

$$1. \quad r = a \cos t + b \sin t$$

$$r' = -a \sin t + b \cos t$$

$$r'' = -a \cos t - b \sin t$$

$$\frac{1}{r} = \frac{a^2 + 2(r')^2 - r r''}{[r^2 + (r')^2]^{3/2}}$$

$$r'' = -r$$

$$\frac{1}{r} = \frac{r^2 + 2(r')^2 - (-r)(r)}{[r^2 + (r')^2]^{3/2}}$$

$$= \frac{2(r^2 + (r')^2)^{3/2}}{(r^2 + (r')^2)^{3/2}} = 2(r^2 + (r')^2)^{-1/2}$$

$$\frac{1}{r} = 2 \left[ (a^2 \cos^2 t + b^2 \sin^2 t + 2ab \sin t \cos t) + (a^2 \sin^2 t + b^2 \cos^2 t - 2ab \sin t \cos t) \right]^{-1/2}$$

$$\frac{1}{r} = 2 \left[ \cos^2 t (a^2 + b^2) + \sin^2 t (a^2 + b^2) \right]^{-1/2}$$

$$= 2(a^2 + b^2)^{-1/2} [1]$$

$$r = \frac{1}{\frac{1}{r}} = \frac{2 \sqrt{a^2 + b^2}}{2}$$

$$2. \quad y = 1/x = x^{-1}$$

$$y' = -1(x)^{-2}$$

$$y'' = 2x^{-3}$$

$$y']_{x=1} = -1$$

$$y'']_{x=1} = 2$$

So,

$$K = \left| \frac{y''}{[1+(y')^2]^{3/2}} \right| = \left| \frac{2}{[1+(-1)^2]^{3/2}} \right|$$

$$K = \frac{2^{1/2}}{2^{3/2}} = 2^{-1/2} = \frac{1}{\sqrt{2}}$$

$$\text{So, } \rho = \frac{1}{K} = \sqrt{2}$$

Centre of curvature:  $(\alpha, \beta)$

$$\alpha = x - \frac{y'}{y''} [1+(y')^2]$$

$$= 1 - \left(-\frac{1}{2}\right)(1+1) = 1+1 = 2$$

$$\beta = y + \left[ \frac{(1+(y')^2)^{3/2}}{y''} \right] = y + \frac{1+(y')^2}{y''}$$

$$\beta = 1 + \frac{1}{2}(1+1) = 1+1 = 2$$

$$(\alpha, \beta) \equiv (2, 2)$$

Circle of curvature is

$$(x-2)^2 + (y-2)^2 = 2.$$

