

Assignment 0

1.1) A.1

$$G_1(s) = \frac{10}{s+10}$$

1) Pole : $s = -10$

$$G_1(0) = \frac{10}{0+10} = 1$$

$$2) G_1(s) = \frac{1}{1 + s/10} = \frac{1}{1 + s/\omega_0} \quad (\omega_0 = 10)$$

Now,

$$G_1(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}} = \frac{1 - j\left(\frac{\omega}{\omega_0}\right)}{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

$$|G_1(j\omega)| = \left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]^{-1/2}$$

$$\arg(G_1(j\omega)) = \tan^{-1}\left(\frac{-\omega}{\omega_0}\right)$$

$$\therefore \text{gain} = 20 \log_{10} \left[1 + \frac{\omega^2}{\omega_0^2}\right]^{-1/2} = -10 \log_{10} \left(1 + \frac{\omega^2}{\omega_0^2}\right)$$

$$\phi = \text{phase} = \tan^{-1}\left(\frac{-\omega}{\omega_0}\right)$$

Now, CI: $\omega < \omega_0$

$$\text{gain} = 0$$

$$\phi = 0$$

CII: $\omega = \omega_0$

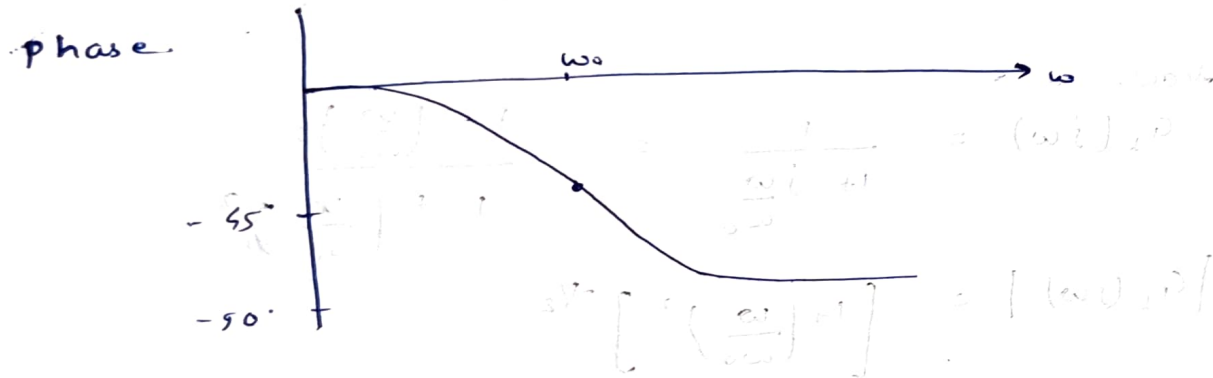
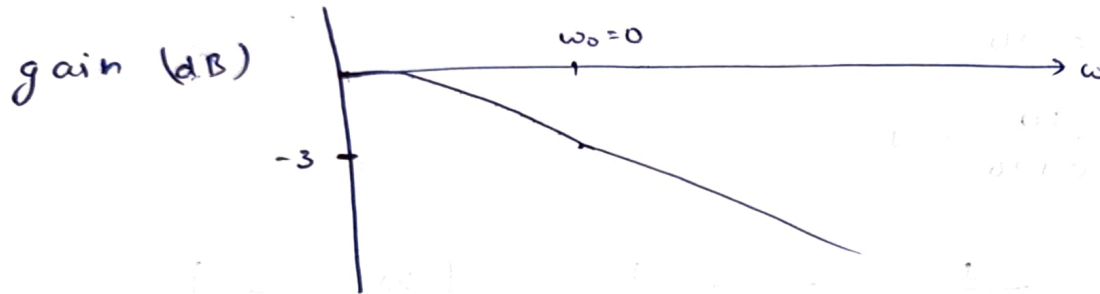
$$\text{gain} = -3$$

$$\phi = -45^\circ$$

C III : $\omega \gg \omega_0$.

