ME2 Computing- Tutorial 8: Complex numbers, complex functions and Fourier Transform

Learning outcomes:

- Being familiar with complex numbers and complex functions in Python
- Being able to plot Bode diagrams
- Being able to calculate and understand DFT and Inverse DFT
- Experiencing basic applications of Signal Processing

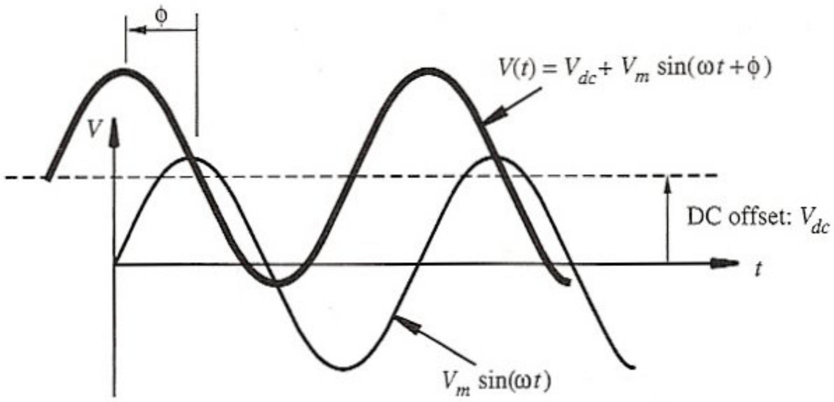
Before you start

In your H drive create a folder *H:\ME2CPT\Tutorial8* and work within it.

Task A: Complex numbers and phasors

i) Plot, in the range $t = [0{:}\,2\pi]$, a cosine wave with amplitude 10 and frequency $f = 0.5Hz$ and another cosine wave with same frequency, amplitude 5 and lagged by 90 degree.

$$y(t) = V_m sin(\omega t + \varphi)$$



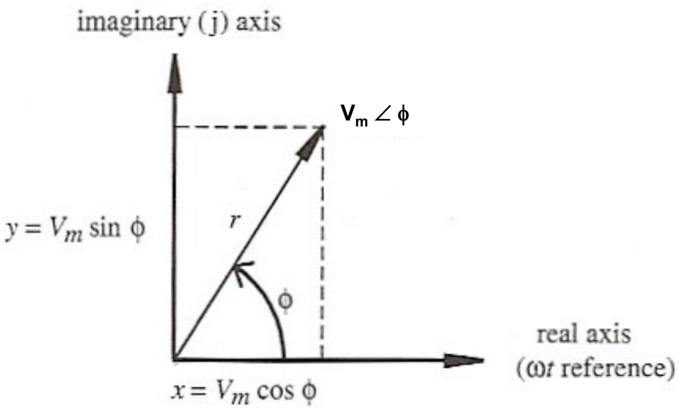
ii) Offset the second signal by 5 (DC component).

iii) Plot, in the range $t = [0{:}\,\pi]$, a cosine wave with amplitude 10 and frequency $f = 0.5Hz$ and another cosine wave with same amplitude, but double frequency.

iv) Plot, in the range $t = [0{:}\,\pi]$, a cosine wave with amplitude 10 and frequency $f = 0.5Hz$ and another cosine wave with same amplitude, double frequency and lagged by 45 degree.

v) Represent the two cosine waves in iv) with phasors and plot them in the complex plane.

$$y = V_m e^{j(\omega t+\varphi)}$$



vi) Add the two signals, both in time domain and as phasors. Plot the corresponding results.

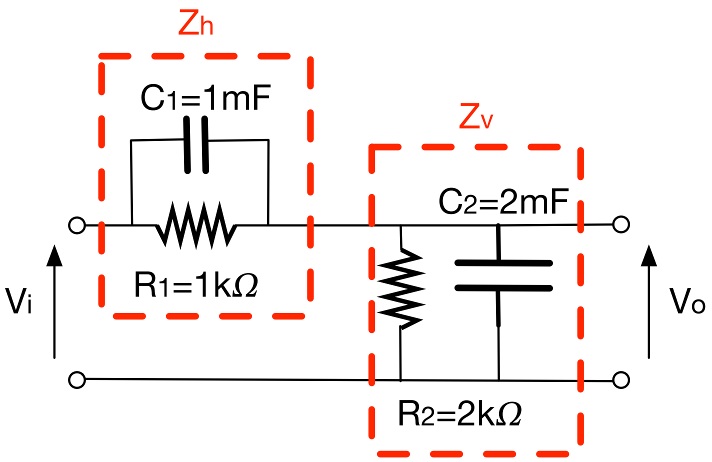
Task B: Complex functions: analogue filters and Bode plots

i) Consider the complex function of ω:

$$H(\omega) = \frac{1}{1 + j0.1\omega}$$

Plot the Bode diagram (with log scale x-axis), for both amplitude and phase, in the range $\omega = [0{:}\,10K]$.
Plot the Bode diagram (with log scale x-axis), with amplitude expressed in dB (decibel).

ii) Determine the gain function, $H(\omega)$, of this electronic linear circuit (make use of impedance concepts):



$$H(\omega) = \frac{v_o}{v_i} = \frac{Z_v}{Z_v + Z_h} = \frac{1}{1 + \frac{Z_h}{Z_v}} = \frac{1}{1 + \frac{Y_h}{Y_v}}$$

Plot the Bode diagram (with log scale x-axis), for both amplitude (in dB) and phase, in the range $\omega = [0.001\colon 10]$.

iii) Express all the components in the above circuit as impedances, in the range $\omega = [0.001\colon 10]$, and determine the numerical equivalent of $H(\omega)$.

