

1. $n = a \cos t + b \sin t$

$$n' = -a \sin t + b \cos t$$

$$n'' = -a \cos t - b \sin t$$

$$k = \frac{a^2 + 2(n')^2 - n''^2}{[n^2 + (n')^2]^{3/2}}$$

$$n'' = -n_2$$

$$k = \frac{n^2 + 2(n')^2 - (-n_1)n_1}{[n^2 + (n')^2]^{3/2}}$$

$$= \frac{2(n^2 + (n')^2)^{1/2}}{(n^2 + (n')^2)^{3/2}} = 2(n^2 + (n')^2)^{-1/2}$$

$$k = 2 \left[(a^2 \cos^2 t + b^2 \sin^2 t + 2ab \sin t \cos t) + \right.$$

$$\left. (a^2 \sin^2 t + b^2 \cos^2 t - 2ab \sin t \cos t) \right]^{-1/2}$$

$$k = 2 \left[\cos^2 t (a^2 + b^2) + \sin^2 t (a^2 + b^2) \right]^{-1/2}$$

$$= 2(a^2 + b^2)^{-1/2} [1]$$

$$n = k^{-1} = \frac{\sqrt{a^2 + b^2}}{2}$$

$$2. \quad y = 1/x = x^{-1}$$

$$y' = -1(x)^{-2} \Rightarrow [y']_{x=1} = -1$$

$$y'' = 2x^{-3} \Rightarrow [y'']_{x=1} = 2$$

so,

$$K = \left| \frac{y''}{[1 + (y')^2]^{3/2}} \right| = \left| \frac{2}{(1 + (-1)^2)^{3/2}} \right|$$

$$K = \frac{2^{1/2}}{2^{3/2}} = 2^{-1/2} = \frac{1}{\sqrt{2}}$$

$$\text{So, } f = \frac{1}{K} = \sqrt{2}$$

Centre of curvature: (α, β)

$$\alpha = x - \frac{y'}{y''} [1 + (y')^2]$$

$$= 1 - \left(-\frac{1}{2}\right)(1+1) = 1+1 = 2$$

$$\beta = y + \left[\frac{(1 + (y')^2)^{3/2}}{y''} \right] = y + \frac{1 + (y')^2}{y''}$$

$$\beta = 1 + \frac{1}{2}(1+1) = 1+1 = 2$$

$$(\alpha, \beta) \equiv (2, 2)$$

Circle of curvature is

$$(x-2)^2 + (y-2)^2 = 2.$$

