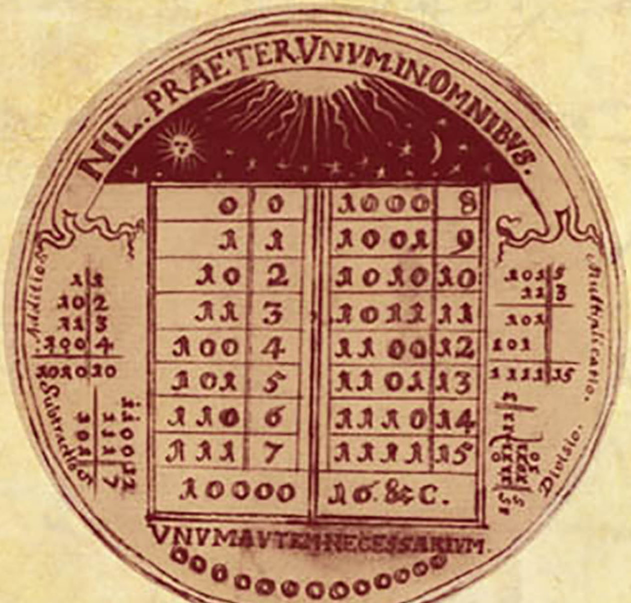
Leibniz on Binary



The Invention of Computer Arithmetic

Lloyd Strickland and Harry Lewis

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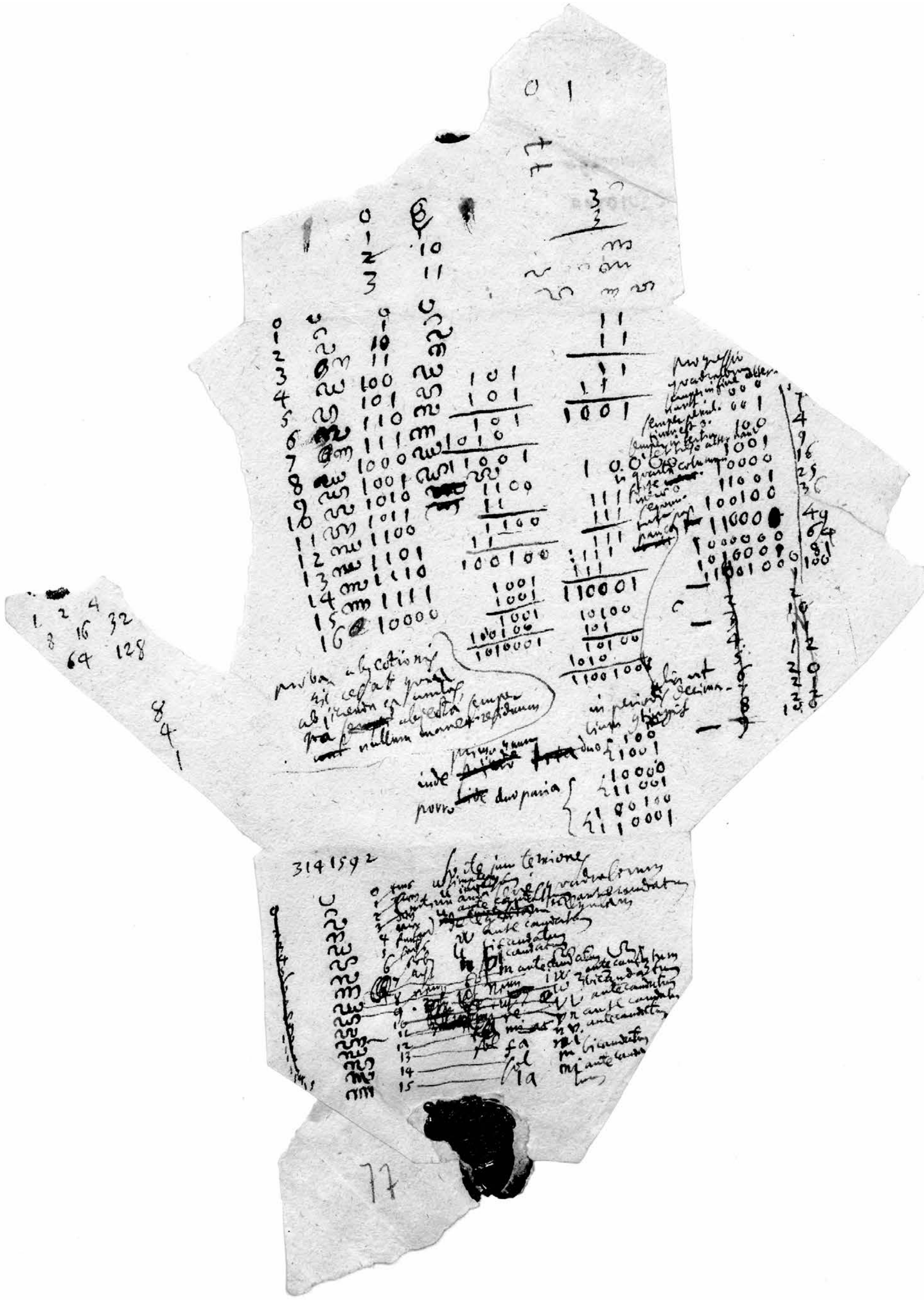

Manuscript of “Sedecimal on an Envelope” (LH 35, 3 B 5 Bl. 77).

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Lloyd Strickland and Harry Lewis

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This book is dedicated
by Lloyd to Vernon Pratt, mentor, friend, and *vir eximie*, as Leibniz would say.
As an indefatigable source of wisdom, inspiration,
and knowledge of where to find good desserts,
you are too good for this world.
by Harry to the memory of his father, Emil Harold Lewis,
who with the 16th General Hospital, from 1943 to 1945,
helped save his father's birthland, free his grandfather's,
and create the enduring Anglo-German-American friendship
of which this book is fruit.
Medio tutissimus ibis.

All programmers are optimists.
—Frederick P. Brooks Jr., *The Mythical Man-Month*

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Abbreviations

A	Leibniz (1923–). Cited by series, volume, and page or item number (N).
DSR	Leibniz (1992).
Dutens	Leibniz (1768).
GM	Leibniz (1849–1863). Reprinted as Leibniz (1977).
Gotha	Manuscript held by Forschungsbibliothek Gotha, Erfurt, Germany.
GP	Leibniz (1875–1890).
LBr	Manuscript held by Gottfried Wilhelm Leibniz Bibliothek, Niedersächsische Landesbibliothek, Hanover, Germany. Cited by shelfmark and Blatt [sheet].
LGR	Leibniz (2016a).
LH	Manuscript held by Gottfried Wilhelm Leibniz Bibliothek, Niedersächsische Landesbibliothek, Hanover, Germany. Cited by shelfmark and Blatt [sheet].
LTS	Leibniz (2011b).
PPL	Leibniz (1969).
SLT	Leibniz (2006b).
W	Leibniz (2006a).
WOC	Leibniz (1994).
Z	Zacher (1973).

