# *Assignment 2.4*

## 1 Singularities

Find the singular points of the following differential equations and classify them.

1. $x^2y'' + (x + x^2)y' + y = 0$
2. $x^2y'' - 5y' + 3x^2y = 0$
3. $(x^2 + x - 2)^2y'' + 3(x + 2)y' + (x - 1)y = 0$
4. $x^4y'' + 4x^3y' + y = 0$
5. $x^3(x^2 - 1)y'' - x(x + 1)y' - (x - 1)y = 0$

## 2 Radius of convergence

Determine the radius of convergence. Show the details of your work.

6. $\sum_{m=0}^{\infty}(m + 1)mx^m$
7. $\sum_{m=0}^{\infty}\frac{(-1)^m}{k^m}x^{2m}$
8. $\sum_{m=0}^{\infty}\frac{x^{2m+1}}{(2m+1)!}$
9. $\sum_{m=0}^{\infty}(\frac{2}{3})^m x^{2m}$

## 3 Series Solutions

For the problem 10-13, apply the power series method. To get a feel for the method, answer the questions like why a series may terminate, or has even powers only, etc. Show the details.

10. $(1 + x)y' = y$
11. $y' = -2xy$
12. $xy' - 3y = k(= constant)$
13. $y'' + y = 0$

Find a power series solution in powers of x. Show the details.

14. $y'' - y' + xy = 0$
15. $y'' - y' + x^2y = 0$

16. $(1-x^2)y'' - 2xy' + 2y = 0$

17. $y'' + (1+x^2)y = 0$

18. $y'' - 4xy' + (4x^2 - 2)y = 0$

Solve the initial value problem by a power series. Graph the partial sums of the powers up to and including $x^5$. Find the value of the sum $s$ (5 digits) at $x_1$.

19. $y'' + 3xy' + 2y = 0, y(0) = 1, y'(0) = 1, x_1 = 0.5$

20. $(1-x^2)y'' - 2xy' + 30y = 0, y(0) = 0, y'(0) = 1.875, x_1 = 0.5$

21. $(x-2)y' = xy, y(0) = 4, x_1 = 2$

# 4 Miscellaneous

22. **Shifting summation indices** is often convenient or necessary in the power series method. Shift the index so that the power under the summation sign is $x^m$. Check by writing the first few terms explicitly.

23. **Information from Graphs of Partial Sums.**
In numerics we use partial sums of power series. To get a feel for the accuracy for various $x$, experiment with $sinx$ . Graph partial sums of the Maclaurin series of an increasing number of terms, describing qualitatively the "breakaway points" of these graphs from the graph of $sinx$. Consider other Maclaurin series of your choice.

# 5 Frobenius Method

Read the section-5.3 of chapter 5, of the book Advanced Engineering Mathematics by Erwin Kreyszig (10th ed.) and answer the following questions.

24. Write a report explaining the difference between the Power series method and Frobenius method. Give simple examples.

Find a basis of solutions by the Frobenius method. Try to identify the series as expansions of known functions. Show the details of your work.

25. $xy'' + 2y' + xy = 0$

26. $xy'' + y = 0$

27. $xy'' + (2x+1)y' + (x+1)y = 0$

28. $y'' + (x-1)y = 0$

29. $2x(x-1)y'' - (x+1)y' + y = 0$

30. $xy'' + (2-2x)y' + (x-2)y = 0$

# Answers:

1. $x = 0, p_1(x) = 1 + x, q_1(x) = -1$, regular singular point.
2. $x = 0, p_1(x) = \frac{-5}{x}, q_1(x) = 3x^2$, irregular singular point.
3. $x = 1$, irregular singular point, $x = -2$, regular singular point.
4. $x = 1$, irregular singular point.
5. $x = 0$, irregular singular point, $x = 1$ and $x = -1$, regular singular point.
7. $\sqrt{|k|}$
9. $\sqrt{\frac{3}{2}}$
11. $y = a_0(1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots)$
13. $y = a_0 + a_1x - \frac{1}{2}a_0x^2 - \frac{1}{6}a_1x^3 + \cdots = a_0cosx + a_1sinx$
15. $a_0(1 - \frac{1}{12}x^4 - \frac{1}{60}x^5 - \dots) + a_1(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{24}x^5 - \dots)$
16. $a_0(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 - \frac{13}{720}x^6 - \dots) + a_1(x - \frac{1}{6}x^3 - \frac{1}{24}x^5 + \frac{5}{1008}x^7 + \dots)$