ME1 Computing- Session 9: Numpy and Matrices

Learning outcomes:

- Getting familiar with numpy library
- Being able to define and generate matrices
- Being able to compute basic matrix operations

Before you start

In your H drive create a folder *H:\ME1MCP\Session9* and work within it.

Task A: Use of numpy

1. Create a range of values *x* between -2.0 and 5.8 with step dx = 0.1.
2. Calculate the values $y = \sin x$ for the above range of x.
3. Plot *y* vs *x*.
4. Create a range of 100 values *x* between -2.0 and 5.8.
5. Calculate the values $y = \frac{e^x - e^{-x}}{2}$ for the above range of x.
6. Plot *y* vs *x*.
7. Generate an array *x* of numbers in the range *[-5 : 5]* with the following steps:

$$\Delta x = 0.5 \text{ in } -5 \le x \le -2$$
$$\Delta x = 0.05 \text{ in } -2 < x < 3$$
$$\Delta x = 0.5 \text{ in } 3 \le x \le 5$$

8. Compute the function: $y = e^{-\frac{x^2}{4}}$ in the above range.
9. Plot *y* vs *x*.

Answer Quiz 1

Task B: Defining and manipulating matrices

Write the following tasks into a new script:

1. Create a matrix **H** of zeros with dimensions 30 x 20

2. Insert values 50 to 69 into the 6^{th} row of **H**.

3. Insert values 100 to 129 into the 8^{th} column of **H**.

4. Generate a a square matrix **S**, of dimension N x N, with the following pattern:

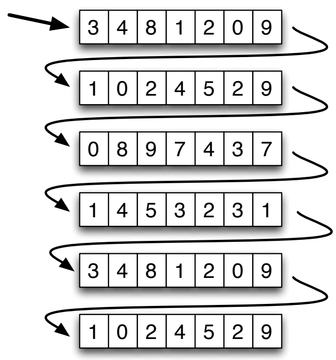
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Task C: Transpose of a matrix

1. Create a matrix **R** of random integer numbers between 1 and 100, with dimensions 10 x 5.
2. Write a function *Transpose*, that receives a matrix **R** and returns its transpose, i.e. **T = R**^{T} (pag. 96 of Maths notes).

Task D: Sum of two matrices

The files *MatA.tx*t and *MatB.txt* contain the values of two matrices, **A** and **B**, of size 60x60. Entries of the matrices are stored in the file one value per line, sequentially as:



1. Read in the numerical values from the two files and form the two matrices **A** and **B** accordingly.

Answer Quiz 2

2. Write a function *MatSum*, that receives two matrices, **A** and **B**, and returns the sum of them, i.e. **C** = **A** + **B** (pag. 91of Maths notes), where the generic element of matrix **C** is $c_{ij} = a_{ij} + b_{ij}$.

Answer Quiz 3

3. Compute the compute the matrix $\boldsymbol{D} = \frac{1}{2}(\boldsymbol{A} + \boldsymbol{A}^{\boldsymbol{T}}) + \frac{1}{2}(\boldsymbol{A} - \boldsymbol{A}^{\boldsymbol{T}})$, and verify, by printing that **D** is the same as **A**, (pag. 97 of Maths notes).

Task E: Matrix-matrix multiplication

Write a function, *MatMat*, that receives two matrices, **A** and **B**, and returns the product of the two matrices, **P** = **AB**, as defined in the Maths lectures (pag. 92), where the generic element of matrix **C** is $c_{ij} = \sum_{k=1}^{N} a_{ik} b_{kj}$.

The function should return the value 0 if the sizes of the two matrices are incompatible for the multiplication.

Verify that $\boldsymbol{AB} \neq \boldsymbol{BA}$**.**

Answer Quiz 4

Task F: Determinant of a matrix

To compute the determinant of a matrix, (pag. 105 of Maths notes) we proceed in two steps:

1) Write a function *Minor*, that receives a matrix A of dimension N x N, and two indices *i* and *j*.
   The function returns a matrix, of dimension (N-1) x (N-1), obtained by matrix A, after removing row *i* and column *j*.
2) Write a **recursive** function *Determinant*, that receives a matrix **A** and returns the value of its determinant, i.e. $|\boldsymbol{A}| = \sum_{k=1}^{N} a_{1k} A_{1k}$, where $A_{ij} = (-1)^{i+j} |M_{ij}|$ is the cofactor of a_{ij} and M_{ij} is the minor matrix of a_{ij}.

Answer Quiz 5

Task G: Inverse of a matrix

To compute the inverse of a matrix, (pag. 107 of Maths notes) we proceed in two steps:

1) Write a function *Adjoint*, that receives a matrix **A** of dimension N x N, and returns its adjointed matrix (*'the adjoint of a matrix is formed by replacing each element by its cofactor and transposing the result'*, cit. Maths notes).
2) Write a function *Inverse*, that receives a matrix A and returns its inverse, i.e. $\boldsymbol{A^{-1}} = \frac{1}{|\boldsymbol{A}|} adjoint(\boldsymbol{A})$.

Answer Quiz 6

Task H: System of linear equations

Given a set of linear equations, of your choice, in matrix form, $\boldsymbol{A} \cdot x = b$:

1) Determine the solution x, by inverting the matrix $\boldsymbol{A}$, i.e., $x = \boldsymbol{A^{-1}} \cdot b$
2) Determine the solution x, by applying Cramer's rule (pag. 108):
   $$x_j = \frac{|\boldsymbol{B}_j|}{|\boldsymbol{A}|}$$
   where the matrix $\boldsymbol{B}_j$ is obtained from matrix $\boldsymbol{A}$ by replacing the column *j* with array b.

Answer Quiz 7