gain $\Rightarrow -\infty$

$\phi = -90^\circ$



1.2. $G_2(s) = \frac{s-2}{s+10}$

HOME PLOTS APPS

New Script New Live Script New Open Import Data Save Workspace Clean Data

FILE VARIABLE

C > Users > Sarwesh Barnwal > Documents > MATLAB

Command Window

New to MATLAB? See resources for [Getting Started](#)

```
>> G1=tf([10][1,10])
G1=tf([10][1,10])
Error: Unexpected '['. Check for missing multiplication operator.
```

[Explain Error](#)

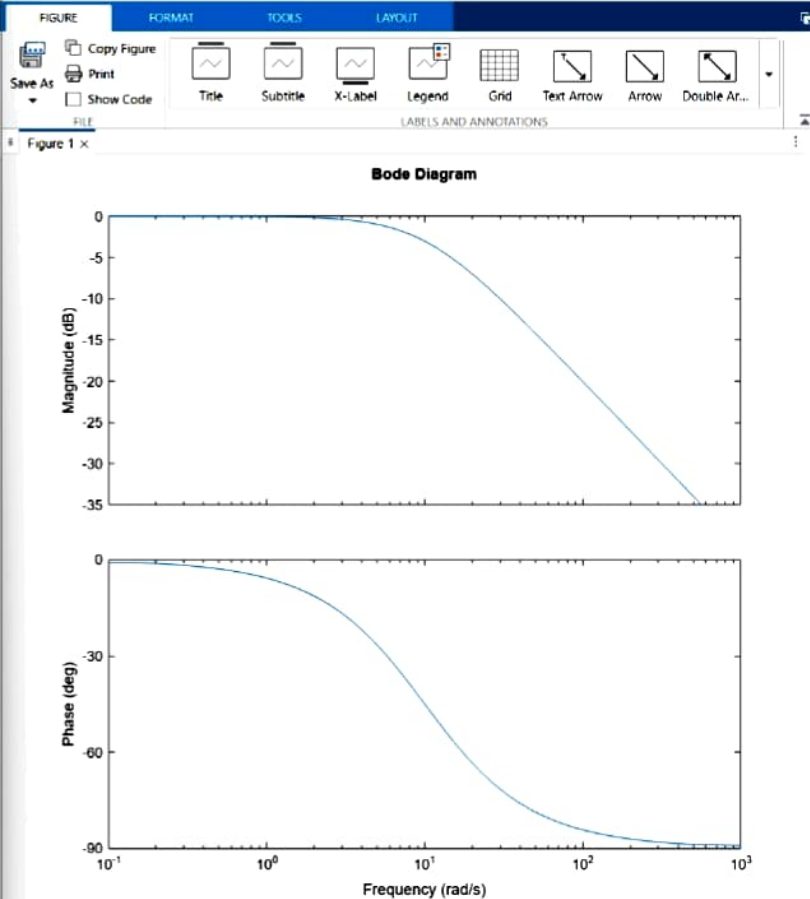
```
>> G1=tf([10],[1 10])

G1 =
    10
-----
    s + 10
```

Continuous-time transfer function.
[Bode Properties](#)

```
>> bode(G1)

>> |
```



$$1.2. \quad G_2(s) = \frac{s-2}{s+10}$$

1. zero: $s=2$

pole: $s=-10$

$$g_2(0) = -1/5$$

$$2. \quad G_2(j\omega) = \frac{j\omega-2}{j\omega+10} \times \frac{-j\omega+10}{-j\omega+10}$$

$$= \frac{(j\omega-2)(10-j\omega)}{\omega^2 - 10j\omega + 10j\omega + 100}$$

$$= \frac{10j\omega - 20 + \omega^2 + 2j\omega}{\omega^2 + 100} = \frac{\omega^2 + 12j\omega - 20}{\omega^2 + 100}$$

$$G_2(s) = \frac{s-2}{s+10} = \frac{-2 \left(1 - \frac{s}{2}\right)}{10 \left(1 + \frac{s}{10}\right)}$$

$$\text{gain} = \text{gain}(-2) + \text{gain}\left(1 - \frac{s}{2}\right) - \text{gain}(10) - \text{gain}\left(1 + \frac{s}{10}\right)$$