Preface

It would be pointless to say that Mr. Leibniz was a mathematician of the first rank, [since] it is through mathematics that he is most generally known.
—Bernard le Bovier de Fontenelle (1718, 108–109)

Despite his original contributions to a host of disciplines, such as law, philosophy, politics, languages, and many areas of science, the fame and renown of the polymath Gottfried Wilhelm Leibniz (1646–1716) has always rested principally on his pioneering mathematical work, in particular his independent invention of the calculus in 1675. Another of his enduring mathematical contributions was his invention of binary arithmetic, though the utility of binary went unrecognized until it became the basis for today's world of digital computing and communications. And in a second flash of foresight, Leibniz also invented the other number system commonly used in computing, namely, base 16, or hexadecimal in modern parlance (Leibniz's own term for it was sedecimal). These two inventions are the focus of this book.

Leibniz's groundbreaking work in mathematics is all the more remarkable given that he did not begin serious study of the subject until he was in his mid-twenties. Born in Leipzig in 1646 to a professor of moral philosophy, Leibniz obtained his bachelor's and master's degrees in philosophy in 1663 and 1664, respectively, before turning to the law, in which he obtained a bachelor's degree in 1665 and a doctorate a year later. By the end of the 1660s, he was at the court of Mainz working alongside the diplomat Johann Christian von Boineburg (1622–1672) on reforming the legal code. In February 1672, Boineburg dispatched him on a diplomatic mission to Paris, where he met some of the foremost mathematicians of the day, most notably Christiaan Huygens (1629–1695). Leibniz later recalled that he had arrived in Paris with "a superb ignorance of mathematics" (GM III, 71), but this was soon rectified by Huygens's tutelage and his own insatiable thirst for inquiry. He initially focused on summing infinite series and the classical Greek problem of squaring the circle, and from his intensive work on both, he eventually arrived at the calculus.[1] In October 1675, he devised the symbols d and $\int$, which are still used today.

Despite his growing reputation as a mathematician, Leibniz was unable to secure a post in Paris. One year after his breakthrough with the calculus, and a little over four and a half years since his arrival in Paris, Leibniz departed for the northern German city of Hanover to take up the post of court counselor, a role he held for the remaining forty years of his life (though in the intervening years, his facility at moving within courtly circles enabled him to add roles in Wolfenbüttel, Berlin, and Vienna to his portfolio). Leibniz later explained that he stayed in Paris as long as he did in order "to stand out a little bit in mathematics" (A II 1, 753).

It was during his Paris years that Leibniz developed his lifelong interest in the automation of calculation. In the 1640s Blaise Pascal (1623–1662) had invented a machine capable of adding and subtracting, but Leibniz wanted to go further, proposing a machine capable of multiplying, dividing, and even (in his initial plans, in 1670, for a "new arithmetic instrument") extracting roots (LH 42, 5 Bl. 16v). His designs were refined over the next three years, but putting into practice his belief that

1. See Probst (2018).

wheels and cranks could perform the labors of the mind proved a challenge. A machine he demonstrated at the Royal Society in London in 1673 was, by all accounts, imperfect.[2] Further refinements were communicated to the craftsman tasked with building the device, but progress was slow. Leibniz several times reported that the machine had been completed and tested,[3] but these successes appear to have been short-lived at best, and it is doubtful that the machine ever worked reliably. Even in the last year of his life, a frustrated Leibniz was firing off instructions for fixing various faults, which he blamed on a craftsman's lack of diligence and precision.[4] Of the three versions of Leibniz's machine built in his lifetime, only the last survives. When it was rediscovered in 1879, Arthur Burkhardt (1857–1918), a leading mechanical engineer, was tasked with making it functional, but he was unsuccessful and concluded that it had probably never worked.[5] Nonetheless Burkhardt soon began manufacturing calculating machines incorporating some of Leibniz's ideas, and in the twentieth century, engineers were able to construct fully-functioning "replicas" of Leibniz's machine,[6] one of which is displayed at the G. W. Leibniz Bibliothek in Hanover.

The G. W. Leibniz Bibliotek also holds the vast majority of Leibniz's surviving writings, which total some 200,000 manuscript pages, a sizable proportion of which have yet to be published. These writings cover a wide variety of topics and disciplines, and range from completed books and journal articles to draft essays, rough jottings, and personal reading notes, as well as letters to and from more than a thousand correspondents. This material offers a window into Leibniz's prodigiously wide-ranging interests, ideas, and inventions, not to mention his various projects. Between 1680 and 1686, he sought to improve the productivity of the Harz silver mines by designing pumps and windmills to drain the floodwaters that made mining impossible after heavy rains.[7] In 1686, he took on the task of writing a history of the House of Guelph (or Welf) in order to enhance his employer's dynastic ambitions (the history was still not complete at the time of his death thirty years later, due to Leibniz's excessive thoroughness and many distractions).[8] From the early 1680s onward, he sought to facilitate a reunion between Catholics and Lutherans and, in the late 1690s, a reunion between Lutherans and Calvinists.[9] In 1700, he lobbied for the formation of the Kurfürstlich Brandenburgische Societät der Wissenschaften [Electoral Brandenburg Society of Sciences], and was immediately installed as its first president.[10] He drew up plans for a universal encyclopedia that would contain everything known, wrote Latin poetry, built a full philosophical system that would have great influence for centuries to come, and undertook pioneering studies on the origin of languages.[11]

Leibniz's scope, reach, and genius were such that he was described as "the ornament of Germany" by the Dutch scientist Herman Boerhaave (1715, 13). The last years of his life were overshadowed by the priority dispute with Sir Isaac Newton (1643–1727) over the invention of the calculus and his frantic but unsuccessful efforts to complete the history of the Guelph house. On 6 November 1716, he became bedridden by an attack of gout and arthritis, and he died on 14 November.

2. For further details, see Beeley (2020).

3. For example, he claimed in July 1674 that his arithmetic machine had been "successfully completed" (A III 1, 119) and in April 1695 that he had "finally had [it] completed" (LH 42, 5 Bl. 63r; English translation: Leibniz 1695a).

4. See Leibniz (1845).

5. See Morar (2015).

6. However, the "replicas" required a little technological license and modern engineering techniques to function. See Stein et al. (2006) and Kopp and Stein (2008).

7. For further information, see Wakefield (2010).

8. For further information, see Antognazza (2018).

9. For further information, see Jordan (1927).

10. For further information, see Rudolph (2018). The institution survives today as the Berlin-Brandenburg Academy of Sciences and Humanities.

