B120054

1. Given $eq^n$ is,

$$y'' + y' = 0$$

Let $y' = v$ then $y'' = \frac{dv}{dx}$

substituting the value of $y'$ and $y''$ in given $eq^n$.

$$\frac{dv}{dx} + v = 0$$

The integrating factor is,

$$I.f = e^{\int dx} = e^x$$

The general $sol^n$ is,

$$v(I.f) = \int 0.(IF)\,dx$$

$$v \times e^x = \int 0\,dx$$

$$y' \cdot e^x = c'$$

$$\int dy = \int c/e^x\,dx$$

$$y = -c_1 e^{-x} + c_2$$

2. The given $eq^n$ is,

$$2xy'' = 3y'$$

$$2xy'' - 3y' = 0$$

Let $y' = v$

$$y'' = dv/dx$$

So, $eq^n$ will become

$$2x\frac{du}{dx} - 3u = 0 \Rightarrow \frac{du}{dx} - \frac{3u}{2x} = 0$$

Here $p = -\frac{3}{2x}$, $q = 0$

The I.F $= e^{\int p\,dx} = e^{\int -3/2x\,dx} = x^{-3/2 \ln x}$

$= x^{-3/2}$

The general $sol^n$ is,

$$u\,(I.F) = \int q\,(IF)\,dx$$

$$u \cdot x^{-3/2} = \int 0\,dx = 0$$

$$y'\,x^{-3/2} = c_1$$

$$y' = c_1\,x^{3/2}$$

Integrating both sides,

$$\int dy = c_1\,x^{3/2}\,dx$$

$$y = \frac{c_1\,x^{3/2+2/2}}{3/2+2/2} = c_1\,x^{5/2} \cdot \frac{2}{5} + c_2$$

3. Given eq$^n$ is, 5

$$yy'' = 3(y')^2$$

$$yy'' - 3y'^2 = 0$$

Let, $y' = v$

$$y'' = \frac{dv}{dx} = \frac{dv}{dy} \times \frac{dy}{dx} = v\frac{dv}{dy}$$

So, eq$^n$ (1) will be,

$$y \cdot \frac{dv}{dx} - 3v^2 = 0$$

$$y \cdot v\frac{dv}{dy} - 3v^2 = 0$$

$$\frac{dv}{dy} - \frac{3}{y}v = 0$$

Here, $P = -3/y$, $Q = 0$

$$I.F = e^{\int P\,dy} = e^{\int -3/y\,dy} = e^{-3\ln y} = y^{-3}$$

The general sol$^n$ is,

$$v \cdot y^{-3} = \int 0\,dy$$

$$v = c_1 y^3$$

$$\frac{dy}{dx} = c_1 y^3$$

$$y = \frac{1}{\sqrt{2}}(-c_1 x - c_2)^{-1/2}$$

5. $x^2y'' - 5xy' + 9y = 0, \quad y_1 = x^3$

$$y = vx^3$$
$$y' = 3x^2v + v'x^3$$
$$y'' = 6vx + 6v'x^2 + x^3v''$$

So, eq$^n$ - (1) is,

$$x^2(6vx + 6v'x^2 + x^3v'') - 5x(3x^2v + v'x^3) + 9vx^3 = 0$$

$$\Rightarrow v' + xv'' = 0$$

Let $v' = z$

$$v'' = \frac{dz}{dx}$$

So, eq$^n$ - (2) is

$$x\frac{dz}{dx} + z = 0$$

$$\frac{dz}{dx} + \frac{1}{x}z = 0$$

$$I.F = e^{\int 1/x\,dx} = x$$

General sol$^n$ is

$$z.x = \int 0.dx$$

$$z.x = c_1 \Rightarrow z = \frac{c_1}{x}$$

$$\frac{dv}{dx} = \frac{c_1}{x} \Rightarrow \int dv = \int \frac{c_1}{x}dx$$

7.

