



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 01



QUESTION [JEE Advanced 2025]



Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y , and z , for which there is a 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$. Then which of the following statements is (are) TRUE?

- A** The determinant of $Q - 2I$ is zero
- B** The determinant of $Q - 6I$ is 12
- C** The determinant of $Q - 3I$ is 15
- D** $yz = 2$

QUESTION [JEE Advanced 2025]

Consider the matrix $P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Let the transpose of a matrix X be denoted by X^T . Then the number of 3×3 invertible matrices Q with integer entries, such that $Q^{-1} = Q^T$ and $PQ = QP$, is

A 32

B 8

C 16

D 24

4. Let S denote the set of all real values of λ such that the system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

is inconsistent, then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to

(A) 12

(B) 2

(C) 4

(D) 6

[Ans. D]

5. Let $x, y, z > 1$ and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$. Then $|\text{adj}(\text{adj}A^2)|$ is equal to

(A) 6^4

(B) 2^8

(C) 4^8

(D) 2^4

[Ans. B]

6. If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2B$, then

(A) $A^2B = I$

(B) $A^2 = I$ or $B = I$

(C) $A^2B = BA^2$

(D) $AB = I$

[Ans. C]

If $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ and B_p is another matrix defined as

$$B_p = (A^p) (\text{adj} (\text{adj} (\dots (\text{adj} A))))$$

where adj is written p times, $p \in N$, then identify the correct statement(s)

A $\text{Tr}(B_1) = 4$

B $\det(B_2) = 8$

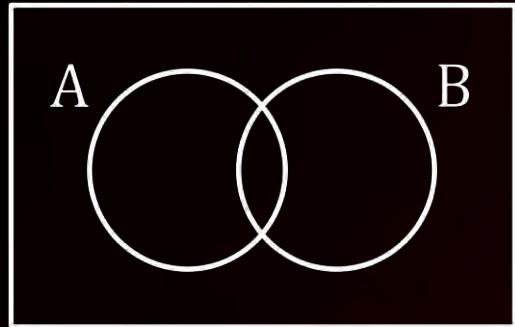
C $\sum_{p=1}^{\infty} \det(B_p^{-1}) = 1$

D $\text{adj} \cdot (\text{adj } B_2) = A^3$

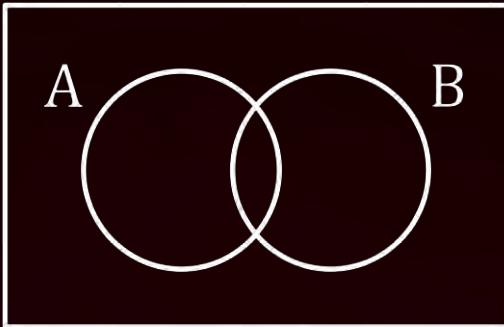
Ans. A, B, D

QUESTION

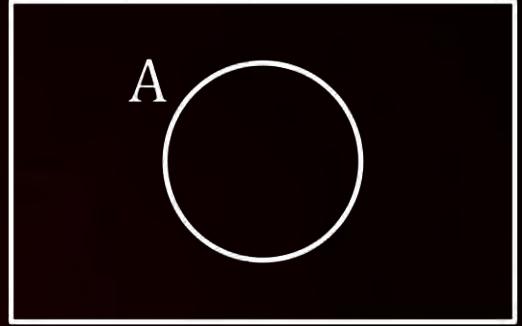
Shade indicated regions.



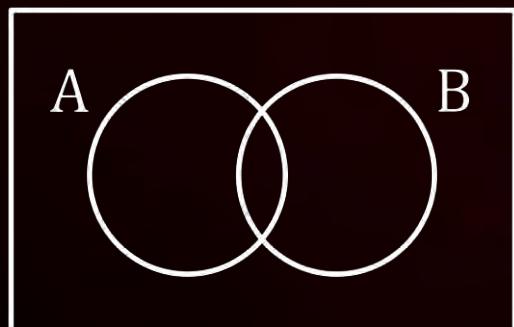
$$A \cup B$$



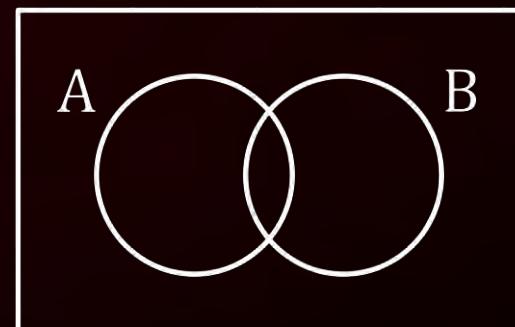
$$A \cap B$$



$$A^c \text{ or } A'$$



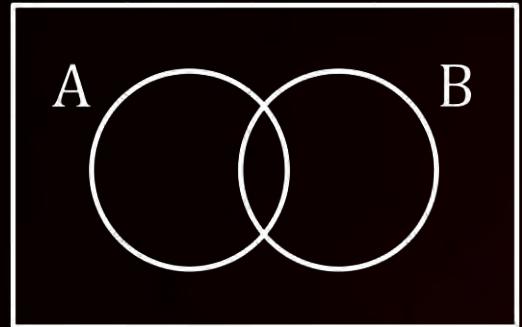
$$A - B$$



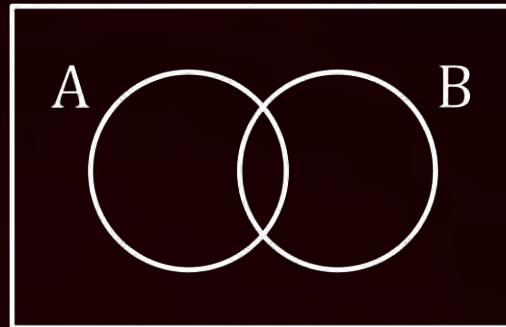
$$B - A$$

QUESTION

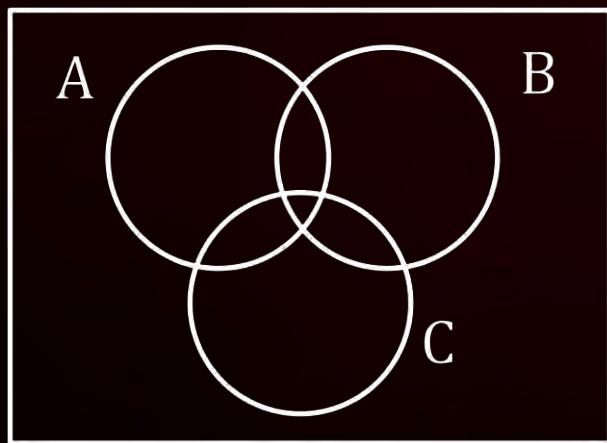
Shade indicated regions.



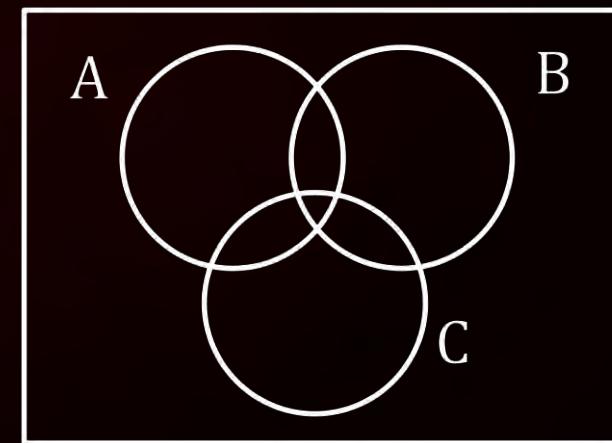
$$A \Delta B$$



$$(A - B) \cup B$$



$$A \cup B \cup C$$



$$A \cap B \cap C$$

QUESTION [BITSAT 2022]



If $A = \{x : x \text{ is a multiple of } 4\}$ and $B = \{x : x \text{ is a multiple of } 6\}$, then $A \cap B$ consists of multiples of

A 16

B 12

C 8

D 4

Ans. B

QUESTION [COMEDK 2024]



Let A and B be two sets then $A - (A \cap B)$ is equal to

A $(A \cap B)'$

B ϕ

C $A - B$

D $B - A$

Ans. C

QUESTION [WB JEE 2009]



For any two sets A and B, $A - (A - B)$ equals

- A** B
- B** $A - B$
- C** $A \cap B$
- D** $A^c \cap B^c$

Ans. C

QUESTION [COMEDK 2023]



If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$ then $(A - B) \times (B \cap C)$ is equal to

A $\{(a, c), (a, d)\}$

B $\{(a, b), (c, d)\}$

C $\{(c, a), (d, a)\}$

D $\{(a, c), (a, d), (b, d)\}$

Ans.

QUESTION



If $A = \{2, 4\}$ and $B = \{3, 4, 5\}$ then $(A \cap B) \times (A \cup B)$ is

A $\{(2, 2), (3, 4), (4, 2), (5, 4)\}$

B $\{(2, 3), (4, 3), (4, 5)\}$

C $\{(2, 4), (3, 4), (4, 4), (4, 5)\}$

D $\{(4, 2), (4, 3), (4, 4), (4, 5)\}$

QUESTION

If $n(A) = 7$, $n(B) = 8$ and $n(A \cap B) = 4$ then match the following columns.

(i)	$n(A \cup B)$	(a)	56
(ii)	$n(A \times B)$	(b)	16
(iii)	$n((B \times A) \times A)$	(c)	392
(iv)	$n((A \times B) \cap (B \times A))$	(d)	96
(v)	$n((A \times B) \cup (B \times A))$	(e)	11

QUESTION

If $n(A) = 5$, $n(B) = 4$, $n(C) = 7$ & $n(A \cap B) = 2$, $n(B \cap C) = 3$, $n(C \cap A) = 3$, then: find:

(i) $n((A \times B) \cap (B \times A)) =$

(ii) $n((A \times B) \cup (B \times A)) =$

(iii) $n((A \times B \times C) \cap (B \times C \times A)) =$

(iv) $n((A \times B \times C) \cap (A \times C \times B)) =$

QUESTION

If the difference between the number of subsets of two sets A and B is 120 then $n(A \times B)$ is equal to

A 21

B 25

C 18

D 24

QUESTION [COMEDK 2022]



If $A = \{3, 5, 7\}$ and $B = \{1, 2, 3, 5\}$ then $A \times B \cap B \times A$ is equal to

A $\{(3, 3), (5, 3), (2, 7), (7, 2)\}$

B $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$

C $\{(3, 3), (5, 5)\}$

D $\{(3, 5), (5, 5), (5, 3)\}$

Ans. B

QUESTION

Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is relation from X to Y

- A** $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$
- B** $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
- C** $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$
- D** $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

QUESTION

Consider set $A = \{-1, 0, 1, 2, 3\}$ and $B = \{1, 2, 4, 5, 0\}$, $R = \{(a, b) : b - a = 1, a \in A, b \in B\}$, then domain of relation R is:

A $\{0, 1, 3\}$

B $\{-1, 0, 1, 3\}$

C $\{1, 0, -1, 2, 3\}$

D $\{1, 0, 2, 3\}$

Ans. B

QUESTION

If $R = \{(x, y) : x^2 + y^2 \leq 4 | \text{ where } x, y \in Z\}$ is a relation on Z then

A Domain of R is $\{0, 1, 2\}$

B Domain of R is $\{-2, -1, 0, 1, 2\}$

C Domain of R = range of R

D $n(R) = 13$

QUESTION [MHT CET 2020 (19 Oct)]



If $R = \{(a, b) / b = a - 1, a \in \mathbb{Z}, 5 < a < 9\}$, then the range of R is

A {7, 8, 9}

B {5, 6, 7}

C {5, 6, 7, 8, 9}

D {6, 7, 8}

Ans. B

QUESTION [MHT CET 2021 (21 Sep)]



Let $A = \{10, 11, 12, 14, 26\}$ and let $f : A \rightarrow N$ be such that $f(a) = \text{highest prime factor of } a$, where $a \in A$, then range of $f =$

A {5, 7, 13}

B {5, 7, 11, 13}

C {3, 5, 7, 11, 13}

D {3, 7, 11, 13}

Ans. C

QUESTION [JEE Mains 2025 (29 Jan)]



Let $S = \mathbb{N} \cup \{0\}$. Define a relation R from S to R by:

$$R = \left\{ (x, y) : \log_e y = x \log_e \left(\frac{2}{5}\right), x \in S, y \in R \right\}.$$

Then, the sum of all the elements in the range of R is equal to:

A $\frac{3}{2}$

B $\frac{10}{9}$

C $\frac{5}{2}$

D $\frac{5}{3}$

Ans. D

QUESTION [COMEDK 2024]



A relation R is defined from $\{2, 3, 4\}$ to $\{3, 6, 7, 10\}$. If $xRy \Leftrightarrow x$ and y are co-prime numbers. Then range of R is

A $\{2, 3, 4\}$

B $\{3, 7\}$

C $\{3, 7, 10\}$

D $\{3, 6, 7, 10\}$

Ans. C

QUESTION

1. Consider set $A = \{-1, 0, 1, 2, 3\}$ and $B = \{1, 2, 4, 5, 0\}$,
 $R = \{(a, b) : b - a = 1, a \in A, b \in B\}$, then domain of relation R is:
(A) $\{0, 1, 3\}$ (B) $\{-1, 0, 1, 3\}$ (C) $\{1, 0, -1, 2, 3\}$ (D) $\{1, 0, 2, 3\}$
- [Ans. B]
2. Let $A = \{2, 4, 5, 7\}$ and $B = \{2, 4, 6\}$ be two sets and let a relation R be a relation from A to B is defined as $R = \{(x, y) : x < y, x \in A, y \in B\}$, then the difference between the sum of elements of domain and range of R is-
(A) 2 (B) 3 (C) 4 (D) 1
- [Ans. D]
3. If given $R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$. Find the domain and range of R.

[Ans. Domain of R = $\{0, 3, 4, 5\}$,
Range of R = $\{5, 4, 3, 0\}\}]$

QUESTION

4. If $A = \{1, 2, 5, 6\}$ and $B = \{1, 2, 3\}$, then $(A \times B) \cap (B \times A)$ is equal to [COMEDK 2021]
- (A) $\{(1, 1), (2, 1), (6, 1), (3, 2)\}$ (B) $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$
(C) $\{(1, 1), (2, 2)\}$ (D) $\{(1, 1), (1, 2), (2, 5), (2, 6)\}$ [Ans. B]
5. If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$ then $(A - B) \times (B \cap C)$ is equal to [COMEDK 2023]
- (A) $\{(a, c), (a, d)\}$ (B) $\{(a, b), (c, d)\}$
(C) $\{(c, a), (d, a)\}$ (D) $\{(a, c), (a, d), (b, d)\}$ [Ans. A]
6. Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is relation from X to Y -
- (A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$
(B) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
(C) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$
(D) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

7. If $A = \{2, 3, 4, 5\}$ and $B = \{3, 6, 7, 10\}$, R is a relation defined by $R = \{(a, b) : a \text{ is relatively prime to } b, a \in A \text{ and } b \in B\}$, then domain of R is
(A) {2, 3, 5} (B) {3, 5} (C) {2, 3, 4} (D) {2, 3, 4, 5}
8. Consider the following statements.
- The relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in Roster form is $\{(3, 27), (5, 125), (7, 343)\}$
 - The range of the relation $R = \{(x + 2, x + 4) : x \in \mathbb{N}, x < 8\}$ is $\{1, 2, 3, 4, 5, 6, 7\}$.
- Choose the correct option.
- (A) Only I is true (B) Only II is true (C) Both are true (D) Both are false



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture - 02



Recap

of previous lecture

1. A relation on set A is a subset of _____ and if $n(A) = 5$ then number of relations on A = _____

2. A relation from A to B is a subset of _____ and if $n(A) = 3$ & $n(B) = 5$ then numbers of relations from A to B is _____

3. $A = \{2, 3, 4, 5, 6\}$, let $R = \{(x, y) : x \mid y, x, y \in A\}$ then domain of R = _____ and range of R = _____

4. $n((A \times B \times C) \cap (B \times C \times A)) =$ _____

Recap

of previous lecture

5. If $n(A) = 3, n(B) = 5$ then number of non empty relations from A to B _____

6. If $S_1 = \{1, 2, 3, \dots, 20\}, S_2 = \{a, b, c, d\}, S_3 = \{b, d, e, f\}$. The number of elements of $(S_1 \times S_2) \cup (S_1 \times S_3)$ is _____

7. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B. _____

8. The Cartesian product $A \times A$ has 9 elements among which two are $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

Recap *of previous lecture*

9. If R is a relation on $A \times B$ then it is a subset of _____

10. $A = \{1, 2, 3\}$ & $B = \{4, 5\}$ then $A \times B =$ _____

also write any two relation on $A \times B$.

$R_1 =$ _____

$R_2 =$ _____

1. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is:

(A) $\frac{1}{\sqrt{5}}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{6}}$

[Ans. C]

2. If $A = \frac{1}{5!6!7!} \begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|\text{adj}(\text{adj}(2A))|$ is equal to

(A) 2^{12}

(B) 2^{20}

(C) 2^8

(D) 2^{16}

[Ans. D]

3. If A is a 3×3 matrix and $|A| = 2$, then $|3 \text{ adj}(|3A|A^2)|$ is equal to

(A) $3^{12} \cdot 6^{10}$

(B) $3^{11} \cdot 6^{10}$

(C) $3^{12} \cdot 6^{11}$

(D) $3^{10} \cdot 6^{11}$

[Ans. B]

QUESTION



Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, \dots, 66, 67\}$. If R be a relation from the set A to the set B defined by

- (i) is square root of
- (ii) is cube root of

Find R and also its domain and range.

Let R be a relation from the set $\{1, 2, 3, \dots, 60\}$ to itself such that

$R = \{(a, b) : b = pq, \text{ where } p, q \geq 3 \text{ are prime numbers}\}.$

Then the number of elements in R is:

A 600

B 660

C 540

D 720

Let $A = \{1, 2, 3, \dots, 20\}$. Let R_1 and R_2 two relation on A such that

$$R_1 = \{(a, b) : b \text{ is divisible by } a\}$$

$$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}.$$

Then, number of elements in $R_1 - R_2$ is equal to

QUESTION [JEE Mains 2023 (6 April)]



Let $A = \{1, 2, 3, 4, \dots, 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times B : 2(a - b)^2 + 3(a - b) \in B\}$ is.

QUESTION

The relation R defined in $A = \{1, 2, 3\}$ by $a R b$ if $|a^2 - b^2| \leq 5$. Which of the following is false?

A $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)\}$

B $R^{-1} = R$

C Domain of R = {1, 2, 3}

D Range of R = {5}

QUESTION

Relation R in the set of A of human beings in a town at a particular time given by

(A) $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$

(B) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$

(C) $R = \{(x, y) : x \text{ is exactly 7 cm taller than } y\}$

(D) $R = \{(x, y) : x \text{ is wife of } y\}$

(E) $R = \{(x, y) : x \text{ is father of } y\}$

Which of the following statements is not correct for the relation R defined by aRb , if and only if b lives within one kilometre from a?

- A** R is reflexive
- B** R is symmetric
- C** R is transitive
- D** Antisymmetric

QUESTION

1. Let $A = \{2, 3, 4, 5\}$, then how many relations can be defined on set A ?
(A) $(16)^2$ (B) $(2)^{16}$ (C) $(16)^4$ (D) $(4)^{16}$

2. Let $A = \{a, e, i, o, u\}$, then how many relations can be defined on power set of A?
(A) $(2)^{10}$ (B) $(2^{10})^2$ (C) $(2)^{2^{10}}$ (D) None of these

3. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$; then the number of relations that can be defined from set A to set B is
(A) $(2)^{12}$ (B) $(12)^2$ (C) $(2)^7$ (D) None of these

4. If $n(A) = m$ and $n(B) = n$ and $m > n$. Let $k =$ number of relations defined from set A to set B and $p =$ number of relations defined from set B to set A, then
(A) $k > p$ (B) $k < p$
(C) $k = p$ (D) Can't be predicted surely

5. Define a relation R on set $A = \{2, 3, 5, 6, 10\}$ as xRy if ' $x < y$ and x divides y ', then the domain of relation is
(A) $\{2, 3, 5, 6, 10\}$ (B) $\{2, 3, 5\}$ (C) $\{2, 3, 5, 6\}$ (D) None of these
6. Define a relation R on set of natural numbers defined as xRy if ' $3x + 5y = 53$ ', then the range set of relation is
(A) $\{1, 4\}$ (B) $\{1, 4, 8\}$ (C) $\{1, 4, 7, 10\}$ (D) None of these
7. Define relations R_1 and R_2 on set $A = \{2, 3, 5, 7, 10\}$ as xR_1y if $x \mid (y - 1)$ and xR_2y if $x + y = 10$, then the relation R given by $R = R_1 \cap R_2$ is
[Note: $a \mid b$ means b is divisible by a]
(A) {} (B) $\{(3, 7)\}$ (C) $\{(3, 7), (5, 5)\}$ (D) None of these

8. Let $A = \{1, 2, 3, 4, 6\}$. Let R is the relation on A defined by $\{(a, b) : a, b \in A, b \text{ is divisible by } a\}$.
- (i) Write R in roster form
 - (ii) Find the domain of R
 - (iii) Find the range of R
9. Determine the domain and range of the following relations on \mathbb{N} .
- (i) $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$
 - (ii) $R = \{(4x + 3, 1 - x) : x \leq 4; x \in \mathbb{N}\}$
 - (iii) $R = \left\{\left(x, \frac{1}{x}\right) : 0 < x < 4 \text{ and } x \text{ is natural number}\right\}$

QUESTION

10. If $n(A) = 4, n(B) = 3, n(A \times B \times C) = 24$, then $n(C)$ is equal to [Kerala CEE]
- (A) 288 (B) 1 (C) 12 (D) 2
11. The number of elements in the set $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in \mathbb{Z}\}$, where \mathbb{Z} is the set of all integers, is [Kerala CEE]
- (A) 2 (B) 4 (C) 8 (D) 12
12. R is a relation on \mathbb{N} given by $R = \{(x, y) : 4x + 3y = 20\}$.
Which of the following belongs to R ? [KCET]
- (A) (-4, 12) (B) (5, 0) (C) (3, 4) (D) (2, 4)



KTK Answers



1. B, C

2. C

3. A

4. C

5. B

6. C

7. B

8. $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}; D_R = R_R = A$

9. (i) $D_R = \{2, 3, 5, 7\}; R_R = \{8, 27, 125, 343\}$

(ii) $D_R = \{\}; R_R = \{\}$

(iii) $D_R = \{\}; R_R = \{\}$

10. D

11. C

12. A, B, D



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 03



Recap *of previous lecture*

1. $n((A \times B) \cap (B \times A)) =$

2. $n((A \times B \times C) \cap (P \times Q \times R)) =$

3. A relation from A to B is _____ while a relation on A is _____ & relation on B is _____

4. Every identity relation is reflexive. (T/F)

5. Every relation which is not symmetric is antisymmetric. (T/F)

Recap *of previous lecture*

6. A relation R on A is

- (i) Reflexive if _____
- (ii) Symmetric if _____
- (iii) Transitive if _____
- (iv) Antisymmetric if _____
- (v) Equivalence if _____

Recap

of previous lecture

7. Let $A = \{2, 3, 4, 5\}$ and let $R = \{(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)\}$ be a relation on A. Then R is _____
8. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is _____
9. Number of relations on A if $n(A) = n$ is _____
10. Number of relations from A to B if $n(A) = n$ & $n(B) = m$ is _____
11. If $n(A) = 5$ then number of elements in power set of A is _____ also number of relations on power set of A is _____ & number of relations from A to power set of A is _____

QUESTION

1. Let $A = \{2, 3, 4, 5\}$, then how many relations can be defined on set A ?
(A) $(16)^2$ (B) $(2)^{16}$ (C) $(16)^4$ (D) $(4)^{16}$

2. Let $A = \{a, e, i, o, u\}$, then how many relations can be defined on power set of A?
(A) $(2)^{10}$ (B) $(2^{10})^2$ (C) $(2)^{2^{10}}$ (D) None of these

3. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$; then the number of relations that can be defined from set A to set B is
(A) $(2)^{12}$ (B) $(12)^2$ (C) $(2)^7$ (D) None of these

4. If $n(A) = m$ and $n(B) = n$ and $m > n$. Let $k =$ number of relations defined from set A to set B and $p =$ number of relations defined from set B to set A, then
(A) $k > p$ (B) $k < p$
(C) $k = p$ (D) Can't be predicted surely

5. Define a relation R on set $A = \{2, 3, 5, 6, 10\}$ as xRy if ' $x < y$ and x divides y ', then the domain of relation is
(A) $\{2, 3, 5, 6, 10\}$ (B) $\{2, 3, 5\}$ (C) $\{2, 3, 5, 6\}$ (D) None of these
6. Define a relation R on set of natural numbers defined as xRy if ' $3x + 5y = 53$ ', then the range set of relation is
(A) $\{1, 4\}$ (B) $\{1, 4, 8\}$ (C) $\{1, 4, 7, 10\}$ (D) None of these
7. Define relations R_1 and R_2 on set $A = \{2, 3, 5, 7, 10\}$ as xR_1y if $x \mid (y - 1)$ and xR_2y if $x + y = 10$, then the relation R given by $R = R_1 \cap R_2$ is
[Note: $a \mid b$ means b is divisible by a]
(A) {} (B) $\{(3, 7)\}$ (C) $\{(3, 7), (5, 5)\}$ (D) None of these

8. Let $A = \{1, 2, 3, 4, 6\}$. Let R is the relation on A defined by $\{(a, b) : a, b \in A, b \text{ is divisible by } a\}$.
- (i) Write R in roster form
 - (ii) Find the domain of R
 - (iii) Find the range of R
9. Determine the domain and range of the following relations on \mathbb{N} .
- (i) $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$
 - (ii) $R = \{(4x + 3, 1 - x) : x \leq 4; x \in \mathbb{N}\}$
 - (iii) $R = \left\{\left(x, \frac{1}{x}\right) : 0 < x < 4 \text{ and } x \text{ is natural number}\right\}$

QUESTION



10. If $n(A) = 4$, $n(B) = 3$, $n(A \times B \times C) = 24$, then $n(C)$ is equal to [Kerala CEE]
- (A) 288 (B) 1 (C) 12 (D) 2
11. The number of elements in the set $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in \mathbb{Z}\}$, where \mathbb{Z} is the set of all integers, is [Kerala CEE]
- (A) 2 (B) 4 (C) 8 (D) 12
12. R is a relation on N given by $R = \{(x, y) : 4x + 3y = 20\}$.
Which of the following belongs to R ? [KCET]
- (A) (-4, 12) (B) (5, 0) (C) (3, 4) (D) (2, 4)

QUESTION [JEE Mains 2023 (6 April)]



Let $A = \{1, 2, 3, 4, \dots, 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times B : 2(a - b)^2 + 3(a - b) \in B\}$ is.

