

Assignment 0

1.1 } A.1

$$G_1(s) = \frac{10}{s+10}$$

1) Pole : $s = -10$

$$G_1(0) = \frac{10}{0+10} = 1$$

$$2) G_1(s) = \frac{1}{1 + s/10} = \frac{1}{1 + s/\omega_0} \quad (\omega_0 = 10)$$

Now,

$$G_1(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}} = \frac{1 - \left(\frac{\omega}{\omega_0}\right)j}{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

$$|G_1(j\omega)| = \left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]^{-1/2}$$

$$\arg(G_1(j\omega)) = \tan^{-1}\left(\frac{-\omega}{\omega_0}\right)$$

So, gain = $20 \log_{10} \left(1 + \frac{\omega^2}{\omega_0^2}\right)^{-1/2} = -10 \log_{10} \left(1 + \frac{\omega^2}{\omega_0^2}\right)$

$$\phi = \text{phase} = \tan^{-1}\left(\frac{-\omega}{\omega_0}\right).$$

Now, CT: $\omega \ll \omega_0$ | CII: $\omega = \omega_0$

$$\text{gain} = 0 \quad | \quad \text{gain} = -3$$

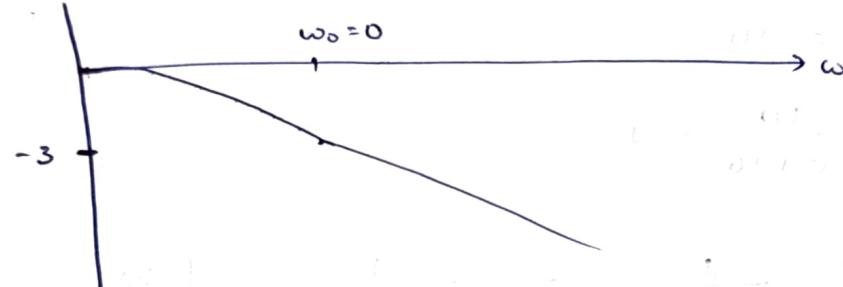
$$\phi = 0 \quad | \quad \phi = -35^\circ$$

(III : $\omega \gg \omega_0$)