11. For additional information on Leibniz's life and work, see Aiton (1985), Belaval (2005), and Antognazza (2009).

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Over the course of writing this book, many debts of gratitude have been incurred, which we are pleased to be able to record here:

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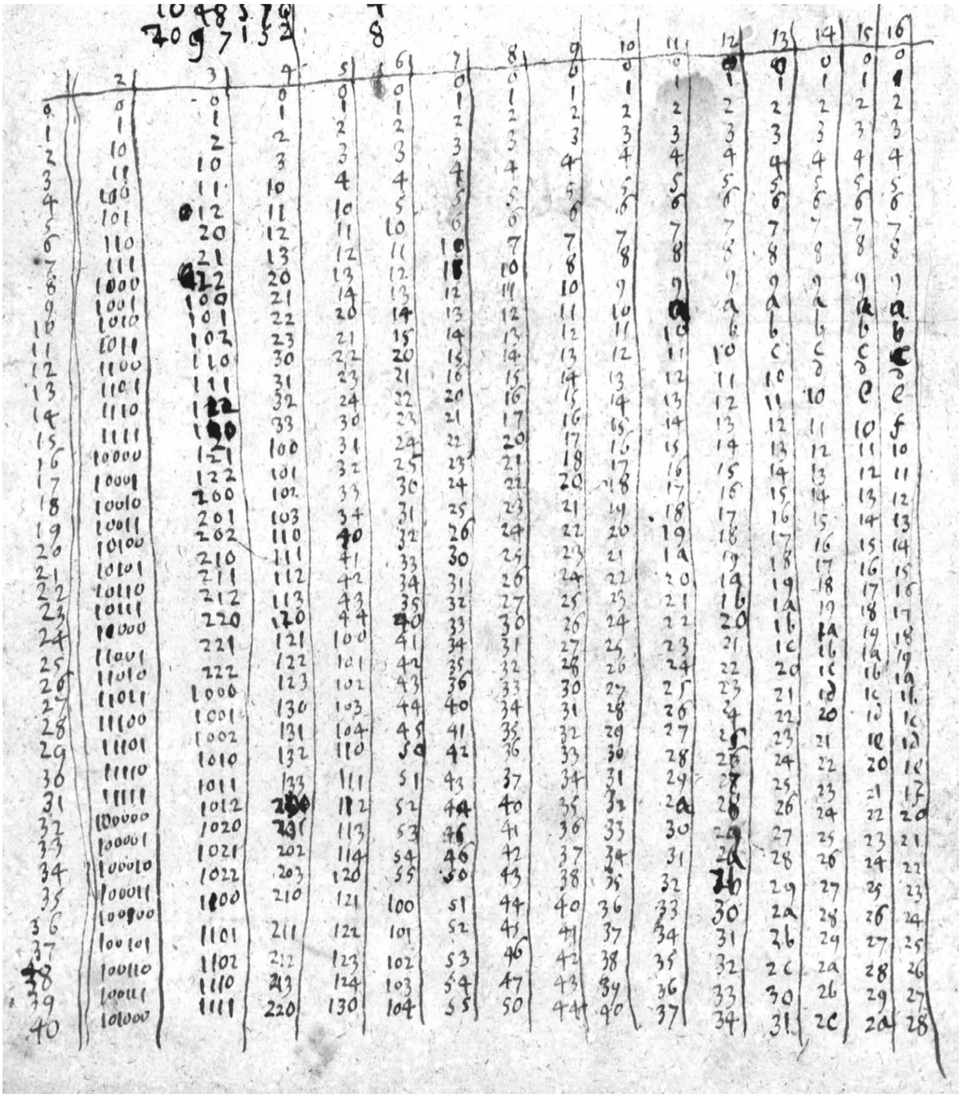


Untitled manuscript from 1703, featuring a table of decimal numbers 0–40 represented in every base from 2 to 16 (LH 35, 3 B 11 Bl. 11v). Note Leibniz's use of the Roman letters a, b, c, d, e, and f for the six extra digits, anticipating the modern convention.

Introduction

Leibniz and Binary: An Overview

Behind every great human invention lies a story. The oft-told story of Leibniz's binary system, a great invention if ever there was one, goes like this:

> Leibniz invented the binary system in 1679, first outlining it in a three-page manuscript written on 15 March of that year. Binary then featured in a couple more texts written in 1679 and was next revisited almost twenty years later, in 1696, when he happened to mention the binary representation of all numbers by 1 and 0 to Duke Rudolph of Brunswick and Lüneburg, who (so the story goes) saw therein an analogy with the Christian doctrine of creation *ex nihilo*, according to which all things were created from nothing by the one God. Excited by its theological potential, in 1697, Leibniz began sending details of the binary system to Christian missionaries in China, hoping that the theological analogy would assist them in converting the Chinese. One such missionary, Joachim Bouvet, was struck by a parallel between binary notation and the hexagrams of the ancient Chinese divinatory system, the Yijing. Each of the sixty-four hexagrams consists of a stack of six horizontal lines, each either broken (a yin) or unbroken (a yang), and by equating the unbroken line with 1 and the broken line with 0, Bouvet suggested that the hexagrams might be nothing more than Leibniz's binary system in alternative notation. Leibniz required little convincing and was so excited by the idea that he had unlocked a piece of ancient Chinese wisdom that he decided to make his invention public without further delay. Within a week he had sent a short paper—"Explanation of Binary Arithmetic"—to the French periodical *Memoires de l'Académie Royale des Sciences*, which published it in 1705.

This story—developed, polished, and rehearsed often over the past fifty years—contains much that is true, at least the part from 1697 onward. The rest is what we might call a work of commentators' fiction, dominated by errors and omissions. Most notably, it says nothing about the most striking by-product of Leibniz's work on binary, namely, his invention of the base-16 number system and experimentation with a variety of symbols to represent what today are called the hexadecimal digits. And this almost 200 years before John W. Nystrom (1825–1885) developed and promoted the base-16 system in a series of articles. The reason the standard story is so distorted is that Leibniz's 200 or so writings on binary remain understudied; even 300 years after his death relatively few have even been published. The thirty-two writings collected in this volume represent the deepest dive to date into Leibniz's work on binary, and together, they tell a story quite different from that sketched above. To provide an overview of the texts in this volume and how they flesh out and correct the oft-told story of Leibniz and binary, it is best to go back to the start of the story or, rather, to what is often believed to be the start.

The earliest dated writing on the binary system in Leibniz's Nachlaß (that is, the collection of manuscripts left after his death) is "On the Binary Progression" (chapter 6), written on 15 March 1679 (that is, 25 March 1679 in the modern Gregorian calendar). The complete manuscript, a remarkably detailed and sophisticated investigation into binary, is divided into two parts. In part I, Leibniz presents tables of binary numeration, offers algorithms for converting decimal numbers into binary, outlines the four basic arithmetic operations in binary, describes how these operations could be carried out by a calculating machine quite different in design from the decimal calculating machine on which he had been working since the early 1670s, and explores the square of a six-digit binary number. In part II, which is more experimental, Leibniz attempts to extract the square root of an eight-digit binary number and examines the repeating binary representation of fractions. The essay closes with rumination about whether arbitrary numbers are expressible using algebraic rules, or whether, to the contrary, some numbers lie beyond such finite control. Given its importance to the history of the binary system and of number systems generally, it is surprising that "On the Binary Progression" is still so little known, or at least known in its entirety.[1] The complete manuscript consists of four folio (i.e., 2^{o}) sheets, that is, eight pages in total, of which seven were used for the text itself.[2] Of these, only the first three are known to scholars due to an unfortunate oversight of previous editors. Hochstetter (1966, 42–47) published a facsimile of only the first three pages (i.e., $1\frac{1}{2}$ of the four folio sheets) along with a German translation. An Italian version of Hochstetter's book followed five years later (see Leibniz 1971), featuring again a facsimile of the first three pages, but this time with an Italian translation. Then, four decades later, Serra (2010b) published a facsimile of the same three pages followed by a French translation. Curiously, no transcription of the original Latin has yet been published,[3] only facsimiles of the first three pages of the manuscript along with translations of those three pages into various European languages. This has created the misleading impression that "On the Binary Progression" consists of only those three pages, which is in fact less than half of the entire text.[4] Our translation in chapter 6 thus marks the first publication of the entire text in any language.