QUESTION



Let L denote the set of all straight lines in a plane. Let a relation R be defined $\alpha R \beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. Then R is

'The relation $R = \{(x, y) : x, y \in \mathbb{Z} \text{ and } x + y \text{ is even}\}$ is:

- A** reflexive and transitive but not symmetric
- B** reflexive and symmetric but not transitive
- C** an equivalence relation
- D** symmetric and transitive but not reflexive

Ans. C

Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by xRy if and only if $\sec^2 x - \tan^2 y = 1$. Then R is :

- A** both reflexive and symmetric but not transitive
- B** both reflexive and transitive but not symmetric
- C** reflexive but neither symmetric nor transitive
- D** an equivalence relation

Ans. D

Let W denote the words in the English dictionary. Define the relation R by:

$R = \{(x, y) \in W \times W \mid \text{the words } x \text{ and } y \text{ have at least one letter in common}\}$. Then R is

- A** reflexive symmetric and not transitive
- B** reflexive symmetric and transitive
- C** reflexive not symmetric and transitive
- D** not reflexive symmetric and transitive

QUESTION [JEE Mains 2022 (28 June)]



Let $R_1 = \{(a, b) \in N \times N : |a - b| \leq 13\}$ and $R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$. Then on N

- A** Both R_1 and R_2 are equivalence relations
- B** Neither R_1 nor R_2 is an equivalence relation
- C** R_1 is an equivalence relation but R_2 is not
- D** R_2 is an equivalence relation but R_1 is not

QUESTION [JEE Mains 2022 (27 July)]



Let R_1 and R_2 be two relations defined on \mathbb{R} by $a R_1 b \Leftrightarrow ab \geq 0$ and $a R_2 b \Leftrightarrow a \geq b$. Then

- A** R_1 is an equivalence relation but not R_2
- B** R_2 is an equivalence relation but not R_1
- C** both R_1 and R_2 are equivalence relations
- D** neither R_1 nor R_2 is an equivalence relation

QUESTION [JEE Mains 2025 (4 April)]

Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and R be a relation on A defined by xRy if and only if $2x - y \in \{0, 1\}$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to:

- A** 17
- B** 18
- C** 15
- D** 16

Ans. A

QUESTION [JEE Mains 2025 (3 April)]

Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $0 \leq x^2 + 2y \leq 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. Then $l + m$ is equal to

- A** 18
- B** 20
- C** 17
- D** 19

Ans. A

QUESTION [JEE Mains 2018 (15 April)]



Consider the following two binary relations on the set $A = \{a, b, c\}$:

$$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\} \text{ and}$$

$$R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)\}.$$

Then:

- A** both R_1 and R_2 are not symmetric.
- B** R_1 is not symmetric but it is transitive.
- C** R_2 is symmetric but it is not transitive.
- D** both R_1 and R_2 are transitive.

Ans. C

QUESTION [JEE Mains 2022 (28 June)]



Let R_1 and R_2 be relations on the set $(1, 2, \dots, 50)$ such that

$R_1 = \{(p, p^n) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$ and

$R_2 = \{(p, p^n) : p \text{ is a prime and } n = 0 \text{ or } 1\}$.

Then, the number of elements in $R_1 - R_2$ is _____

QUESTION [JEE Mains 2020 (2 Sept)]



If $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers \mathbb{Z} , then the domain of R^{-1} is:

- A** {0, 1}
- B** {-2, -1, 1, 2}
- C** {-1, 0, 1}
- D** {-2, -1, 0, 1, 2}

Ans. C

QUESTION [JEE Mains 2024 (6 April)]



Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \dots, 20\}$ be given by

$$R_1 = \{(x, y) : 2x - 3y = 2\} \text{ and } R_2 = \{(x, y) : -5x + 4y = 0\}.$$

If M and N be the minimum number of elements required to be added in R_1 and R_2 , respectively, in order to make the relations symmetric, then $M + N$ equals

A 16

B 12

C 8

D 10

Ans. D

QUESTION [JEE Mains 2023 (13 April)]



Let $A = \{-4, -3, -2, 0, 1, 3, 4\}$ and $R = \{(a, b) \in A \times A : b = |a| \text{ or } b^2 = a + 1\}$ be a relation on A . Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture - 04



Recap *of previous lecture*

Column A		Column B	
(i)	$R: \mathbb{R} \rightarrow \mathbb{R}$: defined as aRb iff $a \leq b$	(a)	Reflexive
(ii)	$R: \mathbb{R} \rightarrow \mathbb{R}$: defined as aRb iff $a \leq b^3$ is	(b)	Symmetric
(iii)	$R: A \rightarrow A$ defined as $R = \{(a, b) : a - b \text{ is even}\}$; $A = \{1, 2, 3, 4, 5\}$ is	(c)	Transitive
(iv)	$R: H \rightarrow H$ where H is set of Human beings defined as $R = \{(x, y) : x \text{ and } y \text{ have same age}\}$ is	(d)	Equivalence
		(e)	Anti-symmetric
		(f)	None of these

Ans. (i) - (a, c, e); (ii) - (f); (iii) - (a, b, c, d); (iv) - (a, b, c, d)

QUESTION [JEE Mains 2022 (27 July)]



Let R_1 and R_2 be two relations defined on \mathbb{R} by $a R_1 b \Leftrightarrow ab \geq 0$ and $a R_2 b \Leftrightarrow a \geq b$. Then

- A** R_1 is an equivalence relation but not R_2
- B** R_2 is an equivalence relation but not R_1
- C** both R_1 and R_2 are equivalence relations
- D** neither R_1 nor R_2 is an equivalence relation

QUESTION [JEE Mains 2022 (28 June)]



Let R_1 and R_2 be relations on the set $(1, 2, \dots, 50)$ such that

$R_1 = \{(p, p^n) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$ and

$R_2 = \{(p, p^n) : p \text{ is a prime and } n = 0 \text{ or } 1\}$.

Then, the number of elements in $R_1 - R_2$ is _____

Let R be a relation on \mathbb{R} given by $R = \{(a, b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$. Then R is

- A** an equivalence relation
- B** reflexive and symmetric but not transitive
- C** reflexive and transitive but not symmetric
- D** reflexive but neither symmetric nor transitive

Among the relations

$$S = \left\{ (a, b) : a, b \in \mathbb{R} - \{0\}; 2 + \frac{a}{b} > 0 \right\} \text{ and } T = \{(a, b) : a, b \in \mathbb{R}; a^2 - b^2 \in \mathbb{Z}\}$$

- A** S is transitive but T is not
- B** both S and T are symmetric
- C** neither S nor T is transitive
- D** T is symmetric but S is not

QUESTION [JEE Mains 2023 (24 Jan)]



The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is :

- A** reflexive but not symmetric
- B** transitive but not reflexive
- C** symmetric but not transitive
- D** neither symmetric nor transitive

QUESTION [JEE Mains 2020]



Let R_1 and R_2 be two relations defined as follows:

$$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\} \text{ and } R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$$

where Q is the set of all rational numbers. Then :

A R_1 is transitive but R_2 is not transitive.

B R_1 and R_2 are both transitive.

C R_2 is transitive but R_1 is not transitive.

D Neither R_1 nor R_2 is transitive.

Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in R$ and $(a, b)R_2(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$. Then:

- A** R_1 and R_2 both are equivalence relations
- B** Only R_1 is an equivalence relation
- C** Only R_2 is an equivalence relation
- D** Neither R_1 nor R_2 is an equivalence relation

Ans. C

Let R be the relation on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) R (c, d)$ if and only if $ad - bc$ is divisible by 5. Then R is

- A** Reflexive and transitive but not symmetric
- B** Reflexive and symmetric but not transitive
- C** Reflexive but neither symmetric nor transitive
- D** Reflexive, symmetric and transitive

Ans. B

Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$. Let R be a relation on $A \times B$ define by $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer. Then the relation R is

- A** reflexive but not symmetric.
- B** transitive but not symmetric.
- C** reflexive and symmetric but not transitive.
- D** an equivalence relation.

QUESTION [JEE Mains 2025 (23 Jan)]



Let $R = \{(1, 2), (2, 3), (3, 3)\}$ be a relation defined on the set $\{1, 2, 3, 4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

- A 9
- B 8
- C 7
- D 10

Ans. C

QUESTION [JEE Mains 2023 (8 April)]



Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y \text{ is odd positive integer or } x - y = 2\}$.

The minimum number of elements that must be added to the relation R , so that it is a symmetric relation, is equal to _____

QUESTION [JEE Mains 2024 (31 Jan)]

Let $A = \{1, 2, 3, \dots, 100\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n . Then, the minimum value of n is

QUESTION [JEE Mains 2025 (24 Jan)]

Let $A = \left\{x \in (0, \pi) - \left\{\frac{\pi}{2}\right\} : \log_{(2/\pi)} |\sin x| + \log_{(2/\pi)} |\cos x| = 2\right\}$

and $B = \{x \geq 0 : \sqrt{x}(\sqrt{x} - 4) - 3|\sqrt{x} - 2| + 6 = 0\}.$

Then $n(A \cup B)$ is equal to:

A 4

B 8

C 6

D 2

Ans. B

Let $A = \{2, 3, 6, 7\}$ and $B = \{4, 5, 6, 8\}$. Let R be a relation defined on $A \times B$ by $(a_1, b_1) R (a_2, b_2)$ if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (1, 4)\}$ be a relation on A . Let S be the equivalence relation on A such that RS and the number of elements in S is n . Then, the minimum value of n is

QUESTION [JEE Mains 2024 (29 Jan)]



Let R is the smallest equivalence relation on the set $\{1, 2, 3, 4\}$ such that $\{(1, 2), (1, 3)\} \subset R$ then the number of elements in R is

A 15

B 10

C 12

D 8

Ans. B

QUESTION [JEE Mains 2024]



Let $S = \{1, 2, 3, \dots, 10\}$. Suppose M is the set of all the subsets of S , then the relation $R = \{(A, B) : A \cap B \neq \emptyset; A, B \in M\}$ is:

- A** Symmetric and reflexive only
- B** Reflexive only
- C** Symmetric and transitive only
- D** Symmetric only

Ans. D



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 05



Recap *of previous lecture*

1. Number of reflexive relations on a set A with P elements = _____ while number of symmetric relations on A is _____

2. Empty relation is _____ & _____ & _____ but not _____

3. If $|A| = 3$ then number of symmetric relations on A is _____ while number of reflexive relations on A _____

4. $R = \{(a, b) : |a - b| > \frac{1}{2}, a, b \in R\}$ then relation is _____

5. $R = \{(a, b) : a \text{ divides } b, a, b \in N\}$ then relation is _____

Recap *of previous lecture*

6. Let $R = \{(3, 3), (5, 5), (9, 9), (12, 12), (5, 12), (3, 9), (3, 12), (3, 5)\}$ be a relation on the set $A = \{3, 5, 9, 12\}$. Then R is _____
7. Number of non empty symmetric relations on a set A where $|A| = 4$ is _____
8. $aRb \Leftrightarrow a = b + 2$ be a relation on A then $a \in \text{_____}$, $b \in \text{_____}$
9. $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ be a relation on $A \times B$ then
 $a \in \text{_____}$, $b \in \text{_____}$, $c \in \text{_____}$ & $d \in \text{_____}$
10. Let M be set of all possible subsets of $A = \{1, 2, 3\}$ and R be a relation on M then elements are of the form = _____ & if $xRy \Leftrightarrow x \cap y = \emptyset$ then $R = \text{_____}$

QUESTION [JEE Mains 2025 (24 Jan)]

Let $A = \left\{x \in (0, \pi) - \left\{\frac{\pi}{2}\right\} : \log_{(2/\pi)} |\sin x| + \log_{(2/\pi)} |\cos x| = 2\right\}$

and $B = \{x \geq 0 : \sqrt{x}(\sqrt{x} - 4) - 3|\sqrt{x} - 2| + 6 = 0\}.$

Then $n(A \cup B)$ is equal to:

A 4

B 8

C 6

D 2

Ans. B

Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$. Let R be a relation on $A \times B$ define by $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer. Then the relation R is

- A** reflexive but not symmetric.
- B** transitive but not symmetric.
- C** reflexive and symmetric but not transitive.
- D** an equivalence relation.

QUESTION

Consider the following equivalence relations defined on $A = \{1, 2, 3, 4, 5\}$.

$$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (2, 1), (1, 2)\}$$

$$R_3 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 3), (3, 1), (2, 4), (4, 2)\}$$

Let $R = \{(P, Q) \mid P \text{ and } Q \text{ are at the same distance from the origin}\}$ be a relation, then the equivalence class of $(1, -1)$ is the set:

- A** $S = \{(x, y) \mid x^2 + y^2 = 4\}$
- B** $S = \{(x, y) \mid x^2 + y^2 = 1\}$
- C** $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$
- D** $S = \{(x, y) \mid x^2 + y^2 = 2\}$

QUESTION [JEE Mains 2025 (22 Jan)]



The number of non-empty equivalence relations on the set $\{1, 2, 3\}$ is:

A 7

B 4

C 5

D 6

Ans. C

QUESTION

Let R and S be two non-void relations on a set A . Which of the following statements is false?

- (a) R and S are transitive $\Rightarrow R \cup S$ is transitive.
- (b) R and S are transitive $\Rightarrow R \cap S$ is transitive.
- (c) R and S are symmetric $\Rightarrow R \cup S$ is symmetric.
- (d) R and S are reflexive $\Rightarrow R \cap S$ is reflexive.
- (e) R and S are symmetric $\Rightarrow R \cap S$ is symmetric.

Which of the following is not correct for relation R on the set of real numbers?

- A** $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.
- B** $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.
- C** $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.
- D** $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric.

Ans. B

QUESTION [JEE Mains 2018 (16 April)]



Let N denote the set of all natural numbers. Define two binary relations on N as

$R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then :

- A** Range of R_1 is $\{2, 4, 8\}$.
- B** Range of R_2 is $\{1, 2, 3, 4\}$.
- C** Both R_1 and R_2 are symmetric relations.
- D** Both R_1 and R_2 are transitive relations.

Ans. B

Let R be the real line. Consider the following subsets of the plane $R \times R$:

$$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$$

$$T = \{(x, y) : x - y \text{ is an integer}\},$$

Which one of the following is true?

- A** Neither S nor T is an equivalence relation on R
- B** Both S and T are equivalence relation on R
- C** S is an equivalence relation on R but T is not
- D** T is an equivalence relation on R but S is not

Ans. D

Define a relation R over a class of $n \times n$ real matrices A and B as
"ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$ ".
Then which of the following is true?

- A** R is reflexive, transitive but not symmetric
- B** R is symmetric, transitive but not reflexive.
- C** R is reflexive, symmetric but not transitive
- D** R is an equivalence relation

QUESTION [JEE Mains 2023 (29 Jan)]



Let R be a relation defined on \mathbb{N} as $a R b$ if $2a + 3b$ is a multiple of 5, $a, b \in \mathbb{N}$. Then R is

- A** an equivalence relation
- B** non reflexive
- C** symmetric but not transitive
- D** transitive but not symmetric



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 06



Recap *of previous lecture*

1. Consider an equivalence relation

$R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 4), (5, 4), (5, 5)\}$

on set $A = \{1, 2, 3, 4, 5\}$ then

$[1] = \underline{\hspace{2cm}}$, $[2] = \underline{\hspace{2cm}}$, $[3] = \underline{\hspace{2cm}}$, $[4] = \underline{\hspace{2cm}}$, $[5] = \underline{\hspace{2cm}}$,

& a partition of A induced by R is $\underline{\hspace{2cm}}$

2. Every equivalence relation on a set gives a $\underline{\hspace{2cm}}$ of the set hence number of equivalence relations on a set is equal to number of ways of $\underline{\hspace{2cm}}$ the set.

Recap *of previous lecture*

3. The Bell number B_n denotes the number of ways of _____ a given set of _____ elements & hence B_n denotes the number of _____ relations on a set with _____ elements.
4. $B_{n+1} = \dots$ where $B_0 = \dots$, $B_1 = \dots$, $B_2 = \dots$
5. $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, c), (b, b), (b, d), (c, a), (c, c), (d, b), (d, d)\}$ then $[a] = \dots$, $[c] = \dots$, $[b] = \dots$ and $[d] = \dots \Rightarrow \dots$ & _____ therefore partition induced by R on set A is _____

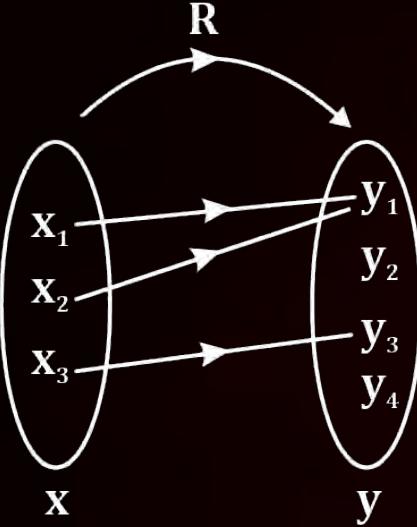
Recap *of previous lecture*

6. If $a \in [b]$ then $b \in [a]$. (T/F)
7. If $(a, b) \in R$ where R is an equivalence relation on a set A then $[a] = [b]$. (T/F)
8. Equivalence classes are either disjoint or identical. (T/F)
9. The equivalence relation induced by the partition $P = \{\{1\}, \{3\}, \{2, 4, 5, 6\}\}$ on $A = \{1, 2, 3, 4, 5, 6\}$ is _____

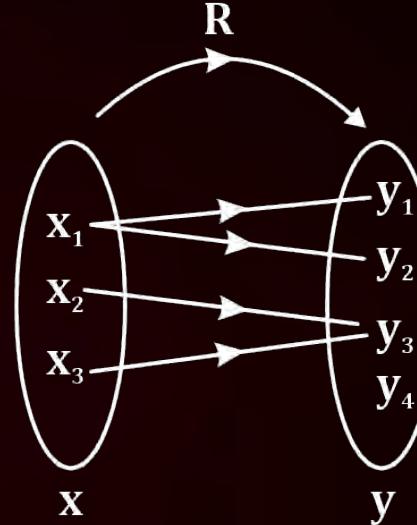
QUESTION

Identify which of the following relations is/are function(s) from $x \rightarrow y$?

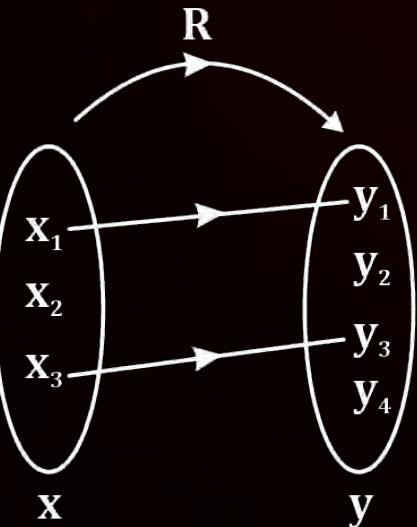
A



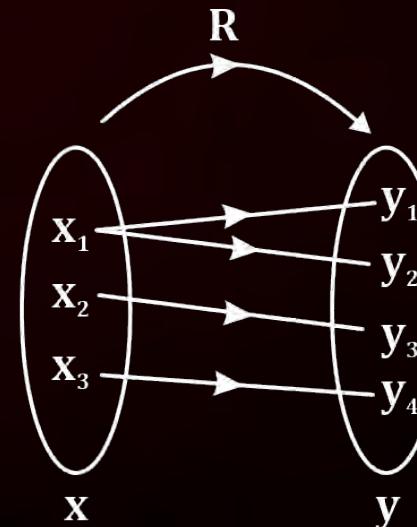
B



C



D



QUESTION

Which of the following relations are function from set X to Y;
where $X = \{1, 3, 5, 7\}$ and set $Y = \{2, 4, 6, 8\}$?

A $\{(3, 2), (3, 4), (5, 4), (7, 4), (1, 8)\}$

B $\{(1, 2), (5, 8), (3, 6)\}$

C $\{(1, 4), (3, 8), (5, 2), (7, 6)\}$

D $\{(1, 4), (3, 4), (5, 8), (7, 8)\}$

QUESTION

Which of the following correspondences can be called a function?

A $f : \{-2, 0, 2\} \rightarrow \{0, 1, 8, 3\}; f(x) = x^3$

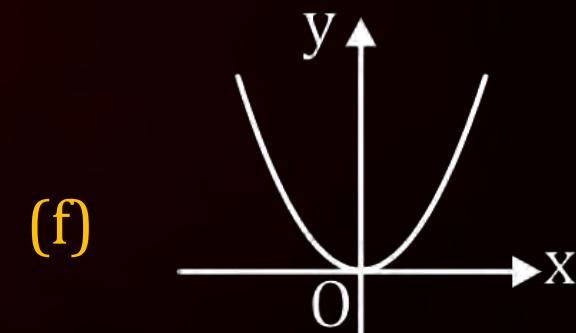
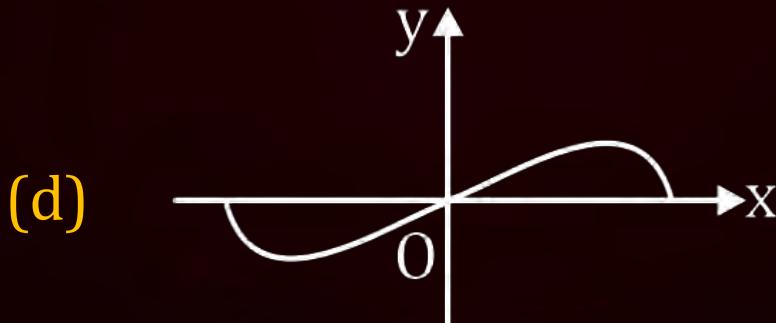
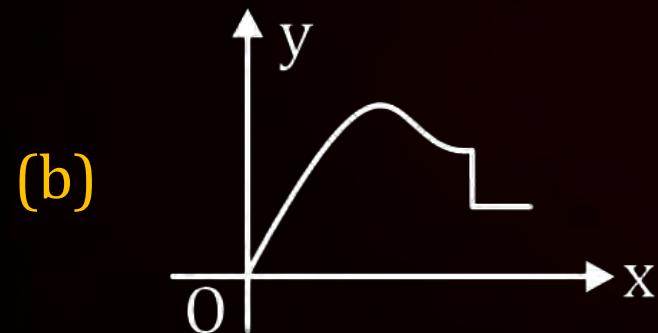
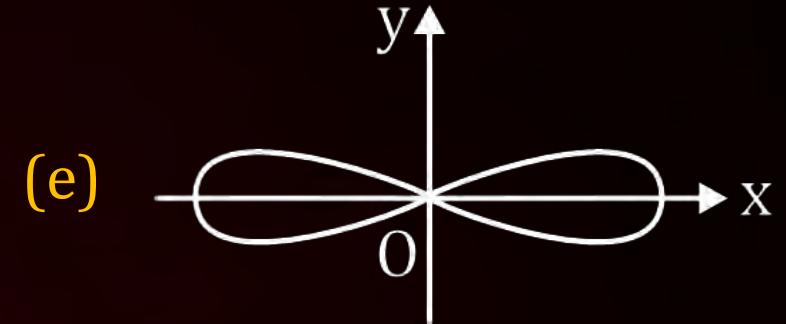
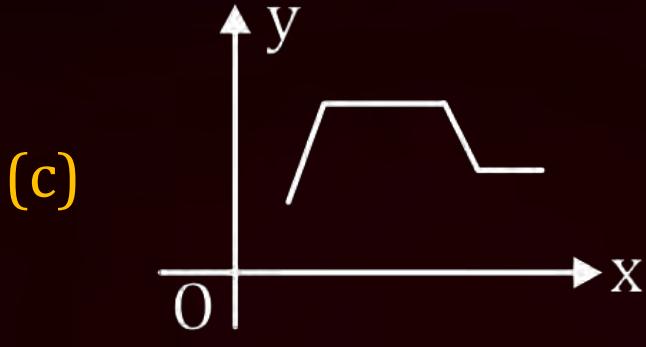
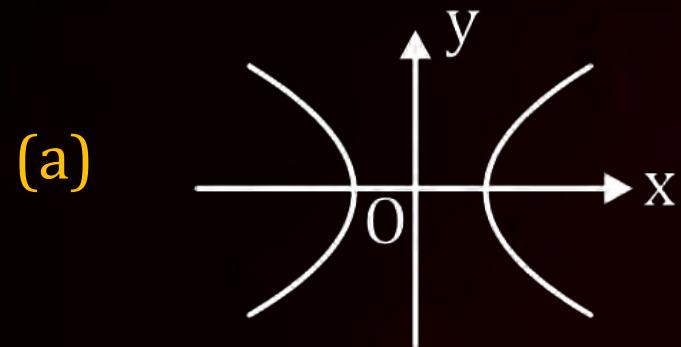
B $f : \{0, 1, 4\} \rightarrow \{-2, -1, 0, 1, 2\}; f(x) = \pm\sqrt{x}$

C $f : \{0, 1, 9\} \rightarrow \{-3, -1, 0, 1, 3\}; f(x) = \sqrt{x}$

D $f : \{0, 1, 9\} \rightarrow \{-3, -1, 0, 1, 3\}; f(x) = -\sqrt{x}$

QUESTION

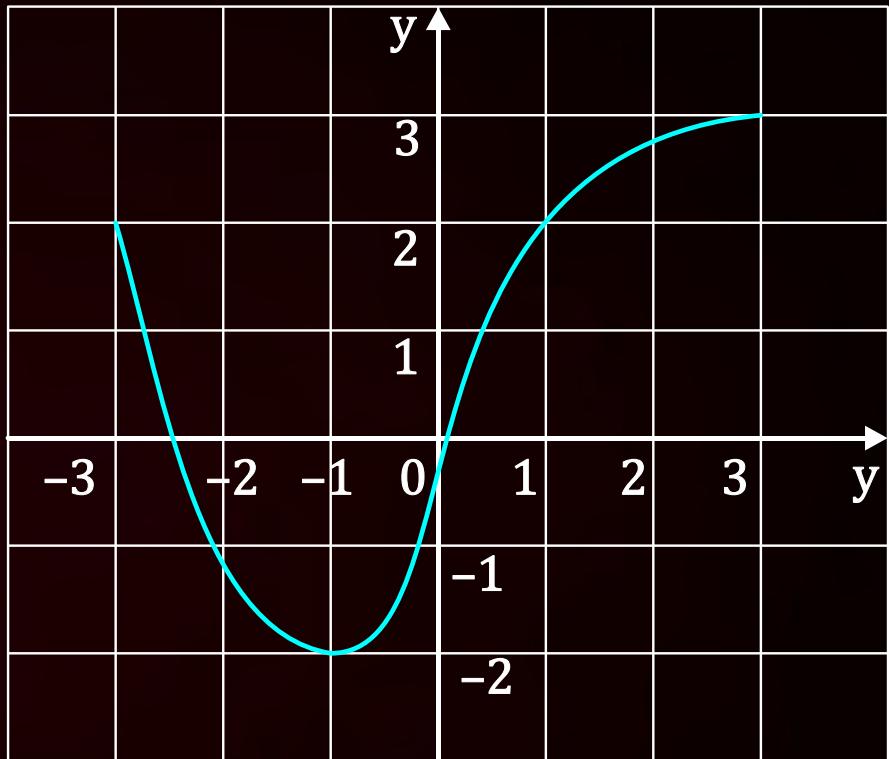
Which of the given graphs is/are Functions?



