MODULE ONE PROBLEM SET

Christian Lloyd

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

In the following question, the domain of **discourse** is a set of male patients in a clinical study. Define the following predicates:

- $P(x):$ x was given the placebo
- $D(x):$ x was given the medication
- $M(x):$ x had migraines

Translate each of the following statements into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

Sample question: Some patient was given the placebo and the medication.

- $\exists x\ (P(x)\ \wedge\ D(x))$
- *Negation:* $\neg\exists x\ (P(x)\ \wedge\ D(x))$
- *Applying De Morgan's law:* $\forall x\ (\neg P(x)\ \vee\ \neg D(x))$
- *English: Every patient was either not given the placebo or not given the medication (or both).*

(a) Every patient was given the medication or the placebo or both.

Given the placebo can be represented as P(x)
Given the medication can be represented as D(x)
Either given the placebo or given the medication can be represented as P(x) D(x)
Every patient implies $\forall x$
Hence the expression will be $\forall x\ (P(x)\ \vee\ D(x))$
Negation: $\neg\forall x\ (P(x)\ \vee\ D(x))$
Applying De Morgan's law: $\exists x\ (\neg P(x)\ \wedge\ \neg D(x))$

(b) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \to q \equiv \neg p \vee q$.)

Took the placebo can be represented as P(x)
Had migraines can be represented as M(x)
Every patient implies $\forall x$
Who took the placebo had migraines can be represented as $P(x)\ \to M(x)\ = \neg P(x)\ \vee\ M(x)$
Hence the expression will be $\forall x\ (P(x)\ \to M(x))\ =\ \forall x\ (\neg P(x) \vee M(x))$
Negation: $\neg\forall x\ (\neg P(x)\ \vee\ M(x))$
Applying De Morgan's law: $\exists x\ (P(x)\ \wedge\ \neg M(x))$

(c) There is a patient who had migraines and was given the placebo.

Took the placebo can be represented as P(x)
Had migraines can be represented as M(x)
Took the medication and had migraines can be represented as $M(x) \wedge P(x)$
There is a patient implies $\exists x$
Hence the expression will be $\exists x\ (M(x)\ \wedge\ P(x))$
Negation: $\neg\exists x\ (M(x)\ \wedge\ P(x))$
Applying De Morgan's law: $\forall x\ (\neg M(x)\ \vee\ \neg P(x))$

Problem 2

Use De Morgan's law for quantified statements and the laws of propositional logic to show the following equivalences:

(a) $\neg\forall x\ (P(x) \wedge \neg Q(x)) \ \equiv\ \exists x\ (\neg P(x) \vee Q(x))$

$(\exists x)\ (\neg P(x)\ \vee\ \neg(\neg Q(x)))\ (\exists x)\ (\neg P(x)\ \vee\ Q(x)))$

(b) $\neg\forall x\ (\neg P(x) \rightarrow Q(x)) \ \equiv\ \exists x\ (\neg P(x) \wedge \neg Q(x))$

$(\exists x)\ \neg(\neg(\neg P(x))\ \vee\ Q(x))\ (\exists x)\ \neg((P(x)\ \vee\ Q(x))\ (\exists x)\ (\neg P(x)\ \wedge\ \neg Q(x))$

(c) $\neg\exists x\ \big(\neg P(x) \vee (Q(x) \wedge \neg R(x))\big) \ \equiv\ \forall x\ \big(P(x) \wedge (\neg Q(x) \vee R(x))\big)$

$(\forall x)\ (\neg(\neg P(x))\ \wedge\ \neg(Q(x)\ \wedge\ \neg R(x)))\ (\exists x)\ (P(x)\ \wedge\ (\neg Q(x)\ \vee\ R(x)))$

Problem 3

The domain of **discourse** for this problem is a group of three people who are working on a project. To make notation easier, the people are numbered 1, 2, 3. The predicate $M(x,\ y)$ indicates whether x has sent an email to y, so $M(2,\ 3)$ is read "Person 2 has sent an email to person 3." The table below shows the value of the predicate $M(x,\ y)$ for each $(x,\ y)$ pair. The truth value in row x and column y gives the truth value for $M(x,\ y)$.

M	1	2	3
1	T	T	T
2	T	F	T
3	T	T	F

Determine if the quantified statement is true or false. Justify your answer.

(a) $\forall x\, \forall y\, (x \neq y) \ \rightarrow \ M(x,\ y))$

TRUE The only two people who haven't sent an email to each other are (2,2) and (3,3). So, if $(x \neq y)$ then $M(x,\ y)$ is true.

(b) $\forall x\, \exists y \ \ \neg M(x,\ y)$

FALSE 1 has sent an email to all persons 1,2,3. So, for x = 1 there doesn't exist any y such that M(x,y).

(c) $\exists x\, \forall y \ \ M(x,\ y)$

TRUE x = 1 is the case where M(x, y) is true for all 1,2,3.

PROBLEM 4

Translate each of the following English statements into logical expressions. The domain of **discourse** is the set of all real numbers.

(a) The reciprocal of every positive number less than one is greater than one.

$\forall x\ (((x > 0) \land (x < 1)) \rightarrow (1/x > 1)$

(b) There is no smallest number.

$\forall x \exists y\ (y < x)$

(c) Every number other than 0 has a multiplicative inverse.

$\forall x\ ((x \neq 0) \rightarrow (\exists y\ (xy = 1)))$

Problem 5

The sets A, B, and C are defined as follows:

$$A = tall, grande, venti$$
$$B = foam, no - foam$$
$$C = non - fat, whole$$

Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

(a) Write an element from the set $A \times B \times C$.

{(tail,foam,non-fat)}

(b) Write an element from the set $B \times A \times C$.

{(foam,tail,non-fat)}

(c) Write the set $B \times C$ using roster notation.

{(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}