use a Cobweb model to investigate a market in which the supply and demand function are given below

$Q = \frac{17.91}{p^{1/2}} - 4.66$

$9S = 5.0\,(P_{-1} - 1)$ Assume the market is clear

$Q = S$

$\frac{17.91}{p^{1/2}} - 4.66 = \frac{5P_{-1} - 5}{9}$

or, $\frac{17.91}{\sqrt{P}} - 4.66\sqrt{P} = \frac{5P_{-1} - 5}{9}$

or, $9 \times 17.91 - 9 \times 4.66\sqrt{P} = 5P_{-1}\sqrt{P} - 5\sqrt{P}$

or, $9 \times 17.91 =$

Queuing ~~Mod~~ Method:

The line ~~waith~~ wait entities or the customer wait is known as queue.

- The combination of all entities in system being served & being waiting for services will be called queing system.

- If the customer after arriving ~~and~~ can enter the service center, it is good otherwise they have to wait for the service & form a queue i.e waiting line they remain in queue till they are provided the services.

- Characteristics of elements of queeing system

1) Calling population
   Finite   Infinite
2) System Capacity
   - Limited   Unlimited
3) Arrival process
   Random arrival   Scheduled arrival
4) Queue Behaviour & Discipline
5) Service Time & Service Mechanism

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a) Arrival process of pattern

Any ~~few~~ queue ~~systing~~ system must work ~~work~~ on something customers, parts, patients, orders, etc. We generally called them as entities or customer. Before entities can be processed or subject to waiting they must first entery the system depending on the environment, entities can arrive smoothly or in unpredictable fashion. They can arrive ~~smoothly~~ ~~or~~ ~~in un~~ one at a time or in ~~dumbs~~ They can arrive independently or accoundantly to some kind of coorelation. Eg: Where this occurs are phone calls arriving at an exchange, customer arriving at a fast food restaurant, hits on the website & many others.

(b) Service process

Once the entities has entered the system they must be ~~solved~~ served. The physical meaning of service depend on the system, customer may go the checkout process.

- Parts may go through matching patients, & it may go through medical treatment & so on.
- From modeling stand point, the operation characteristics of service matter more than the physical characteristic. Specially, we care abt whether the service time are long or short & whether they are regularly or highly available.
- We care abt whether the entities are processing first come first serve (FCFS) order or according to some kind of parity rule. We care abt ~~the~~ whether the entities are serviced through single server or by multiple server, working in paraller

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Queueing Behaviour & Discipline

Customer may balk join the queue when it is too long (eg:- cars pass off a drive through restaurant if there are too many cars already waiting) is known as the bulking.

Customer may also exit the system due to the impatience (eg customer kept waiting to long at a bank decide to leave without service or perishability (eg:- sample waiting for testing at a lab spoiled after some time period). It is known as reneging

When there is more than one line forming for the same service or server, the action of moving customer from 1 line to another line because they think that they have choose slow line, it is known as Jockeying.

Queue Discipline:

There are various queue Discipline

→ FIFO (First in First Out):

According to this rule, services is offered on the basis of arrival time of customer, the customer who come first will get the service 1^{st}, so in other word, the customer who get the service next will be determine on the basis of longest waiting time.

→ LIFO (Last in First Out):

It is usually abbribated as a LIFO, that ~~occurs~~ occurs when service is next offered to the customer that arrive recently or which have the waiting time least. In the crowded train, the passanger gain in or out from the train is the eg of LIFO

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c) # Service in random order (SIRO)

It means that the random choice is made between the waiting customer at a service time is offered i.e a customer is picked up randomly from the waiting queue ~~from~~ the service.

D # (~~Certain Time First~~) Shortest Processing Time First:

It means that the customer will shortest service time will be choosen 1st for the service i.e the shortest service time, customer will get the priority in the selection process

E) Priority:

A special nor is assigned to each customer in the waiting line. it is called priority. Then according to the number, the customer is choosen for the service.