Although "On the Binary Progression" is Leibniz's earliest *dated* writing on the binary system, is it really likely to be his *earliest writing* on the subject, as many scholars have claimed?[5] Arguably, no: given how well developed are the ideas in that text, it seems much more likely that it was preceded by other, more preliminary studies, over the course of which Leibniz developed his understanding of binary to the high standard found in "On the Binary Progression." Surprisingly, at the time of writing, only one attempt has been made to establish how the binary system developed in Leibniz's hands, almost fifty years ago by Hans Zacher, although his studies are partial and incomplete. Zacher (1973, 10–11) did, however, discover "Notes on Algebra, Arithmetic, and Geometric Series" (chapter 1), at the bottom of which Leibniz wrote this:

The binary progression

0	1	2	3	4	5	6	7	8	0	0	0	0	0
0	1	10	11	100	101	110	111	1000	.		.		.

$$\begin{array}{r} 111 \\ \underline{101} \\ 111 \\ \underline{1110\,.} \\ 100011 \end{array}$$

The manuscript itself has now been dated to October 1674, which places it during Leibniz's four-year stay in Paris (from March 1672 to October 1676), a time when he studied alongside leading

1. Scholars have only just started to explore part II of "On the Binary Progression"; see Brancato (2021).

2. The remaining page contains the wage tables for the Harz silver mines for the final quarter (quarthall luciae = quarter of St. Lucy) of 1678. Between 1680 and 1686, Leibniz attempted to build wind machines to drain the mines, but his involvement with the mines began in the fall of 1678. See Wakefield (2010).

3. One will eventually appear in series 7 (devoted to Leibniz's mathematical writings) of *Sämtliche Schriften und Briefe* (abbreviated throughout this book as A).

4. See, for example, Serra (2010a).

5. See, for example, Mungello (1977, 51–52), Pappas (1991, 125), Ryan (1991, 31), Antognazza (2009, 247 and 275n255), Bauer (2010, 14), and Ingaliso (2017, 112n12). Implicit in the claim seems to be the implausible suggestion that Leibniz thought of the binary system, and developed it to the high standard found in that text, in a single day. Despite Leibniz's undoubted genius, one can find no other example in his writings of him having an idea and developing it to a high degree of sophistication in a single day. It is therefore much more likely that Leibniz first had the idea for binary numeration before "On the Binary Progression" and then developed it.

mathematicians such as Christiaan Huygens and made a number of important mathematical advances, among them the invention of the infinitesimal calculus.[6] Could binary be another of Leibniz's mathematical inventions from his Paris years? Such a conclusion is difficult to draw with absolute certainty, since the material on binary in "Notes on Algebra, Arithmetic, and Geometric Series" was written in different ink from everything else on the sheet and therefore might have been added later, with no way to tell whether this was later the same day or days, weeks, months, or even years later. However, given that the manuscript consists of rough notes on undated scrap paper, it seems unlikely that Leibniz would return to it a long time after to add the material on binary, making it reasonable to suppose that the material does date from the latter months of 1674. (As we shall see later in this introduction, around this time, Leibniz was also working on a different number system, the base-60 or sexagesimal system; see LH 35, 3 A 26 Bl. 17 and LH 35, 8, 30 Bl. 27.)

The only other candidate text identified by Zacher as possibly written prior to "On the Binary Progression" is "Thesaurus mathematicus" [Mathematical Thesaurus], a wide-ranging piece covering various topics in arithmetic, geometry, and mechanics.[7] In a passage near the end, Leibniz first outlines how positional notation works in the decimal and duodecimal number systems before identifying binary as an alternative:

> From this outline it is clear that only these ten digits are needed: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Those in the first position signify the equivalent number of 1s, namely no 1s, one 1, two 1s, three, four, five, six, seven, eight, nine 1s. Those in the second position signify the equivalent number of 10s, that is, 1s taken ten times; in the third position, the equivalent number of 100s, that is, 10s taken ten times, or the squares of 10; in the fourth position, the equivalent number of 1000s, that is, 100s taken ten times, or the cubes of 10, and so on. And in place of 10 one would be able to put any other number, for example, 12. For just as when the base a is 10, the square a^2 signifies 100 and the cube a^3 signifies 1000, so when a is 12, a^2 will be 12 times 12, that is, 144, and a^3 will be 12 times 144. But on this method, instead of the digits mentioned above—0, 1 etc. 9—two new digits would be needed in addition, one which would represent ten, the other which would represent eleven; but [the digits] 10 would signify twelve, and 100 would signify one hundred and forty four. And there are some who prefer to use this method of calculating over the common method, because 12 can be divided by 2, 3, 4, and 6; in addition, a calculation is completed with fewer digits. But the difference is not so great as to be worth abandoning the decimal progression. If anyone should want to use the binary progression, he would not need any digits except 0 and 1. Hence he would not need the Pythagorean table,[8] of which [we shall speak] later. But the calculation would be longer, albeit easier.[9] (LH 35, 1, 25 Bl. 3v)

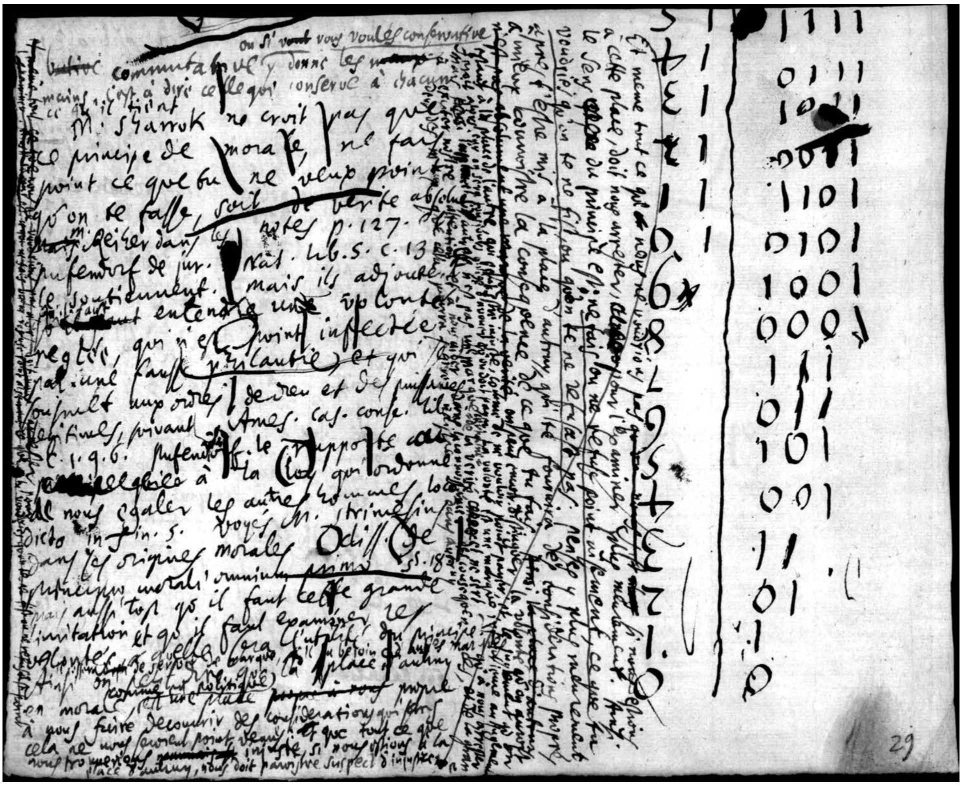
On the basis of its watermark, "Thesaurus Mathematicus" was likely written in 1678 or 1679, making it one of Leibniz's earliest writings on binary numeration and possibly one that predates "On the Binary Progression."