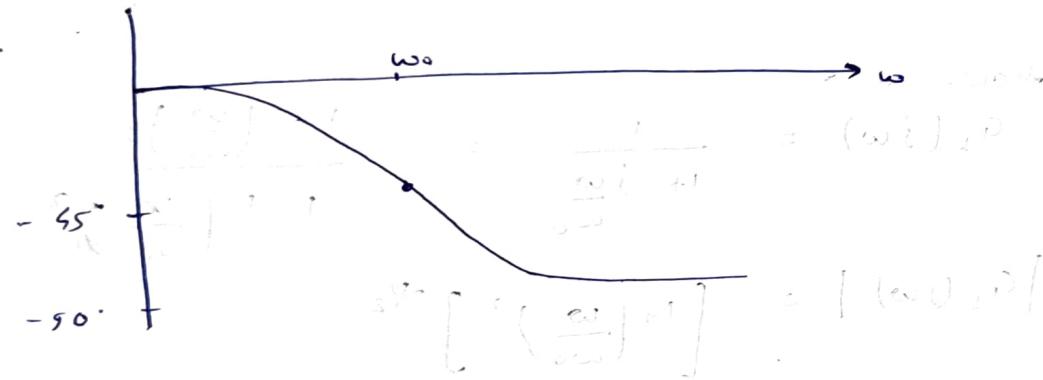
gain $\rightarrow -\infty$

$$\phi = -90^\circ$$

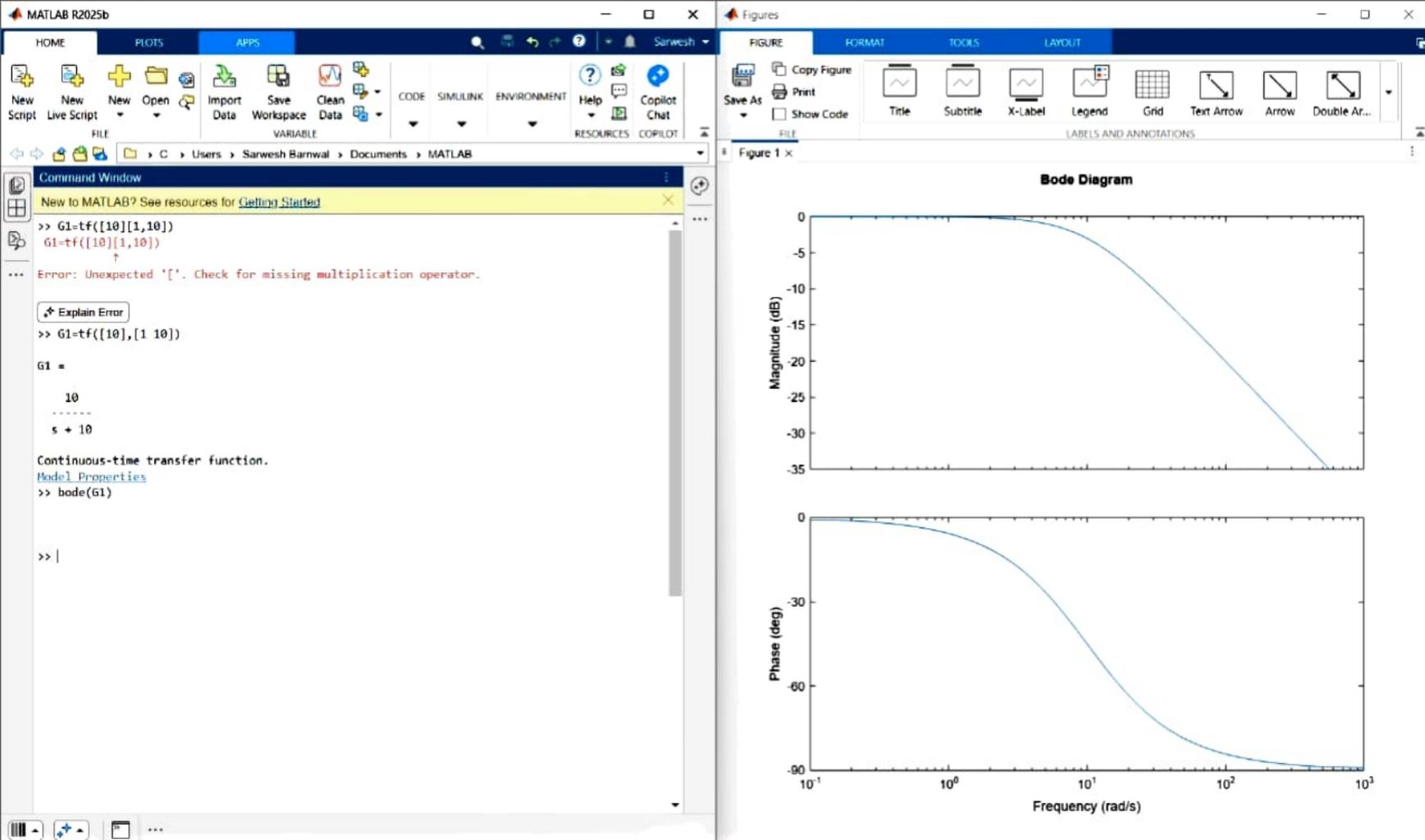
gain (dB)



phase



1.2. $G_2(s) = \frac{s - 2}{s + 10}$ $\left(\frac{10}{s+10}\right)^2 \approx 100 \approx (100 \cdot 0) \cdot e^{j0^\circ}$



$$1.2. \quad G_2(s) = \frac{s-2}{s+10}$$

$\stackrel{\text{pole } s=2}{\cancel{\text{zero } s=-10}}$

$$1. \text{ zero. } s=2$$

pole $s = -10$

$$G_2(0) = -1/5$$

~~$$2. \quad G_2(j\omega) = \frac{j\omega - 2}{j\omega + 10} \times \frac{-j\omega + 10}{-j\omega + 10}$$~~