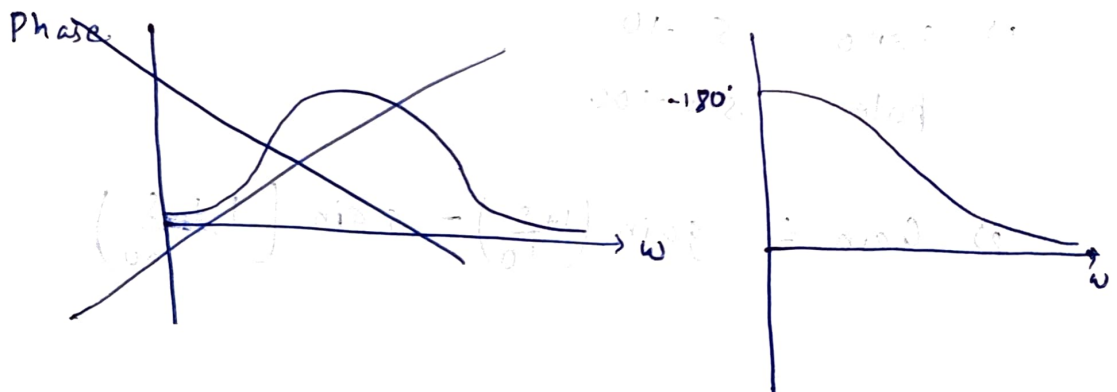
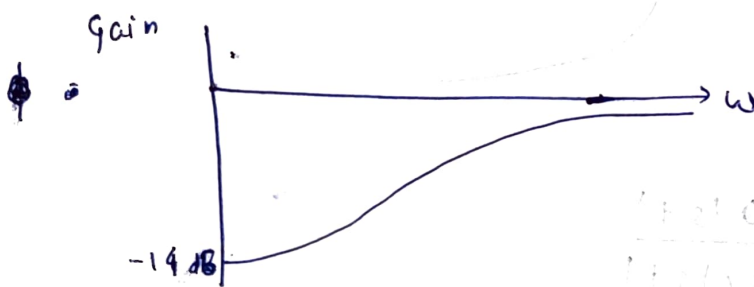
$$= 6 + 20 \log \sqrt{1 + \frac{\omega^2}{4}} - 20 - 20 \log \sqrt{1 + \frac{\omega^2}{100}}$$

$$= -14 + 20 \log \frac{\sqrt{1 + \frac{\omega^2}{4}}}{\sqrt{1 + \frac{\omega^2}{100}}}$$

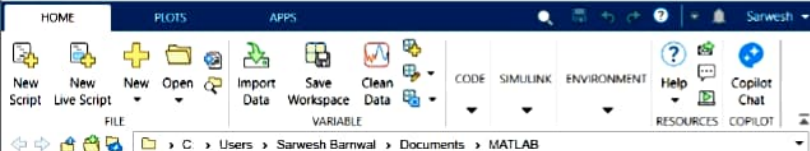
$$= -14 + 10 \log \frac{1 + \omega^2/4}{1 + \omega^2/100}$$