QUESTION

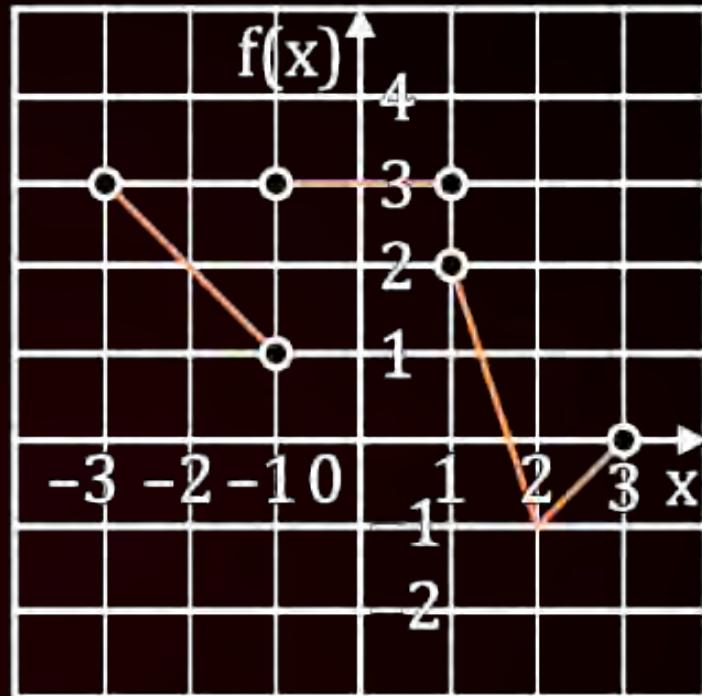
The graph of a function f is given:

- (a) State the value of $f(-1)$.
- (b) For what values of x is $f(x) = 2$.
- (c) State the domain and range of f .
- (d) On what interval is f increasing?



QUESTION

Find domain & range of f .



QUESTION



The domain of the function $f(x) = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$ is

- A** $[-1, 2) \cup [3, \infty)$
- B** $(-1, 2) \cup [3, \infty)$
- C** $[-1, 2] \cup [3, \infty)$
- D** None of these

QUESTION



The domain of function $\frac{1}{\sqrt[3]{(x-1)(x-2)(x-4)}}$

A $(1, 2) \cup (4, \infty)$

B $\mathbb{R} - \{1, 2, 4\}$

C \mathbb{R}

D None of these

QUESTION



The domain of function $\frac{1}{\sqrt{x(x-2)(x-3)}}$

A $(0, 2)$

B $\mathbb{R} \sim \{0, 2, 3\}$

C $(0, 2) \cup (3, \infty)$

D None of these

QUESTION



Find Domain of $f(x) = \sqrt{x^2 - 3x + 2} + \frac{1}{\sqrt{x^2 - 3x - 4}}$



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 07



Recap *of previous lecture*

1. Every relation is a function (T/F)

2. Every function is a relation (T/F)

3. If $n(A) = p$, $n(B) = q$ then number of functions from A to B is _____ & number of functions from B to A is _____

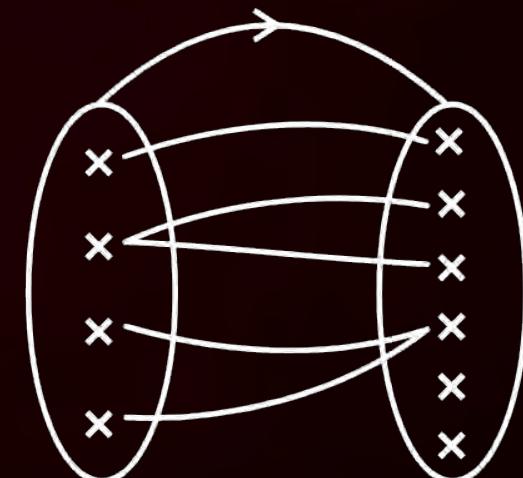
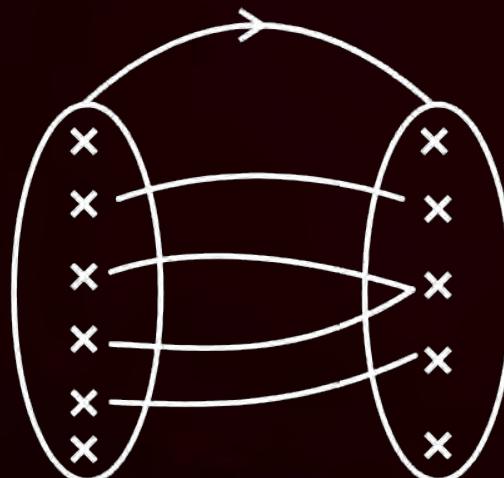
4. If a line parallel to y-axis intersects the graph in two or more points then the graph represents a _____

Recap

of previous lecture

5. A relation from A to B is a function if _____ & _____

6.



Recap

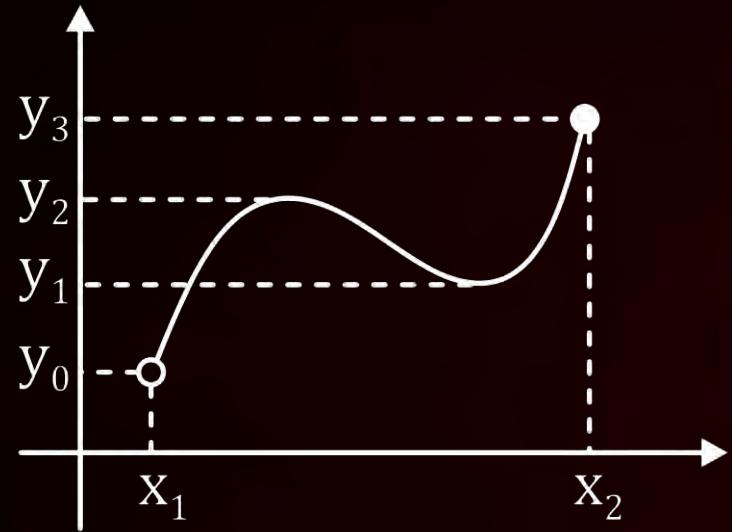
of previous lecture

7. Range of a continuous function is the interval from _____ to _____
8. Range is always a _____ of codomain.
9. If codomain is not given then it is taken to be _____

Recap

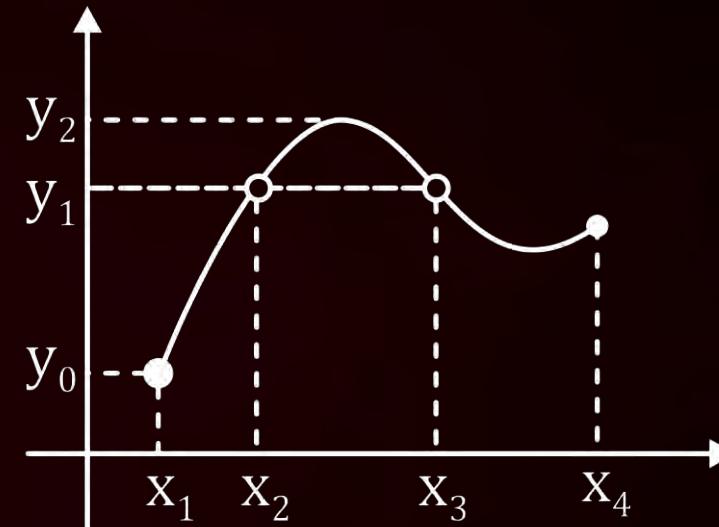
of previous lecture

10.



Domain =

Range =



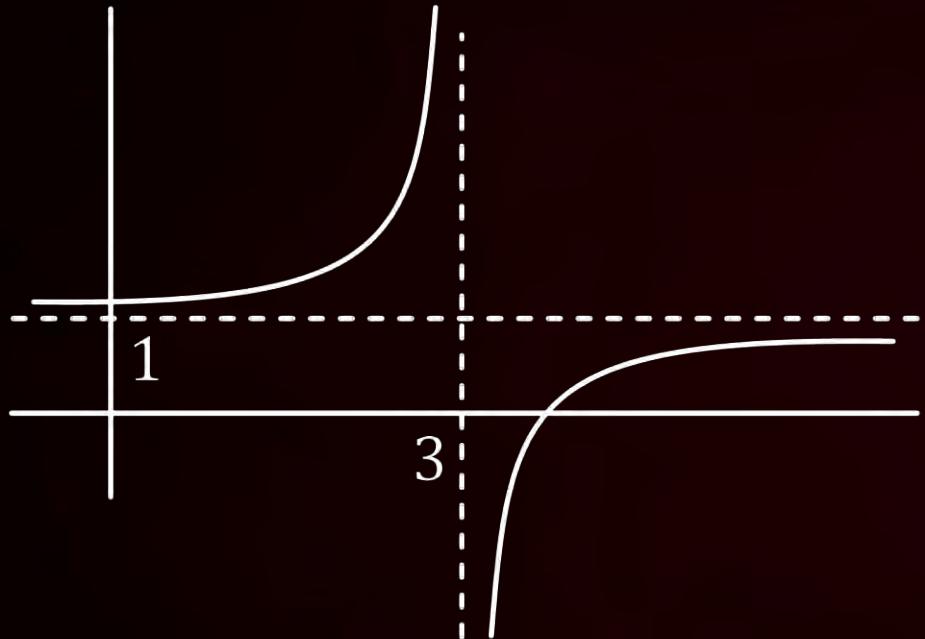
Domain =

Range =

Recap

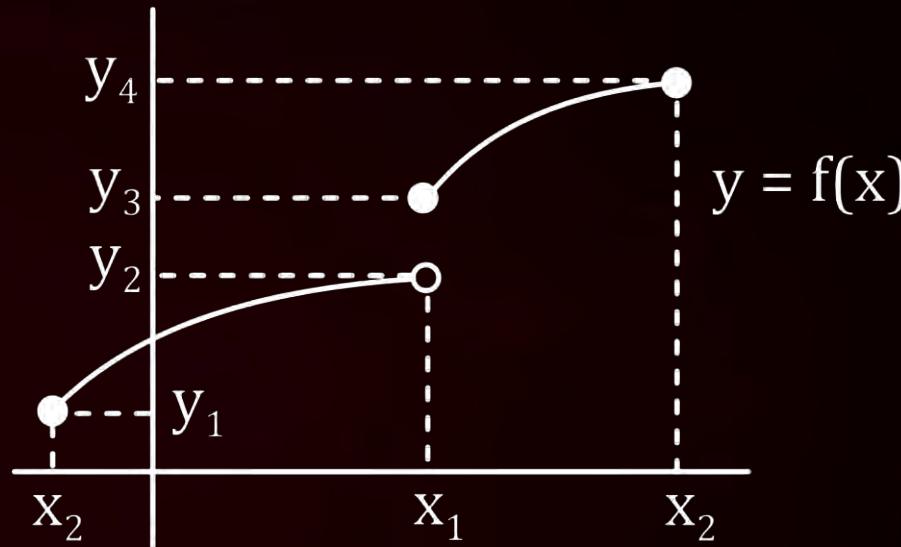
of previous lecture

11.



Domain =

Range =



$$f(x_1) =$$

Domain =

Range =

Recap *of previous lecture*

12. If a line parallel to x-axis intersects the graph of a relation in atmost one point then it is a function. (T/F)

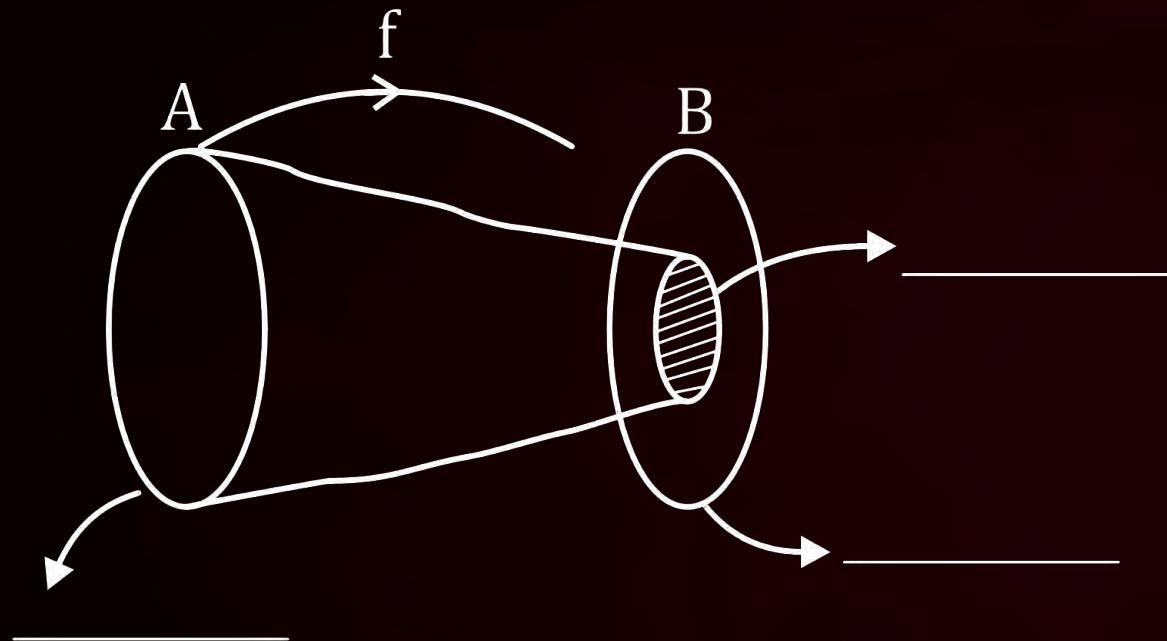
13. If a line parallel to y-axis intersects the graph of a relation in two or more points then it is not a function. (T/F)

14. If a line parallel to x-axis intersects the graph of a relation in two or more points then it is not a function. (T/F)

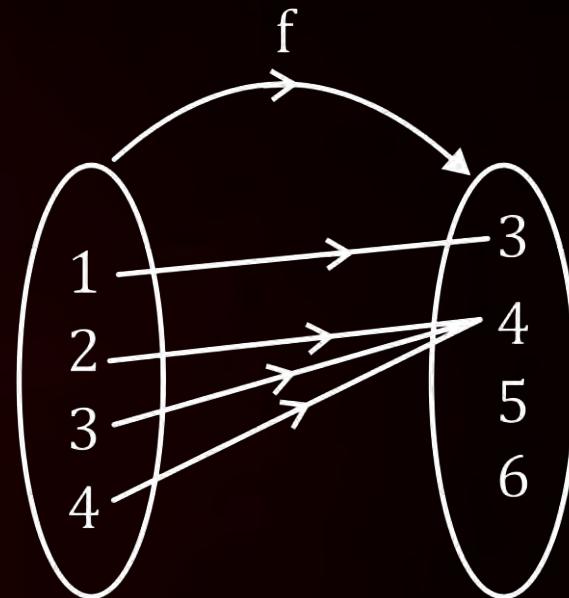
Recap

of previous lecture

15.



16.



1. $f(2) = \underline{\hspace{2cm}}$

2. $f(4) = \underline{\hspace{2cm}}$

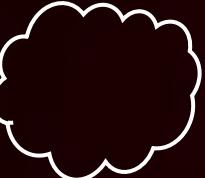
3. Image of 3 under f $\underline{\hspace{2cm}}$

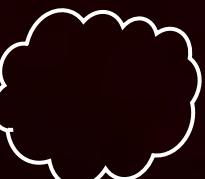
4. Preimages of 4 under f $\underline{\hspace{2cm}}$

Recap

 of previous lecture

$$17. f(x) = \sqrt[2n]{\text{cloud}}$$
 →  _____

$$f(x) = \sqrt[2n+1]{\text{cloud}}$$
 →  _____

$$18. f(x) = \frac{1}{\text{cloud}}$$
 →  _____

QUESTION



Function	Domain	Range
$\sin x$		
$\cos x$		
$\tan x$		
$\sec x$		
$\operatorname{cosec} x$		
$\cot x$		

QUESTION

Find the domain of following functions :

(i) $y = \sqrt{5 - 2x}$

(ii) $y = \frac{1}{\sqrt{x-|x|}}$

Bumper Practice Questions



Find the domain of definition of the given functions :

(i) $y = \sqrt{-px}$ ($p > 0$)

(ii) $y = \frac{1}{x^2+1}$

(iii) $y = \frac{1}{x^3-x}$

(iv) $y = \frac{1}{\sqrt{x^2-4x}}$

(v) $y = \sqrt{x^2 - 4x + 3}$

(vi) $y = \frac{x}{\sqrt{x^2-3x+2}}$

(vii) $y = \sqrt{1 - |x|}$

(i) $-\infty < x \leq 0$

(ii) $x \in \mathbb{R}$

(iii) $x \in \mathbb{R} - \{-1, 0, 1\}$

(iv) $-\infty < x < 0 \text{ } \& \text{ } 4 < x < \infty$

(v) $-\infty < x \leq 1 \text{ and } 3 \leq x < \infty$

(vi) $-\infty < x < 1 \text{ and } 2 < x < \infty$

(vii) $-1 \leq x \leq 1$

QUESTION

If a polynomial function 'f' satisfies the relation

$$\log_2(f(x)) = \log_2\left(2 + \frac{2}{3} + \frac{2}{9} + \dots + \infty\right) \cdot \log_3\left(1 + \frac{f(x)}{f\left(\frac{1}{x}\right)}\right) \text{ and } f(10) = 1001,$$

then the value of $f(20)$ is

A 2002

B 7999

C 8001

D 16001

QUESTION [JEE Mains 2025 (3 April)]



Let $A = \{-2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $y = \max\{x, 1\}$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to

- A** 11
- B** 12
- C** 14
- D** 13

Ans. B

Let the set of all relations R on the set $\{a, b, c, d, e, f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by S .
Then the number of elements in S is

QUESTION [JEE Mains 2024 (30 Jan)]



The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is _____

Ans. 960

QUESTION [JEE Mains 2025 (23 Jan)]

Let $X = R \times R$. Define a relation R on X as: $(a_1, b_1)R(a_2, b_2) \Leftrightarrow b_1 = b_2$

Statement I: R is an equivalence relation.

Statement II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)R(a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the correct answer from the options given below:

A Both Statement I and Statement II are true

B Statement I is true but Statement II is false

C Both Statement I and Statement II are false

D Statement I is false but Statement II is true

Ans. B

Let N be the set of natural numbers and a relation R on N be defined by $R = \{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$. Then the relation R is

- A** Reflexive but neither symmetric nor transitive
- B** An equivalence relation
- C** Symmetric but neither reflexive nor transitive
- D** Reflexive and symmetric, but not transitive



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 08



Recap *of previous lecture*

1. Range is the set of _____ of functions and it is always a subset of _____

2. $|x| > x \Leftrightarrow x \in \underline{\hspace{2cm}}$, $|x| = x \Leftrightarrow x \in \underline{\hspace{2cm}}$
 $|x| \leq a, a \in R^+ \Rightarrow x \in \underline{\hspace{2cm}}$, $|x| \geq a, a \in R^+ \Rightarrow x \in \underline{\hspace{2cm}}$

3. Range of odd degree polynomial defined over R is also _____

4. An even degree polynomial can never have range equal to _____ it is always a _____ of R .

Recap *of previous lecture*

5. If for a polynomial function f , we have $f(x) + f(1/x) = f(x) \cdot f(1/x)$ then $f(x)$ can be _____ or _____ or _____ $n \in I^+$
6. $\sqrt{\log_{g(x)} f(x)}$ is defined if _____
7. $\frac{1}{\sqrt{\log f(x)}}$ is defined if _____
8. $\frac{1}{f(x)}$ is defined if _____

Recap *of previous lecture*

9. $\frac{1}{\sqrt{f(x)}}$ is defined if _____

10. $\sqrt{f(x)}$ is defined if _____

11. $\frac{1}{\log f(x)}$ is defined if _____

Let the set of all relations R on the set $\{a, b, c, d, e, f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by S . Then the number of elements in S is

QUESTION [JEE Mains 2024 (30 Jan)]



The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is _____

Ans. 960

QUESTION [JEE Mains 2025 (3 April)]



If the domain of the function $f(x) = \log_7(1 - \log_4(x^2 - 9x + 18))$ is $(\alpha, \beta) \cup (\gamma, \infty)$, then $\alpha + \beta + \gamma + \hat{o}$ is equal to

A 17

B 15

C 16

D 18

Ans. D

QUESTION

The domain of function $f(x) = \log_3(-(\log_3 x)^2 + 5(\log_3 x) - 6)$ is

A $(9, 27)$

B $[9, 27]$

C $(0, 9) \cup (27, \infty)$

D $(0, 9] \cup [27, \infty)$

QUESTION

Find domain of : $f(x) = \log_7 \log_5 \log_3 \log_2(2x^3 + 5x^2 - 14x)$

QUESTION

If $f(x)$ is defined on $(0, 1)$, the domain of definition of $g(x) = f(e^x) + f(\ln |x|)$ is

A $(0, 1)$

B \mathbb{R}

C $(-e, -1)$

D $(-e, e) - [-1, 1]$

QUESTION

If the domain of $g(x)$ is $[3, 4]$, then the domain of $g(\log_2(x^2 + 3x - 2))$ is

- A** $[-4, -1] \cup [2, 7]$
- B** $[-3, 2]$
- C** $[-6, -5] \cup [2, 3]$
- D** $\left[\frac{3}{2}, 5\right]$

QUESTION

If $y = \sqrt{\operatorname{sgn}(x^2 - 2(k+1)x + 4)}$ is defined for all $x \in R$ then find number of integral values of k .

[Note: $\operatorname{sgn}(k)$ denotes signum function of k .]

QUESTION



For $f(x) = \frac{1}{[x]}$ find domain and range of f .

QUESTION [WB JEE 2009]

The domain of definition of the function $f(x) = \sqrt{1 + \log_e(1 - x)}$ is

A $-\infty < x \leq 0$

B $-\infty < x \leq \frac{e - 1}{e}$

C $-\infty < x \leq 1$

D $x \geq 1 - e$

Ans. B

QUESTION [WB JEE 2022]

Domain of $y = \sqrt{\log_{10} \frac{3x - x^2}{2}}$ is

A $x < 1$

B $2 < x$

C $1 \leq x \leq 2$

D $2 < x < 3$

Ans. C

QUESTION [WB JEE 2020]



The domain of $f(x) = \sqrt{\left(\frac{1}{\sqrt{x}} - \sqrt{x+1}\right)}$ is

A $x > -1$

B $(-1, \infty) \setminus \{0\}$

C $\left(0, \frac{\sqrt{5}-1}{2}\right]$

D $\left[\frac{1-\sqrt{5}}{2}, 0\right)$

Ans. C

QUESTION [BITSAT 2021]



If $f(x) = 4x - x^2$, $x \in \mathbb{R}$, and $f(a + 1) - f(a - 1) = 0$, then a is equal to

A 0

B 2

C 1

D 3

Ans. B

Let R be the set of real numbers.

Statement I : $A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R .

Statement II : $B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}$ is an equivalence relation on R .

- A** Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
- B** Statement I is true, Statement II is false.
- C** Statement I is false, Statement II is true.
- D** Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

Ans. B

Find the domain of definition of the given functions :

(viii) $y = \log_x 2$

(ix) $y = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

(x) $y = \sqrt{x} + \sqrt[3]{\frac{1}{x-2}} - \log_{10}(2x-3)$

(xi) $y = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$

(xii) $y = \frac{1}{\sqrt{\sin x}} + \sqrt[3]{\sin x}$

(xiii) $y = \log_{10}(\sqrt{x-4} + \sqrt{6-x})$

(xiv) $y = \log_{10}[1 - \log_{10}(x^2 - 5x + 16)]$

- (viii) $0 < x < 1$ and $1 < x < \infty$
- (ix) $-2 \leq x < 0$ and $0 < x < 1$
- (x) $\frac{3}{2} < x < 2$ and $2 < x < \infty$
- (xi) $-1 < x < 0$ and $1 < x < 2$; $2 < x < \infty$
- (xii) $2k\pi < x < (2k + 1)\pi$, where k is an integer
- (xiii) $4 \leq x \leq 6$
- (xiv) $2 < x < 3$



LAKSHYA

JEE 2026 BATCH

Relations and Functions

Mathematics

**By – Ashish Agarwal Sir
(IIT Kanpur)**

Lecture – 09



Recap *of previous lecture*

1. (a) $[x + n] = \underline{\hspace{2cm}}$ where $n \in I$

(b) $[x] + [-x] = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

(c) $f(x) = [x]$ is a _____ function.

(d) $[x + [x + [x + [x]]]] = \underline{\hspace{2cm}}$

(e) $[x] = x \Leftrightarrow \underline{\hspace{2cm}}$

(f) $\underline{\hspace{1cm}} < x < \underline{\hspace{1cm}}$

2. $\operatorname{sgn}(\operatorname{sgn}(\operatorname{sgn}(x))) = \underline{\hspace{2cm}}$

3. $f(x) = \log_a x$ is increasing if _____ & decreasing if _____

Recap

of previous lecture

4. If $[x] \geq 3 \Rightarrow x \in \underline{\hspace{2cm}}$

5. If $[x] \leq -3 \Rightarrow x \in \underline{\hspace{2cm}}$

6. If $[x] > 4 \Rightarrow x \in \underline{\hspace{2cm}}$

7. If $[x] < -4 \Rightarrow x \in \underline{\hspace{2cm}}$

8. If $[x] \leq 5 \Rightarrow x \in \underline{\hspace{2cm}}$

9. If $[x] \geq -6 \Rightarrow x \in \underline{\hspace{2cm}}$

Recap *of previous lecture*

10. If $[x] < -\sqrt{6} \Rightarrow x \in \underline{\hspace{2cm}}$

11. If $[x] \geq \sqrt{10} \Rightarrow x \in \underline{\hspace{2cm}}$

12. If $[x] \in [-\sqrt{5}, \sqrt{17}] \Rightarrow x \in \underline{\hspace{2cm}}$

13. $[x]^2 - 5[x] + 6 = 0 \Rightarrow x \in \underline{\hspace{2cm}}$

Find the domain of definition of the given functions :

$$(xii) \quad y = \frac{1}{\sqrt{\sin x}} + \sqrt[3]{\sin x}$$

$$(xiii) \quad y = \log_{10}(\sqrt{x-4} + \sqrt{6-x})$$

Let R be the set of real numbers.