~~$$= \frac{(j\omega - 2)(10 - j\omega)}{\omega^2 + 10j\omega + 100}$$~~

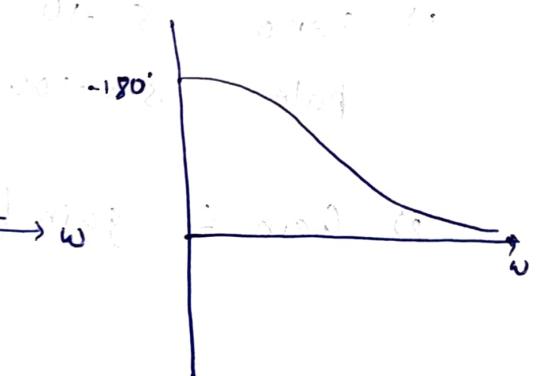
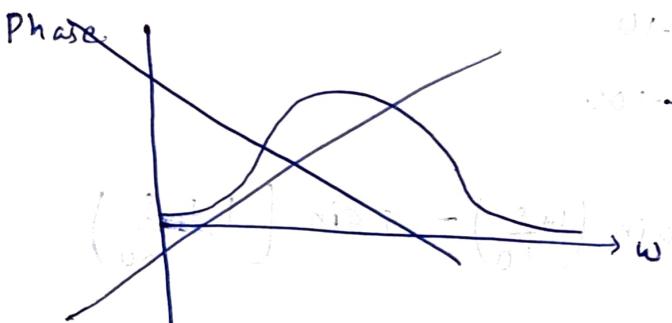
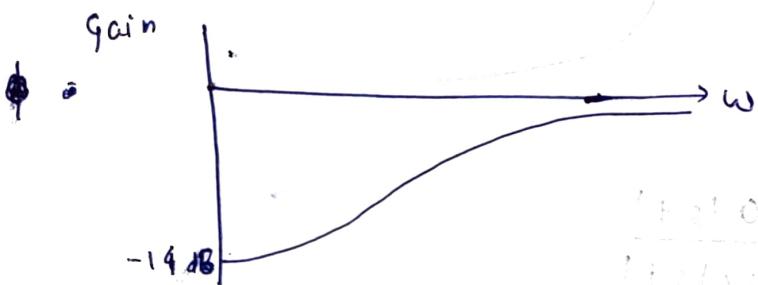
~~$$= \frac{10j\omega - 20 + \omega^2 + 2j\omega}{\omega^2 + 100} = \frac{\omega^2 + 12j\omega - 20}{\omega^2 + 100}$$~~

$$G_2(s) = \frac{s-2}{s+10} = \frac{-2\left(1-\frac{s}{2}\right)}{10\left(1+\frac{s}{10}\right)}$$