$$v = c_1 \ln|x| + c_2$$

$$\frac{1}{x^3}\frac{dy}{dx} = c_1 \ln x + c_2$$

$$\int dy = \int c_1 \ln x \, dx \, x^3 + \int c_2 dx \, x^3$$

$$y = c_1 \int \ln x \cdot x^3 . dx + c_2 \int x^3 dx$$

$$y = c_1 \left( \ln x . \frac{x^4}{4} - \frac{x^4}{16} \right) - c_2 \frac{x^4}{4} + c_3$$

6. $y'' = 2y'$

$$y'' - 2y' = 0$$

$$D^2 - 2D = 0 \Rightarrow D = 0, 2$$

General solⁿ.

$$y = c_1 y_1 + c_2 y_2$$

$$y_1 = e^{0 \cdot x} = 1, \quad y_2 = e^{2x}.$$

Hence, particular solⁿ is

$$y = c_1 + c_2 e^{2x}$$

It is the required eqⁿ of curve.

7. $y'' = k\sqrt{1+(y')^2}$ , $k = 1$

$y'' = \sqrt{1+(y')^2}$

$(y'')^2 + (y')^2 = 1$

$(y'')^2 - (y')^2 - 1 = 0$

Let, $v = y' \Rightarrow y'' = \frac{dv}{dx}$

$\frac{dv}{dx} = \sqrt{1+v^2} \Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int dx$

$\ln|v + \sqrt{1+v^2}| = x + c$

$v + \sqrt{1+v^2} = (e^{x+c}) - v$

Squaring both sides,

$1 + v^2 = (e^{x+c})^2 + v^2 - 2v \cdot e^{x+c}$

$e^{x+c} - 2v = e^{-(x+c)}$

$v = \frac{e^{x+c} - e^{-(x+c)}}{2}$

$v = \sinh(x+c)$

$\frac{dy}{dx} = \sinh(x+c)$

$\int dy = \int \sinh(x+c)\, dx$

$y = -\cosh(x+c) + c_2$

$y = \cosh(x+c+\pi) + c_2$

$y = \cosh(x+c_1) + c_2$

Now $f(x) = y$ passes through $(-1,0)$ & $(1,0)$

$$0 = c_2 + \frac{e^{(-1+c_1)} + e^{-(-1+c_1)}}{2} \quad \text{——— (2)}$$

$$\& \quad c_2 + \frac{e^{(1+c_1)} + e^{-(+1+c_1)}}{2} = 0 \quad \text{——— (1)}$$

Equating eq$^n$ (1) & (2)

$$e^{-(-1+c_1)} + e^{-(-1+c_1)} = e^{(1+c_1)} + e^{-(1+c_1)}$$

$$\frac{e^{c_1}}{e} + \frac{e}{e^{c_1}} = e \cdot e^{c_1} + \frac{1}{e \cdot e^{c_1}}$$

$$\frac{(e^{c_1})^2 + e^2}{e \cdot \cancel{e^{c_1}}} = \frac{(e \cdot e^{c_1})^2 + 1}{e \cdot \cancel{e^{c_1}}}$$

$$e^{2c_1} - e^2 \cdot e^{2c_1} = 1 - e^2$$

$$e^{2c_1}(1-e^2) - (1-e^2) = 0$$

$$(1-e^2)(e^{2c_1} - 1) = 0$$

Now, $1 - e^{\ } = 0 \Rightarrow e^{\ } = 1$.