Statement I : $A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R .

Statement II : $B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}$ is an equivalence relation on R .

- A** Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
- B** Statement I is true, Statement II is false.
- C** Statement I is false, Statement II is true.
- D** Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

Ans. B

QUESTION



If $y = \sqrt{\operatorname{sgn}(x^2 - 2(k+1)x + 4)}$ is defined for all $x \in \mathbb{R}$ then find number of integral values of k .

[Note: $\operatorname{sgn}(k)$ denotes signum function of k .]

QUESTION [JEE Mains 2019]



Let $[x]$ denote the greatest integer less than or equal to x . Then the values of $x \in \mathbb{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lies in the interval:

A $\left[0, \frac{1}{e}\right)$

B $[\log_e 2, \log_e 3)$

C $[1, e)$

D $[0, \log_e 2)$

QUESTION



$$f(x) = \sqrt{\log_2 \left(\frac{5x-x^2}{4} \right)} \text{ or } \sqrt{\log_{\frac{1}{2}} \frac{5x-x^2}{4}}$$

QUESTION



The domain of the function, $f(x) = \frac{\sqrt{\sin x}}{\sqrt{(x-2)(8-x)}}$ is

- A** $[0, \pi] \cup [2\pi, 8)$
- B** $(2, \pi] \cup [2\pi, 8)$
- C** $(2, 8)$
- D** $(0, 8)$

QUESTION



$$f(x) = \sqrt{\sin x} + \sqrt{16 - x^2}$$

QUESTION

Find the domain of the function

$$f(x) = \frac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6 + 35x - 6x^2}}$$

QUESTION [JEE Mains 2024 (9 April)]



If the domain of the function $f(x) = \sin^{-1} \left(\frac{x-1}{2x+3} \right)$ is $\mathbb{R} - (\alpha, \beta)$, then $12\alpha\beta$ is equal to

A 40

B 36

C 24

D 32

Ans. D

QUESTION [JEE Mains 2024 (30 Jan)]



If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to

- A 9
- B 12
- C 11
- D 10

Ans. B

QUESTION

Find domain of following functions.

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $h(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

(iii) $f(x) = \log_{100x} \left(\frac{2 \log_{10} x + 1}{-x} \right)$

(iv) $f(x) = \sqrt{\frac{1 - 5^x}{7^{-x} - 7}}$

QUESTION [JEE Mains 2019 (9 April)]



The domain of the definition of the function

$$f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$$

- A** $(-1, 0) \cup (1, 2) \cup (2, \infty)$
- B** $(-2, -1) \cup (-1, 0) \cup (2, \infty)$
- C** $(1, 2) \cup (2, \infty)$
- D** $(-1, 0) \cup (1, 2) \cup (3, \infty)$

Ans. A

QUESTION [AIEEE 2004]



The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$.

A [1, 2]

B [2, 3)

C [1, 2)

D [2, 3]

Ans. B

QUESTION [JEE Mains 2025 (7 April)]



If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$, $x \neq 1, 2$, is $(-\infty, \alpha] \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to:

- A** 188
- B** 192
- C** 190
- D** 194

Ans. D

QUESTION [JEE Mains 2025 (2 April)]

If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b) , then $(1+a)^2 + b^2$ is equal to:

A 29

B 30

C 25

D 26

Ans. D

QUESTION [JEE Mains 2025 (29 Jan)]



If the domain of the function $\log_5(18x - x^2 - 77)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ) , then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

- A** 186
- B** 179
- C** 195
- D** 174

Ans. A



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 10



Recap *of previous lecture*

1. $a \leq |x| \leq b$ where $a, b \in R^+$ then $x \in \underline{\hspace{10cm}}$

2. $|x| \leq a$ where $a \in R^+$ then $x \in \underline{\hspace{10cm}}$, whereas if $|x| < a, a \in R^+$, then $x \in \underline{\hspace{10cm}}$

3. $|x| \geq a$ where $a \in R^+$ then $x \in \underline{\hspace{10cm}}$, whereas if $|x| > a, a \in R^+$, then $x \in \underline{\hspace{10cm}}$

4. $[x] \in [-\sqrt{3}, \sqrt{26}]$ then $x \in \underline{\hspace{10cm}}$

Recap

of previous lecture

5. If $x \in [-2, 3]$ then $\frac{1}{x} \in \underline{\hspace{2cm}}$
 $x^2 \in \underline{\hspace{2cm}}$
 $2x + 7 \in \underline{\hspace{2cm}}$
6. If $x \in [-1, 5]$ then range of $f(x) = x^2 - 2x + 3$ is $\underline{\hspace{2cm}}$
7. If $x \in [-2, 3]$ then range of $f(x) = 2x^2 + 3x + 1$ is $\underline{\hspace{2cm}}$
8. $\log_{1/2} x < -1$ then $x \in \underline{\hspace{2cm}}$
9. Range of $f(x) = 2 \tan x \cdot \cos x$ is $\underline{\hspace{2cm}}$

Recap

of previous lecture

10. If domain of $f(x)$ is $[-2, 5]$ then domain of $g(x) = f(2x + 3)$ is

11. Function

Range

$$2 \sin x$$

$$\sin 2x$$

$$2 \sin x + 5$$

$$\cos(2x + 3)$$

$$\left(\sin x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

Recap *of previous lecture*

12. $f(x) = \{x\}$ has domain = _____ and range = _____ also $x = \{x\}$ is a _____ function with period _____.

13. (a) $\{x + n\} = \text{_____}$ $n \in \mathbb{I}$

(b) $\{x\} + \{-x\} = \text{_____}$

= _____

(c) $[\{x\}] = \text{_____}$

(d) $\{[x]\} = \text{_____}$

(e) $\{x\} = 0 \Rightarrow x \in \text{_____}$

(f) $\{x\} = \frac{1}{2}$ if $x = \text{_____}$

(g) $\{x\} = \frac{1}{3}$ if $x = \text{_____}$

QUESTION

Find domain of following functions.

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $h(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

(iii) $f(x) = \log_{100x} \left(\frac{2 \log_{10} x + 1}{-x} \right)$

(iv) $f(x) = \sqrt{\frac{1 - 5^x}{7^{-x} - 7}}$

QUESTION

Identify the equal function

(i) $f(x) = \log_x e; g(x) = \frac{1}{\log_e x}$

(ii) $f(x) = \log_e x; g(x) = \frac{1}{\log_x e}$

(iii) $f(x) = \sqrt{x^2 - 1}; g(x) = \sqrt{x - 1}\sqrt{x + 1}$

(iv) $f(x) = \log(x + 2) + \log(x - 3); g(x) = (x^2 - x - 6)$

(v) $f(x) = x|x|; g(x) = x^2 \operatorname{sgn} x$

(vi) $f(x) = \frac{1}{1+\frac{1}{x}}; g(x) = \frac{x}{1+x}$

(vii) $f(x) = [\{x\}]; g(x) = \{[x]\}$

QUESTION



$f(x) = \ln(\sqrt{x^2 - 5x - 24} - x - 2)$, find Domain of f.

QUESTION



$$f(x) = \sqrt{(x^2 - 3x - 10) \cdot \ln^2(x - 3)}$$

QUESTION [JEE Mains 2023 (29 Jan)]



The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2 \log_e x} - (2x+3)}$, $x \in \mathbb{R}$ is

- A** $(-1, \infty) - \{3\}$
- B** $\mathbb{R} - \{-1, 3\}$
- C** $(2, \infty) - \{3\}$
- D** $\mathbb{R} - \{3\}$

Ans. C

QUESTION



Domain of the function $f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$ is

A [0, 2]

B [-1, 1]

C [-1, 2]

D [1, 2]

Ans. C

QUESTION [JEE Mains 2021]



Let $[x]$ denote the greatest integer $\leq x$, where $x \in \mathbb{R}$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a < b < c$, then the value of $a + b + c$ is :

- A** 8
- B** 1
- C** -2
- D** -3

QUESTION [IIT-JEE 1994]

Let $\{x\}$ & $[x]$ denotes the fractional and integral part of a real number x respectively.
Solve $4\{x\} = x + [x]$.

QUESTION

Given that $y = f(x)$ is a function whose domain is $[4, 7]$ and range is $[-1, 9]$. Find the range and domain of

a. $g(x) = \frac{1}{3}f(x)$

b. $h(x) = f(x - 7)$

QUESTION [AIEEE 2002]

The domain of $\sin^{-1} \left[\log_3 \left(\frac{x}{3} \right) \right]$ is

A [1, 9]

B [-1, 9]

C [9, 1]

D [-9, -1]

Ans. A

QUESTION [AIEEE 2011]



The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is

A $(0, \infty)$

B $(-\infty, 0)$

C $(-\infty, \infty) - \{0\}$

D $(-\infty, \infty)$

QUESTION [JEE Mains 2024 (30 Jan)]



If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to

- A 9
- B 12
- C 11
- D 10

Ans. B

QUESTION [JEE Mains 2023 (11 April)]

The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is :

(where $[x]$ denotes the greatest integer less than or equal to x)

A $(-\infty, -2) \cup [6, \infty)$

B $(-\infty, -3] \cup [6, \infty)$

C $(-\infty, -2) \cup (5, \infty)$

D $(-\infty, -3] \cup (5, \infty)$

Ans. A

QUESTION [JEE Mains 2024 (30 Jan)]



If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta] - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to:

A 11

B 12

C 9

D 8

Ans. A

Bumper Practice Problems



1. Find the domain of function : $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$

2. The domain of the function $f(x) = \sqrt{\frac{x-1}{(x-2)(x-3)}}$ is:
(A) $(1, 2) \cup (3, \infty)$ (B) $[1, 2) \cup (3, \infty)$
(C) $(1, \infty)$ (D) None of these

3. The domain of the function $f(x) = \sqrt{(x-1)} + \sqrt{(5-x)} + \frac{1}{2x-5}$ is:
(A) $[1, 5]$ (B) $[1, 5] - \left\{\frac{5}{2}\right\}$ (C) $(1, 5)$ (D) $\left(1, \frac{3}{2}\right)$

4. The domain of the function $f(x) = \sqrt{(x^2 - 1)} + \sqrt{(9 - x^2)}$ is:
(A) $(-3, -1) \cup (1, 3)$ (B) $[-3, -1] \cup [1, 3]$
(C) $[-1, 1]$ (D) $[-3, 3]$

Bumper Practice Problems



5. Domain of $f(x) = \frac{1}{\sqrt{13x-x^2-36}}$ is:

(A) $x \in [4, 9]$

(B) $x \in (-\infty, 4) \cup (9, \infty)$

(C) $x \in (4, 9)$

(D) $x \in \mathbb{R}$

6. Find the domain of

(1) $f(x) = \log_{(x^2-3)} x$

(2) $f(x) = \log_x(x^2 - 3)$

(3) $f(x) = \frac{1}{\ln(x^2-3)}$

(4) $f(x) = \log_{x-5}(x^2 - 5x + 6)$

1. $D_f = (-3, -\infty) - \{-2, -1\}$
2. B
3. B
4. B
5. C
6. (i) $D_f = (\sqrt{3}, \infty) - \{2\}$
(ii) $D_f = (\sqrt{3}, \infty)$
(iii) $D_f = (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty) - \{-2, 2\}$
(iv) $D_f = (5, \infty) - \{6\}$



LAKSHYA

JEE 2026 BATCH

Relations and Functions

Mathematics

**By – Ashish Agarwal Sir
(IIT Kanpur)**

Lecture – 11



Recap

of previous lecture

Find Range of following

$$1. \quad f(x) = x^2$$

$$2. \quad f(x) = |x|$$

$$3. \quad f(x) = |x - 3|$$

$$4. \quad f(x) = \sqrt{x}$$

$$5. \quad f(x) = \ln x$$

$$6. \quad f(x) = \frac{1}{x}$$

$$7. \quad f(x) = 3 + 5x^2$$

$$8. \quad f(x) = 7 - x^2$$

$$9. \quad f(x) = \frac{2}{3+x^2}$$

$$10. \quad f(x) = \sqrt{x}$$

Recap *of previous lecture*

Find Range of following

$$11. \ f(x) = \sqrt{x - 4}$$

$$12. \ f(x) = \sqrt{25 - x^2}$$

$$13. \ f(x) = \sqrt{x^2 - 9}$$

$$14. \ f(x) = \log_2(3 \sin x + 4 \sin x + 7)$$

$$15. \ f(x) = \log_3(12 \cos x + 5 \sin x + 14)$$

$$16. \ f(x) = \log_{\sqrt{2}}(2 - x^2)$$

$$17. \ \text{Find } x \text{ if } \{x\} \geq \frac{1}{3}$$

QUESTION [JEE Mains 2024 (30 Jan)]



If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta] - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to:

A 11

B 12

C 9

D 8

Ans. A

QUESTION

Find Range of

$$f(x) = \frac{2e^x}{3e^x + 5}$$

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

QUESTION



Find range of $f(x) = \frac{|x| + 4}{7 - |x|}$

- A** $(-\infty, -1)$
- B** $\left[\frac{4}{7}, \infty\right)$
- C** $\left(-1, \frac{4}{7}\right]$
- D** $(-\infty, -1) \cup \left[\frac{4}{7}, \infty\right)$

QUESTION



Range of $f(x) = \frac{3^x - 3^{-x}}{3^x + 3^{-x}}$ is

A $(0, \infty)$

B $(-\infty, 0)$

C $(-1, 1)$

D $[-1, 1]$

QUESTION

Find Range of following functions:

$$(a) \ f(x) = e^{(x-1)^2}$$

$$(b) \ f(x) = 2^{x^2} + 1$$

$$(c) \ f(x) = \frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}$$

QUESTION [IIT-JEE 2010]



Find the maximum & minimum value of :

$$y = \frac{1}{\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta}$$

QUESTION

Find Range of

$$f(x) = \log_2 \left(2 - \log_{\sqrt{2}}(16 \sin^2 x + 1) \right)$$

QUESTION [JEE Mains 2024 (6 April)]



Let $f(x) = \frac{1}{7 - \sin 5x}$ be a function defined on \mathbb{R} . Then the range of the function $f(x)$ is equal to :

A $\left[\frac{1}{8}, \frac{1}{5} \right]$

B $\left[\frac{1}{7}, \frac{1}{6} \right]$

C $\left[\frac{1}{7}, \frac{1}{5} \right]$

D $\left[\frac{1}{8}, \frac{1}{6} \right]$

Ans. D

QUESTION [JEE Mains 2023 (25 Jan)]



Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m , such that the range of f is $[0, 2]$. Then the value of m is

A 4

B 3

C 5

D 2

QUESTION [JEE Mains 2024 (9 April)]



Let the range of the function $f(x) = \frac{1}{2+\sin 3x+\cos 3x}$, $x \in \mathbb{R}$ be $[a, b]$. If α and β are respectively the A.M. and the G.M. of a and b , then $\frac{\alpha}{\beta}$ is equal to

- A π
- B $\sqrt{\pi}$
- C $\sqrt{2}$
- D 2

Ans. C

QUESTION

Find the range of the function $f(x) = \sqrt{3} \sin x + \cos x + 4$.

QUESTION

Find the range of the function

$$f(x) = \frac{1}{2 \cos^2 x + 4 \sin x \cos x + 4}$$

QUESTION



Find range of :

$$(1) \quad f(x) = \frac{2x-3}{x-1}$$

$$(2) \quad f(x) = \frac{x+3}{2-5x}$$

$$(3) \quad f(x) = \frac{6}{4x+7}$$

$$(4) \quad f(x) = \frac{7x+5}{3}$$

Bumper Practice Problems



1. Find the range of the following functions

(a) $y = \frac{1}{3 \sin x + 4 \cos x + 7}$

(b) $y = \frac{1}{3 \sin x + 4 \cos x + 2}$

(c) $y = 6 \tan x \cos x$

(d) $f(x) = \frac{1}{8 - 3 \sin x}$

(e) $f(x) = \frac{1}{2 - 3 \sin x}$

2. Find the range of the following function:

(a) $y = \ln x$

(b) $y = \frac{1}{\ln x}$

(c) $y = \log_x 2$

(d) $y = \log_3(1 + 26 \sin^2 x)$

(e) $y = \log_3(1 - 26 \sin^2 x)$

(f) $y = \ln(3 \sin x + 4 \cos x)$

(g) $y = \ln(3 \sin x + 4 \cos x + 7)$

1. (a) Range = $\left[\frac{1}{12}, \frac{1}{2}\right]$ (b) Range = $(-\infty, -\frac{1}{3}] \cup \left[\frac{1}{7}, \infty\right)$
- (c) Range = $(-6, 6)$ (d) Range = $\left[\frac{1}{5}, \frac{1}{11}\right]$
- (e) Range = $(-\infty, -1] \cup \left[\frac{1}{5}, \infty\right)$
2. (a) Range = R
(b) Range = $R - \{0\} = (-\infty, 0) \cup (0, \infty)$
(c) Range = $R - \{0\} = (-\infty, 0) \cup (0, \infty)$
(d) Range = $[0, 3]$ (e) Range = $(-\infty, 0]$
(f) Range = $(-\infty, \log_2 5]$ (g) Range = $[\log_{1/2} 12, -1]$



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 12



Recap *of previous lecture*

1. Check it following functions are identical or non identical.

(a) $f(x) = e^{\ln e^x}; g(x) = e^x$

(b) $f(x) = \sqrt{\frac{1-\cos 2x}{2}}; g(x) = \sin x$

(c) $f(x) = \sqrt{x^2}; g(x) = (\sqrt{x})^2$

(d) $f(x) = \frac{1}{1+\frac{1}{x}}; g(x) = \frac{x}{1+x}$

2. If $\frac{1}{2} < \{x\} < \frac{1}{3}$ then $x \in \underline{\hspace{2cm}}$

Recap

of previous lecture

3. If $x \in (-2, 5)$ then $|x| \in \underline{\hspace{2cm}}$

$x^2 \in \underline{\hspace{2cm}}$

$\frac{1}{x+1} \in \underline{\hspace{2cm}}$

4. Range of (a) $f(x) = 3 \tan x \cdot \cos x$ is $\underline{\hspace{2cm}}$

(b) $f(x) = 3 - 2^x$ is $\underline{\hspace{2cm}}$

(c) $f(x) = \frac{2x+3}{1-x}$ is $\underline{\hspace{2cm}}$

(d) $f(x) = \log_{\sqrt{5}}\{\sqrt{2}(\sin x - \cos x) + 3\}$ is $\underline{\hspace{2cm}}$

(e) $f(x) = \frac{\sin^2 x + 4 \sin x + 5}{2 \sin^2 x + 8 \sin x + 8}$ is $\underline{\hspace{2cm}}$

Recap *of previous lecture*

5. The sum $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{2000}\right] + \left[\frac{1}{2} + \frac{2}{2000}\right] + \left[\frac{1}{2} + \frac{3}{2000}\right] + \cdots + \left[\frac{1}{2} + \frac{1999}{2000}\right]$ is equal to
6. A non zero polynomial function f satisfying $f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right) \forall x \in R_0$ is ____
7. Solve for x :
- (a) $\{x + 1\} + 2x = 4[x + 1] - 6$
- (b) $4\{x\} = x + [x]$
8. Find range of
- (a) $f(x) = \sin(\sin x)$ (b) $f(x) = \sin(2 \sin x)$
- (c) $f(x) = \cos(\sin x)$ (d) $f(x) = \cos(2 \sin x)$

QUESTION [JEE Mains 2023 (25 Jan)]



Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m , such that the range of f is $[0, 2]$. Then the value of m is

A 4

B 3

C 5

D 2

Bumper Practice Problems



1. Find the range of the following functions

(a) $y = \frac{1}{3 \sin x + 4 \cos x + 7}$

(b) $y = \frac{1}{3 \sin x + 4 \cos x + 2}$

(c) $y = 6 \tan x \cos x$

(d) $f(x) = \frac{1}{8 - 3 \sin x}$

(e) $f(x) = \frac{1}{2 - 3 \sin x}$

2. Find the range of the following function:

(a) $y = \ln x$

(b) $y = \frac{1}{\ln x}$

(c) $y = \log_x 2$

(d) $y = \log_3(1 + 26 \sin^2 x)$

(e) $y = \log_3(1 - 26 \sin^2 x)$

(f) $y = \ln(3 \sin x + 4 \cos x)$

(g) $y = \ln(3 \sin x + 4 \cos x + 7)$

1. (a) Range = $\left[\frac{1}{12}, \frac{1}{2}\right]$ (b) Range = $(-\infty, -\frac{1}{3}] \cup \left[\frac{1}{7}, \infty\right)$
(c) Range = $(-6, 6)$ (d) Range = $\left[\frac{1}{5}, \frac{1}{11}\right]$
(e) Range = $(-\infty, -1] \cup \left[\frac{1}{5}, \infty\right)$
2. (a) Range = R
(b) Range = $R - \{0\} = (-\infty, 0) \cup (0, \infty)$
(c) Range = $R - \{0\} = (-\infty, 0) \cup (0, \infty)$
(d) Range = $[0, 3]$ (e) Range = $(-\infty, 0]$
(f) Range = $(-\infty, \log_2 5]$ (g) Range = $[\log_e 2, \log_e 12]$

QUESTION



Find the range of $f(x) = \frac{x^2 - 5x + 6}{x^2 - 6x + 8}$.

QUESTION



Find the range of $f(x) = \frac{x^2 - 5x + 4}{x^2 + 2x - 3}$

QUESTION



Find the range of $f(x) = \frac{x - 1}{x^2 - 3x + 2}$

QUESTION



Find domain & range of $f(x) = \frac{x^2+x+1}{x^2-x+1}$.

QUESTION



Find domain & range of $f(x) = \frac{2x^2+2x+3}{x^2+x+1}$.

QUESTION [JEE Mains 2025 (7 April)]



If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$, $x \neq 1, 2$, is $(-\infty, \alpha] \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to:

- A** 188
- B** 192
- C** 190
- D** 194

Ans. D

QUESTION [JEE Mains 2023 (31 Jan)]



Let $f: \mathbb{R} - \{2,6\} \rightarrow \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

- A** $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$
- B** $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$
- C** $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$
- D** $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

QUESTION



Find the range of $f(x) = \frac{x - 3}{2x + 1}$

A $R - 2$

B $R - \left\{-\frac{1}{2}\right\}$

C $R - \left\{\frac{1}{2}\right\}$

D None of these

Ans. C

QUESTION [JEE Mains 2019 (11 Jan)]



Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in \mathbb{R}$. Then the range of f is :

- A** $\left[-\frac{1}{2}, \frac{1}{2}\right]$
- B** $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$
- C** $(-1, 1) - \{0\}$
- D** $\mathbb{R} - [-1, 1]$

Ans. A

QUESTION [JEE Mains 2023 (31 Jan)]



If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2, 6)$. then its range is

A $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

B $\left(\frac{5}{37}, \frac{2}{5}\right]$

C $\left(\frac{5}{26}, \frac{2}{5}\right]$

D $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

3. Find the Domain and Range of the following functions:

(i) $f(x) = \ln(2x - x^2)$

(ii) $f(x) = \sin(\ln(5x^2 - 8x + 4))$

(iii) $f(x) = \log_2 \left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}} \right)$

QUESTION



$$f(x) = \frac{1}{[x]} + \log_{1-[x]}(x^2 - 3x + 10) + \frac{1}{\sqrt{2 - |x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$$

QUESTION

Given $f(x)$ is a polynomial function of x , $f(x) \cdot f(y) = f(x) + f(y) + f(xy) - 2$ for all $x, y \in R$ and that $f(2) = 5$ Then $f(3)$ is equal to

A 10

B 24

C 15

D none

QUESTION

Let f be a polynomial function which satisfies the relation

$$f(x) + f\left(\frac{x}{y^2}\right) + f\left(\frac{x}{y}\right) = f(x) \cdot f\left(\frac{1}{y}\right) - \frac{1}{y^3} + \frac{x^3}{y^6} + 2 \quad \forall x \in \mathbb{R} - \{0\}, f(1) \neq 1 \text{ and } f(2) = 9.$$

The value of $\sum_{r=1}^{100} f(r)$ equals

A 5050

B $(5050)^2$

C $100 + (5050)^2$

D $100 + (5050)^3$

QUESTION [JEE Mains 2023 (30 Jan)]



The range of the function $f(x) = \sqrt{3 - x} + \sqrt{2 + x}$ is :

A $[2\sqrt{2}, \sqrt{11}]$

B $[\sqrt{5}, \sqrt{13}]$

C $[\sqrt{2}, \sqrt{7}]$

D $[\sqrt{5}, \sqrt{10}]$

QUESTION

Find Range of

$$(a) \quad f(x) = \frac{1}{\sin^4 x + \cos^4 x}$$

$$(b) \quad f(x) = |\sin x| + |\cos x|$$

Bumper Practice Questions



Find the range of the following functions :

$$(i) \quad f(x) = \frac{x-1}{x+2}$$

$$(ii) \quad f(x) = \frac{2}{x}$$

$$(iii) \quad f(x) = \frac{1}{x^2-x+1}$$

$$(iv) \quad f(x) = \frac{x^2-x+1}{x^2+x+1}$$

$$(v) \quad f(x) = e^{(x-1)^2}$$

$$(vi) \quad f(x) = x^3 - x^2 + x + 1$$

$$(vii) \quad f(x) = \log(x^8 + x^4 + x^2 + 1)$$

$$(viii) \quad f(x) = \sin^2 x - 2 \sin x + 4$$

$$(ix) \quad f(x) = \sin(\log_2 x)$$

$$(x) \quad f(x) = 2^{x^2} + 1$$

$$(xi) \quad f(x) = \frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}$$

$$(xii) \quad f(x) = \frac{1}{8-3 \sin x}$$

(i) $\mathbb{R} - \{1\}$

(ii) $\mathbb{R} - \{0\}$

(iii) $\left(0, \frac{4}{3}\right]$

(iv) $\left[\frac{1}{3}, 3\right]$

(v) $[1, \infty)$

(vi) \mathbb{R}

(vii) $[0, \infty)$

(viii) $[3, 7]$

(ix) $[-1, 1]$

(x) $[2, \infty)$

(xi) $\left[\frac{1}{3}, 1\right)$

(xii) $\left[\frac{1}{11}, \frac{1}{5}\right]$

Bumper Practice Problems (BPP)