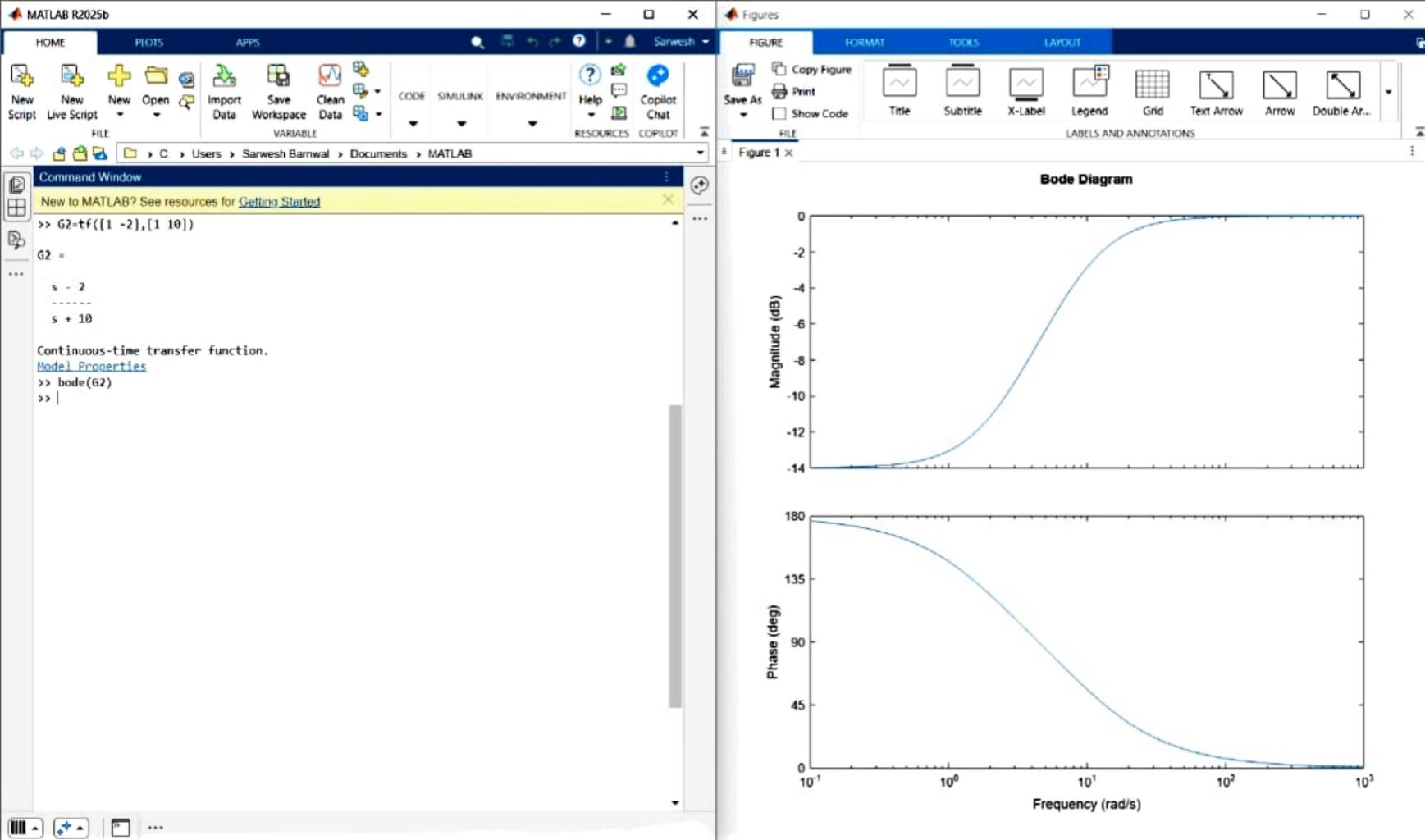
$$\begin{aligned} \text{gain} &= \text{gain}(-2) + \text{gain}\left(1-\frac{s}{2}\right) - \text{gain}(0) - \text{gain}\left(1+\frac{s}{10}\right) \\ &= 6 + 20 \log \sqrt{1+\frac{\omega^2}{4}} - 20 - 20 \log \sqrt{1+\frac{\omega^2}{100}} \\ &= -14 + 20 \log \frac{\sqrt{1+\omega^2}}{\sqrt{1+\frac{\omega^2}{100}}} \\ &= -14 + 10 \log \frac{1+\omega^2}{1+\omega^2/100} \end{aligned}$$

at $\omega=0$ gain = -14

$\omega \rightarrow \infty$ gain = 0.