Hence, $e^{2c_1} - 1 = 0 \Rightarrow 2c_1 = \ln 1$

$2c_1 = 0 \Rightarrow c_1 = 0$

Putting $c_1 = 0$ in eq$^n$ (2),

$$c_2 = \frac{e + e^{-1}}{2} = \cosh$$

Particular sol$^n$ : $y = \cosh(x) + \cosh(1)$

8. Let the pos$^n$ of the body be described as

$$y = f(t)$$

$\Rightarrow$ velocity $\Rightarrow$ $v = \frac{dy}{dt} = y'$

acceleration : $a = y'' = \frac{d^2y}{dt^2}$

According to question,

$$y'' + y' = c_1$$

Let $v = y' \Rightarrow y'' = \frac{dv}{dt}$

$$\frac{dv}{dt} + v = c_1$$

$$\int \frac{dv}{(c_1 - v)} = \int dt$$

$$\int \frac{dv}{(v-c_1)} = -\int dt$$

$$\ln|v-c_1| + \ln|c| = -t$$

$$\frac{v-c_1}{c} = e^{-t}$$

$$v = e^{-t}\cdot c + c_1$$

$$\frac{dy}{dt} = \frac{c}{e^t} + c_1$$

$$y = -e^{-t}c + c_1 t + c_2$$

9. The posⁿ be defined as

$$y = f(t)$$

Hence, velocity, $v = \frac{dy}{dt} = y'$

acceleration, $a = \frac{dv}{dt} = y''$

According to the question,

$$v = \frac{1}{a} \Rightarrow y' = \frac{1}{y''}$$

$$y''\cdot y' = 1 \Rightarrow y''\cdot y' - 1 = 0$$

Let, $v = y'$, $y'' = v' = \frac{dv}{dt}$

$$v\cdot\frac{dv}{dt} - 1 = 0$$

$$\int u \cdot du = \int dt$$

$$\frac{u^2}{2} = t + c_1$$

$$u = \pm\sqrt{2(t+c_1)}$$

$$\frac{dy}{dt} = \left|\sqrt{2(t+c_1)}\right|$$

$$\int dy = \int \sqrt{2(t+2c_1)}\, dt$$

$$\int dy = \int \sqrt{2t+c_2}\, dt$$

$$y = \frac{(2t+c_2)^{3/2}}{3} + c_3$$

10. $4y'' + 25y = 0$

$y'(0) = -2.5$, $y(0) = 3$

Characteristic eqn: $4D^2 + 25 = 0$

$$D = \pm \tfrac{5}{2} i$$

General sol: $y = c_1 y_1 + c_2 y_2$

$$y_1 = e^{0 \cdot x} \cos(2.5x)$$

$$y_2 = e^{0 \times x} \sin(2.5x)$$

Hence, $y = c_1 \cos(2.5x) + c_2 \sin(2.5x)$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos(2.5x) & \sin(2.5x) \\ -2.5\sin(2.5x) & 2.5\cos(2.5x) \end{vmatrix}$$

$$= 2.5 \neq 0$$

Hence, the given functions are linearly independent.

Now, $y(0) = 3 \Rightarrow 3 = c_1\cos(0) + c_2\sin(0)$

$\Rightarrow 3 = c_1$

$y' = -2.5c_1\sin(2.5x) + 2.5c_2\cos(2.5x)$

$y'(0) = -2.5$

$-2.5 = -(2.5)c_1\sin(0) + 2.5c_2\cos 0$

$-2.5 = 2.5c_2$

$c_2 = -1$

$y = 3\cos(2.5x - 1\sin(2.5x))$

11. $y'' + 0.6y' + 0.09y = 0$

$y(0) = 2.2 \Rightarrow y'(0) = 0.14$

Characteristic eq$^n$ : $D^2 + 0.6D + 0.09 = 0$

$D = -0.3$

12. $y = c_1 y_1 + c_2 y_2$

$y_1 = e^{-0.3x}$

$y_2 = x e^{-0.3x}$

Hence, $y = c_1 e^{-0.3x} + c_2 x e^{-0.3x}$

$$w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{-0.3x} & x e^{-0.3x} \\ -0.3 e^{-0.3x} & -0.3 e^{-0.3x} \cdot x + e^{-0.3x} \end{vmatrix}$$

$$= \left(e^{-0.3x}\right)^2 \begin{vmatrix} 1 & x \\ -0.3 & 1 - 0.3 \end{vmatrix}$$

$$= e^{-0.6x}$$

$y_1$ & $y_2$ are linearly independent.