Find the domains of definition of the following function

(i) $f(x) = \sqrt{4x - x^3}.$

Ans. $D(f) = (-\infty, -2] \cup [0, 2]$

(ii) $y = \sqrt{5 - x - \frac{6}{x}}.$

Ans. $D(y) = (-\infty, 0) \cup [2, 3]$

(iii) $f(x) = \sqrt{\frac{(x-1)(7-x)(x-5)^4}{x(x-5)^2}}$

Ans. $x \in (-\infty, 0) \cup [1, 7] - \{5\}$

(iv) $f(x) = \sqrt{4 - \sqrt{5 + 2x}}$

Ans. $\left[\frac{-5}{2}, \frac{11}{2} \right]$

(v) $y = \sqrt{\frac{\sqrt{17-15x-2x^2}}{x+3}}.$

Ans. $D(y) = (-3, 1]$



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 13



Recap *of previous lecture*

1. If 'f' is an increasing function and $g(x)$ has range $[a, b]$ then range of $f(g(x)) = \underline{\hspace{2cm}}$ i.e. range of $\log_2(x^2 + 2x + 3)$ is $\underline{\hspace{2cm}}$ while range $e^{\sin x}$ is $\underline{\hspace{2cm}}$.
2. If f is a decreasing function and $g(x)$ has range $[a, b]$ then range of $f(g(x))$ is $\underline{\hspace{2cm}}$ i.e. range of $\log_{\frac{1}{3}}(x^2 + 2x + 4)$ is $\underline{\hspace{2cm}}$ while range of $\left(\frac{1}{2}\right)^{x^2+4}$ is $\underline{\hspace{2cm}}$.
3. If $g(x)$ has range $[a, b]$ and $f(x)$ is neither increasing nor decreasing then range of $f(g(x))$ is $[f(a), f(b)]$. (True/False)

Recap *of previous lecture*

4. $f(x) = \sin(x^2 + 4)$, $x \in [-2, 2]$ then range of $f(x) = [\sin 0, \sin 16]$. (True/False)

5. Range of $\cos(\{x\})$ is _____

$\sin(\{x\})$ is _____

$\ln \{x\}$ is _____

$e^{\{x\}}$ is _____

$\sin(e^x)$ is _____

($\{\cdot\}$ denotes fractional parts function)

Recap *of previous lecture*

6. Draw following graphs:

(i) $y = e^{-x}$

(ii) $y = \max(1 - x, 1 + x)$

(iii) $y = \min(e^x, e^{-x})$

(iv) $y = \min(\sin x, 1/2)$

Recap *of previous lecture*

7. $\cos x = \frac{1}{2} \Rightarrow x = \underline{\hspace{2cm}}$

$\sin x = \frac{1}{\sqrt{2}} \Rightarrow x = \underline{\hspace{2cm}}$

$\tan x = -\sqrt{3} \Rightarrow x = \underline{\hspace{2cm}}$

$\cos x = -\frac{1}{\sqrt{2}} \Rightarrow x = \underline{\hspace{2cm}}$

8. $\frac{(x-1)(x-2)^2(x-3)^4}{(x-4)(x-5)(x-3)} \geq 0$ then $x \in \underline{\hspace{2cm}}$

9. $\frac{(x-1)(x-5)^2(x-6)}{x^2(x-3)(x-2)} \leq 0$ then $x \in \underline{\hspace{2cm}}$

Recap *of previous lecture*

10. $1 - \cos \theta = \underline{\hspace{2cm}}$

$1 + \cos \theta = \underline{\hspace{2cm}}$

$\cos 2\theta = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

$\sin 2\theta = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

$\tan 2\theta = \underline{\hspace{2cm}}$

11. Range of $f(x) = \frac{x^3}{3} - 2x^2 + 3x + 4$. where $x \in [-1, 6]$.

3. Find the Domain and Range of the following functions:

(i) $f(x) = \ln(2x - x^2)$

(ii) $f(x) = \sin(\ln(5x^2 - 8x + 4))$

(iii) $f(x) = \log_2 \left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}} \right)$

Bumper Practice Problems (BPP)



Find the domains of definition of the following function

(i) $f(x) = \sqrt{4x - x^3}.$

Ans. $D(f) = (-\infty, -2] \cup [0, 2]$

(ii) $y = \sqrt{5 - x - \frac{6}{x}}.$

Ans. $D(y) = (-\infty, 0) \cup [2, 3]$

(iii) $f(x) = \sqrt{\frac{(x-1)(7-x)(x-5)^4}{x(x-5)^2}}$

Ans. $x \in (-\infty, 0) \cup [1, 7] - \{5\}$

(iv) $f(x) = \sqrt{4 - \sqrt{5 + 2x}}$

Ans. $\left[\frac{-5}{2}, \frac{11}{2} \right]$

(v) $y = \sqrt{\frac{\sqrt{17-15x-2x^2}}{x+3}}.$

Ans. $D(y) = (-3, 1]$

QUESTION

Find whether the following functions are even or odd or none

(a) $f(x) = \log(x + \sqrt{1 + x^2})$

(b) $f(x) = \frac{x(a^x+1)}{a^x-1}$

(c) $f(x) = \sin x + \cos x$

(d) $f(x) = x \sin^2 x - x^3$

(e) $f(x) = \sin x - \cos x$

(f) $f(x) = \frac{(1+2^x)^2}{2^x}$

(g) $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$

(h) $f(x) = [(x+1)^2]^{1/3} + [(x-1)^2]^{1/3}$

QUESTION [JEE Mains 2023 (6 April)]



Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. Then among the statements

- (S1): $A \cap B = (1, \infty) - \mathbb{N}$ and
(S2): $A \cup B = (1, \infty)$

- A** only (S2) is true
- B** only (S1) is true
- C** neither (S1) nor (S2) is true
- D** both (S1) and (S2) are true

Ans. B

Bumper Practice Problems (BPP)



1. Find the range of the following function: $f(x) = \frac{2e^x}{3e^x + 5}$ Ans. $f(x) \in \left(0, \frac{2}{3}\right)$

2. Range of $f(x) = \frac{\{x\}-2}{1-3\{x\}}$ (where $\{.\}$ represents FPF) Ans. $(-\infty, -2] \cup \left(\frac{1}{2}, \infty\right)$

3. Find the range of $f(x)$:
 - (i) $f(x) = 2x^2 - 3x + 2$ Ans. $\left[\frac{7}{8}, \infty\right)$
 - (ii) $f(x) = 2x^2 - 3x + 2, x \in [0, 2]$ Ans. $\left[\frac{7}{8}, 4\right]$
 - (iii) $f(\theta) = 2 \cos^2 \theta - 6 \sin \theta + 1$ Ans. $[-5, 7]$

QUESTION**KTK 1**

Find domain of $f(x) = \sqrt{\log_{1/3}(\log_4([x]^2 - 5))}$ (Where $[.]$ denotes G.I.F.)

Ans. $[-3, -2] \cup [3, 4]$

The range of the function $f(x) = 2|\sin x| - 3|\cos x|$ is :

A $[-2, \sqrt{13}]$

B $[-2, 3]$

C $[3, \sqrt{13}]$

D $[-3, 2]$

Ans. D

The function $f(x)$ is defined on the interval $[0, 1]$. Find the domain of definition of the functions.

(a) $f(\sin x)$

(b) $f(2x + 3)$

Ans. (a) $2K\pi \leq x \leq 2K\pi + \pi$ where $K \in \mathbb{Z}$;
(b) $[-3/2, -1]$

QUESTION**KTK 4**

Find the value of $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{1000}\right] + \cdots + \left[\frac{1}{2} + \frac{2946}{1000}\right]$ where $[.]$ greatest integer function?

Ans. 4341

Let $f(x) = \sqrt{\log_2 \left(\frac{10x-4}{4-x^2} \right)} - 1$. Then sum of all integers in domain of $f(x)$ is

A -15

B -16

C -17

D -18

Ans. C

Number of real roots of the equation $6x - 7[x] = 2$ is

[Note: $[k]$ denotes greatest integer function less than or equal to k]

A 6

B 5

C 8

D 7

Ans. A



LAKSHYA

JEE 2026 BATCH

Relations and Functions

Mathematics

**By – Ashish Agarwal Sir
(IIT Kanpur)**

Lecture – 14



Recap *of previous lecture*

1. Graphs of odd functions are symmetric about _____ while graphs of even functions are symmetric about _____

2. If an odd function 'f' is defined at $x = 0$ then $f(0) = _____$

3. Every function is odd or even. (T/F)

4. Function defined on entire number line which is odd as well as even is _____

5. Every constant function defined on \mathbb{R} is _____

6. A function 'f' defined on a symmetric domain is even if _____ or _____ and is odd if _____ or _____

Recap *of previous lecture*

7. If for an odd function $f(2) = 3, f(3) = -2$, then value of $f(f(-2)) + f(f(-3)) = \underline{\hspace{10cm}}$
8. If $f(x) = \max\left\{x^3, x^2, \frac{1}{64}\right\} \forall x \in [0, \infty)$, then
- (A) $f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ x^3, & x > 1 \end{cases}$
- (B) $f(x) = \begin{cases} \frac{1}{64}, & 0 \leq x \leq \frac{1}{4} \\ x^2, & \frac{1}{4} < x \leq 1 \\ x^3, & x > 1 \end{cases}$
- (C) $f(x) = \begin{cases} \frac{1}{64}, & 0 \leq x \leq \frac{1}{8} \\ x^2, & \frac{1}{8} < x \leq 1 \\ x^3, & x > 1 \end{cases}$
- (D) None of these

QUESTION**(KTK 2)**

The range of the function $f(x) = 2|\sin x| - 3|\cos x|$ is :

A $[-2, \sqrt{13}]$

B $[-2, 3]$

C $[3, \sqrt{13}]$

D $[-3, 2]$

Ans. D

QUESTION**(KTK 4)**

Find the value of $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{1000}\right] + \cdots + \left[\frac{1}{2} + \frac{2946}{1000}\right]$ where $[.]$ greatest integer function?

Ans. 4341

QUESTION

Find whether the following functions are even or odd or none

(a) $f(x) = \log(x + \sqrt{1 + x^2})$

(b) $f(x) = \frac{x(a^x+1)}{a^x-1}$

(c) $f(x) = \sin x + \cos x$

(d) $f(x) = x \sin^2 x - x^3$

(e) $f(x) = \sin x - \cos x$

(f) $f(x) = \frac{(1+2^x)^2}{2^x}$

(g) $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$

(h) $f(x) = [(x+1)^2]^{1/3} + [(x-1)^2]^{1/3}$

Bumper Practice Problems (BPP)



1. Find the range of the following function: $f(x) = \frac{2e^x}{3e^x + 5}$ Ans. $f(x) \in \left(0, \frac{2}{3}\right)$

2. Range of $f(x) = \frac{\{x\}-2}{1-3\{x\}}$ (where $\{.\}$ represents FPF) Ans. $(-\infty, -2] \cup \left(\frac{1}{2}, \infty\right)$

3. Find the range of $f(x)$:
 - (i) $f(x) = 2x^2 - 3x + 2$ Ans. $\left[\frac{7}{8}, \infty\right)$
 - (ii) $f(x) = 2x^2 - 3x + 2, x \in [0, 2]$ Ans. $\left[\frac{7}{8}, 4\right]$
 - (iii) $f(\theta) = 2 \cos^2 \theta - 6 \sin \theta + 1$ Ans. $[-5, 7]$

QUESTION**(KTK 6)**

Number of real roots of the equation $6x - 7[x] = 2$ is

[Note: $[k]$ denotes greatest integer function less than or equal to k]

A 6

B 5

C 8

D 7

Ans. A

QUESTION**(KTK 1)**

Find domain of $f(x) = \sqrt{\log_{1/3}(\log_4([x]^2 - 5))}$ (Where $[.]$ denotes G.I.F.)

Ans. $[-3, -2] \cup [3, 4]$

QUESTION**(KTK 3)**

The function $f(x)$ is defined on the interval $[0, 1]$. Find the domain of definition of the functions.

(a) $f(\sin x)$

(b) $f(2x + 3)$

Ans. (a) $2K\pi \leq x \leq 2K\pi + \pi$ where $K \in \mathbb{Z}$;
(b) $[-3/2, -1]$

QUESTION**(KTK 5)**

Let $f(x) = \sqrt{\log_2 \left(\frac{10x-4}{4-x^2} \right)} - 1$. Then sum of all integers in domain of $f(x)$ is

A -15**B** -16**C** -17**D** -18

Ans. C

QUESTION

Let $f(x)$ be an odd function defined on \mathbb{R} such that

$f(1) = 2, f(3) = 5$ and $f(-5) = -1$. The value of $\frac{f(f(f(-3))) + f(f(0))}{3f(1) - 2f(3) - f(5)}$ is

A $\frac{-2}{5}$

B $\frac{-2}{3}$

C $\frac{2}{5}$

D $\frac{2}{3}$

Ans. C

QUESTION



If $f(x) = (a - 2)x + 3a - 4$ is even/odd. Find 'a'.

QUESTION [JEE Mains 2019]



Let $f(x) = a^x$ ($a > 0$) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals is :

A $2f_1(x + y)f_2(x - y)$

B $2f_1(x)f_1(y)$

C $2f_1(x + y)f_1(x - y)$

D $2f_1(x)f_2(y)$

QUESTION

Suppose that $f(x)$ is a function of the form $f(x) = \frac{ax^8+bx^6+cx^4+dx^2+15x+1}{x}$ ($x \neq 0$).

If $f(5) = 2$ then the value of $f(-5)$ is equal to

A -2

B 28

C 13

D -13

Ans. B

QUESTION

If $h(x) = Ax^5 + B \sin x + C \ln\left(\frac{1+x}{1-x}\right) + 7$, where A, B, C are non-zero real constants and $h\left(\frac{-1}{2}\right) = 6$, then find the value of $h\left(\frac{\operatorname{sgn}(e^{-x})}{2}\right)$.

QUESTION

The smallest natural number k for which $f(x) = \ln(x^3 + \sqrt{x^6 + 1}) + \sin 5x + \left[\frac{x^2}{k} \right]$ is an odd function $\forall x \in [-2\pi, 2\pi]$, is ($[y]$ denotes largest integer $\leq y$)

A 38

B 39

C 40

D 41

QUESTION

Let $f(x) = \begin{cases} x, & -2 \leq x \leq -1 \\ x^2 + 2x, & -1 < x \leq 0 \\ 2x - x^2, & 0 < x \leq 1 \\ 2 - x, & 1 < x \leq 2 \end{cases}$. Find the number of integers in the range of $f(x)$.

QUESTION

$$\text{Let } f(x) = \begin{cases} 2x^2 - 10x, & -\infty < x \leq -5 \\ x^2 - 5, & -5 < x < 3 \\ x^2 + 1, & 3 \leq x < \infty \end{cases} .$$

Number of negative integers in the range of the function $f(x)$ is

- A** 6
- B** 5
- C** 4
- D** 3

QUESTION [JEE Mains 2019]



For $x \in \mathbb{R}$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series

$$\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \cdots + \left[-\frac{1}{3} - \frac{99}{100}\right] \text{ is:}$$

A -153

B -133

C -135

D -131

If range of function $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \frac{1}{2}f(2x + 1)$ is equal to :

A $[-2, 4]$

B $[-1, 2]$

C $[-3, 9]$

D $[-2, 2]$

Ans. B

If domain of $y = f(x)$ is $[-3, 2]$, then domain of $f(|[x]|)$ is equal to

[Note: $[k]$ denotes greatest integer function less than or equal to k]

A $[-3, 2]$

B $[-2, 3)$

C $[-3, 3]$

D $[-2, 3]$

Ans. B

If the range of $f(\theta) = \frac{\sin^4 \theta + 3 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$, $\theta \in \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to

The range of the function,

$$f(x) = \log_{\sqrt{5}} \left(3 + \cos \left(\frac{3x}{4} + x \right) + \cos \left(\frac{\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} - x \right) - \cos \left(\frac{3\pi}{4} - x \right) \right)$$
 is

A $(0, \sqrt{5})$

B $[-2, 2]$

C $\left[\frac{1}{\sqrt{5}}, \sqrt{5} \right]$

D $[0, 2]$

Ans. D

Let $f: (1, 3) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is

A $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{4}, \frac{4}{5}\right]$

B $\left(\frac{3}{5}, \frac{4}{5}\right)$

C $\left(\frac{2}{5}, \frac{4}{5}\right]$

D $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

Ans. A

Draw the graph of (1 to 5)

1. $f(x) = \min\{(1 - x, 1 + x, 2\}$

2. $f(x) = \min\{x, x^2, 1\}$

3. $f(x) = \max\{\sin x, \cos x\}, \forall x \in \mathbb{R}$

4. $f(x) = \min\{\tan x, \cot x\}$

5. $f(x) = \max\{1 + x, 1 - x, 2\}$

6. Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \dots + [\sqrt{120}]$ is equal to [JEE Mains 2023]



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 15



Recap *of previous lecture*

1. $f(x) = 4|\sin x| - 3|\cos x|$ has range = _____

2. If domain & range of $f(x)$ are $[-1, 3]$ and $[6, 12]$ then domain & range of $g(x) = \frac{1}{2} \cdot f(3x + 1)$ are _____ and _____ respectively.

3. Graph of $y = f(x) = \max(\tan x, \cot x)$

$$\text{Graph of } y = f(x) = \begin{cases} x + 2 & x < 0 \\ x^2 - 5x + 6 & 0 \leq x \leq 4 \\ 2x - 6 & x > 4 \end{cases}$$

Recap *of previous lecture*

4. Range of $f(x) = \log_3(5 + 4x - x^2)$ is _____

5. Range $f(x) = \frac{\{x\}}{\{x\}+2}$ is _____

6. Range of $f(x) = \sqrt{\frac{x-1}{2-x}}$ is _____

7. Classify in to Odd or Even functions.
 - (a) $f(x) = \log_e\left(\frac{1-x}{1+x}\right) + e^x - e^{-x}$
 - (b) $g(x) = \cos(\sin x)$
 - (c) $f(x) = \sin(\sin(\sin(x)))$

QUESTION

Find the range of the function

$$f(x) = x^4 + 4x^2 + \frac{1}{x^4 + 4x^2 + 9} + 10$$

Let $f: (1, 3) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is

A $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{4}, \frac{4}{5}\right]$

B $\left(\frac{3}{5}, \frac{4}{5}\right)$

C $\left(\frac{2}{5}, \frac{4}{5}\right]$

D $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

Ans. A

If the range of $f(\theta) = \frac{\sin^4 \theta + 3 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$, $\theta \in \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to

Check if following functions are one-one or many-one:

1. All even degree polynomials defined over R
2. All odd degree polynomials defined over R
3. All even functions
4. All odd functions
5. $f: \mathbb{R} \rightarrow \mathbb{R}$ $f(x) = x^2 \sin x$
6. $f: \mathbb{R} \rightarrow \mathbb{R}$ $f(x) = x + \cos x$
7. $f: \mathbb{R} \rightarrow \mathbb{R}$ $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$

QUESTION



Check if following functions are one-one or many-one:

$$1. \quad f(x) = 2x^3 - 9x^2 + 12x + 18$$

$$2. \quad f(x) = (x^2 + x + 5)(x^2 + x + 3)$$

Draw the graph of (1 to 5)

1. $f(x) = \min\{1 - x, 1 + x, 2\}$

2. $f(x) = \min\{x, x^2, 1\}$

3. $f(x) = \max\{\sin x, \cos x\}, \forall x \in \mathbb{R}$

4. $f(x) = \min\{\tan x, \cot x\}$

5. $f(x) = \max\{1 + x, 1 - x, 2\}$

6. Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \dots + [\sqrt{120}]$ is equal to [JEE Mains 2023]

If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1 - x}$, then what is the common domain of the following functions: $f + g, f - g, f/g, g/f, g - f$ where

$$(f \pm g)(x) = f(x) \pm g(x), (f/g)(x) = \frac{f(x)}{g(x)}.$$

- A** $0 \leq x < 1$
- B** $0 < x < 1$
- C** $0 < x \leq 1$
- D** $0 \leq x \leq 1$

If the domain of the function

$$f(x) = \log_e(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x + 6}{3}\right)$$

is $(\alpha, \beta]$, then $36|\alpha + \beta|$ is equal to

A 45

B 72

C 54

D 63

Ans. D

The value of $f(x) = 3 \sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lie in the interval _____

The domain of definition of $f(x) = \log_{(x^2-x+1)}(2x^2 - 7x + 9)$ is :

A R

B $R - \{0\}$

C $R - \{0, 1\}$

D $R - \{1\}$

Which of the following is not an odd function?

- A $\ln \left(\frac{x^4 + x^2 + 1}{(x^2 + x + 1)^2} \right)$
- B $\operatorname{sgn}(\operatorname{sgn}(x))$
- C $\sin(\tan x)$
- D $f(x)$, where $f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right) \forall x \in \mathbb{R} - \{0\}$ and $f(2) = 33$

1. Which of the following functions is odd?
(A) $\operatorname{sgn} x + x^{2000}$ (B) $|x| - \tan x$ (C) $x^3 \cot x$ (D) $\operatorname{cosec} x^{55}$ Ans. D
2. Range of the function $f(x) = \frac{\{x\}}{1+\{x\}}$ where $\{x\}$ denotes the fraction part function is
(A) $[0, 1)$ (B) $\left[0, \frac{1}{2}\right]$ (C) $\left[0, \frac{1}{2}\right)$ (D) $\left(0, \frac{1}{2}\right)$ Ans. C
3. The domain of the function $f(x) = \frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}}$ is
(A) $(1, 4)$ (B) $(-2, 4)$ (C) $(2, 4)$ (D) $[2, \infty)$ Ans. D
4. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is
(A) $(1, 2)$ (B) $(-1, 0) \cup (1, 2)$
(C) $(1, 2) \cup (2, \infty)$ (D) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ Ans. D

Bumper Practice Problems (BPP)



5. Range of $f(x) = \ln(3x^2 - 4x + 5)$ is
(A) $\left[\ln\frac{11}{3}, \infty\right)$ (B) $[\ln 10, \infty)$ (C) $\left[\ln\frac{11}{6}, \infty\right)$ (D) $\left[\ln\frac{11}{12}, \infty\right)$ Ans. A
6. Range of $f(x) = 4^x + 2^x + 1$ is
(A) $(0, \infty)$ (B) $(1, \infty)$ (C) $(2, \infty)$ (D) $(3, \infty)$ Ans. B
7. Range of function $f(x) = \frac{x-2}{x^2-4x+3}$
(A) $(-\infty, 0)$ (B) R (C) $(0, \infty)$ (D) $R - \{0\}$ Ans. B



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 16





Transformation Involving Negative Sign



$$1. \quad y = -f(x)$$

$$2. \quad y = f(-x)$$



Transformation Involving Addition/Subtraction of constant

1. $y = f(x) + a, a > 0$

2. $y = f(x) - a, a > 0$



Transformation Involving Addition/Subtraction of constant

3. $y = f(x + a)$, $a > 0$

4. $y = f(x - a)$, $a > 0$



Transformation Involving Multiplication of constant



1. $y = af(x), a > 0$

2. $y = f(ax), a > 0$



Transformation Involving Modulus



$$1. \quad y = |f(x)|$$

$$2. \quad y = f(|x|)$$



Transformation Involving Modulus

3. $|y| = f(x)$





Max - Min Function



1. $y = \max \{f(x), g(x)\}$

2. $y = \min \{f(x), g(x)\}$



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 17



Recap *of previous lecture*

1. A continuous function which is increasing or decreasing on its domain is _____
2. A function which is not one-one is _____
3. A function which is not onto _____
4. If the derivative of a function $dy/dx = f'(x) \geq 0$ on an interval where equality to 0 holds at some discrete points of the interval not forming a subinterval then the function is _____ on the interval.

Recap *of previous lecture*

5. If the derivative of a function $dy/dx = f'(x) \leq 0$ on an interval where equality to 0 holds at some discrete points of the interval not forming a subinterval then the function is _____ on the interval.

6. If for the function $f(x_1) = f(x_2)$ where $x_1 \neq x_2$ then the function is _____

7. Even & periodic functions are _____

8. Number of one-one functions from A to B + number of many one functions from A to B = _____

Recap *of previous lecture*

9. If every horizontal line intersect the graph of $y = f(x)$ in at most one point then the function is _____ whereas if it intersect the graph in two or more points, the function is _____
10. (a) Every function which is increasing or decreasing in its, entire domain is one-one. (T/F)
(b) Every function is either 1-1 or many-one. (T/F)
(c) Every function, which has a local, maximum or local minimum is many-one. (T/F)
(d) Every even degree polynomial is many-one. (T/F)
(e) Every odd degree polynomial is 1-1. (T/F)
(f) $f(x) = \text{Sgn}(x)$ is an odd function. (T/F)

QUESTION

Find the range of the function

$$f(x) = x^4 + 4x^2 + \frac{1}{x^4 + 4x^2 + 9} + 10$$

QUESTION

Find the domain & range of the following functions:

$$(1) \quad f(x) = \frac{x}{1+|x|}$$

$$(2) \quad f(x) = \frac{\sqrt{x+4}-3}{x-5}$$

QUESTION

Classify the following functions $f : \mathbb{R} \rightarrow \mathbb{R}$

(a) $f(x) = e^x + e^{-x}$

(b) $f(x) = \sqrt{1 + x^2}$

(c) $f(x) = x^3$

(d) $f(x) = |x| \operatorname{Sgn} x$

(e) $f(x) = x^3 - 2x^2 + 5x + 13$

(f) $f(x) = 2x^3 - 6x^2 - 18x + 17$

(g) $f(x) = \frac{x^2+4x+30}{x^2-8x+18}$

The function $f : N - \{1\} \rightarrow N$; defined by $f(n) =$ the highest prime factor of n , is

- A one-one only
- B neither one-one nor onto
- C onto only
- D both one-one and onto

Ans. B

QUESTION [JEE Mains 2024 (6 April)]



The function $f(x) = \frac{x^2+2x-15}{x^2-4x+9}$, $x \in \mathbb{R}$ is

- A** both one-one and onto.
- B** onto but not one-one.
- C** neither one-one nor onto.
- D** one-one but not onto.