4. It causes a phase lag.



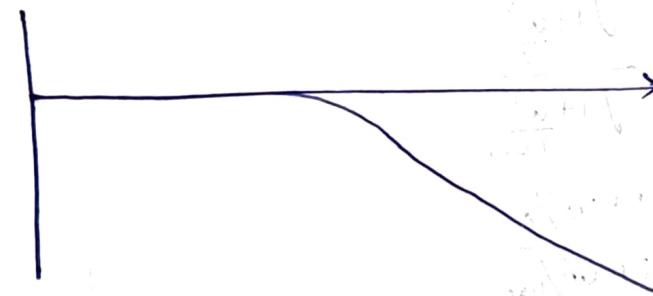
$$1.3. \quad G_g(s) = \frac{100}{s^2 + 10s + 100}$$

1. poles : $s = -5 \pm 5\sqrt{3}j$

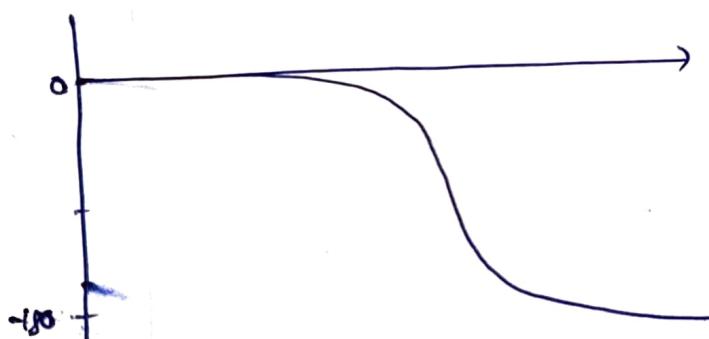
2. $G_p(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$