6. Leibniz's later pronouncements do not enable us to date his invention of binary with any greater accuracy. In 1698, he told Johann Christian Schulenburg that he had been thinking about binary for "twenty years and more" (A II 3, 451), and in 1703, he explained to Joachim Bouvet that his ideas about binary went back "well over 20 years" (chapter 29). Both claims loosely point to Leibniz first having the idea during the latter half of his stay in Paris or shortly afterward.

7. In fact, Zacher (1973, 11, 18–19) did identify one other manuscript that he believed was written prior to 1679. The manuscript in question contains some rudimentary material on binary (namely, a table showing the values of the decimal numbers 0–8 in binary along with some binary sums), leading Zacher to claim that it was probably from 1676 and therefore from Leibniz's Paris years. However, the manuscript, a set of notes recording mathematical ideas made during a meeting with the Dutch mathematician Johann Jakob Ferguson (1630–1706), has since been dated to spring 1680 and is therefore not one of Leibniz's earliest forays into the binary system at all. See A III 3, 138.

8. The Pythagorean table is a simple multiplication table, which in Leibniz's time went up to 9×9.

9. In a marginal comment, Leibniz added, "We shall say more things about the binary progression below," though this promise is not fulfilled and the text says nothing further on the subject.



Reverse manuscript page of “The Place of Others” (LH 34 Bl. 29r).

Although not mentioned by Zacher, various other texts may have been written prior to “On the Binary Progression.” For example, after having written a short text entitled “La place d’autruy” [The Place of Others] about the importance of putting ourselves in the place of others, Leibniz added in the empty margin of the reverse manuscript page a table of binary numbers from 0 to 15, while on top of his words of the text, he scrawled these two sums in binary (see LH 34 Bl. 29r; manuscript reproduced above).

$$\begin{array}{r} 11 \\ 1 \\ \hline 100 \end{array} \qquad \begin{array}{r} 101 \\ 1 \\ \hline 110 \\ 1 \\ \hline 111 \end{array}$$

Another text, consisting of half a page torn from a larger sheet, finds Leibniz using division to work out the binary representation of the fractions $\frac{1}{13}$ and $\frac{1}{17}$ (see LH 35, 13, 2B Bl. 155). Unfortunately, neither of these two manuscripts bears a watermark, nor do they contain any internal evidence that would enable a reliable dating. However, their embryonic treatments of binary suggest that they were written earlier than “On the Binary Progression,” though how much earlier is impossible to determine. Nevertheless, they probably capture some of Leibniz’s earliest explorations of binary, since one might reasonably expect his initial ventures into binary to take the form found in these manuscripts, namely, tables of binary numbers, simple binary calculations, and brief descriptions of key features of binary numeration.

Four other texts that likely capture Leibniz’s early exploration of binary—or at least earlier than “On the Binary Progression”—are included in this volume. In “The Series of All Numbers, and on Binary Progression" (chapter 2), Leibniz outlines binary notation and gestures at algorithms for the basic arithmetic operations on binary numbers, while in "Binary Progression" (chapter 3), he calculates the binary representations of various fractions, such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and so on. In "Geometric Progressions and Positional Notation" (chapter 4), Leibniz notes a property of the double geometric progression (1, 2, 4, 8, 16, 32, and so on) at the heart of binary numeration, namely, that the sum of any three consecutive terms is always divisible by 7; in a passage that was subsequently deleted, he also looks to binary numeration for insight into perfect numbers (positive integers equal to the sum of their divisors other than themselves). And last, in "Binary Arithmetic Machine" (chapter 5), Leibniz sketches out a design for a binary calculating machine. Since the design was based on his preexisting decimal calculating machine, this text likely captures Leibniz's first thoughts on how to mechanize binary calculations, with the very different design outlined in "On the Binary Progression" capturing a later, more considered view. All four of these texts were undated and bear no watermarks, but they probably precede "On the Binary Progression" (15/25 March 1679), since the latter contains similar ideas in more developed form.

Other texts in this volume shed new light on Leibniz's work on binary after "On the Binary Progression." Heretofore, Leibniz was known to have touched on binary only twice between 1679 and the mid-1690s. In "On the Organon or Great Art of Thinking" (chapter 10), he fuses mathematics and philosophical theology by drawing a parallel between the binary system, in which all numbers derive from 1 and 0, and the idea that in the universe, there may be only one thing conceived through itself, namely, God, aside from whom there is nothing (privation). In his other previously known early text containing material on binary, "Summum calculi analytici fastigium" [The Highest Peak of Analytical Calculation], he observes that when an arithmetic progression is expressed in binary, the digits in any given column recur periodically. For example, the decimal sequence of odd numbers 1, 3, 5, 7, 9, 11, 13, 15, and 17 is expressed in binary thus:

$$\begin{array}{c} 00001 \\ 00011 \\ 00101 \\ 00111 \\ 01001 \\ 01011 \\ 01101 \\ 01111 \\ 10001 \end{array}$$

Leibniz notes that "while the digits of the first [column counting from the right] are formed only of 1s, the digits of the second [column] are alternately 0 and 1, of the third alternately 00 and 11, of the fourth are alternately 0000 and 1111, the digits of the fifth are 0000 0000 and 1111 1111, and so on in a geometric progression"[10] (LH 35, 13, 3 Bl. 21v; Zacher 1973, 223). The column periodicity of binary numbers would prove to be a source of fascination for Leibniz in the decades that followed, convinced as he was that the binary system, by returning to first principles (namely, 0 and 1), could reveal mathematical truths inaccessible by other means.

