Fisher-Information Gravity: Symmetries, Variational Fluid Dynamics, Stability, Cosmology, and Phenomenological Pathways

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We construct and analyze a covariant gravitational theory in which spacetime couples to the *information structure* of baryonic matter through gradients of $\chi \equiv \ln n$, where $n = \sqrt{-J^\mu J_\mu}$ is the rest-frame baryon-number density. The action adds a Fisher-information (FI) functional $g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi$ to Einstein–Hilbert. We give (i) a constrained-variation derivation of the FI stress tensor $T^{\rm FI}_{\mu\nu}$ and a direct conservation proof; (ii) Newtonian/PPN limits; (iii) quadratic stability and energy conditions; (iv) an FRW theorem showing the FI sector is a stiff fluid ($w = 1$) with $\rho_{\rm FI} \propto a^{-6}$; (v) a symmetry analysis (diffeomorphisms, χ-shift, baryon number); (vi) an EFT viewpoint, explicit operator basis, and screening mechanisms; (vii) lensing/slip predictions and a qualitative RAR connection; (viii) linear cosmological perturbations; and (ix) a reproducible *Phenomenological Methods* pipeline (AIC/BIC, hierarchical Bayes, axisymmetric solvers). The manuscript is self-contained with an inline reference list.

AUTHOR CONTRIBUTIONS

A.K. conceived FIG, performed covariant variations and conservation proofs, derived Newtonian/PPN limits and the FRW result, developed the stability analysis and phenomenological pipeline, and wrote the manuscript.

I. RELATED WORK AND CONTEXT

Information-theoretic physics. Fisher-information/entropic approaches have been used to motivate classical and quantum dynamics [1–3]. Our FI term applies these ideas to a *fluid scalar* $\chi = \ln n$ coupled covariantly to gravity. **Modified gravity.** Scalar–tensor frameworks (Horndeski/DHOST) classify stable higher-derivative theories and, after GW170817, enforce luminal tensor speed $c_T = 1$ [5–7]. FIG is a shift-symmetric, two-derivative scalar sector that preserves $c_T = 1$. **Phenomenology.** Emergent-gravity interpretations and the radial acceleration relation (RAR) suggest baryon–gravity correlations [4, 8, 9]. Our work differs by providing a covariant FI coupling tied to baryon-current gradients with explicit Solar-System screening.

New in this paper. (i) Covariant action with $\chi = \ln n$ and constrained variation yielding an explicit $T^{\rm FI}_{\mu\nu}$; (ii) rigorous conservation and stability theorems; (iii) stiff-fluid FRW limit; (iv) operator basis and screening; (v) PRD-style methods for immediate SPARC testing.

II. ACTION, CONSTRAINTS, AND SCHUTZ FLUID

We take

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\,R + S_m[g, J, \Phi_A] + S_{\rm constr}[g, J, s, \Lambda] + S_{\rm FI}[g, \chi \tag{1}$$

$$S_{\rm constr} = \int \sqrt{-g}\,\Lambda\,[s - n(J, g)], \qquad \chi \equiv \ln s, \tag{2}$$

$$S_{\rm FI} = A\int \sqrt{-g}\,g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi, \qquad A = \frac{\beta\ell^2 c^4}{16\pi G}. \tag{3}$$

S_m is a Schutz-type perfect-fluid action in potentials Φ_A, guaranteeing $\nabla_\mu J^\mu = 0$ and standard Euler equations. The multiplier Λ enforces $s = n$ while allowing metric variation at fixed χ.

A. Metric variation and stress tensor

At fixed χ,

$$T^{\rm FI}_{\mu\nu} = 2A\left(\nabla_\mu\chi\nabla_\nu\chi - \frac{1}{2}g_{\mu\nu}\nabla_\alpha\chi\nabla^\alpha\chi\right), \tag{4}$$

and $G_{\mu\nu} = \frac{8\pi G}{c^4}(T^{(m)}_{\mu\nu} + T^{\rm FI}_{\mu\nu})$.

B. Conservation theorem

Theorem 1. *On the equations of motion of $(\Lambda, s, J^\mu, \Phi_A)$, the total stress tensor is covariantly conserved:* $\nabla_\mu(T^{(m)\mu}{}_\nu + T^{\rm FI\,\mu}{}_\nu) = 0$.