$$= -0.3c_1 e^{-0.3x} + c_2 e^{-0.3x}(-0.3x+1)$$

$y'(0) = 0.14$

$\Rightarrow$

$0.14 = -0.3c_1 + c_2 \cdot e^{-0.3(0)}$

$0.14 = -0.3c_1 + c_2$

$-0.3c_1 + c_2 = 0.14$

$y(0) = 2.2$

$2.2 = c_1 + 0 \Rightarrow c_1 = 2.2$

$-0.3 \times 2.2 + c_2 = 0.14$

$c_2 = 0.8$

Particular sol$^n$ is, $y = 2.2e^{-0.3x} + 0.8xe^{-0.3x}$

12. $4x^2y'' - 3y = 0$ ; $y(1) = -3$

$y'(1) = 0$

Let, $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

So, the given eq$^n$ will be

$4 \cdot x^2 \cdot m(m-1)x^{m-2} - 3x^m = 0$

$4(m^2-m)x^m - 3x^m = 0$

$$x^m[4m^2-4m-3] = 0$$

$$4m^2-4m-3 = 0$$

$$m = -1/2, 3/2$$

$$y = c_1 x^{-1/2} + c_2 x^{3/2}$$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x^{-1/2} & x^{3/2} \\ -1/2\, x^{-3/2} & 3/2\, x^{1/2} \end{vmatrix}$$

$$= 2 \neq 0$$

$y_1$ & $y_2$ are linearly independent.

$$y = c_1 x^{-1/2} + c_2 x^{3/2}$$

$$y' = -1/2\, c_1 x^{-3/2} + 3/2\, c_2 x^{1/2}$$

$$y(1) = -3$$

$$-3 = c_1 (1)^{-1/2} + c_2 (1)^{3/2}$$

$$c_1 + c_2 = -3$$

$$y'(1) = 0 \Rightarrow 0 = -1/2\, c_1 + 3/2\, c_2$$

$$\Rightarrow 3c_2 - c_1 = 0$$

By equation eq$^n$ 1 & 2.

$$c_1 = -0.75, \quad c_2 = -2.25$$

Particular solⁿ: $-0.75x^{-1/2} - 2.25x^{3/2}$

13(i) $y'' - y = 0$

a) Characteristic eqⁿ is,

$$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$$

we get real and distinct roots,

So, general solⁿ is,

$$y = c_1 e^x + c_2 e^{-x}$$

b) To solve ODE by hyperbolic function, reduce it to simplified form.

Let $z = y'$, $y$ is independent variable

$$y'' = \frac{d^2}{dx^2}[y(x)] = \frac{d}{dx}\left(\frac{dy(x)}{dx}\right) = \frac{dz}{dx}$$