at $\omega = 0$ gain = -14

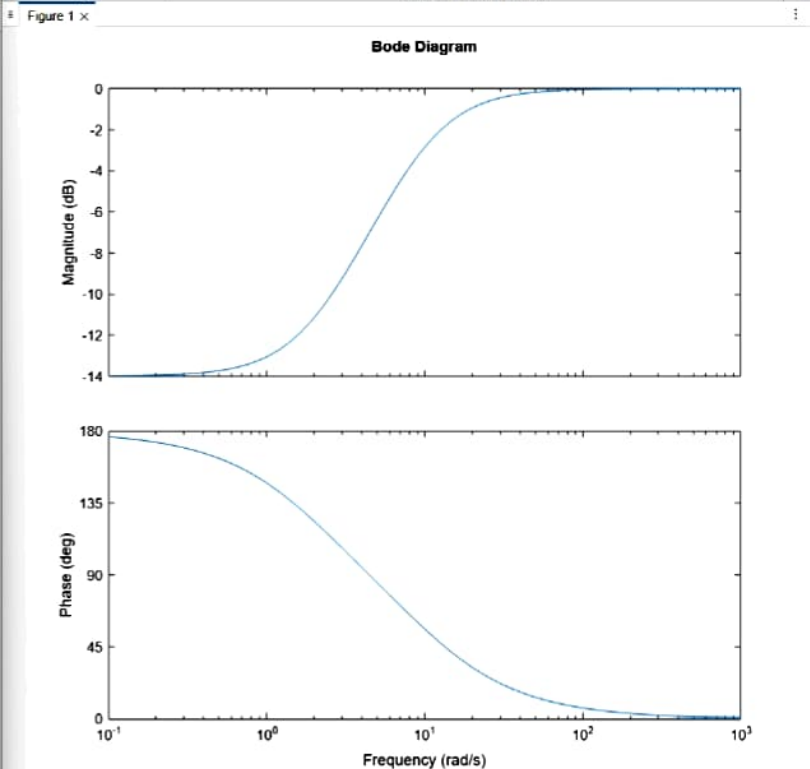
$\omega \rightarrow \infty$ gain = 0.



4. It causes a phase lag.



```
Command Window
New to MATLAB? See resources for Getting Started
>> G2=tf([1 -2],[1 10])
G2 =
    s - 2
    -----
    s + 10
Continuous-time transfer function.
Model Properties
>> bode(G2)
>> |
```



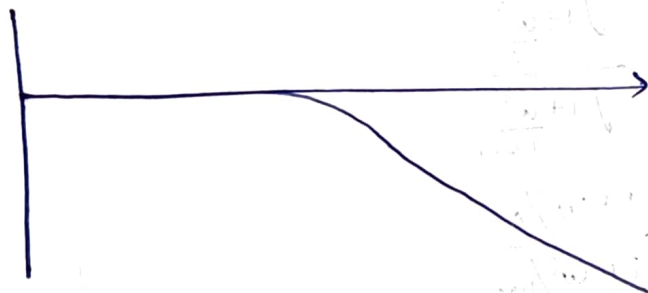
1.3. $G(s) = \frac{100}{s^2 + 10s + 100}$

1. poles : $s = -5 \pm 5\sqrt{3}j$

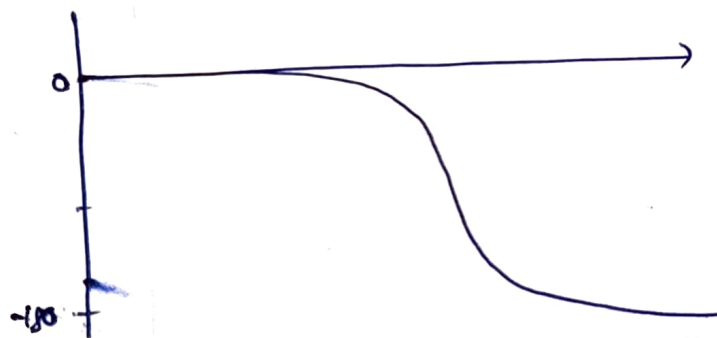
2. $G(s) = \frac{\omega_0^2}{s^2 + 2z\omega_0 s + \omega_0^2}$