Ans. C

QUESTION [JEE Mains 2025 (24 Jan)]



The function $f: (-\infty, \infty) \rightarrow (-\infty, 1)$, defined by $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$ is:

- A** One-one but not onto
- B** Onto but not one-one
- C** Both one-one and onto
- D** Neither one-one nor onto

QUESTION [JEE Mains 2023 (29 Jan)]



Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then

- A** $f(x)$ is many-one in $(-\infty, -1)$
- B** $f(x)$ is one-one in $(-\infty, \infty)$
- C** $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$
- D** $f(x)$ is many-one in $(1, \infty)$

Ans. C



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 18



Recap *of previous lecture*

1. A function $f: A \rightarrow B$ is onto if _____
2. A function which is not onto is _____
3. $ax^2 + bx + c > 0 \forall x \in R$ if _____
4. $ax^2 + bx + c < 0 \forall x \in R$ if _____
5. $ax^2 + bx + c \geq 0 \forall x \in R$ if _____
6. $ax^2 + bx + c \leq 0 \forall x \in R$ if _____
7. Sum, difference and product of two continuous functions is also _____

Recap *of previous lecture*

8. If f & g are continuous functions then $\frac{f}{g}$ is also continuous provided _____
9. $F(x) = \frac{x^2 - 5x + 7}{x^2 - 6x + 15}$ is _____ & _____
10. $f(x) = \frac{x^2 - 3x + 2}{x^2 - 5x + 4}$ is _____ & _____
11. $f(x) = \frac{x^2 - 6x + 8}{x^2 - 5x + 6}$ is _____ & _____

Recap *of previous lecture*

12. If f, g are continuous function on \mathbb{R} and

$$\min(f(x)) = a \quad \text{at } x = p$$

$$\min(g(x)) = b \quad \text{at } x = p$$

$$\max(f(x)) = c \quad \text{at } x = q$$

$$\max(g(x)) = d \quad \text{at } x = q$$

then $f(x) + g(x)|_{\min} = \underline{\hspace{10cm}}$

$$f(x) + g(x)|_{\max} = \underline{\hspace{10cm}}$$

$$\text{Range of } f(x) + g(x) = \underline{\hspace{10cm}}$$

13. $f(x) = 4^x + 2^x + 1$ is &

QUESTION

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A$ to B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto?

- A** one-one but not onto
- B** both one-one and onto
- C** onto but not one-one
- D** neither one-one nor onto

Ans. B

Let $A = \{x \in \mathbb{R} : x \text{ is not a positive integer}\}$. Define a function $f : A \rightarrow \mathbb{R}$ as $f(x) = \frac{2x}{x-1}$, then f is :

- A** neither injective nor surjective
- B** not injective
- C** injective but not surjective
- D** surjective but not injective

QUESTION [JEE Mains 2025 (28 Jan)]

Let $f: [0, 3] \rightarrow A$ be defined by $f(x) = 2x^3 - 15x^2 + 36x + 7$ and $g: [0, \infty) \rightarrow B$ be defined by $g(x) = \frac{x^{2025}}{x^{2025} + 1}$. If both the functions are onto and $S = \{x \in \mathbb{Z}; x \in A \text{ or } x \in B\}$, then $n(S)$ is equal to:

- A** 29
- B** 31
- C** 30
- D** 36

Ans. C

QUESTION [IIT-JEE 2012]



The function $f : [0, 3] \rightarrow [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

- A one-one and onto.
- B onto but not one-one.
- C one-one but not onto.
- D neither one-one nor onto.

Ans. B

QUESTION



The function $f : [2, \infty) \rightarrow Y$ defined by $f(x) = x^2 - 4x + 5$ is both one-one and onto if:

A $Y = \mathbb{R}$

B $Y = [1, \infty)$

C $Y = [4, \infty)$

D $[5, \infty)$

QUESTION [JEE Mains 2022 (28 June)]



Let a function $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \dots \\ n - 1, & n = 3, 7, 11, 15, \dots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \dots \end{cases}$ then, f is

- A** one-one but not onto
- B** onto but not one-one
- C** neither one-one nor onto
- D** one-one and onto

QUESTION

If numbers of ordered pairs (p, q) from the set $S = \{1, 2, 3, 4, 5\}$ such that the function $f(x) = \frac{x^3}{3} + \frac{p}{2}x^2 + qx + 10$ defined from \mathbb{R} to \mathbb{R} is injective, is n then n is divisible by

A 3

B 5

C 7

D 11

QUESTION

If functions $f(x)$ and $g(x)$ are defined on $\mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x) = \begin{cases} x + 3, & x \in \text{ rational} \\ 4x, & x \in \text{ irrational} \end{cases}, g(x) = \begin{cases} x + \sqrt{5}, & x \in \text{ irrational} \\ -x, & x \in \text{ rational} \end{cases}, \text{ then } (f - g)(x) \text{ is}$$

- A** one-one and onto
- B** neither one-one nor onto
- C** one-one but not onto
- D** onto but not one-one

QUESTION



15 different books to be divided in to

- (a) 3 groups of sizes 5, 7, 3
- (b) 4 groups of sizes 3, 5, 2, 7
- (c) 4 groups of sizes 3, 4, 6, 2
- (d) 4 groups of sizes 5, 5, 2, 3
- (e) 3 groups of sizes 5, 5, 5

QUESTION



12 different books to be distributed among

- (a) 3 people getting 2, 6, 4 books
- (b) 3 people each getting 4 books
- (c) 4 people getting 4, 4, 2, 2 books

QUESTION



A function f is called strictly increasing if $a > b \Rightarrow f(a) > f(b)$, for all a, b . Find the number of strictly increasing functions $f: \{1, 2, 3\} \rightarrow \{1, 2, \dots, 5\}$.

QUESTION [JEE Mains 2022 (25 July)]



The number of bijective functions $f: \{1, 3, 5, 7, \dots, 99\} \rightarrow \{2, 4, 6, 8, \dots, 100\}$, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq \dots \geq f(99)$, is

- A ${}^{50}P_{17}$
- B ${}^{50}P_{33}$
- C $33! \times 17!$
- D $\frac{50!}{2}$

Ans. B

QUESTION [JEE Mains 2024 (9 April)]

Let $A = \{(x, y) : 2x + 3y = 23, x, y \in \mathbb{N}\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____

QUESTION [JEE Mains 2024 (5 April)]



Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \rightarrow B$, such that $f(1) + f(3) = 14$, is:

- A** 480
- B** 240
- C** 120
- D** 180

Ans. B

If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b) , then $(1+a)^2 + b^2$ is equal to :

A 29

B 30

C 25

D 26

Ans. D

If the domain of the function $f(x) = \log_7(1 - \log_4(x^2 - 9x + 18))$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $\alpha + \beta + \gamma + \delta$ is equal to

A 17

B 15

C 16

D 18

Ans. D

$f: \mathbb{R} \rightarrow [0, 3]$

$f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$ then f is

- A** one-one & onto
- B** many-one & into
- C** one-one & into
- D** many-one & onto

For real x , let $f(x) = x^3 + 5x + 1$, then

- A** f is one-one but not onto \mathbb{R}
- B** f is onto \mathbb{R} but not one-one
- C** f is one-one and onto \mathbb{R}
- D** f is neither one-one nor onto \mathbb{R}

Ans. C

The function $f: \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is

- A invertible
- B injective but not surjective
- C surjective but not injective
- D neither injective nor surjective

The function $f : N \rightarrow N$ defined by $f(x) = x - 5 \left[\frac{x}{5} \right]$, Where N is the set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x , is:

- A one-one and onto
- B one-one but not onto
- C onto but not one-one
- D neither one-one nor onto

Ans. D



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 19



Recap *of previous lecture*

1. A continuous function which is increasing or decreasing in whole domain is _____

2. A function which is continuous on \mathbb{R} and one-one then it is _____ or _____ on \mathbb{R} .

3. If function which is defined at discrete points is one-one then it should be increasing or decreasing. (T/F)

4. If a function defined at discrete points is increasing or decreasing then it is one-one. (T/F)

5. A function is onto if _____

Recap *of previous lecture*

6. $y = ax^2 + bx + c > 0 \forall x \in \mathbb{R}$ if _____

7. $y = ax^2 + bx + c < 0 \forall x \in \mathbb{R}$ if _____

8. $f : A \rightarrow B$ if $n(A) = m, n(B) = n$ then

(a) Number of functions from A to B _____

(b) Number of one-one function from A to B if $m > n$ is _____

(c) Number of one-one functions from A to B if $m < n$ _____ while number of onto functions = _____

QUESTION [JEE Mains 2024 (5 April)]



Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \rightarrow B$, such that $f(1) + f(3) = 14$, is:

- A** 480
- B** 240
- C** 120
- D** 180

Ans. B

The function $f: \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is

- A invertible
- B injective but not surjective
- C surjective but not injective
- D neither injective nor surjective

The function $f : N \rightarrow N$ defined by $f(x) = x - 5 \left[\frac{x}{5} \right]$, Where N is the set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x , is:

- A one-one and onto
- B one-one but not onto
- C onto but not one-one
- D neither one-one nor onto

Ans. D

QUESTION [JEE Mains 2020 (5 Sep)]

Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f: A \rightarrow B | 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is

QUESTION [JEE Mains 2025 (22 Jan)]



Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16\}$. Then the number of many-one functions $f : A \rightarrow B$ such that $1 \in f(A)$ is equal to:

A 151

B 139

C 163

D 127

Ans. A

QUESTION [JEE Mains 2022 (July)]



If $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, \dots, 6\}$. Find the total number of functions possible such that $f(1) + f(2) = f(3)$.

Ans. 90

QUESTION



Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = (a^2 - 1)(a^2 - 4)x^3 + (a^2 - 1)(a + 2)x^2 + (a + 1)(a + 2)x + a + 5.$$

If $f(x)$ is into then number of possible values of 'a' are

- A** 1
- B** 2
- C** 3
- D** more than 3

QUESTION



Draw following graphs:

(i) $y = x^2 - 7x + 12$

(ii) $y = |x^2 - 7x + 12|$

(iii) $y = x^2 - 7|x| + 12$

(iv) $y = |x^2 - 7|x|+12|$

QUESTION

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = x^3 + 2x^2 + 7x + 5 + 3 \sin x - 4 \cos x$ be a function then $f(x)$ is

- A** one-one and onto
- B** one-one but not onto
- C** onto but not one-one
- D** neither one-one nor onto

QUESTION



Draw following graphs:

1. $y = -e^{-x}$
2. $y = ||x - 4| - 3|$
3. $y = e^{|x|}$
4. $y = e^{-|x|}$
5. $y = |e^{|x|} - 3|$
6. $y = |\ln|x||$
7. $y = \sin \pi x$
8. Find number of solutions of $|\ln|x|| = \sin \pi x$
9. $y = -\ln(-x)$
10. $y = |\sin|x||$

QUESTION

Let $g(x) = \begin{cases} x^2 - 2, & -\infty < x < 0 \\ x, & 0 \leq x < 2 \\ (x - 2)^2, & 2 \leq x < 4 \\ x - 4, & 4 \leq x < \infty \end{cases}$. If the equation $g(x) = k$ has four real and distinct roots, then find the sum of all possible integral values of k .

QUESTION



Find the range of k for which $\left| |x^2 - 6x + 8| - 12 \right| = k$ has

1. No solution
2. Exactly 2 real solutions
3. Exactly 4 real solutions
4. Exactly 6 real solutions

QUESTION [AIEEE 2004]

If $f : \mathbb{R} \rightarrow S$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$, is onto, then the interval of S is

A $[-1, 3]$

B $[-1, 1]$

C $[0, 1]$

D $[0, 3]$

Ans. A



Home Challenge - 08



Let f be a real valued function defined by $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$, range of f is $[a, b)$, then find the value of $(5a + 4b)$.

QUESTION

Let $p, q, r \in R$ such that $3q > p^2$. Then the function $g: R \rightarrow R$ given by $g(x) = x^3 + px^2 + qx + r$, is

- A** one-one and onto
- B** onto but not one-one
- C** one-one but not onto
- D** neither one-one nor onto

Ans. A

QUESTION



Let $f : [0, 2] \rightarrow [2, 5]$ be defined as $f(x) = 3x^2 - 6x + 5$, then $f(x)$ is

- A** injective but not surjective
- B** surjective but not injective
- C** injective as well as surjective
- D** neither injective nor surjective

Ans. B

QUESTION

Which one of the following function is **surjective** but **not injective**?

A $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 + x + 1$

B $f : [0, \infty) \rightarrow (0, 1]; f(x) = e^{-|x|}$

C $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 + 2x^2 - x + 1$

D $f : \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = \sqrt{1 + x^2}$

Ans. C

QUESTION

Range of $y = \frac{\sin^2 x + 4 \sin x + 6}{\sin^2 x + 4 \sin x + 5}$ is given by

A $\left[\frac{3}{2}, 2\right]$

B $\left[\frac{11}{10}, 2\right]$

C $\left[\frac{6}{5}, \frac{3}{2}\right]$

D $\left[\frac{11}{10}, \frac{3}{2}\right]$

Ans. D

QUESTION

Let $f : R - \{0\} \rightarrow [0, \infty)$ be a function defined by $f(x) = \frac{|x-1|}{x^2}$. Then

- A** $f(x)$ is injective in $(2, \infty)$
- B** $f(x)$ is one-one in $(0, 1)$
- C** $f(x)$ is surjective
- D** $f(x)$ is many-one in $(-\infty, 0)$

Ans. A, B, C

Find the domain and range of the following functions:

(i) $f(x) = 3 - 4 \sin x$

(ii) $f(x) = \frac{1}{2-5 \sin x}$

(iii) $f(x) = \frac{1}{\sqrt{\sqrt{3}-\tan x}}$

(iv) $f(x) = \sqrt{2 \sin x - 1}$

Answer:

(i) $D_f = \mathbb{R}, R_f = [-1, 7]$

(ii) $D_f = \mathbb{R} - \left\{2n\pi + \sin^{-1} \frac{2}{5}\right\}, R_f = \left(-\infty, -\frac{1}{3}\right] \cup \left[\frac{1}{7}, \infty\right)$

(iii) $D_f = \left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{3}\right), R_f = (0, \infty)$

(iv) $D_f = \left[2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right], R_f = [0, 1]$



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 20



Recap *of previous lecture*

1. A function ' f ' is periodic if there exist a _____ number T such that _____
2. If there is a repeating pattern in graph of a function than it is _____
3.

Functions	Period
(a) $\{x\}$	
(b) $ \sin x $	
(c) $ \cos x $	
(d) $\max(\tan x, \cot x)$	

Recap *of previous lecture*

4. If $f(x) = \begin{cases} x + 1 & x < 2 \\ x^2 + 5x + 7 & x \geq 1 \end{cases}$ and $g(x) = \begin{cases} x^2 + 2x + 7 & x < 1 \\ x + 3 & x \geq 2 \end{cases}$ then

$$f(x) + g(x) = \underline{\hspace{10cm}}$$

$$\underline{\hspace{10cm}}$$

$$\underline{\hspace{10cm}}$$

$$f(x) - g(x) = \underline{\hspace{10cm}}$$

$$\underline{\hspace{10cm}}$$

$$\underline{\hspace{10cm}}$$

$$\frac{f(x)}{g(x)} = \underline{\hspace{10cm}}$$

$$\underline{\hspace{10cm}}$$

$$\underline{\hspace{10cm}}$$

Recap *of previous lecture*

5. (a) If f, g are both one-one then $f(x) + g(x)$ is also one-one. (T/F)

- (b) If f, g are both one-one then $f(x) - g(x)$ is also one-one. (T/F)

- (c) If f, g are both one-one then $f(x) \cdot g(x)$ is also one-one. (T/F)

- (d) If a function is odd then values of function at $x = 0$ is 0. (T/F)

QUESTION [JEE Mains 2025 (22 Jan)]



Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16\}$. Then the number of many-one functions $f : A \rightarrow B$ such that $1 \in f(A)$ is equal to:

A 151

B 139

C 163

D 127

Ans. A

QUESTION

Let $f : R - \{0\} \rightarrow [0, \infty)$ be a function defined by $f(x) = \frac{|x-1|}{x^2}$. Then

- A** $f(x)$ is injective in $(2, \infty)$
- B** $f(x)$ is one-one in $(0, 1)$
- C** $f(x)$ is surjective
- D** $f(x)$ is many-one in $(-\infty, 0)$

Ans. A, B, C

QUESTION

Find the period of the following function.

- (a) $f(x) = \cos \frac{2x}{3} - \sin \frac{4x}{5}$ Ans. 15π
- (b) $f(x) = \cos(\sin x)$ Ans. π
- (c) $f(x) = \sin(\cos x)$ Ans. 2π
- (d) $f(x) = \sin^4 x + \cos^4 x$ Ans. $\left(\frac{\pi}{2}\right)$
- (e) $f(x) = x - [x] = \{x\}$ Ans. 1
- (f) $f(x) = 2 \cos\left(\frac{x-\pi}{5}\right)$ Ans. $p = 10\pi$
- (g) (i) $\tan\left(\frac{\pi}{2}[x]\right)$ (ii) $\tan\left(\frac{\pi}{4}[x]\right);$
(iii) $\sin\left(\frac{\pi}{2}[x]\right)$ (iv) $\sin\left(\frac{\pi}{4}[x]\right)$

If $f(x)$ is defined on $(0, 1)$, then the domain of definition of $f(e^x) + f(\ln|x|)$ is

A $(-e, -1)$

B $(-e, -1) \cup (1, e)$

C $(-\infty, -1) \cup (1, \infty)$

D $(-e, e)$

Ans. A

If the function $f : \mathbb{R} - \{1, -1\} \rightarrow A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then **A** is equal to

- A** $\mathbb{R} - (-1, 0)$
- B** $\mathbb{R} - \{-1\}$
- C** $\mathbb{R} - [-1, 0)$
- D** $[0, \infty)$

Ans. C

The function $f : [0, \infty) \rightarrow [2, \infty)$ defined by $f(x) = x^4 + 2x^3 + 3x^2 + 2x + 2$ is

- A** one-one but not onto
- B** onto but not one-one
- C** both one-one and onto
- D** neither one-one nor onto

Ans. C

If $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = (p^2 - 9)(p^2 - 4)x^5 + (p^2 - 9)(p - 2)x^3 + (p^2 - 4)(p + 3)x^2 - (p + 7)(p - 6)$$

is into, then number of possible integral value(s) of p is

A 4

B 3

C 2

D 1

Ans. B

Let $f(x)$ be an odd function defined on $(-\infty, \infty)$ and $g(x)$ be an even function defined on $(-\infty, \infty)$. If $f(x) - g(x) = x^2 + 5x + 7$, then $g(2)$ is equal to

A -13

B 9

C 7

D -11

Ans. D

Bumper Practice Problems



1. $f(x) = x + \frac{1}{x}$, range of $f(x)$ when
 - (a) $x \in (0, 2)$ is _____
 - (b) $x \in (1, 5)$ is _____
 - (c) $x \in (-1, 1)$ is _____
 - (d) $x \in (-2, 3)$ is _____
2. Find the range of the function $f(x) = x^4 + 4x^2 + \frac{1}{x^4+4x^2+9} + 10$.
3. Range of $f(x) = e^x + e^{-x}$ is _____
4. Range of $f(x) = \sec x + \cos x$ is _____
5. Range of $f(x) = \tan^2 x + \cot^2 x$ is _____



LAKSHYA

JEE 2026 BATCH



Mathematics

Relations and Functions

Lecture - 21

By – Ashish Agarwal Sir
(IIT Kanpur)

Recap *of previous lecture*

1. If $f(x), g(x)$ both are periodic then $f(x) + g(x)$ is also periodic. (T/F)
2. If $f(x), g(x)$ both are aperiodic then $f(x) + g(x)$ can be periodic. (T/F)
3. If $f(x), g(x)$ both have period T then $f(x) + g(x)$ also has period T. (T/F)
4. If period of f is T_1 while that of g is T_2 the period of $f \pm g, f \cdot g, \frac{f}{g}$ is _____
provided there does not exist a _____ number for which value _____
5. Discontinuities of a periodic function also repeat _____

Recap *of previous lecture*

6. If f is periodic with period 4, $f(2) = 3$, $f(1) = 5$, then the value of

$$f(6) = \underline{\hspace{2cm}}$$

$$f(5) = \underline{\hspace{2cm}}$$

$$f(-2) = \underline{\hspace{2cm}}$$

$$f(-3) = \underline{\hspace{2cm}}$$

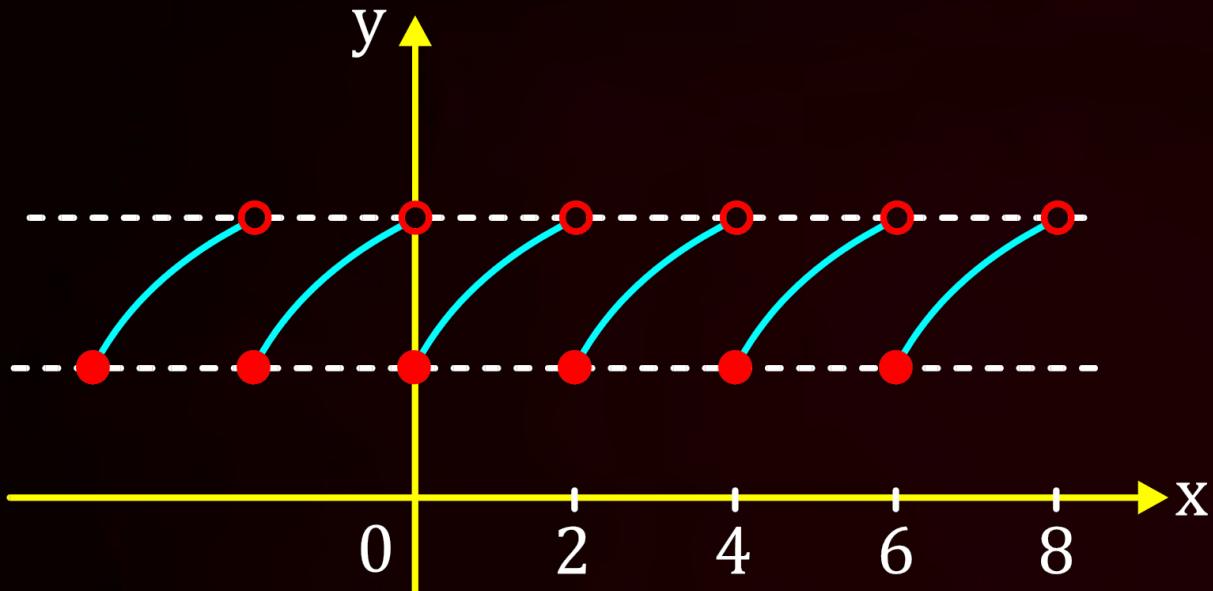
$$f(10) = \underline{\hspace{2cm}}$$

$$f(9) = \underline{\hspace{2cm}}$$

Recap

of previous lecture

7.



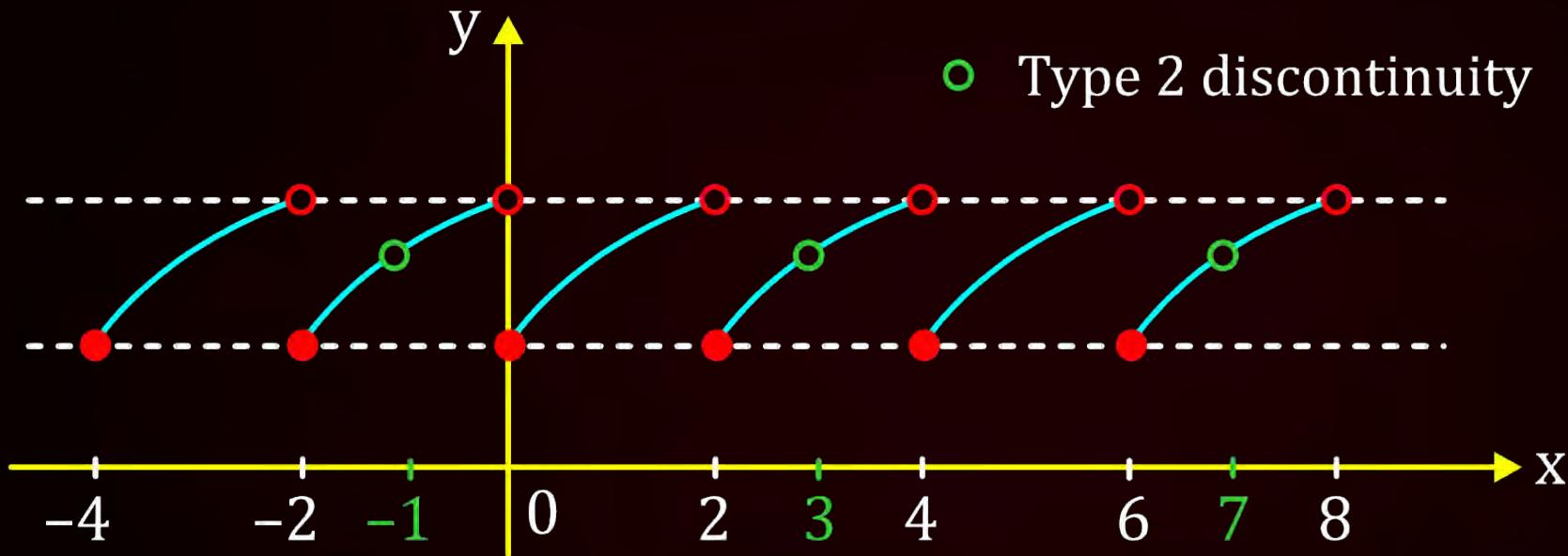
Period of discontinuities = _____

Period of above function = _____

Recap

of previous lecture

8.



Period of discontinuity of type 1 = _____

Period of discontinuity of type 2 = _____

Period of above function = _____

The function $f : [0, \infty) \rightarrow [2, \infty)$ defined by $f(x) = x^4 + 2x^3 + 3x^2 + 2x + 2$ is

- A** one-one but not onto
- B** onto but not one-one
- C** both one-one and onto
- D** neither one-one nor onto

Ans. C

The function $f: \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is

- A invertible
- B injective but not surjective
- C surjective but not injective
- D neither injective nor surjective

QUESTION

Find the range of k for which $\left| |x^2 - 6x + 8| - 12 \right| = k$ has

1. No solution
2. Exactly 2 real solutions
3. Exactly 4 real solutions
4. Exactly 6 real solutions

QUESTION



The period of the function $\cos(\sin x) + \cos(\cos x)$ is

A $\frac{\pi}{3}$

B $\frac{\pi}{2}$

C π

D 2π

QUESTION



The period of the function $\frac{|\sin x + \cos x|}{|\sin x| + |\cos x|}$

A $\frac{\pi}{3}$

B $\frac{\pi}{2}$

C π

D 2π

QUESTION



Period of $f(x) = \{x\} + \left\{x + \frac{1}{3}\right\} + \left\{x + \frac{2}{3}\right\}$ is equal to
(where $\{\cdot\}$ denotes fractional part function)

- A** 1
- B** $2/3$
- C** $1/2$
- D** $1/3$

QUESTION



The period of the function $\frac{\sin x + \sin 5x}{\cos x + \cos 5x}$ is

A $\frac{\pi}{3}$

B $\frac{\pi}{2}$

C π

D 2π

QUESTION



The period of the function $\frac{\sin x + \sin 17x}{\cos x + \cos 17x}$

A $\frac{\pi}{9}$

B $\frac{\pi}{8}$

C π

D 2π

QUESTION



The period of the function $\frac{\sin 12x + \sin 6x}{\cos 12x + \cos 6x}$

A $\frac{\pi}{9}$

B $\frac{\pi}{3}$

C π

D 2π

QUESTION

Let $f(x) = \sin^2 x + \cos^4 x + 2$ and $g(x) = \cos(\cos x) + \cos(\sin x)$. Also let period of $f(x)$ and $g(x)$ be T_1 and T_2 respectively then

A $T_1 = 2T_2$

B $2T_1 = T_2$

C $T_1 = T_2$

D $T_1 = 4T_2$

QUESTION



Consider those functions f that satisfy $f(x + 4) + f(x - 4) = f(x)$ for all real x .
Prove that all such functions are periodic.