$$= \frac{dz}{dy} \times \frac{dy}{dx} = z'y'$$

$y'' = z'z \Rightarrow$ So, $y'' - y = 0$

$z'z - y = 0 \Rightarrow \int z\,dz = \int y\,dy$

$z^2 = y^2 + c$

Put $z = y'$,

$$(y')^2 = y^2 + c$$

$$\left(\frac{dy}{dx}\right)^2 = y^2 + c$$

$$\int \frac{dy}{\pm\sqrt{y^2+c}} = \int dx$$

$$x + C_1 = \pm \int \frac{dy}{\sqrt{y^2+c}}$$

$$x + c_1 = \pm \sinh^{-1}\left(\frac{y}{\sqrt{c}}\right)$$

$$y = \pm \sqrt{c}\, \sinh(x + c_1)$$

Now, $\sinh(x+y) = \sinh x . \cos y + \cos x . \sinh y$

$$y = \sqrt{c}\, \sinh(x + c_1)$$

$$= \sqrt{c_1} \cosh x + \sqrt{c_2} \sinh x$$

$$c_1 e^{+x} + c_2 e^{-x} = \sqrt{c_2} \sinh x + \sqrt{c_2} \cosh x$$

$$\cosh x = (e^x + e^{-x}) / 2$$

$$\sinh x = (e^x - e^{-x}) / 2$$

So, $\sinh x + \cosh x = e^x$

$$-\sinh x + \cosh x = e^{-x}$$

By comparing,

$$c_1(\sinh x + \cosh x) = c_1 e^x$$

$$c_1 \sinh x + c_1 \cosh x = c_1 e^x$$

Similarly,

$$-c_2 \sinh x + c_2 \cosh x = c_2 e^{-x}$$

$$c_1 e^x + c_2 e^{-x} = c_2 e^{-x}$$

$$c_1 e^x + c_2 e^{-x} = c_1 \sinh x + c_1 \cosh x - c_2 \sinh x + c_2 \cosh x$$

$$= \sinh x (c_1 - c_2) + \cosh x (c_1 + c_2)$$

(ii) $$W = \begin{vmatrix} \sinh x (c_1 - c_2) & \cosh x (c_1 + c_2) \\ \cosh x (c_1 - c_2) & -(c_1 + c_2) \sinh x \end{vmatrix}$$

$$W = -(c_1 + c_2)(c_1 - c_2) \sinh^2 x - \cosh^2 x (c_1 + c_2)(c_1 - c_2)$$

$$= -(c_1 + c_2)(c_1 - c_2) = -(c_1^2 - c_2^2) \neq 0$$

So, it's not linearly depedendent.

18. $y_1 = e^{0x}$, $y_2 = e^{-2x}$

Let the 2nd order ODE be,

$$ay'' + by' + cy = 0$$

$$ar^2 + br + c = 0$$

Let the roots be $r_1$ & $r_2$.

$y_1 = e^{r_1 x}$ , $y_2 = e^{r_2 x}$

Comparing the above with given eq$^n$,

$$r_1 = 0, \quad r_2 = -2$$

$$(r-0)(r+2) = 0$$

$$r^2 + 2r = 0$$

So, $a = 1$, $b = 2$, $c = 0$

Hence, the ODE : $y'' + 2y' = 0$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_1' \\ y_2 & y_2' \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ e^{-2x} & -2e^{-2x} \end{vmatrix}$$

$$= -2e^{-2x} \neq 0$$

Hence the functions are linearly independent

$$y = c_1 y_1 + c_2 y_2$$

$$y = c_1 + c_2 e^{-2x}$$

$$y(0) = c_1 + c_2 = 1 \quad , \quad y' = -2c_2 e^{-2x} \text{ and}$$

$$y'(0) = -2c_2 = -1 \Rightarrow c_2 = 1/2$$

Particular sol$^n$ : $y = 0.5 + 0.5\, e^{-2x}$

19.

$$y^{v} - 5y^{3} + 4y' = 0$$

$$y(0) = 3, \quad y'(0) = -5$$

$$y''(0) = 11, \quad y'''(0) = -23, \quad y^{iv}(0) = 47$$

$$D^5 - 5D^3 + 4D = 0$$

$$D = 0, \pm 1, \pm 2$$

$$y = c_1 + c_2 e^{2x} + c_3 e^{-2x} + c_4 e^{x} + c_5 e^{-x}$$

$$y' = 2c_2 e^{2x} - 2c_3 e^{-2x} + c_4 e^{x} - c_5 e^{-5}$$

From given eq$^n$ of $y(0)$, $y'(0)$, $y''(0)$, $y'''(0)$ and $y^{iv}(0)$, we get