Such confidence did lead to some early missteps, however. For example, in "Attempted Expression of the Circle in Binary Progression" (chapter 7), Leibniz seeks to convert into binary his alternating convergent series for the quadrature, i.e., $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$ etc., albeit without success, while in "Binary Ancestral Calculations" (chapter 11), he uses the double geometric progression and binary notation to try to determine the number of ancestors a person alive today would have from 3000 years ago, again unsuccessfully. Even more ambitiously, in an untitled text from August 1680, Leibniz attempts to elicit an algorithm for determining prime numbers from examples of the periodicity of binary fractions, the failure of the attempt sitting uneasily with the boast found in the opening

10. That is, the length of each column's pattern is twice that of its neighbor to the right.

line of the text: "I am gradually obtaining access to the innermost secrets of numbers" (LH 35, 13, 3 Bl. 33).[11]

Much more fruitful was Leibniz's invention of the other number system that has come to prominence in our computer age, namely, base-16, or sedecimal in Leibniz's terminology (hexadecimal in modern parlance). Although long believed to be an invention of the engineer John W. Nystrom in the mid-nineteenth century, Leibniz's first writing on the subject dates from 1679, with others following shortly thereafter. In "Sedecimal Progression" (chapter 8), Leibniz develops an algorithm for converting decimal numbers to sedecimal and offers two distinct ways of completing the sedecimal character set by identifying the six extra digits 10–15, one using the names of musical notes—ut, re, mi, fa, sol, la—and the other using the Roman letters m, n, p, q, r, and s (today, the standard is A, B, C, D, E, and F). In the shortest text in this volume, "Binary Progression Is for Theory, Sedecimal for Practice" (chapter 9), Leibniz develops novel symbols for all sixteen sedecimal digits, and a different set of symbols in "Sedecimal on an Envelope" (chapter 12, see also frontispiece), in which he also identifies the binary equivalents of sedecimal digits, thus linking together his two chief inventions in number theory. Although between them, these three early writings on sedecimal could comfortably fit on two or three typeset pages, this should not be taken to imply a lack of interest on Leibniz's

11. The pieces in this volume are focused mainly on the significance of binary notation and its close relative sedecimal, patterns in sequences of binary numbers, and how to carry out arithmetic calculations in binary or sedecimal. Elsewhere, Leibniz used the simplicity of the binary system to seek out more general mathematical truths. In a 1680 manuscript (LH 35, 13, 3 Bl. 33–34), for example, he proved a restricted version of a famous theorem of Euler from the next century, that $a^{\phi(n)} \equiv 1 \pmod{n}$ whenever a and n are coprime ($\phi(n)$ being Euler's totient function, the number of positive integers less than n which are coprime with n). Leibniz's special case is that for any odd n, there is a k such that $2^k \equiv 1 \pmod{n}$. (Leibniz does not mention the obvious fact that n must be odd or that similar restrictions apply when the base is 10 or another number.)

To derive his result, Leibniz begins by reasoning that a fraction such as $\frac{1}{n}$ in any base is periodic, since when n is divided into $1.000\ldots$, the division to produce a digit of the quotient results in a remainder $<n$, which is multiplied by the base (by appending a 0) to begin the next iteration. When the value of the remainder repeats, the subsequent digits of the quotient will repeat. If the base is 2 and the denominator is $2^k - 1$ for some $k > 0$, then

$$\frac{1}{2^k - 1} = \sum_{i=1}^{\infty} 2^{-ik},$$

which has a 1 bit each k bit positions starting with the kth to the right of the binary point and 0s elsewhere.

Now (continuing with the base 2 case), let n be any odd integer. Suppose $\frac{1}{n}$ consists of an initial block of bits $c_1 \ldots c_\ell$ ($\ell \geq 0$) immediately to the right of the binary point followed by indefinite repetitions of a block $b_1 \ldots b_k$ ($k > 0$). The binary sequences $c_1 \ldots c_\ell$ and $b_1 \ldots b_k$ represent nonnegative integers $C < 2^\ell$ and $B < 2^k$. Then

$$0.b_1 \ldots b_k b_1 \ldots b_k \cdots = B \times 0.\overbrace{00\ldots01}^{k}\overbrace{00\ldots01}^{k}\cdots = B \times \sum_{i=1}^{\infty} 2^{-ik} = B \times \frac{1}{2^k - 1},$$

and, since multiplying by $2^{-\ell}$ shifts bits ℓ positions to the right,

$$\frac{1}{n} = 2^{-\ell}\left(C + \frac{B}{2^k - 1}\right).$$

Therefore

$$\frac{2^\ell(2^k - 1)}{n} = C(2^k - 1) + B, \text{which is an integer.}$$

Since n, being odd, has no factor in common with 2^ℓ, n must divide $2^k - 1$. Hence, Leibniz's "wonderful theorem" that for any odd n, some multiple of n is of the form $2^k - 1$. And in fact, the nonrepeating initial segment $c_1 \ldots c_\ell$ is empty, that is, shifting the bits of $\frac{1}{n}$ left by k positions and taking the fractional part results in $\frac{1}{n}$ once again. For let A be the integer $\frac{2^k-1}{n}$, so that $2^k = An + 1$. Multiplying $\frac{1}{n}$ by 2^k shifts the bits left k positions, but $2^k \cdot \frac{1}{n} = (An + 1) \cdot \frac{1}{n}$, of which the fractional part is $\frac{1}{n}$. (And therefore $B = A$.)

A direct way of finding a multiple of odd n of the form $2^k - 1$ considers the n remainders when $2^1, 2^2, \ldots, 2^n$ are divided by n (Astin 1984). There can be at most $n-1$ different remainders, since a remainder of 0 is impossible when dividing an even number by an odd. Then if $1 \leq i < j \leq n$ and dividing 2^i and 2^j by n leave the same remainder, then $2^j - 2^i = 2^i(2^{j-i} - 1)$ is divisible by n, so letting $k = j - i$ yields the desired result.

part. In fact, Leibniz would return occasionally to sedecimal, developing a third form of notation for sedecimal digits in a 1701 letter to Joachim Bouvet (chapter 22), a fourth way of completing the character set using the Roman letters a, b, c, d, e, and f in an untitled text from 1703 (see LH 35, 3 B 11 Bl. 11v, reproduced facing the introducton of this book), and recommending the sedecimal system over the decimal for any future reform of common practice (see chapters 13, 22, and 30).

Contrary to the popular account that Leibniz's investigations into binary hit a long barren patch after "On the Binary Progression," the texts in this volume show that Leibniz filled many pages with such investigations between 1679 and the early 1680s. Curiously, however, he appears to have said relatively little about it to others during this time. As far as we can tell, he first mentioned the binary system to the Dutch mathematician Johann Jakob Ferguson (1630–1706) during a meeting in spring 1680; near the end of Leibniz's notes from that meeting, there is a table showing the values of the decimal numbers 0–8 in binary along with some binary sums (see A III 3, 138). In writing, Leibniz first mentioned the binary system in a letter to the German mathematician Detlev Clüver (1645–1708) of 31 August/10 September 1680: "I have quite often fruitfully used binary logistics of progression in which there occur no other digits besides 0 and 1, nor is that for common use but for the sake of theory, and so it does not seem the common method of counting should be disturbed without good reason" (A III 3, 263). Less than two years later, Leibniz mentioned the binary system to another mathematician acquaintance of his, Ehrenfried Walter von Tschirnhaus (1651–1708), in a letter written at the end of June 1682: "The binary progression would be particularly useful for expressions of quantity in numbers, because it is basic and simplest, and I do not doubt that there would be many harmonies to be found therein, not likewise to be found in other progressions" (A III 3, 655–656). Although Leibniz failed to stimulate any interest in binary in any of these interlocutors, this would eventually change in 1696.

