ME2 Computing- Session 4: Advanced Numerical Integration and Differentiation

Learning outcomes:

- Being able to compute Simpson and Adaptive Simpson Integration
- Being able to compute K-th order derivative
- Being able to combine numerical derivative and interpolation

Before you start:

In your H drive create a folder *H:\ME2MCP\Session4* and work within it.

Office Contact hours: see Blue slots marked as CPT2 here:

Task A: Simpson Integration and Adaptive Simpson Integration

1. Write a function, *Simpson*, to integrate over a uniformly distributed set of nodal point.
   Compute the integral:
   $$\int_0^1 \frac{1}{1+x^2}\,dx$$
   with steps $\mathrm{d}x = 0.01$ and $\mathrm{d}x = 0.001$.

2. Write an adaptive script to integrate numerically a given function *f(x)*, over a prescribed domain, until a desired tolerance is achieved.
   Compute the above integral, with tolerances $\varepsilon = 10^{-2}$ and $\varepsilon = 10^{-6}$.

Note the analytical solution:

$$\int \frac{1}{1+x^2}\,dx = \operatorname{atan}(x) + c$$

Answer Quizzes 1 and 2

Task B: K-th order derivative

Write a function, *Derivative*, to compute the *k-th* order derivative for a given set of uniformly distributed nodal points (choose yourself to apply either the forward or backward scheme).

Task C: Smoothing derivatives with polynomial interpolation. Launch of a rocket.

A rocket is launched into space. After reaching the top point, the rocket descends back, with the deployment of a parachute.
The trajectory of the rocket is depicted in Figure C. The actual values of the altitude are provided in the file *Rocket.txt*, taken at time intervals of 100sec.

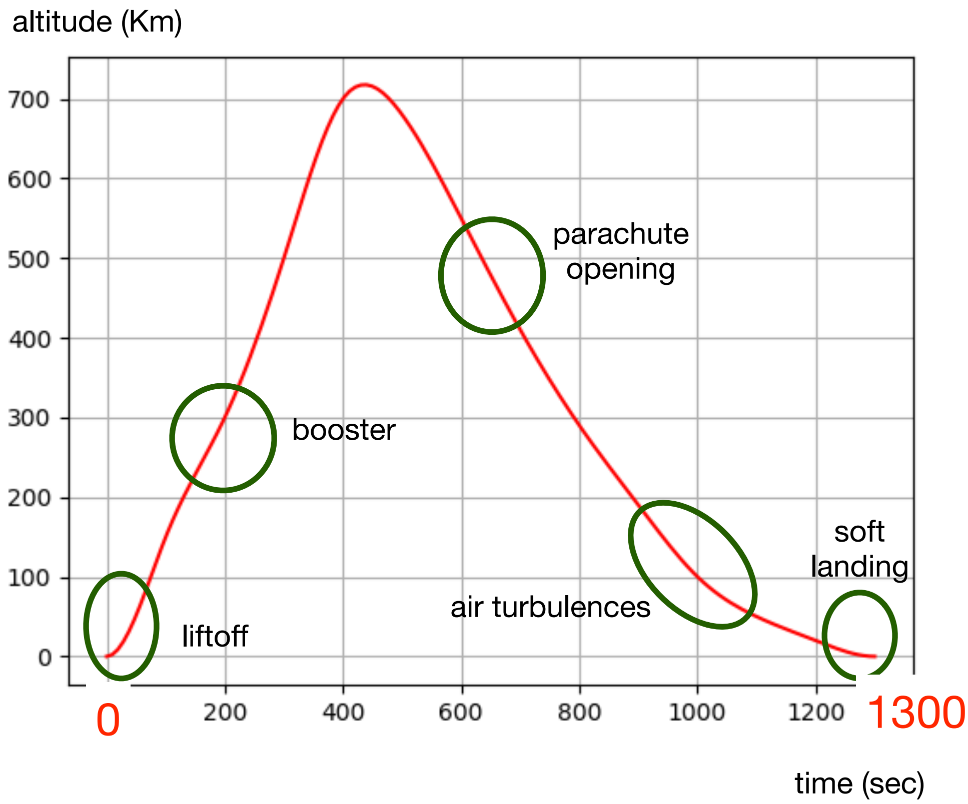

Figure C: Trajectory of a rocket

1. Compute the velocity and the acceleration of the rocket at the given nodal points, by using the function *Derivative* from Task B.

2. Interpolate, with splines, the nodal points (x_n, y_n), for 140 points over the same time domain. Recompute the velocity and the acceleration of the rocket at the interpolated points.
(You need to use splines to avoid the Runge's phenomenon. In fact, if you use Lagrangian interpolation, and you can verify this yourself, the interpolated trajectory will suffer of the Runge's problem).

Answer Quiz 3

Task D: Gauss integration

The Gauss-Legendre Quadrature nodes and weights are coded in file *Gauss.py*, into the two array variables *tg* and *wg*, for nodes $n = 1, 2, 3, 4, 5$.

1) Write a script to integrate a function $f(x)$ with Gauss quadrature.
Compute the integral:

$$\int_0^1 \frac{1}{1+x^2}\,dx$$