$$\omega_0 = 10, 2\zeta\omega_0 = 10 \Rightarrow \zeta = \frac{1}{2}$$

gain

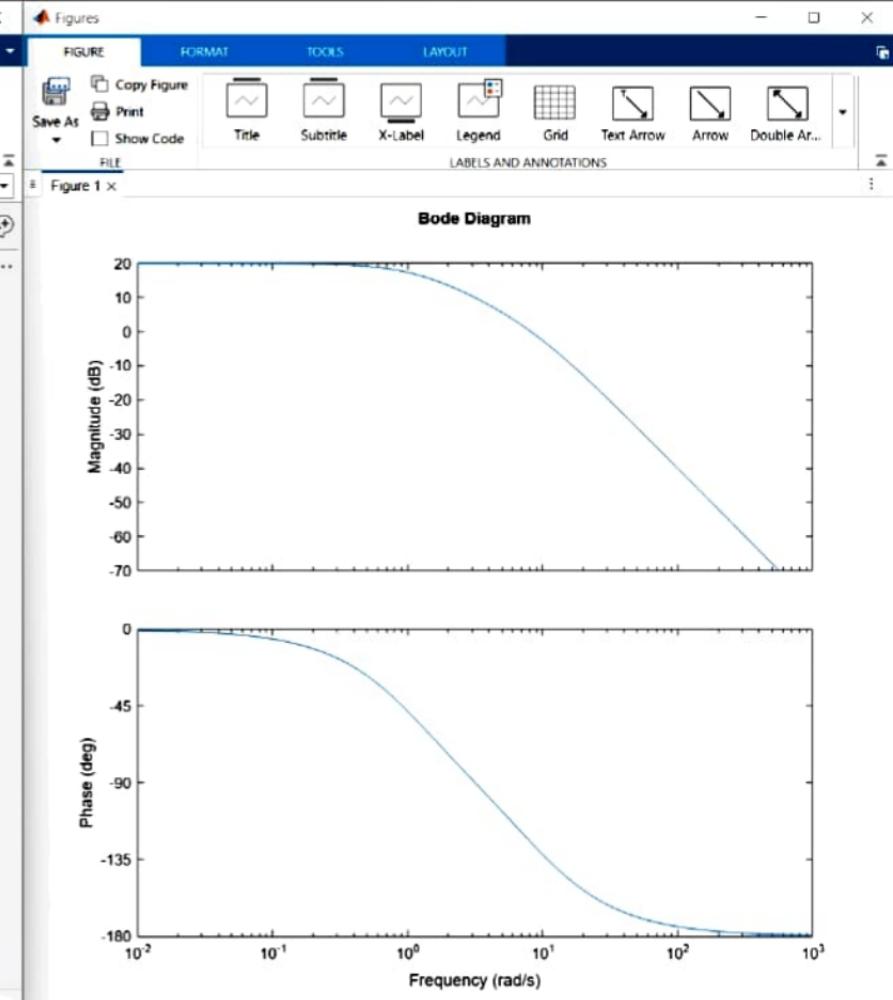
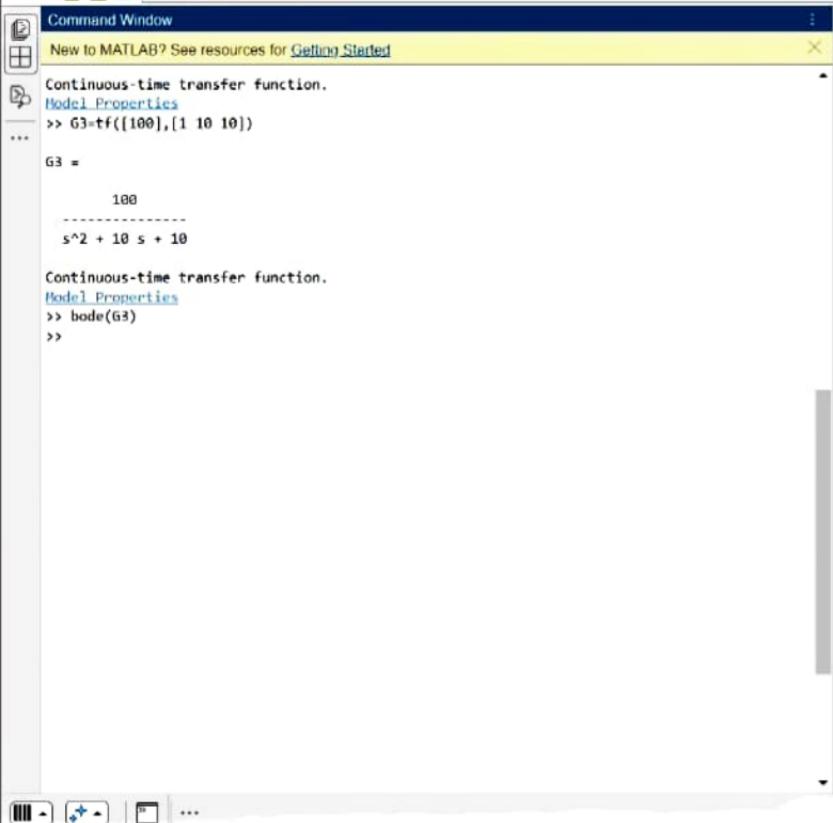