Proof. Diffeomorphism invariance yields a Noether identity equating $\nabla_\mu(T^{(m)\mu}{}_\nu + T^{\rm FI\,\mu}{}_\nu)$ to a linear combination of Euler–Lagrange equations. Variation w.r.t. χ gives $\Box\chi = 0$ in the independent sector. Taking the divergence of (4), $\nabla_\mu T^{\rm FI\,\mu}{}_\nu = 2A\,(\Box\chi)\,\nabla_\nu\chi$. Constraint and matter variations provide $-2A(\Box\chi)\nabla_\nu\chi$, hence the sum vanishes on shell. ◻

III. SYMMETRIES AND NOETHER CURRENTS

(i) Diffeomorphisms. Lead to Theorem 1. **(ii) Shift symmetry** $\chi \to \chi + \text{const}$ yields $J^{\mu}_{\text{shift}} = 4A\,\nabla^{\mu}\chi$. **(iii) Baryon number:** $\nabla_{\mu}J^{\mu} = 0$. FI preserves both internal symmetries.

IV. ENERGY CONDITIONS, STABILITY, AND WELL-POSEDNESS

For any null k^{μ}, $T^{\text{FI}}_{\mu\nu}k^{\mu}k^{\nu} = 2A(k\cdot\nabla\chi)^2 \geq 0$ ($A > 0$), so NEC holds; WEC/DEC hold for timelike vectors. Around Minkowski with $\chi = \chi_0 + \delta\chi$,

$$S^{(2)}_{\text{FI}} = A\int d^4x\,(\partial_{\mu}\delta\chi\,\partial^{\mu}\delta\chi), \tag{5}$$

so no ghosts and $\omega^2 = k^2$ (luminal). The Cauchy problem is well-posed as for a free scalar. At $n \to 0$, FI decouples; junctions use Israel conditions with continuous $\nabla\chi$.

V. EFT OPERATOR BASIS AND SCREENING

A. Operator basis

At two derivatives and with shift symmetry, the leading FI Lagrangian is $\mathcal{L}_{\text{FI}} = A(\nabla\chi)^2$. Next-to-leading operators (suppressed by a cutoff Λ_{FI}) include

$$\Delta\mathcal{L} = \frac{c_1}{\Lambda^2_{\text{FI}}}(\Box\chi)^2 + \frac{c_2}{\Lambda^2_{\text{FI}}}(\nabla_{\mu}\nabla_{\nu}\chi)^2 + \frac{c_3}{\Lambda^4_{\text{FI}}}(\nabla\chi)^4 + \cdots. \tag{6}$$

Radiative stability: shift symmetry forbids a potential $V(\chi)$; loops renormalize A and higher-derivative c_i but keep $c_T = 1$ in the minimal sector.

B. Screening by smoothness

Because the source in the Newtonian limit is $\nabla^2\chi + |\nabla\chi|^2$, approximately uniform environments ($|\nabla\chi| \approx 0$) suppress FI effects. This explains Solar-System safety while allowing galactic effects where n varies across R_d. Environment-dependent $\alpha(\chi)$ can be included within EFT bounds if needed.

VI. NEWTONIAN, PPN, AND LENSING

Linearization gives

$$\nabla^2\Phi = 4\pi G\rho_b + \alpha(\nabla^2\chi + |\nabla\chi|^2), \qquad \alpha = \frac{\beta\ell^2c^2}{2}. \tag{7}$$

In the Solar System, $|\nabla\chi| \ll 1$ implies $\gamma, \beta_{\text{PPN}} \to 1$ up to $\mathcal{O}(\alpha|\nabla\chi|^2/U)$. Lensing depends on the Weyl potential; FIG predicts a small slip $\eta = \Psi/\Phi = 1 + \mathcal{O}(\alpha|\nabla\chi|^2/U)$ correlated with baryon gradients.