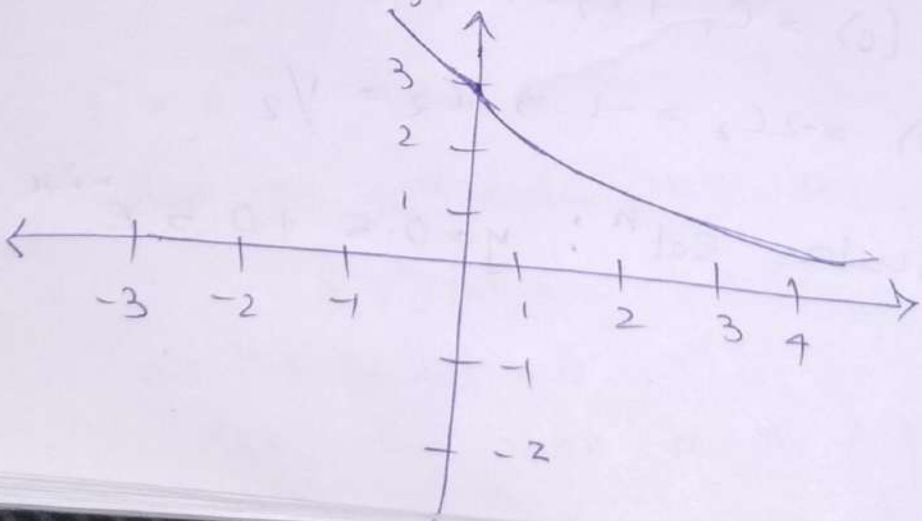
$c_1 = 1 \qquad c_2 = 0$

$c_3 = 3 \qquad c_4 = 0$

$c_5 = -1$

So particular sol$^n$ is,

$$y = 0.5e^{5x} + 0.5e^{-5x} - \cos 4x$$



For $[-1, \infty]$, the graph is decreasing.

20. $5x^2y'' + 23xy' + 16.2y = 0$

$y = x^m$

$y' = mx^{m-1} \Rightarrow y'' = m(m-1)x^{m-2}$

$5x^2[m(m-1)x^{m-2}] + 23x[mx^{m-1}] + 16.2x^m = 0$

$5m(m-1) + 23m + 16.2$

$m = -1.8$

$y = (c_1 + c_2 \ln x)x^m$

$y = (c_1 + c_2 \ln x)x^{-1.8}$

21. $xy'' + 2y' = 0$, let $y = x^m$

$y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$x[m(m-1)x^{m-2}] + 2mx^{m-1} = 0$

$x^{m-1}[m(m-1)] + 2mx^{m-1} = 0$

$m^2 - m + 2m = 0$

$m^2 + m = 0 \Rightarrow m = 0, -1$

$y = c_1y_1 + c_2y_2$

$y_1 = x^{m_1} = x^0 = 1$

$y_2 = x^{m_2} = x^{-1} = 1/x \Rightarrow y = c_1 + c_2$

22. $(x^2D^2 - xD + 5 1)y = 0$

$x^2(D^2y) - x(Dy) + 5(Iy) = 0$

$x^2y'' - xy' + 5y = 0$

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$x^2[m(m-1)x^{m-2}] - x \cdot m\, x^{m-1} + 5x^m = 0$

$m(m-1) - m + 5 = 0$

$m^2 - 2m + 5 = 0$

$m = 1 \pm 2i$

According to Euler-Cauchy relation, the gen. soln. is.

$y = x[c_1 \cos(2\ln x) + c_2 \sin(2\ln x)]$

23. $x^2y'' + xy' + 9y = 0$

$y(1) = 0, \; y'(1) = 2.5$

$y = x^m, \; y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$$x^2\left[m(m-1)x^{m-2}\right] + x\left[m\, x^{m-1}\right] + 9x^m = 0$$