QUESTION



Let $f : \mathbb{R} \rightarrow \mathbb{R} - \{3\}$ be a function with the property that there exist $T > 0$ such that $f(x + T) = \frac{f(x)-5}{f(x)-3}$ for every $x \in \mathbb{R}$. Prove that $f(x)$ is periodic.

Paragraph

If the function $f(x)$ satisfies the equation $f(x + 1) + f(x - 1) = \sqrt{3}f(x)$ for all $x \in \mathbb{R}$,

Period of $f(x)$ is

A 12

B 3

C 18

D 6

Paragraph

If the function $f(x)$ satisfies the equation $f(x + 1) + f(x - 1) = \sqrt{3}f(x)$ for all $x \in \mathbb{R}$,

If $f(2) = 9$, find the sum $\sum_{r=0}^9 f(2 + 12r)$.

A 90

B 91

C 99

D 81

QUESTION [JEE Mains 2023 (24 Jan)]



Let $f(x)$ be a function such that $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{N}$.

If $f(1) = 3$ and $\sum_{k=1}^n f(k) = 3279$, then the value of n is

- A** 9
- B** 7
- C** 6
- D** 8

Ans. B

QUESTION [JEE Mains 2025 (28 Jan)]

If $f(x) = \frac{2^x}{2^x + \sqrt{2}}$, $x \in \mathbb{R}$, then $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to

A 82

B $81\sqrt{2}$

C 41

D $\frac{81}{2}$

Ans. D

QUESTION [JEE Mains 2023 (24 Jan)]

If $f(x) = \frac{2^{2x}}{2^{2x}+2}$, $x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \dots + f\left(\frac{2022}{2023}\right)$ is equal to

A 2011

B 2010

C 1010

D 1011

Ans. D

QUESTION [JEE Mains 2020]



Suppose that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(1) = 3$.

$$f(1) = 3. \text{ If } \sum_{i=1}^n f(i) = 363, \text{ then } n \text{ is equal to } \underline{\hspace{2cm}}.$$

QUESTION [JEE Mains 2024 (9 April)]



If a function f satisfies $f(m + n) = f(m) + f(n)$ for all $m, n \in \mathbb{N}$ and $f(1) = 1$, then the largest natural number λ such that

$$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2 \text{ is equal to}$$

Ans. 1010

QUESTION [JEE Mains 2025 (3 April)]



Let f be a function such that $f(x) + 3f\left(\frac{2^4}{x}\right) = 4x, x \neq 0$. Then $f(3) + f(8)$ is equal to

A 13

B 11

C 10

D 12

Ans. B

QUESTION [JEE Mains 2021]



If $a + \alpha = 1$, $b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \neq 0$, then the value of expression

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$$
 is



Home Challenge - 09



Let $f(x) = (x + 2)e^{\ln(x+2)}$ and $g(x) = \frac{e^{\left(\frac{1}{-\log_x e}\right)}}{x} - \frac{2x}{e^{-\ln 2}}$.

If $h(x) = f(x) + g(x)$, then the smallest positive integer in the range of $h(x)$ is

A 2

B 3

C 6

D 7

Which of the following is not an odd function?

A $g(x) - g(-x)$

B $(g(x) - g(-x))^3$

C $\log\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right)$

D $xg(x) \cdot g(-x) + \tan(\sin x)$

Ans. C

Let $f(x)$ be an odd function defined on \mathbb{R} such that $f(1) = 2$, $f(3) = 5$ and $f(-5) = -1$.

The value of $\frac{f(f(f(-3)))+f(f(0))}{3f(1)-2f(3)-f(5)}$ is

A $\frac{-2}{5}$

B $\frac{-2}{3}$

C $\frac{2}{5}$

D $\frac{2}{3}$

Ans. C

If $f : \mathbb{N} \rightarrow \mathbb{N}$ where $f(x) = x - (-1)^x$, then f is

- A** one-one and into
- B** many-one and into
- C** one-one and onto
- D** many-one and onto

Ans. C

$f(x) = x^3 - 12x + 1$ is

- A** non-injective on \mathbb{R}
- B** injective on $(-\infty, -2]$ and on $[2, \infty)$
- C** injective on $[-2, 2]$
- D** non-injective on $[-2 - k, -2 + k]$ and on $[2 - k, 2 + k]$ for each $k > 0$

Ans. A, B, C, D

The function $f(x) = x^4 - 4x^3 + 4x^2 - 1$ is

- A** injective on \mathbb{R}
- B** non-injective on $(1 - k, 1 + k)$ for each $k > 0$
- C** non-injective on $[2, \infty)$
- D** non-injective on $(-\infty, 0]$

Ans. D



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 22



Recap

of previous lecture

1. If $f(x), g(x)$ are both periodic then $f(x) \pm g(x)$ is also periodic. (T/F)

	Function	Period		Function	Period
(i)	$\sin^3 x$	_____	(ii)	$\cos^2 x$	_____
(iii)	$\sin^4 x$	_____	(iv)	$ \cos x $	_____
(v)	$ \sin x $	_____	(vi)	$\left\{ \frac{2}{3}x + 5 \right\}$	_____
(vii)	$\sin(2x + 3)$	_____			
(viii)	$ \sin x + \cos x $	_____			
(ix)	$\{x\} + \left\{ x + \frac{1}{2} \right\} + \{x\} + \left\{ x + \frac{1}{4} \right\} + \left\{ x + \frac{1}{2} \right\} + \left\{ x + \frac{3}{4} \right\}$	_____			

Recap *of previous lecture*

3. If f and g are both periodic with period T_1 & T_2 respectively then period of $f(x) \pm g(x)$, $f(x)/g(x)$, $f(x) \cdot g(x)$ is $T = \text{_____}$ provided there exists no real numbers less than T for which value repeats.

4. If $g(x)$ has period T then period of $f(ax + b)$ is _____

Recap *of previous lecture*

5. If $0 \in D_f$ then _____ = $f(0)$ = _____

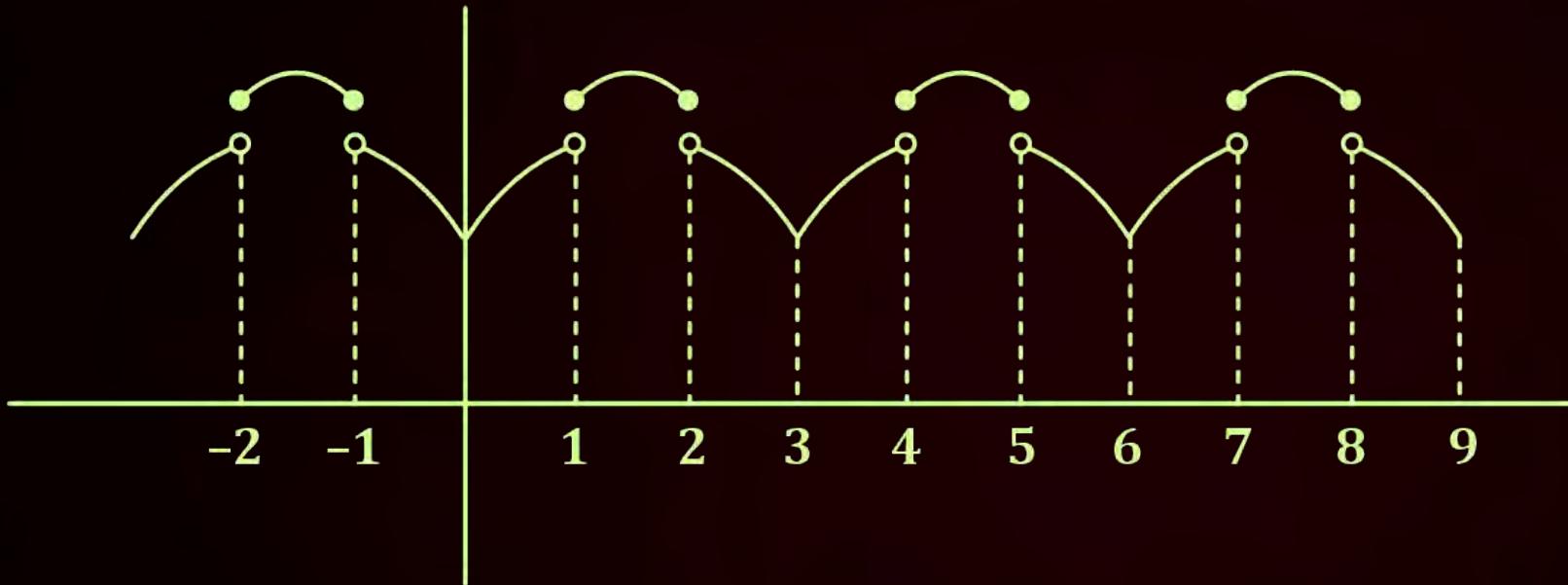
6. $f(x + nT) =$ _____

$f(x - nT) =$ _____ $n \in \mathbb{N}$, where T is period of function.

7. If f is periodic then f is _____

8. Discontinuities of a periodic function also repeat _____

Recap

of previous lecture

Period of function is _____

QUESTION

Let $f(x) = \sin^2 x + \cos^4 x + 2$ and $g(x) = \cos(\cos x) + \cos(\sin x)$. Also let period of $f(x)$ and $g(x)$ be T_1 and T_2 respectively then

A $T_1 = 2T_2$

B $2T_1 = T_2$

C $T_1 = T_2$

D $T_1 = 4T_2$



Home Challenge - 09



Let $f(x) = (x + 2)e^{\ln(x+2)}$ and $g(x) = \frac{e^{\left(\frac{1}{-\log_x e}\right)}}{x} - \frac{2x}{e^{-\ln 2}}$.

If $h(x) = f(x) + g(x)$, then the smallest positive integer in the range of $h(x)$ is

A 2

B 3

C 6

D 7

QUESTION [JEE Advanced 2024 (Paper 2)]

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, and $g : \mathbb{R} \rightarrow (0, \infty)$ be a function such that $g(x + y) = g(x)g(y)$ for all $x, y \in \mathbb{R}$.

If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$ is

QUESTION [JEE Mains 2025 (24 Jan)]

Let $f(x) = \frac{2^{x+2} + 16}{2^{2x+1} + 2^{x+4} + 32}$. Then the value of $8\left(f\left(\frac{1}{15}\right) + f\left(\frac{2}{15}\right) + \dots + f\left(\frac{59}{15}\right)\right)$ is equal to

- A** 108
- B** 92
- C** 118
- D** 102

Ans. C

QUESTION [AIEEE 2005]



A real valued function $f(x)$ satisfies the function equation

$f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where a is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to

A $f(1) + f(a - x)$

B $f(-x)$

C $-f(x)$

D $f(x)$

QUESTION



Let $f(x) = \sqrt{x}$; $g(x) = \sqrt{2 - x}$, find the domain of

A fog

B gof

C fof

D gog

QUESTION



If $f(x) = \left(\frac{2x}{4x-1}\right)$, then find the domain of $f(f(x))$.

- A** $R - \left\{\frac{1}{4}\right\}$
- B** $R - \left\{\frac{1}{4}, -\frac{1}{4}\right\}$
- C** $R - \left\{\frac{1}{4}, -\frac{1}{4}, \frac{1}{12}\right\}$
- D** None of these

Ans. B

QUESTION



If $f(x) = \left(\frac{2x}{4x-1}\right)$, then find the domain of $f(f(f(x)))$.

- A** $R - \left\{\frac{1}{4}\right\}$
- B** $R - \left\{\frac{1}{4}, -\frac{1}{4}\right\}$
- C** $R - \left\{\frac{1}{4}, -\frac{1}{4}, \frac{1}{12}\right\}$
- D** None of these

Ans. C

QUESTION [JEE Mains 2025 (23 Jan)]

Let $f(x) = \log_e x$ and $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$. Then the domain of $f \circ g$ is

A $(0, \infty)$

B $[1, \infty)$

C \mathbb{R}

D $[0, \infty)$

Ans. C

QUESTION [JEE Mains 2024 (27 Jan)]



Let $f : R - \left\{ \frac{-1}{2} \right\} \rightarrow R$ and $g : R - \left\{ \frac{-5}{2} \right\} \rightarrow R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then, the domain of the function fog is :

A $R - \left\{ -\frac{7}{4} \right\}$

B R

C $R - \left\{ -\frac{5}{2}, -\frac{7}{4} \right\}$

D $R - \left\{ -\frac{5}{2} \right\}$

Ans. D

QUESTION

Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$. Match the composite function given in Column-I with their respective domains given in Column-II.

Column-I		Column-II	
(A)	fog	(P)	$R - \{-2, -5/3\}$
(B)	gof	(Q)	$R - \{-1, 0\}$
(C)	fof	(R)	$R - \{0\}$
(D)	gog	(S)	$R - \{-2 - 1\}$

QUESTION



Let $f(x) = \frac{1}{1-x}$, $x \neq 0, 1$. Find $f^{2023}(2024)$, where $f^n = f \circ f \circ f \dots n \text{ times}$.

QUESTION [JEE Mains 2016 (9 April)]

For $x \in \mathbb{R}, x \neq 0$, Let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2, \dots$

Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to:

- A** $\frac{8}{3}$
- B** $\frac{5}{3}$
- C** $\frac{4}{3}$
- D** $\frac{1}{3}$

Ans. B

QUESTION [JEE Mains 2022 (26 June)]

Let $f(x) = \frac{x-1}{x+1}$, $x \in \mathbb{R} - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all $n \in \mathbb{N}$, then $f^6(6) + f^7(7)$ is equal to :

- A** $\frac{7}{6}$
- B** $-\frac{3}{2}$
- C** $\frac{7}{12}$
- D** $-\frac{11}{12}$

Ans. B

QUESTION [JEE Mains 2024 (4 April)]

Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of $f, (\underbrace{f \circ f \circ f \circ \dots \circ f}_{10 \text{ times}})(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha + 1}$ is equal to



Home Challenge - 10



Let $f(x) = x^2 + 3x + 2$, then number of solutions to

- A** $f(|x|) = 2$ is 1
- B** $f(|x|) = 2$ is 3
- C** $|f(x)| = 0.125$ is 4
- D** $|f(|x|)| = 0.125$ is 8

Ans. A, C

Let the function $f : [0, 1] \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{4^x}{4^x + 2}$.

Then the value of $f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \cdots + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right)$ is

The number of solutions of the equation $\left| |x^2 - 4x + 3| - 2 \right| = 1$, is

A 5

B 6

C 7

D 8

Ans. A



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

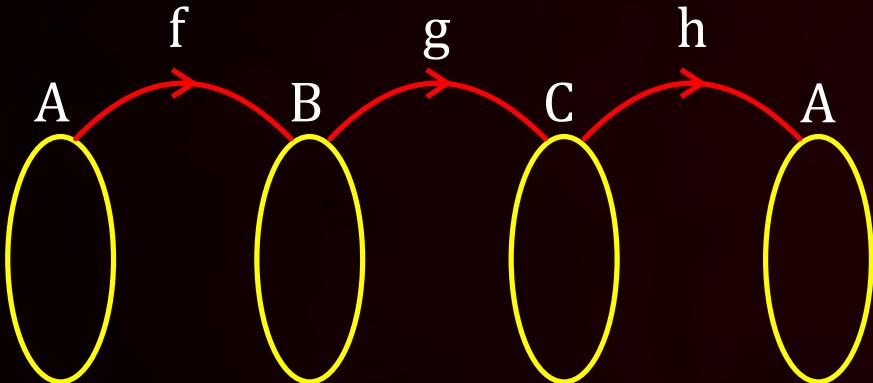
Lecture – 23



Recap *of previous lecture*

1. If $f : A \rightarrow B$ & $g : B \rightarrow C$ then $g \circ f : \underline{\hspace{2cm}}$, Domain of $g \circ f = \underline{\hspace{2cm}}$ &
Range of $g \circ f : \underline{\hspace{2cm}}$

2.



$g \circ f : \underline{\hspace{2cm}}$, $h \circ g : \underline{\hspace{2cm}}$, $h \circ g \circ f : \underline{\hspace{2cm}}$

3. $f \circ g = g \circ f$ always. (T/F)

Recap *of previous lecture*

4. If $f(x) = \frac{\sin 7x + \sin 3x}{\cos 7x + \cos 3x}$ then period of $f(x)$ is _____

5. If $f : A \rightarrow A$ is one-one then it is also onto. (T/F)

6. If $f : A \rightarrow A$ is onto then it is also one-one. (T/F)

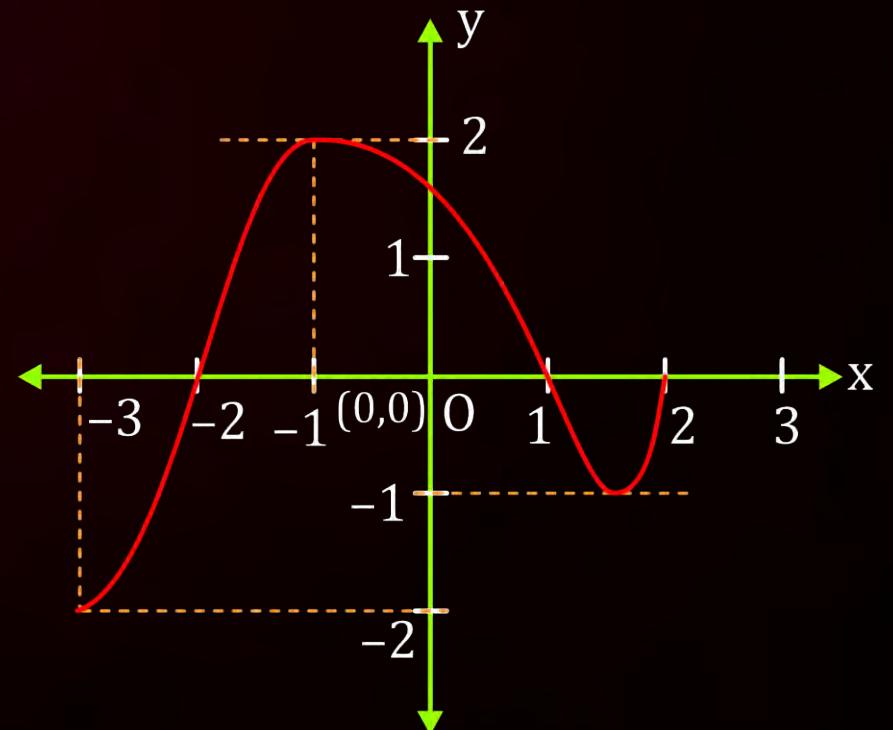
7. If $f : A \rightarrow A$ where A is a finite sets and f is one-one then f onto. (T/F)

8. If $f : A \rightarrow A$ where A is a finite sets and f is onto then f one-one. (T/F)

Recap

 of previous lecture

9. To draw $y = f(-x)$ we should _____ the graph of $y = f(x)$ about _____ axis.
10. To draw $y = -f(x)$ we should _____ the graph of $y = f(x)$ about _____ axis.
11. The figure illustrates the graph of the function $y = f(x)$ defined in $[-3, 2]$. Then draw the graph of $y = f(-|x|)$ and hence find its domain & range.



QUESTION [JEE Mains 2025 (23 Jan)]

Let $f(x) = \log_e x$ and $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$. Then the domain of $f \circ g$ is

A $(0, \infty)$

B $[1, \infty)$

C \mathbb{R}

D $[0, \infty)$

Ans. C

QUESTION [JEE Mains 2024 (4 April)]

Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of $f, (\underbrace{f \circ f \circ f \circ \dots \circ f}_{10 \text{ times}})(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha + 1}$ is equal to

QUESTION

If $g(x) = \left(4 \cos^4 x - 2 \cos 2x - \frac{1}{2} \cos 4x - x^7\right)^{\frac{1}{7}}$, then the value of $g(g(100))$ is equal to

A -1

B 0

C 1

D 100

QUESTION [JEE Mains 2022 (25 June)]



Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)(2 + x^{25})\right)^{\frac{1}{50}}$. If the function $g(x) = f(f(f(x))) + f(f(x))$, then the greatest integer less than or equal to $g(1)$ is

Ans. 2

QUESTION

If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then find $(g \circ f)(x)$.

QUESTION



Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions and $g \circ f : A \rightarrow C$. Which of the following statements is true?

- A** If $g \circ f$ is one-one then f and g both are one-one.
- B** If $g \circ f$ is one-one then f is one-one.
- C** If $g \circ f$ is a bijection then f is one-one and g is onto.
- D** If f and g are both one-one then $g \circ f$ is one-one.

QUESTION

If $f(x) = \log_{100x} \left(\frac{2 \log_{10} x + 2}{-x} \right)$ and $g(x) = \{x\}$. If the function $(f \circ g)(x)$ exists then find the largest possible range of $g(x)$.

$$\text{Ans. } \left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{10}\right)$$

QUESTION



If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$, find a function f such that $f \circ g = h$.

Ans. $f(x) = x^2 + 6$

QUESTION [JEE Mains 2020 (7 Jan)]

If $g(x) = x^2 + x - 1$ and $(g \circ f)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to:

- A** $\frac{1}{2}$
- B** $\frac{3}{2}$
- C** $-\frac{1}{2}$
- D** $-\frac{3}{2}$

Ans. C

QUESTION [JEE Mains 2022 (29 June)]



Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively.

If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is

QUESTION [JEE Mains 2022 (26 June)]



Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x - 1$ and $g : \mathbb{R} - \{1, -1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$.

Then the function $f \circ g$ is:

- A** one-one but not onto
- B** onto but not one-one
- C** both one-one and onto
- D** neither one-one nor onto

Ans. D

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 2x - 1$ and $g : \mathbb{R} - \{1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x-1}{x-2}$. Then the composition function $f(g(x))$ is :

- A** one-one but not onto
- B** onto but not one-one
- C** both one-one and onto
- D** neither one-one nor onto

Ans. A

QUESTION



$$f(x) = \begin{cases} 1 + x & \text{if } 0 \leq x \leq 2 \\ 3 - x & \text{if } 2 < x \leq 3 \end{cases}$$

Find $(f \circ f)$.

$$\text{Ans. } (f \circ f)(x) = \begin{cases} x + 2 & 0 \leq x \leq 1 \\ 2 - x & 1 < x \leq 2 \\ 4 - x & 2 < x \leq 3 \end{cases}$$

QUESTION [JEE Mains 2024 (29 Jan)]

If $f(x) = \begin{cases} 2 + 2x, & -1 \leq x < 0 \\ 1 - \frac{x}{3}, & 0 \leq x \leq 3 \end{cases}$; $g(x) = \begin{cases} -x, & -3 \leq x \leq 0 \\ x, & 0 < x \leq 1 \end{cases}$, then range of $(f \circ g)(x)$ is

A $[0, 1)$

B $[0, 3)$

C $(0, 1]$

D $[0, 1]$

Ans. D

QUESTION

$f(x) = \begin{cases} 1-x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$ and $g(x) = \begin{cases} -x & \text{if } x < 1 \\ 1-x & \text{if } x \geq 1 \end{cases}$ find $(f \circ g)(x)$ and $(g \circ f)(x)$.

$$\text{Ans. } (g \circ f)(x) = \begin{cases} x & \text{if } x \leq 0 \\ -x^2 & \text{if } 0 < x < 1; \\ 1-x^2 & \text{if } x \geq 1 \end{cases}$$

$$(f \circ g)(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1+x & \text{if } 0 \leq x < 1 \\ x & \text{if } x \geq 1 \end{cases}$$

QUESTION



Let $f: \mathbb{R} \rightarrow [1, \infty)$ be defined as $f(x) = \log_{10}(\sqrt{3x^2 - 4x + k + 1} + 10)$. If $f(x)$ is surjective, then

A $k = 1/3$

B $k < 1/3$

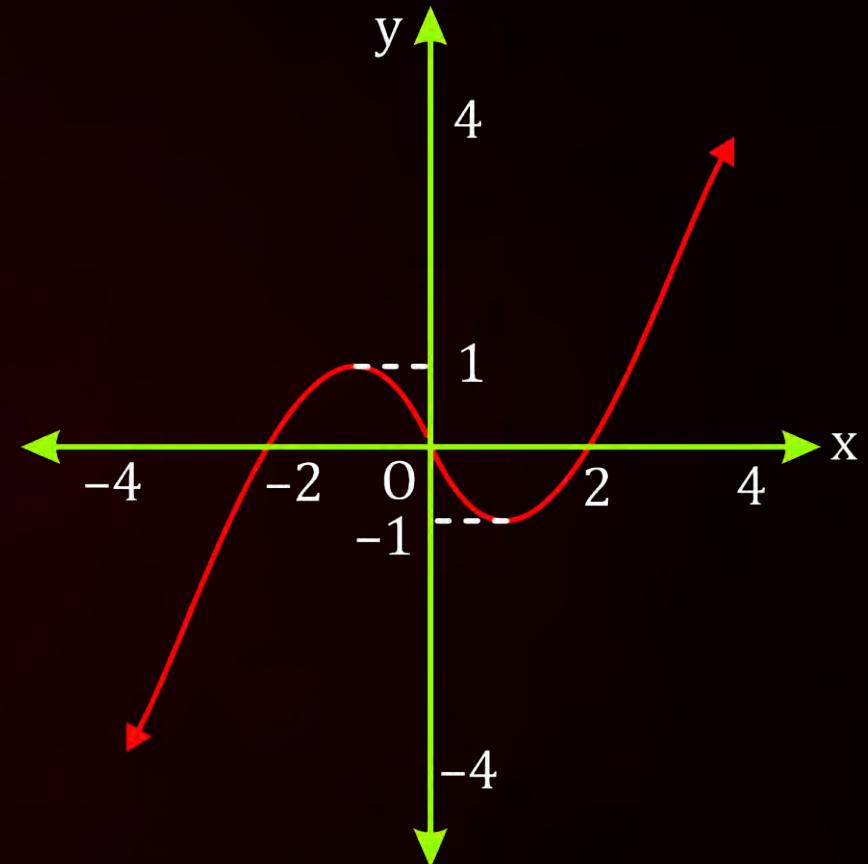
C $k > 1/3$

D $k = 1$

If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function such that $f(x) + f(x + 2) = \sqrt{2}f(x + 1)$, for all x . Show that f is periodic.