In May of that year, Leibniz explained the binary system in person to Rudolph August, Duke of Brunswick and Lüneburg (1627–1704), or at least explained enough of it to draw an analogy between the representation of all numbers by 1 and 0 and the theologically orthodox doctrine of creation *ex nihilo*, that is, the creation of all things out of nothing by (one) God. While it is often claimed that the binary-creation analogy was the duke's idea,[12] the analogy appears in "Remarks on Weigel" (chapter 13), a text Leibniz wrote at least a year before his discussion with the duke. In that text, Leibniz makes a series of critical observations on the work of his old mathematics tutor, Erhard Weigel (1625–1699), who had advocated the adoption of a quaternary, or base-4, system some years before. After noting Weigel's preference for base 4, Leibniz insists that binary is superior for matters of theory and sedecimal for matters of practice, before outlining the binary-creation analogy he would later relate to Duke Rudolph August.

By all accounts, the duke himself was delighted with the analogy and this prompted Leibniz to draft a short paper outlining the basics of the binary system (chapters 15 and 16), along with a short covering letter (chapter 14). At the start of 1697, Leibniz wrote again to the duke, this time suggesting that the binary-creation analogy be commemorated in the form of a medal, to be struck at the duke's command (chapter 17). Although no such medal was ever struck, Leibniz later learned that the duke had commemorated the binary-creation analogy in a different way, by having a set of wax seals designed (chapter 18). Now convinced of the theological value of the binary-creation analogy, particularly its potential to illuminate the doctrine of creation *ex nihilo*, in early 1697 Leibniz wrote to a Jesuit missionary in China, Claudio Filippo Grimaldi, presenting the analogy and some of the mathematics underpinning the binary system (chapter 19). Undeterred by the lack of reply, in 1701, he repeated the attempt with another Jesuit missionary in China, Joachim Bouvet (chapter 22).

The discovery of the binary-creation analogy, and the duke's royal approval, prompted Leibniz to communicate the binary system to a series of mathematical acquaintances from the late 1690s onward. A recurring theme in this correspondence was the column-periodicity of number sequences expressed in binary. In truth, this had exercised Leibniz for some time, though with little progress to show for the many manuscript pages he had devoted to it. However, there were some occasional

12. See Zacher (1973, 36–37), Aiton (1985, 206), and Antognazza (2009, 357).

advances, such as his discerning in “Periods” (chapter 20) the “general rule” that the first half of any sequence of digits of a period is the complement (i.e., the opposite) of the second half. Realizing that he could devote insufficient time to determining other such rules, Leibniz attempted to enlist other mathematicians, such as Philippe Naudé (1654–1729), by outlining what he had achieved thus far (chapter 21). His only success in this regard was with Pierre Dangicourt (1664–1727), who composed an essay on the topic in 1701, “On the Periodic Intervals of Binaries in the Columns of Natural Numbers” (A III 8, 534–545).

Eventually, Leibniz cast his net wider than his own mathematical acquaintances. In early 1701, he composed a treatise entitled “Essay on a New Science of Numbers” for the Académie Royale des Sciences [Royal Academy of Sciences] in Paris, which had elected him as a foreign member the previous year (chapter 23). By his own admission, the mathematically rich treatise, which captured the key advances he had made thus far, was designed to inspire other members of the Académie to develop binary even further. In this it did not succeed, and a disappointed Leibniz eventually declined the Académie’s offer to publish the paper in its annual proceedings, the *Histoire de l’Académie Royale des Sciences* [*History of the Royal Academy of Sciences*]. However, later in 1701, he did find the time to make some further advances himself. In “Binary Addition” (chapter 24), he developed a simplified method for adding together many binary numbers. His investigations into the column periodicity of binary numbers led him to experiment with simplified notation for long strings of digits in “Periods and Powers” (chapter 26), though ultimately the notation was abandoned. In “Periods in Binary” (chapter 25), he determined that a “summatrix” column, formed by summing initial terms from another column, is itself periodic. He formalized this finding in “Demonstration That Columns of Sequences Exhibiting Powers of Arithmetic Progressions, or Numbers Composed from These, Are Periodic” (chapter 27), which marked the last of Leibniz’s mathematical advances in binary.

Perhaps the most intriguing chapter in the Leibniz-binary story began on 1 April 1703, when Leibniz received a response to the letter he had sent the Jesuit missionary Joachim Bouvet almost two years before. Bouvet was a long-time student of the ancient Chinese divination system, the Yijing, or Book of Changes, and upon reading Leibniz’s account of binary arithmetic, he was struck by a correlation between Leibniz’s binary numeration and the sixty-four hexagrams of the Yijing. Each hexagram consists of six stacked horizontal lines, each line being either unbroken (representing yin) or broken (representing yang), and by construing the unbroken line as 1 and the broken line as 0, Bouvet concluded that the hexagrams were a form of binary numeration (chapter 28). Thrilled by the idea that binary was the key to deciphering an ancient Chinese enigma that had baffled scholars for millennia, Leibniz wrote an enthusiastic reply (chapter 29) in which he floated the suggestion that the hexagrams, understood as an encoded binary system, may also have been intended to represent the creation of all things from nothing by God, giving a new twist to an idea he had entertained since the mid-1690s. Buoyed by Bouvet’s hypothesis that binary was the key to deciphering the secrets of the hexagrams of the Yijing, Leibniz wrote a new paper for the Académie Royale des Sciences to replace the one he had withdrawn. This new paper, “Explanation of Binary Arithmetic” (chapter 30), opens with an introduction to the binary system, with Leibniz explaining binary notation, giving examples of all four standard arithmetic operations, and presenting the periodicity in the columns of natural numbers expressed in binary as evidence of the wonderful order to be found in his new calculus. The paper then turns, about halfway through, to outlining Bouvet’s hypothesis that the binary system was foreshadowed in the sixty-four hexagrams of the Yijing.

“Explanation of Binary Arithmetic” was eventually published by the Académie Royale in 1705, but despite Leibniz’s efforts to determine the fate of his paper, he never did learn of its publication. Despite or perhaps because of this, he continued to promote the binary system whenever the opportunity presented itself. Sometimes this was with the long-held aim of encouraging others to investigate it further; such was the purpose of “On Binary” (chapter 32), Leibniz’s last systematic presentation of the binary system, intended for the mathematician Jakob Hermann (1678–1733). His overtures to other mathematicians met with a range of responses, usually measured enthusiasm or polite indifference, but sometimes the response was a little more challenging. César Caze (1641–1719), for example, took it upon himself to throw doubt on the suggestion of a link between binary and the hexagrams of the Yijing and also put it to Leibniz that other mathematicians had invented binary before him, forcing him on to the defensive in his reply (chapter 31).

The last decade of Leibniz's life was marked by much less activity in both investigating and promoting his binary system. Perhaps most significantly, he helped Dangicourt rework the essay he had initially composed back in 1701, with it ultimately being published in 1710 as "On the Periods of Columns in a Binary-Expressed Sequence of Numbers of an Arithmetic Progression" (see Dangicourt 1710). Leibniz also managed to induce another correspondent, Theobald Overbeck (1672–1719), to investigate column periodicity of binary numbers (see LH 35, 12, 1 Bl. 190–191). But much of Leibniz's attention, so far as binary was concerned, was focused on celebrating the supposed correlation between binary numeration and the hexagrams of the Yijing. In "Discourse on the Natural Theology of the Chinese," a lengthy essay written in the last year of his life, Leibniz noted previous attempts to explain the hexagrams mystically or philosophically, before claiming that "actually, the 64 figures represent a Binary Arithmetic which apparently this great legislator [Fu Xi] possessed, and which I have rediscovered some thousands of years later" (WOC 132).