$$m(m-1) + m + 9 = 0$$

$$m = \pm 3i$$

$$y = c_1 \cos(3\ln x) + c_2 \sin(3\ln x)$$

$$y(1) = 0$$

$$c_1 \cos(0) + c_2 \sin(0) = 1$$

$$c_1 = 0$$

$$y' = -3/x \sin(3\ln x) + \frac{3}{x} c_2 \cos(3\ln x)$$

$$y'(1) = 0 + 3c_2 \cos(0) = 2.5$$

$$c_2 = 2.5/3$$

$$y = 2.5/3 \sin(3\ln x)$$

24. $(x^2D^2 + xD + I)\,y = 0$ ; $y(1) = 0$

$$x^2(D^2y) + x(Dy) + (Iy) = 0 \quad ; \; y'(1) = 0$$

$$x^2y'' + xy' + y = 0$$

$$y = x^m ,$$

$$y' = mx^{m-1} ;$$

$$y'' = m(m-1)\,x^{m-2}$$

$$x^2\left[m(m-1)x^{m-2}\right] + x\left[mx^{m-1}\right] + x^m = 0$$

$$m = \pm i$$

According to Euler-Cauchy relation, the general sol$^n$,

$$y = x^{\alpha}[c_1 \cos(\beta \ln x) + c_2 \sin(\beta \ln x)]$$

$$y = x^0 [c_1 \cos(\ln x) + c_2 \sin(\ln x))$$

$$y(1) = c_1 \cos(0) + c_2 \sin(0)$$

$$\Rightarrow c_1 = 1$$

$$y'(1) = -\frac{c_1}{x} \sin(\ln x) + \frac{c_2}{x} \cos(\ln x)$$

$$= -c_1 \sin(0) + c_2 \cos(0)$$

$$\Rightarrow c_2 = 0$$

Particular sol$^n$ : $y = \cos(\ln x)$

25. $$\left(D^2 + 2D + 3\, I/_4\right) y = 3e^x + \frac{9x}{2}$$

$$D^2 y + 2(Dy) + \frac{3}{4}(Iy) = 3e^x + \frac{9x}{2}$$

$$y'' + 2y' + \frac{3}{4} y = 3e^x + \frac{9x}{2}$$

General sol$^n$ : $y = y_c + y_p$

$$y_c = c_1 y_1 + c_2 y_2$$

$$D^2 + 2D + 3/4 = 0$$

$$D = -0.5, -1.5$$

$$y_c = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

$$y_c = c_1 e^{-0.5x} + c_2 e^{-1.5x}$$

$$y_p = c_1(x)\, y_1 + c_2(x)\, y_2$$

Choice of $y_p$ : $y_{p_1} = c\, e^{ax}$

$$y_{p_2} = k_1 x + k_p$$

$$c_1 e^{-0.5x} + c_2 e^{-1.5x} + 4e^2 + \frac{9x}{2} - 8$$

29. $(x^2 D^2 - 3xD + 3I)\, y = 3\ln x - 4$

$y(1) = 0 , \quad y'(1) = 1$

$y_p = \ln x$

General soln : $y = y_c + y_p$

Now, $y_c = c_1 y_1 + c_2 y_2$

$$x^2 y'' - 3y'x + 3y = 0$$

$y = x^m , \quad y' = m x^{m-1} , \quad y'' = m(m-1)x^{m-2}$

$$m(m-1) - 3(m) + 3 = 0$$

$$m^2 - 4m + 3 = 0$$

$$m = 1, 3$$

$$y_c = c_1 x + c_2 x^3$$

$y = c_1 x + c_2 x^3 + \ln x$

$y(1) = c_1 + c_2 + 0 = 0$

$\Rightarrow c_1 + c_2 = 0$

$y' = c_1 + 3c_2 x^2 + \frac{1}{x}$

$y'(1) = c_1 + 3c_2 + 1 = 0$

$c_1 + 3c_2 = 0$

By equating $eq^n$ (1) and (3),

$c_1 = 0, \ c_2 = 0$

$y = \ln x$

particular $sol^n$ is,

$y = \ln x$