$\omega_0 = 10$, $2z\omega_0 = 10 \Rightarrow z = \frac{1}{2}$

gain



phase





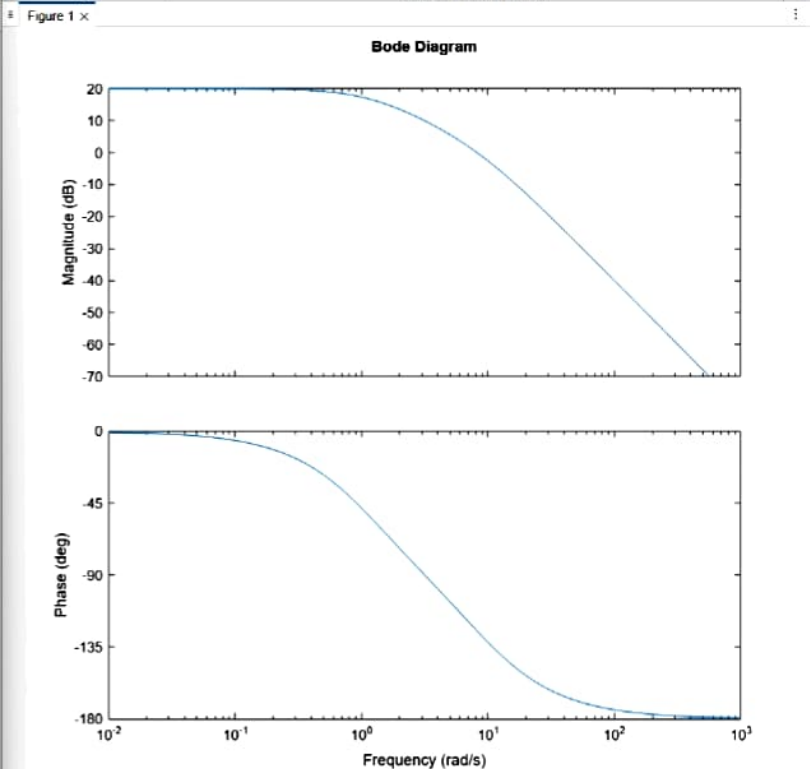
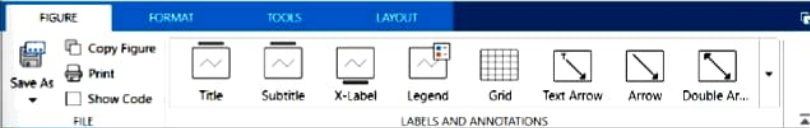
```
Command Window
New to MATLAB? See resources for Getting Started

Continuous-time transfer function.
Model Properties
>> G3=tf([100],[1 10 10])

G3 =

      100
-----
s^2 + 10 s + 10

Continuous-time transfer function.
Model Properties
>> bode(G3)
>>
```

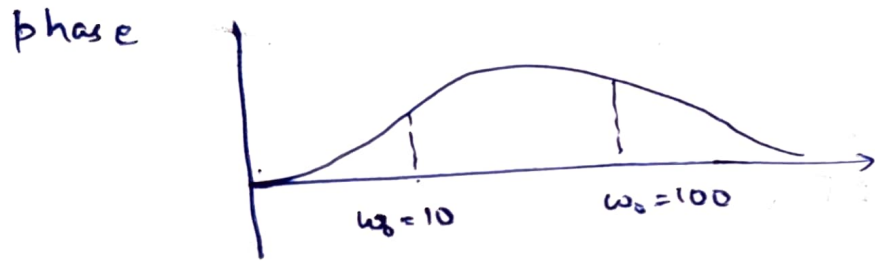
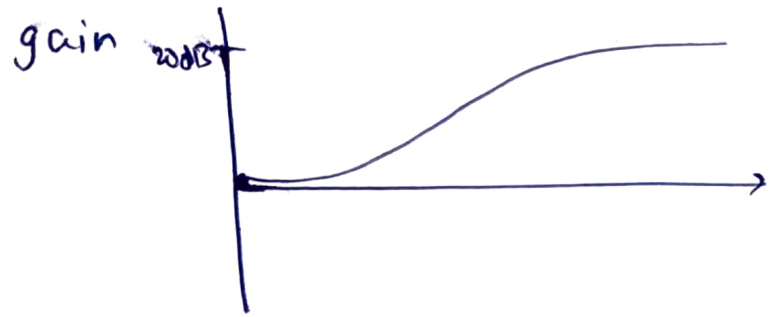