Queue Theory

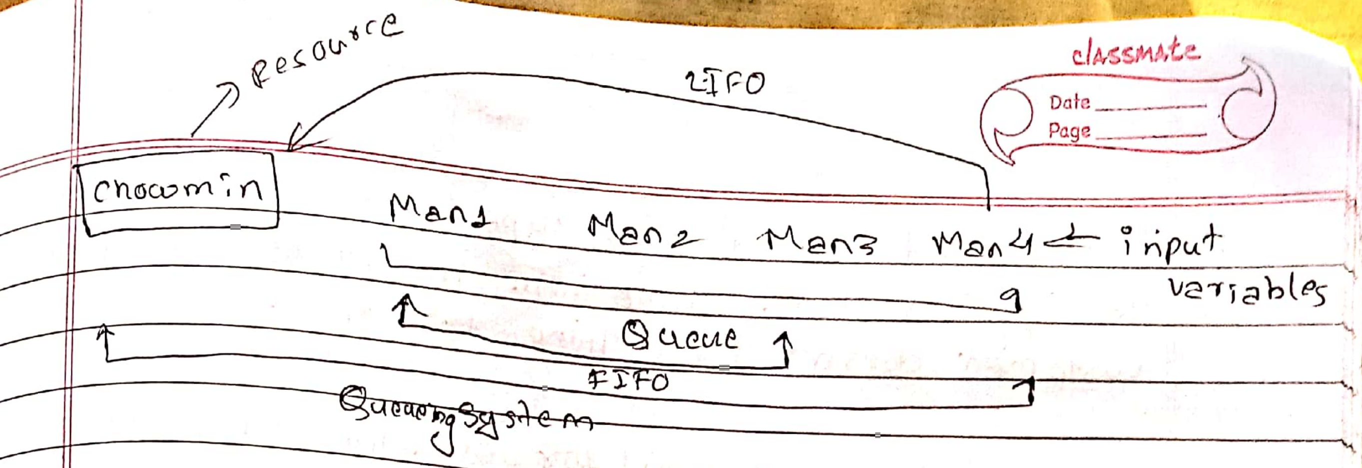
1) Queue
2) Queueing Theory
2) Reason of Queue

(1) Queue:

A line of people or vehicles awaiting for their turn is known as Queue.

(2) Queueing Theory:

It is a mathematical theory of waiting time/queue, this technique provides the basic decision making about the resource needed to provide a service.



Reason of Queue

A Queue is formed when a customer is made to wait due to the fact that number of customer are more than a service provider.

OR

The serving time to a customer is more than the arrival time of the customer.

Resson to study Queue

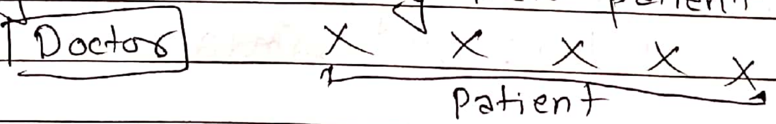
- To find out the cost of offering the service.
- To find out the cost in occured ~~about~~ due to the delay of occuring service.
- To allocate the resource. (eg:- from the above fig the system contains (Chowmin.

Structure / Component of Queueing System (4 type)

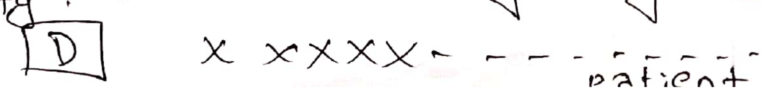
1) Calling Population: (2 type)

i) Size of population: The calling popⁿ means the number of customer waiting for their turn (2 type)

a) Finite: A limit on the customer for waiting the queue. eg:- Doctor serving their patient in clinic.



b) Infinite: Therea is no limit on the customer. eg Customer arriving at a bank, student arriving to gate, Admission at University.

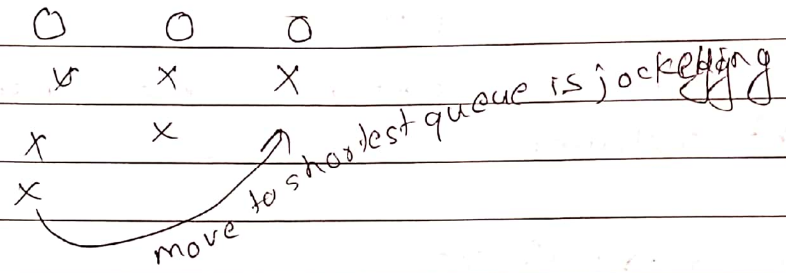


ii) Behaviour of Arrival (3) type

a) Balking : Custome don't join the queue may be the customer doesn't have time to wait.

b) Renging : customer wait for sometime in queue but leave before being served.

c) Jockeging : Customer move from one queue to another hoping to receive service more quickly.