The graph of the function $y = g(x)$ is shown.
The number of solutions of the equation
 $|g(x)| - 1| = \frac{1}{2}$, is

- A 4
- B 5
- C 6
- D 8



Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = 2f(x)f(y)$ for natural numbers x and y . If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1) \text{ holds, is:}$$

A 2

B 3

C 4

D 6



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 24



Recap *of previous lecture*

1. If $f: A \rightarrow B$ & $g: B \rightarrow C$ then $gof : \underline{\hspace{2cm}}$ also if gof is one-one then $\underline{\hspace{2cm}}$ is $\underline{\hspace{2cm}}$ and if gof is onto then $\underline{\hspace{2cm}}$ is $\underline{\hspace{2cm}}$
2. If f is one-one then gof is one-one. (T/F)
3. If g is onto then gof is onto. (T/F)
4. If f, g are both one-one then gof is also one-one. (T/F)

Recap *of previous lecture*

5. If $f(2x + 1) + f(2x + 3) = 5$ the f is _____ & $T =$ _____
6. If $g(x) = \sqrt{1 + f(x)}$ where $f^3(x) = x^2 + 3x f(x) + 1$ where $x \in \mathbb{R}$ & $(f(1) \neq -1)$
then domain of $g(x) =$ _____
7. $f(x) = |\sin 2x| - |\cos 2x|$ has period _____

QUESTION



Let $f: \mathbb{R} \rightarrow [1, \infty)$ be defined as $f(x) = \log_{10}(\sqrt{3x^2 - 4x + k + 1} + 10)$. If $f(x)$ is surjective, then

A $k = 1/3$

B $k < 1/3$

C $k > 1/3$

D $k = 1$

Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = 2f(x)f(y)$ for natural numbers x and y . If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1) \text{ holds, is:}$$

A 2

B 3

C 4

D 6

QUESTION

$$f(x) = \begin{cases} 1-x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases} \text{ and } g(x) = \begin{cases} -x & \text{if } x < 1 \\ 1-x & \text{if } x \geq 1 \end{cases} \text{ find } (f \circ g)(x) \text{ and } (g \circ f)(x).$$

$$\text{Ans. } (g \circ f)(x) = \begin{cases} x & \text{if } x \leq 0 \\ -x^2 & \text{if } 0 < x < 1; \\ 1-x^2 & \text{if } x \geq 1 \end{cases}$$

$$(f \circ g)(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1+x & \text{if } 0 \leq x < 1 \\ x & \text{if } x \geq 1 \end{cases}$$

QUESTION



$$f(x) = \begin{cases} 1 - x & \text{if } 0 \leq x \leq 1 \\ x + 2 & \text{if } 1 < x < 2 \\ 4 - x & \text{if } 2 \leq x \leq 4 \end{cases}$$

Find (fof).

$$\text{Ans. (fof)(x)} = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } 1 < x < 2 \\ 6 - x & \text{if } 2 < x < 3 \\ x - 3 & \text{if } 3 \leq x \leq 4 \\ 2 & \text{if } x = 2 \end{cases}$$

QUESTION [JEE Advanced 2025 (Paper 1)]



Let \mathbb{N} denote the set of all natural numbers, and \mathbb{Z} denote the set of all integers. Consider the functions $f: \mathbb{N} \rightarrow \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{N}$ defined by

$$f(n) = \begin{cases} \frac{(n+1)}{2} & \text{if } n \text{ is odd,} \\ \frac{(4-n)}{2} & \text{if } n \text{ is even,} \end{cases} \quad \text{and } g(n) = \begin{cases} 3 + 2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$. Then which of the following statements is (are) TRUE?

- A** $g \circ f$ is NOT one-one and $g \circ f$ is NOT onto
- B** $f \circ g$ is NOT one-one but $f \circ g$ is onto
- C** g is one-one and g is onto
- D** f is NOT one-one but f is onto

Ans. A, D

QUESTION

Compute inverse of following functions.

(a) $f : \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = 10^{x+1}$

(b) $f : (-2, \infty) \rightarrow \mathbb{R}, f(x) = 1 + \ln(x + 2)$

(c) $f : [0, \infty) \rightarrow [1, \infty), f(x) = \frac{e^x + e^{-x}}{2}$

QUESTION



If $f : [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals

A

$$\frac{x + \sqrt{x^2 - 4}}{2}$$

B

$$\frac{x}{1 + x^2}$$

C

$$\frac{x - \sqrt{x^2 - 4}}{2}$$

D

$$1 + \sqrt{x^2 - 4}$$

QUESTION

Let f be a function defined as, $f : (0, e^{-\frac{3}{2}}] \rightarrow \left[\frac{-1}{4}, \infty\right)$, $f(x) = (\ln x)^2 + 3 \ln x + 2$ then $f^{-1}(x)$ equals

A $\log\left(\frac{-3+\sqrt{4x+1}}{2}\right)$

B $\log\left(\frac{-3-\sqrt{4x+1}}{2}\right)$

C $e^{\frac{-3+\sqrt{4x+1}}{2}}$

D $e^{\frac{-3-\sqrt{4x+1}}{2}}$

QUESTION [JEE Mains 2021 (17 March)]



The inverse of $y = 5^{\log x}$ is:

A $x = 5^{\log y}$

B $x = y^{\frac{1}{\log 5}}$

C $x = 5^{\frac{1}{\log y}}$

D $x = y^{\log 5}$

Ans. B

QUESTION [JEE Mains 2018 (15 April)]

Let $f: A \rightarrow B$ be a function defined as $f(x) = \frac{x-1}{x-2}$, where $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. Then f is :

- A** invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$
- B** invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$
- C** invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$
- D** not invertible

Ans. B

QUESTION [JEE Mains 2021]



Let $f: R - \{3\} \rightarrow R - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g: R \rightarrow R$ be given as $g(x) = 2x - 3$.

Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

- A** 7
- B** 2
- C** 5
- D** 3

QUESTION [JEE Mains 2020 (8 Jan)]



The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1, 1)$, is :

A $\frac{1}{4} \log_e \left(\frac{1-x}{1+x} \right)$

B $\frac{1}{4} (\log_8 e) \log_e \left(\frac{1-x}{1+x} \right)$

C $\frac{1}{4} (\log_8 e) \log_e \left(\frac{1+x}{1-x} \right)$

D $\frac{1}{4} \log_e \left(\frac{1+x}{1-x} \right)$

Ans. C

QUESTION



$$f(x) = \begin{cases} x & \text{if } x < 1 \\ x^2 & 1 \leq x \leq 4 \\ 8\sqrt{x} & x > 4 \end{cases}, \text{ find } f^{-1}(x).$$

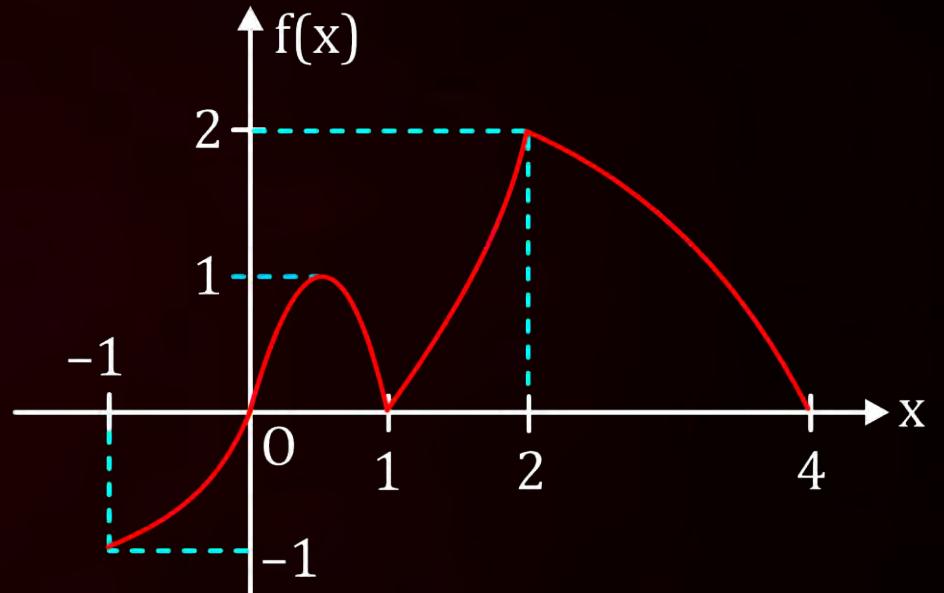
QUESTION

Let f be a real valued invertible function such that $f\left(\frac{2x-3}{x-2}\right) = 5x - 2, x \neq 2$. Find $f^{-1}(13)$.

QUESTION

If graph of a function $f(x)$ which is defined in $[-1, 4]$ is shown in the adjacent figure then identify the correct statement(s).

- A** domain of $f(|x| - 1)$ is $[-5, 5]$
- B** range of $f(|x| + 1)$ is $[0, 2]$
- C** range of $f(-|x|)$ is $[-1, 0]$
- D** domain of $f(|x|)$ is $[-3, 3]$



QUESTION

Draw the graph of the following functions:

$$f(x) = \begin{cases} 2x + 2 & \text{if } x < -1 \\ x^2 - 1 & \text{if } -1 \leq x \leq 1 \\ 2x - 3 & \text{if } x > 1 \end{cases}$$

QUESTION

Given $f(x) = \sqrt{\frac{8}{1-x} + \frac{8}{1+x}}$ and $g(x) = \frac{4}{f(\sin x)} + \frac{4}{f(\cos x)}$ then $g(x)$ is

- A** periodic with period $\pi/2$
- B** periodic with period π
- C** periodic with period 2π
- D** aperiodic

Ans. A

Bumper Practice Problems



1. Find the period of the following functions:

(i) $f(x) = \left| \cos^5 \left(\frac{2x}{3} \right) \right|$ Ans. $3\pi/2$

(ii) $f(x) = \sin(\pi x) + [x] + 2 - x$ Ans. 2

(iii) $f(x) = x - [x - b]$ Ans. 1

2. Find the fundamental period of the following functions:

(i) $f(x) = \sin 3x + \cos 2x + |\tan x|$ Ans. 2π

(ii) $f(x) = [\sin 3x] + |\cos 6x|$ Ans. $2\pi/3$

(iii) $f(x) = \frac{\sin 12x}{1+\cos^2 6x}$ Ans. $\pi/6$

(iv) $f(x) = \sec^3 x + \operatorname{cosec}^3 x$ Ans. 2π



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 25



Recap

of previous lecture

1. f is invertible if & only if f is a _____

2. A bijective function may have more than one inverse. (T/F)

3. Every periodic function is non-invertible.

4. Composite of two bijections is a bijection. (T/F)

5. Range of $g \circ f$ is always a _____

Recap *of previous lecture*

6. Only _____ functions are invertible.
7. If $f : A \rightarrow B$ is a bijection then $f^{-1} : \underline{\hspace{2cm}}$ is also a _____
8. If f is a linear function such that $f(1) = 2$ & $f(2) = 3$ then $f^{-1}(7) = \underline{\hspace{2cm}}$
9. Period of $f(x) = [\sin x]$ is _____
10. Domain of $f = \underline{\hspace{3cm}}$ of f^{-1} . Range of $f = \underline{\hspace{3cm}}$ of f^{-1}
11. Consider a function $f(x) = \frac{ax+b}{cx-a}$ then
domain of $f(x) = \underline{\hspace{3cm}}$, range of $f(x) = \underline{\hspace{3cm}}$ & $f_0 f(x) = \underline{\hspace{3cm}}$

QUESTION

Let f be a function defined as, $f : (0, e^{-\frac{3}{2}}] \rightarrow \left[\frac{-1}{4}, \infty\right)$, $f(x) = (\ln x)^2 + 3 \ln x + 2$ then $f^{-1}(x)$ equals

A $\log\left(\frac{-3+\sqrt{4x+1}}{2}\right)$

B $\log\left(\frac{-3-\sqrt{4x+1}}{2}\right)$

C $e^{\frac{-3+\sqrt{4x+1}}{2}}$

D $e^{\frac{-3-\sqrt{4x+1}}{2}}$

QUESTION [JEE Mains 2021 (17 March)]



The inverse of $y = 5^{\log x}$ is:

A $x = 5^{\log y}$

B $x = y^{\frac{1}{\log 5}}$

C $x = 5^{\frac{1}{\log y}}$

D $x = y^{\log 5}$

Ans. B

QUESTION [JEE Advanced 2025 (Paper 1)]



Let \mathbb{N} denote the set of all natural numbers, and \mathbb{Z} denote the set of all integers. Consider the functions $f: \mathbb{N} \rightarrow \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{N}$ defined by

$$f(n) = \begin{cases} \frac{(n+1)}{2} & \text{if } n \text{ is odd,} \\ \frac{(4-n)}{2} & \text{if } n \text{ is even,} \end{cases} \quad \text{and } g(n) = \begin{cases} 3 + 2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$. Then which of the following statements is (are) TRUE?

- A** $g \circ f$ is NOT one-one and $g \circ f$ is NOT onto
- B** $f \circ g$ is NOT one-one but $f \circ g$ is onto
- C** g is one-one and g is onto
- D** f is NOT one-one but f is onto

Ans. A, D

Consider function $f : A \rightarrow B$ and $g : B \rightarrow C$, ($A, B, C \subseteq \mathbb{R}$) such that $(gof)^{-1}$ exists, then:

- A** f and g both are one-one
- B** f and g both are onto
- C** f is one-one and g is onto
- D** f is onto and g is one-one

QUESTION

Let $f : R - \left\{ \frac{1}{5\sqrt{2}} \right\} \rightarrow R - \left\{ \frac{1}{5\sqrt{2}} \right\}$ be a function defined as $f(x) = \left(\frac{x-2\sqrt{5}}{5\sqrt{2}x-1} \right)$ then which of the following is(are) correct?

- A** $f\left(f\left(f\left(f(2017)\right)\right)\right) = 2017$
- B** $f(x) = f^{-1}(x)$ has more than 3 real roots
- C** $f(x)$ is a bijective function
- D** $f\left(f\left(f(x)\right)\right) = \left(f\left(f\left(f\left(f(x)\right)\right)\right)\right) \forall x \neq \frac{1}{5\sqrt{2}}$

QUESTION [JEE Mains 2024 (31 Jan)]



If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ and $(f \circ f)(x) = g(x)$, where $g : \mathbb{R} - \left\{\frac{2}{3}\right\} \rightarrow \mathbb{R} - \left\{\frac{2}{3}\right\}$, then $(gogog)(4)$ is equal to

- A** -4
- B** $\frac{19}{20}$
- C** $-\frac{19}{20}$
- D** 4

Ans. D

QUESTION [JEE Mains 2021]



Let $f : R - \left\{ \frac{\alpha}{6} \right\} \rightarrow R$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of α for which $(f \circ f)(x) = x$, for all $x \in R - \left\{ \frac{\alpha}{6} \right\}$, is:

A No such α exists

B 5

C 8

D 6

QUESTION

Let $f(x) = \frac{3}{2} + \sqrt{x - \frac{7}{4}}$ and $g(x)$ be the inverse function of $f(x)$ then the value of $(f^{-1}og^{-1})(17)$ equal to

- A** $\frac{3 + \sqrt{61}}{2}$
- B** 242
- C** 17
- D** $\frac{3 - \sqrt{61}}{2}$

QUESTION

If $f : R \rightarrow R, f(x) = x^3 + 3$, $g : R \rightarrow R, g(x) = 2x + 1$, then $f^{-1} \circ g^{-1}(23)$ equals

A 2

B 3

C $(23)^{1/3}$

D $(14)^{1/3}$

QUESTION

Let f be a one-one function with domain (x, y, z) and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false.

$f(x) = 1$; $f(y) \neq 1$; $f(z) \neq 2$. Determine $f^{-1}(1)$.

Let a function f defined from $R \rightarrow R$ as: $f(x) = \begin{cases} x + m & \text{for } x \leq 1 \\ 2mx - 1 & \text{for } x > 1 \end{cases}$. If the function is surjective on R then must lie in the interval

A $(0, 2]$

B $(-\infty, 0]$

C $(-\infty, 0)$

D $(0, \infty)$

Let a function f defined from $R \rightarrow R$ as $f(x) = \begin{cases} x + p^2, & \text{for } x \leq 2 \\ px + 5, & \text{for } x > 2 \end{cases}$

If the function is surjective, then find the sum of all possible integral values of p in $[-100, 100]$.

Determine the inverse function and its domain of definition if $f(x) = \begin{cases} x; & -\infty < x < 1 \\ x^2; & 1 \leq x \leq 4 \\ 2^x; & 4 < x < \infty \end{cases}$

$$\text{Ans. } f^{-1}(x) = \begin{cases} x; & -\infty < x < 1 \\ \sqrt{x}; & 1 \leq x \leq 16 \\ \log_2 x; & 16 < x < \infty \end{cases}$$

If $f : [1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f'(x)$ is given as

A $\frac{1}{2} [1 - \sqrt{1 + 4 \log_2 x}]$

B $\frac{1}{2} [1 + \sqrt{1 + 4 \log_2 x}]$

C $[1 - \sqrt{1 + 4 \log_2 x}]$

D $[1 - \sqrt{1 + 4 \log_2 x}]$

Ans. B

Let f be a real valued function such that $f(x) + 2f\left(\frac{2002}{x}\right) = 3x$ for all $x > 0$. The value of $f(2)$, is

A 1000

B 2000

C 3000

D 4000

Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in \mathbb{N}$, and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is

A 60

B 58

C 61

D 59

Suppose f is a function satisfying $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{N}$ and

$$f(1) = \frac{1}{5}. \text{ If } \sum_{n=1}^m \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}, \text{ then } m \text{ is equal to}$$

Bumper Practice Problems



1. If $f : (-\infty, 1] \rightarrow (-\infty, 1]$ such that $f(x) = x(2 - x)$, then $f^{-1}(x)$ is given as
(A) $1 - \sqrt{1 - x}$ (B) $1 + \sqrt{1 - x}$ (C) $-1 - \sqrt{1 - x}$ (D) $-1 + \sqrt{1 - x}$

Ans. A

2. If $f(x) = 3x - 5$ and let $g(x) = f^{-1}(x)$, then $g(4)$ equals
(A) 7
(B) 3
(C) $g(x)$ does not exist as $f(x)$ is not one-one
(D) $g(x)$ does not exist as $f(x)$ is not onto

Ans. B



LAKSHYA

JEE 2026 BATCH

Mathematics

Relations and Functions

By – Ashish Agarwal Sir
(IIT Kanpur)

Lecture – 26



Recap *of previous lecture*

1. If $f : N \rightarrow Z$ & $g : Z \rightarrow N$ then $f \circ g : \underline{\hspace{10cm}}$ & $g \circ f : \underline{\hspace{10cm}}$

2. If $f : A \rightarrow B$ is a bijection and g be its inverse then $g : \underline{\hspace{10cm}}$, $D_g = \underline{\hspace{10cm}}$,
 $R_g = \underline{\hspace{10cm}}$, $g \circ f : \underline{\hspace{10cm}} = \underline{\hspace{10cm}}$ and $f \circ g : \underline{\hspace{10cm}} = \underline{\hspace{10cm}}$

3. If $f : A \rightarrow A$ be self inverse that is $f^{-1} = \underline{\hspace{10cm}}$ then $(f \circ f)(x) = \underline{\hspace{10cm}}$,
 $\underbrace{(f \circ f \circ f \circ \dots \circ f)}_{2024 \text{ times}}(x) = \underline{\hspace{10cm}}$ and $\underbrace{(f \circ f \circ f \circ \dots \circ -f)}_{2025 \text{ times}}(x) = \underline{\hspace{10cm}}$

4. If $f : A \rightarrow B$ & $g : B \rightarrow C$ are both bijections then $g \circ f : \underline{\hspace{10cm}}$ is also a $\underline{\hspace{10cm}}$
and $(g \circ f)^{-1} = \underline{\hspace{10cm}}$

Recap *of previous lecture*

5. If $f : A \rightarrow B$ be a bijection and g be its inverse then

$$f \circ g(x) = \underline{\hspace{2cm}} \quad \forall x \in \underline{\hspace{2cm}} \text{ and } g \circ f(x) = \underline{\hspace{2cm}} \quad \forall x \in \underline{\hspace{2cm}}$$

6. If g is inverse of f then $f \circ g$ and $g \circ f$ are both identity functions. (T/F)

7. A constant function is always many-one. (T/F)

8. An identity function on a set is always bijective. (T/F)

9. If $f : A \rightarrow B$ be a bijection and $g : B \rightarrow A$, $g \circ f(x) = x \quad \forall x \in A$ then g will be inverse of f from B to A . (T/F)

Recap *of previous lecture*

10. If $f : A \rightarrow A$ & $(f \circ f)(x) = x$ then f is invertible & $f^{-1} = f$. (T/F)

11. If $f : A \rightarrow B$ be a bijection and $g : B \rightarrow A$ be such that
 $(f \circ g)(x) = x \forall x \in B$ then $g = f^{-1}$. (T/F)

Suppose f is a function satisfying $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{N}$ and

$$f(1) = \frac{1}{5}. \text{ If } \sum_{n=1}^m \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}, \text{ then } m \text{ is equal to}$$

Let a function f defined from $R \rightarrow R$ as: $f(x) = \begin{cases} x + m & \text{for } x \leq 1 \\ 2mx - 1 & \text{for } x > 1 \end{cases}$. If the function is surjective on R then must lie in the interval

A $(0, 2]$

B $(-\infty, 0]$

C $(-\infty, 0)$

D $(0, \infty)$

Let a function f defined from $R \rightarrow R$ as $f(x) = \begin{cases} x + p^2, & \text{for } x \leq 2 \\ px + 5, & \text{for } x > 2 \end{cases}$

If the function is surjective, then find the sum of all possible integral values of p in $[-100, 100]$.

QUESTION

$$f(x) = [x - 1] + \{x\}^{[x]}, x \in (1, 3), \text{ then } f^{-1}(x) \text{ is}$$

(where $[.]$ denotes greatest integer function and $\{.\}$ denotes fractional part function)

A
$$\begin{cases} x + 1 & x \in (1, 2) \\ 2 + \sqrt{x - 1} & x \in [2, 3) \end{cases}$$

B
$$\begin{cases} x - 1 & x \in (1, 2) \\ 2 - \sqrt{x - 1} & x \in [2, 3) \end{cases}$$

C
$$\begin{cases} x - 1 & x \in (0, 1) \\ 2 - \sqrt{x - 1} & x \in [1, 2) \end{cases}$$

D
$$\begin{cases} x + 1 & x \in (0, 1) \\ 2 + \sqrt{x - 1} & x \in [1, 2) \end{cases}$$

QUESTION



A function $f : \left[\frac{3}{2}, \infty\right) \rightarrow \left[\frac{7}{4}, \infty\right)$, defined as $f(x) = x^2 - 3x + 4$.

Solve the equation $f(x) = f^{-1}(x)$.

QUESTION



$f: [2, \infty) \rightarrow (-\infty, 4]$, where $f(x) = x(4 - x)$ then find $f^{-1}(x)$ and solve the equation, $f^{-1}(x) = f(x)$.

QUESTION

$f: \left[\frac{1}{2}, \infty\right) \rightarrow \left[\frac{3}{4}, \infty\right)$ is the function given by $f(x) = x^2 - x + 1$. Show that f is bijective, Find f^{-1} . Hence solve for real x , $x^2 - x + 1 = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$

QUESTION

Which of the following function is not homogeneous?

A $f(x, y) = \frac{x - y \cos x}{y \sin x + x}$

B $f(x, y) = \frac{x}{y} \ln \frac{y}{x} + \frac{y}{x} \ln \frac{x}{y}$

C $\frac{xy}{x^2 + y^2}$

D $\frac{2x - y + 1}{2y - x + 1}$

QUESTION

Which of the following function is implicit function?

A $y = \frac{x^2 + e^x + 5}{\sqrt{1 - \cos^{-1} x}}$

B $y = x^2$

C $xy - \sin(x + y) = 0$

D $y = \frac{x^2 \log x}{\sin x}$

QUESTION

Write explicitly, functions of y defined by the following equations and also find the domains of definition of the given implicit functions :

(a) $10^x + 10^y = 10$

(b) $x + |y| = 2y$

$$\text{Let } f(x) = \begin{cases} x & ; \quad x < 0 \\ 1-x & ; \quad x \geq 0 \end{cases} \text{ & } g(x) = \begin{cases} x^2 & ; \quad x < -1 \\ 2x+3 & ; \quad -1 \leq x \leq 1 \\ x & ; \quad x > 1 \end{cases}$$

On the basis of above information, answer the following questions:

1. Range of $f(x)$ is -
(A) $(-\infty, 1]$ (B) $(-\infty, \infty)$ (C) $(-\infty, 0]$ (D) $(-\infty, 2]$

[Ans. A]

2. Range of $g(f(x))$ is -
(A) $(-\infty, \infty)$ (B) $[1,3) \cup (3, \infty)$ (C) $[1, \infty)$ (D) $[0, \infty)$

[Ans. C]

Let f be a function such that $f(x) + 3f\left(\frac{2^4}{x}\right) = 4x, x \neq 0$. Then $f(3) + f(8)$ is equal to

A 13

B 11

C 10

D 12

Ans. B

Let $f, g: (1, \infty) \rightarrow \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $fog: [2, 4] \rightarrow \mathbb{R}$ is $[\alpha, \beta]$, then $\frac{1}{\beta-\alpha}$ is equal to

- A 56
- B 2
- C 29
- D 68

Ans. A

Let $f: \mathbb{R} - 0, 1 \rightarrow \mathbb{R}$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to

A $\frac{9}{4}$

B $\frac{7}{4}$

C $\frac{7}{3}$

D $\frac{9}{2}$

Ans. A

Consider a function $f: \mathbb{N} \rightarrow \mathbb{R}$, satisfying

$f(1) + 2f(2) + 3f(3) + \dots + x f(x) = x(x+1) f(x)$; $x \geq 2$ with $f(1) = 1$. Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to

- A** 8000
- B** 8400
- C** 8100
- D** 8200

Ans. C



THANK
You