phase





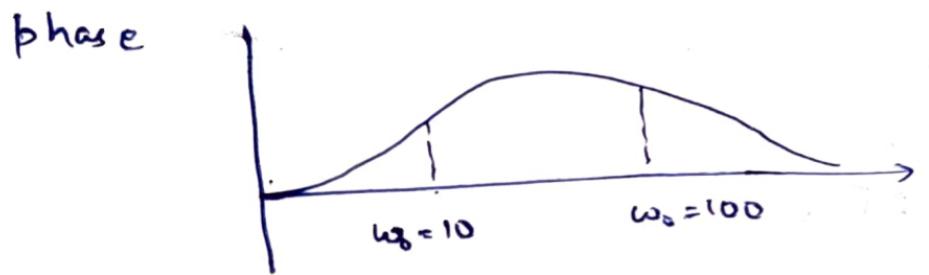
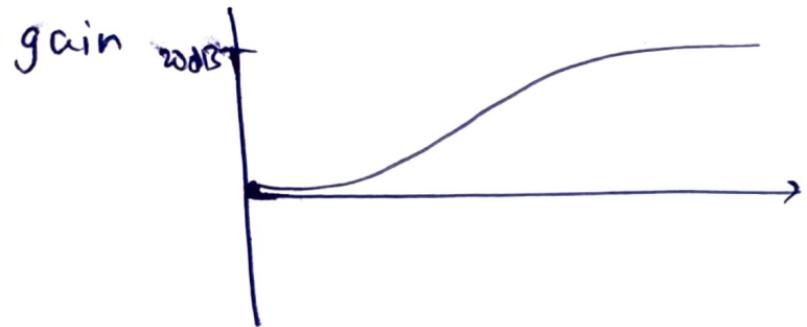
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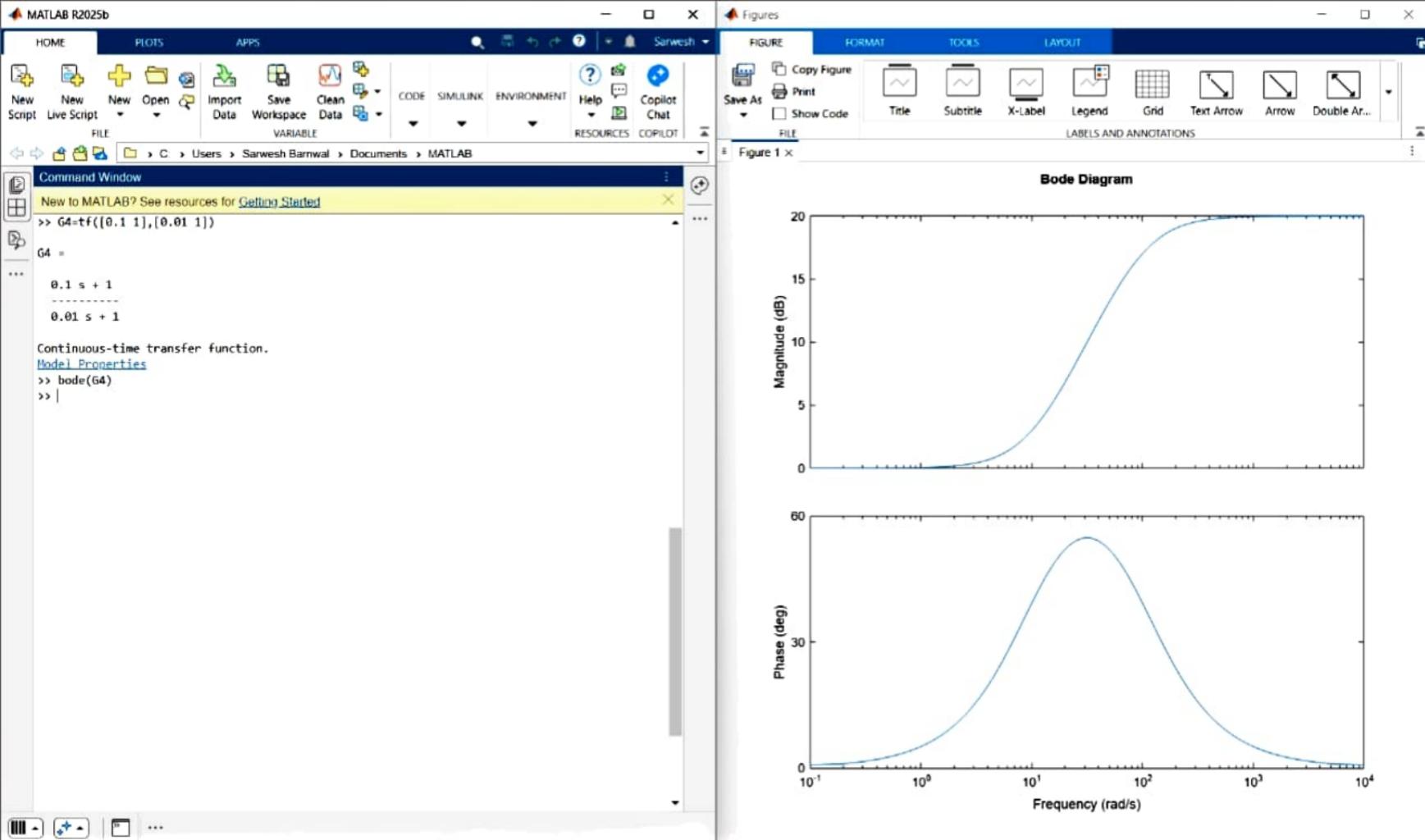
1.4. $G_2(s) = \frac{0.1s+1}{0.01s+1}$