1.4.

$$G_z(s) = \frac{0.1s+1}{0.01s+1}$$

1) zero $s = -10$
pole $s = -100$

2) Gain $\hat{=}$ gain $\left(1 + \frac{s}{10}\right) - \text{gain} \left(1 + \frac{s}{100}\right)$

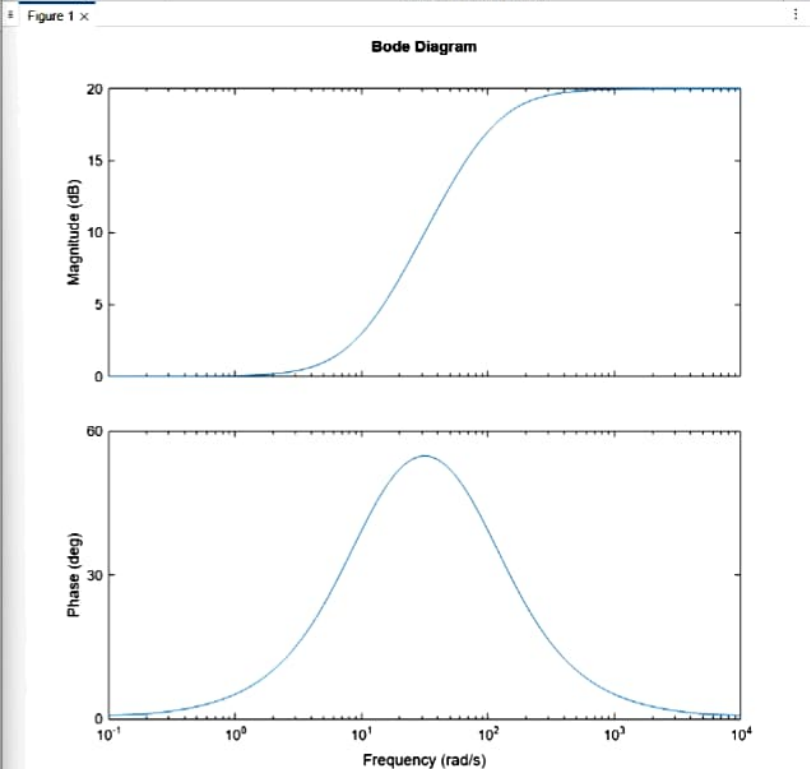


4) Between zero and pole $G_4(s)$ tends to add +ve phase as seen from the plots.

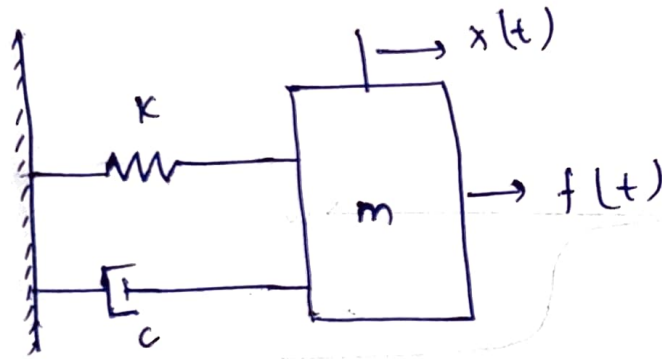


```
Command Window
New to MATLAB? See resources for Getting Started
>> G4=tf([0.1 1],[0.01 1])
G4 =
    0.1 s + 1
    -----
    0.01 s + 1

Continuous-time transfer function.
Model Properties
>> bode(G4)
>> |
```