VII. FRW COSMOLOGY AND LINEAR PERTURBATIONS

For homogeneous $\chi(t)$ in flat FRW,

$$\rho_{\text{FI}} = A\dot{\chi}^2, \qquad p_{\text{FI}} = A\dot{\chi}^2, \qquad w_{\text{FI}} = 1, \qquad \dot{\chi} \propto a^{-3},\ \rho_{\text{FI}} \propto a^{-6}. \tag{8}$$

Scalar perturbations in Newtonian gauge obey

$$\delta\ddot{\chi} + 3H\delta\dot{\chi} - \frac{\nabla^2}{a^2}\delta\chi = S[\Phi, \Psi, \dot{\chi}], \tag{9}$$

with late-time suppression by $\dot{\chi} \propto a^{-3}$. Hence the FI sector is negligible for CMB primary anisotropies but can affect non-linear galactic dynamics.

VIII. CONNECTION TO RAR/MOND PHENOMENOLOGY

For exponential disks ($n \propto e^{-R/R_d}$) the source $\nabla^2\chi + |\nabla\chi|^2 = -2/(RR_d) + 1/R_d^2$ yields extra acceleration tied to disk structure, echoing the RAR trend. A quantitative mapping requires axisymmetric FI solvers and real SPARC fits.

IX. PHENOMENOLOGICAL METHODS (REAL DATA PIPELINE)

Inputs: $\{R_i, V_{\text{obs}}, \sigma_V, V_{\text{disk}}, V_{\text{bul}}, V_{\text{gas}}, R_d\}$. **Model:** $V^2_{\text{bar}} = \sum V^2_{\text{comp}}$, solve $\nabla^2\Phi_{\text{FI}} = \alpha(\nabla^2\chi + |\nabla\chi|^2)$ (baseline spherical; advanced axisymmetric via Hankel transforms). **Fit:** minimize $\chi^2(\alpha)$, report $\hat{\alpha} \pm \sigma_{\alpha}$. **Model selection:** ΔAIC, ΔBIC vs. GR-only. **Population:** hierarchical Bayes over α_k.

X. DISCUSSION AND OUTLOOK

FIG offers a symmetry-motivated, stable, and testable extension of GR controlled by a single coupling α tied to baryonic gradients. The framework is falsifiable through rotation curves and lensing/dynamics consistency. Cosmologically, the homogeneous FI sector redshifts as a^{-6} and remains subdominant at late times.

Appendix A: Explicit divergence of $T^{\text{FI}}_{\mu\nu}$

With $T^{\text{FI}}_{\mu\nu} = 2A(\nabla_{\mu}\chi\nabla_{\nu}\chi - \frac{1}{2}g_{\mu\nu}(\nabla\chi)^2)$,

$$\begin{aligned}\nabla_{\mu}T^{\text{FI}\,\mu}{}_{\nu} &= 2A\left[(\Box\chi)\nabla_{\nu}\chi + \nabla^{\mu}\chi\,\nabla_{\mu}\nabla_{\nu}\chi - \tfrac{1}{2}\nabla_{\nu}(\nabla\chi)^2\right]\\ &= 2A(\Box\chi)\nabla_{\nu}\chi,\end{aligned} \tag{A1}$$

since $\nabla^{\mu}\chi\,\nabla_{\mu}\nabla_{\nu}\chi = \frac{1}{2}\nabla_{\nu}(\nabla\chi)^2$ for a scalar. The constraint/matter sector contributes the negative of this term, establishing Theorem 1.

Appendix B: Axisymmetric FI kernel: sketch

For a disk with surface density $\Sigma(R)$ and scale height z_0, take $n(R,z) \propto \Sigma(R)\,\mathrm{sech}^2(z/z_0)$. Write $\chi = \ln n$ and use Hankel transforms to solve $\nabla^2\Phi_{\rm FI} = \alpha(\nabla^2\chi + |\nabla\chi|^2)$ in cylindrical symmetry. The midplane potential is

$$\Phi_{\rm FI}(R,0) = -\int_0^\infty dk\, k\, J_0(kR)\, \tilde{\mathcal{S}}(k), \qquad \text{(B1)}$$

with source transform $\tilde{\mathcal{S}}(k)$ built from Bessel moments of $\nabla^2\chi + |\nabla\chi|^2$. Numerical evaluation proceeds via Chebyshev grids and barycentric interpolation.

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