1) zero $s = -10$
pole $s = -100$

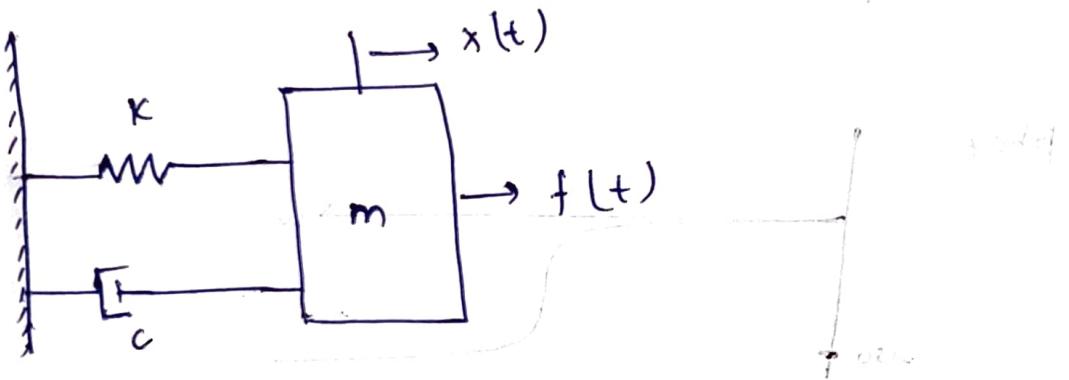
2) Gain \equiv gain $(1 + \frac{s}{10})$ - gain $(1 + \frac{s}{100})$



- 4) Between zero and pole $g_s(s)$ tends to add +ve phase as seen from the plots.



B. 1. 1



$$f(t) - k u(t) - c \frac{du}{dt}(t) = m \frac{d^2 u}{dt^2}(t)$$

2. $F(s) = ms^2 x(s) + cs x(s) + k x(s)$

$$F(s) = x(s) [ms^2 + cs + k]$$

3. $\frac{x(s)}{F(s)} = \frac{1}{ms^2 + cs + k} = G(s)$

B.2

$$m = 1 \text{ kg.}$$

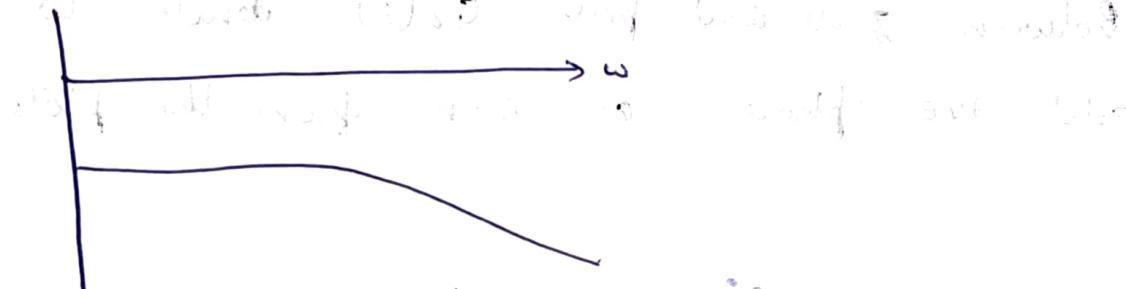
$$c = 4 \cdot N \cdot s/m$$

$$k = 16 \text{ N/m}$$

2) $G(s) = \frac{1}{s^2 + 4s + 16}$

2) poles : $s = -2 \pm 2\sqrt{3}i$

Gain



phase