Pattern of arrival:

- The customer can arrive to the patches individually. Eg Family or patches
- The customer may be arrived in schedule or unschedule time ; eg: chef in the resturant may come inside or outside 9 o clock.
- The arrival of time is calculated using Poission Distribution, Exponential Distribution

Poission Distribution

The probabilities for the number of customer that may arrive in the any specific interval of time λ (lamda)

The propability distribution of x for the Poission Distribution is given below

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

Exponential Distributions

The probability for the times gap between two consecutive arrivals i.e (μ)

2) Queuing Process (structure wala ko 2nd)

The queue process refers to the no. of queue & their respective length.

Eg:

Cinema line	~~xxxxx~~
Tax office	x ~~xxxxxxxx~~ xx
ATM	x 0000000

~~3) Queue discipline~~

a) Finite Source Queue

This source are provided to a limited no. of customer. Eg: Cinema, Admission, Restaurant

b) Infinite source Queue

The services are provided to all the customer who come at random time;

Eg: order receiving to the sales department,

Eg. Queue in the tax office.

3) Queue discipline.

Queue discipline are the ~~bu~~ rules formed to serve the customer

1) First in First Out, or First come First out
2) Last come First out

3) Served in random order
4) Priority based order i.e emergency

④ Service provider
Service process means the process of man or machine to solve the customer.
eg :- Token in a bank for service to provide the customer.

Queuing Notation

Kendall's Notation define the following queing theory notation i.e A/B/C/N/K
where,
A = arrival pattern
B = service pattern
C = Number of servers
N = System capacity
K = calling population

The above symbols used for the probability distribution for inter arrival times & service time as D for Deterministic, M for Exponential or (Markov) & E_k for Erlang.
If the capacity is not specified as it is taken is infinite & if calling population is not specified, it is assumed to unlimited or infinite.
For eg : M/D/2/5/∞
It stands for the queing system having exponential or Markov chain arrival time in a deterministic service time which contains two (waiter) servers & 5 customers & infinite calling popⁿ.

eg: M/D/2

It stands for exponential arrival times deterministic service time two servers, capacity of infinite customer & infinite calling population

Central Limit Theorem (H/w) → Normal Distribution with formula

Measure of Queues

We have already define the mean inter arrival time ~~(EA)~~ T_a & mean Suffic time T_s & the corresponding rates

Arrival rate $\lambda = \frac{1}{T_a}$.

Service rate $\mu = \frac{1}{T_s}$.

The following measures are used in the analysis of queue system

(i) Traffic intensity
(ii) Server utilization

imp

Traffic intensity

The ratio of mean service time to mean interarrival time is called traffic intensity

$$\mu = \frac{T_s}{T_a}$$

If there is any bulking or renging anot all the arrival entities get solved. It is necessary therefore distinguish between the actual arrival rate & the arrival rate of entities that get served. λ denotes all the arrivals included bulking or renging.

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iii) Server utilization
It consists of only the arrival that get served. It is denoted by ρ (row) & define as

$$\rho = \lambda \times T_s = \frac{\lambda}{\mu}$$

* Server utilization for single server
- This is also the average number of customer in the service facility. ~~The pro~~
- The probability of finding service founder free is $(q = 1 - P)$

Poission's Arrival pattern (Single server queue mechanism)
The condition is which arrival of customer is completely random means that an arrival can occur at any time & the time of next arrival is independent of previous arrival. With this assumption it is possible to show that the distribution of interarrival time is exponential. This is equivalent to saying that the no. of arrival for unit time is random variable with a poission distribution.

This distribution is used when chance occurance of event out of large sample is small i.e if X = no. of arrival for unit time then probability distribution function for arrival is given by

$$f(x) = P(X = x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$\therefore \lambda > 0$

$x = 0, 1, 2, 3 \ldots$

where λ is an average number of arrivals per

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$\rho = \lambda \times T_s$

$= \frac{\lambda}{\mu}$

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$\therefore \lambda > 0$

$x = 0, 1, 2, 3 \ldots$

where λ is an average number of arrivals per unit time $\left(\frac{1}{\tau}\right)$ τ = Tau

x = number of customer per unit time

The pattern of arrival is called Poission's arrival pattern

eg:- In a simple form, service station vehicle arrived for ~~fill~~ fueling with an average of 5 mints between arrivals. If a hour is taken as a unit of time, cars arrive according to the Poission's process $\lambda = 12$ cars per hour. Distribution of number of arrivals per hour is $\frac{e^{-12}\lambda^x}{x!}$