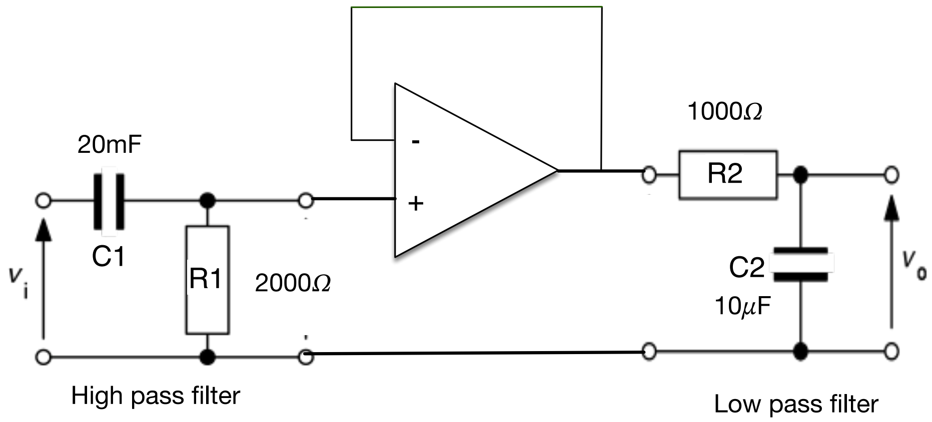
Plot the Bode diagram (with log scale x-axis), for both amplitude (in dB) and phase, in the range $\omega = [0.001\colon 10]$.

iv) Consider the two gain functions of a passive low pass filter and a passive high pass filter:

$$H_{LP}(\omega) = \frac{1}{1 + j0.01\omega}$$

$$H_{HP}(\omega) = \frac{j40\omega}{1 + j40\omega}$$

Cascade the two filters, decoupling them with an op-amp voltage follower buffer:



Plot the Bode diagram (with log scale x-axis), for both amplitude (in dB) and phase, in the range $\omega = [0.0001\colon 10000]$, for the two individual filters and the cascaded filter. Plot a point correspondingly to the corner frequencies of the two individual filters.

Task C: Fourier Series

i) Evaluate the Fourier series, representing a saw function, by using different numbers of terms, i.e., $N = [2,6,50]$:

$$y(t) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{N}\frac{sin\left(\frac{2n\pi}{T}t\right)}{n}$$

Plot the results in the range $t = [0: 2T]$, where T is the chosen period for the saw wave.

ii) Evaluate the Fourier series, representing a square function, by using different numbers of terms, i.e., $N = [2,6,50]$:

$$y(t) = \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{N} \frac{sin\left(\frac{2n\pi}{T} t\right)}{n}$$

Plot the results in the range $t = [0: 2T]$, where T is the chosen period for the square wave.

Task D: Discrete Fourier Transform

i) Write a function *DFT()*, that receives a set of numerical values, y_n,and returns another set of values, FT_k, as the Discrete Fourier Transform of y_n:

$$FT_k = \sum_{n=0}^{N-1} y_n e^{-\frac{2\pi jkn}{N}}$$

$$with\ k = 0, 1, 2, \dots N-1$$

ii) Write a function *DFTInv()*, that receives a set of numerical values, FT_k,and returns another set of values, y_n, as the Inverse Discrete Fourier Transform of FT_k:

$$y_n = \frac{1}{N} \sum_{k=0}^{N-1} FT_k e^{\frac{2\pi jkn}{N}}$$

$$with\ n = 0, 1, 2, \dots N-1$$

iii) Create a discrete function, in the range $t = [0: 6\pi]$, with the following cases:

a) $y_n = \sin(t_n)$
b) $y_n = \sin(t_n) + sin(3t_n)$
c) $y_n = \sin(t_n) + sin(3t_n) + sin(6t_n)$

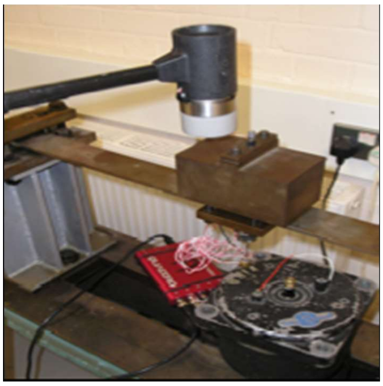
d) $y_n = \exp\left(-\frac{(t_n-5)^2}{0.5}\right)$
e) $y_n = \exp\left(-\frac{(t_n-5)^2}{4}\right)$

For each case, determine the Discrete Fourier Transform of $y_n(t_n)$, and plot it vs frequency.

iv) For each of the Discrete Fourier Transform found in iii), reconstruct the signal by computing the Inverse Discrete Transform.

Task E: Signal processing

i) The file *Vibration.txt* contains the temporal response of a vibrating beam, excited by a hammer bang, with vibrating signal sampled every 0.01 sec. Determine the resonant frequency of the beam.



ii) The file *Noisy.txt* contains a Gaussian signal, disturbed by a superimposed random noise. The signal has been sampled with a sampling rate of 20 sample per sec.
Apply a numerical filter to cut off the disturbing noise.