B.1.1



$$f(t) - k u(t) - c \frac{d}{dt}(u(t)) = m \frac{d^2}{dt^2}(u(t))$$

2.

$$F(s) = m s^2 X(s) + c s X(s) + k X(s)$$

$$F(s) = X(s) \cdot [m s^2 + c s + k]$$

3.

$$\frac{X(s)}{F(s)} = \frac{1}{m s^2 + c s + k} = G(s)$$

B.2

$$m = 1 \text{ kg.}$$

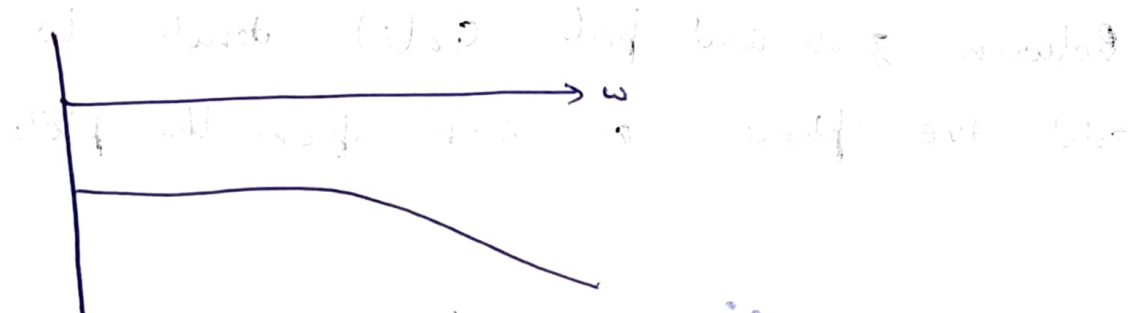
$$c = 4 \text{ N.s/m}$$

$$k = 16 \text{ N/m}$$

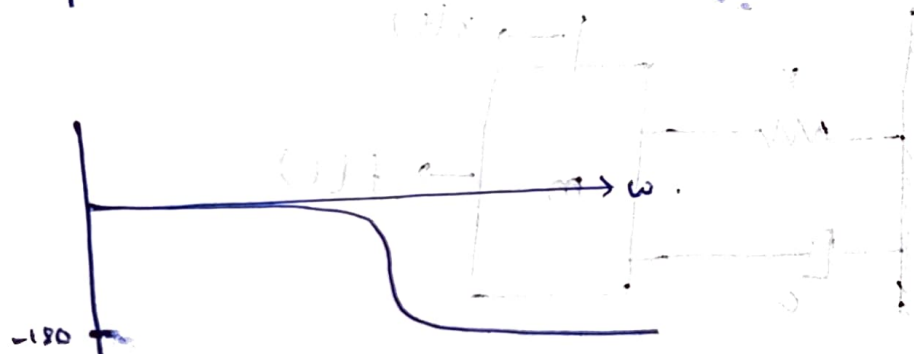
$$2) \quad G(s) = \frac{1}{s^2 + 4s + 16}$$

$$2) \quad \text{poles: } s = -2 \pm 2\sqrt{3}i$$

Gain



phase



$$\left(\frac{1}{16} \right) \frac{1}{s^2 + 4s + 16} = \left(\frac{1}{16} \right) \frac{1}{s^2 + 4s + 16} = \left(\frac{1}{16} \right) \frac{1}{s^2 + 4s + 16}$$



```
Command Window
New to MATLAB? See resources for Getting Started
>> G5=tf([ 1],[1 4 16])
G5 =
    1
-----
s^2 + 4 s + 16
Continuous-time transfer function.
Model Properties
>> bode(G5)
>> |
```