While Bouvet's hypothesis of a correlation between binary notation and the hexagrams of the Yijing ultimately proved untenable,[13] Leibniz's suggestion here that he had not so much discovered binary arithmetic as rediscovered it turns out to have been prescient after all, since as we shall see, the history of binary extends back far beyond the seventeenth century, to ancient Egypt.

A Brief History of Binary and Other Nondecimal Systems

Certain reckoning methods used in antiquity are best, if anachronistically, explained in terms of the binary system. For example, the Egyptian method of multiplication, illustrated in the Rhind papyrus of c. 1700 BCE, computes a product $m \times n$ by repeatedly doubling n—a process known as *duplation*—and then adding up certain of the resulting values $2^0 n, 2^1 n, 2^2 n, \ldots$. Alongside the duplation of n, a parallel duplation process is begun starting with 1, yielding $2^0, 2^1, 2^2, \ldots$, stopping before the result exceeds m. A unique subset of these powers of 2 sums to m—namely, 2^i for those i such that the ith bit of the binary representation of m is 1—and $m \times n$ is then the sum of $2^i n$ for those values of i. That is, if the binary representation of m is $a_k \ldots a_0$, where each a_i is 0 or 1, then $m \times n$ is computed as

$$\left(\sum_{i=0}^{k} a_i 2^i\right) \times n = \sum_{i=0}^{k} \left(a_i 2^i n\right).$$

For example, 23×27 is computed by repeatedly doubling 1 (resulting in 1, 2, 4, 8, 16 and stopping there since $32 > 23$), while repeatedly doubling 27 (27, 54, 108, 216, 432) and then adding up $27 + 54 + 108 + 432$ to get the result 621, those terms having been selected since $1 + 2 + 4 + 16 = 23$ (Newman 1956, I: 172). So this method works because every positive integer has a unique binary representation ($23 = 10111_2$ in this case).[14]

The so-called Russian peasant algorithm for multiplication (Gimmestad 1991) works similarly, except that in place of the duplation of 1, the multiplier m is repeatedly halved—a process called *mediation*—until it is reduced to 1. The terms $2^i n$ that are summed up to yield the product are those for which the mediated multiplier is odd. So to continue with the same example, the mediated multipliers are 23, 11, 5, 2, 1, so the product is the sum of the duplated terms $2^i n$ corresponding to the odd numbers in this sequence, namely (as before), $27 + 54 + 108 + 432$.

Use of pan balances led naturally to systems of measurement with a binary structure. Old English and Scottish units, some still in use in the United States, were in powers-of-2 ratios to one another. Swinton (1789, 29), for example, includes a table of Scottish measures for liquids then in use (4 gills to the mutchkin, 2 mutchkins to the chopin, 2 chopins to the pint, 2 pints to the quart, 4 quarts to the gallon, 8 gallons to the barrel), complete with all the powers of 2 needed to convert any unit to any other (for example, 1024 gills to the barrel). In the same spirit, weighing an item on a pan balance

13. For details on why Bouvet's hypothesis is incorrect, see Sypniewski (2005) and Zhonglian (2000).

14. This method should not be confused with binary multiplication—the multiplier and multiplicand themselves are represented in a nonpositional form of decimal notation (Peet 1923, 11–13).

using weights in the proportions 1:2:4:8 … amounts to determining the binary representation of the item's weight. For example, if the available weights are 1, 2, 4, 8, and 16 ounces, an item weighing 23 ounces in one pan will be counterbalanced by placing the 1-, 2-, 4-, and 16-ounce weights (but not the 8-ounce) in the other pan (23 being 10111_2).

Such methods of reckoning and measuring, while dependent on the principle of unique binary representation of numbers, lack any suggestion of a binary positional notation. But nondecimal positional systems do have a long and independent history, dating back more than four millennia. Ifrah (1998, chap. 12) shows how Sumerian children learned to do sexagesimal arithmetic (base 60), not just sums but division (the exercise $1,152,000 \div 7 = 164,571$ with a remainder of 3 is worked out on a tablet from c. 2650 BCE). Base 60 may have been derived from duodecimal: Ifrah (1998, chap. 9) conjectures that duodecimal counting started with one-handed counting, using the thumb to count on the twelve phalanges of the other four fingers.[15] Duodecimal survives in the merchandizing of goods by the dozen and gross and in hours on the dial; sexagesimal in minutes, seconds, and degrees of the circle.

And yet the principle that numbers can be represented positionally using arbitrary bases seems to have remained unnoticed by mathematicians until Blaise Pascal (1665, 42) mentioned it quite offhandedly.[16] His initial subject was the rule for "casting out 9s" to determine whether an integer represented in decimal notation was divisible by 9, but he generalizes the rule to other divisors and also to other bases. The method works, he says, "not just in our decimal system of numeration (which has been established not as a result of natural necessity, as the common man thinks, but as a result of human custom, and quite foolishly, to be sure), but in a system of numeration based on any progression whatsoever." The only nondecimal example he presents is duodecimal, showing (Pascal 1665, 47–48) that casting out nines would then become casting out elevens. Having spent enough time on this "novelty," he ends with "I'll stop here lest I become tiresome to the reader by going through too many details."

But by this time, a form of binary reckoning, without binary notation, had been detailed by John Napier (1617) in his *Rabdologiæ* (English translation: Napier 2017, 649–749). In the final part of this work, "Location Arithmetic," Napier details a chessboard-style instrument for numerical calculations, cautioning in the Preface: "There is one small difficulty in working with it, and that is that the numbers it uses differ from ordinary numbers, so that one must begin by expressing ordinary numbers in the new form and end by reducing them to common form" (Napier 2017, 727). In Napier's representation, each power of 2 is associated with a Roman letter—*a* for 1, *b* for 2, *c* for 4, *d* for 8, *e* for 16, and so on—and numbers are written in a form reminiscent of Roman numerals by concatenating the letters corresponding to what we would now call the 1 bits in its binary representation. So $23 = 10111_2$, for example, would be written as *abce*. Or to be precise, that would be the canonical representation of 23; any other sequence of letters that sum up to 23 would be equivalent—*abcdd*, for example, or *aaace*—and these various forms could be derived from one another by replacing a *c* by two *b*s or two *b*s by a *c*, for example. Napier's multiplication method involves starting with a checkerboard (of any size) labeled along both its bottom edge (right to left) and right edge (bottom to top) *a*, *b*, *c*, …; putting tokens on those two margins to represent what are essentially the binary representations of the two operands; putting a token at the intersection of any row and column that both have tokens at the margin (the intersection between the row representing 2^i and the column representing 2^j being the square representing 2^{i+j}); removing from the margins the tokens representing the operands; and finally accumulating at the bottom margin the tokens on the various northeast-to-southwest diagonals (as all interior squares on any one such diagonal have the same value of $i + j$ and

15. His conjecture that duodecimal began with one-handed counting (a technique he describes as still in use in the Middle East) would imply that the factorization of 12 as 4×3 was part of the origin of duodecimal, with the factor of 5 to reach 60 having a more obvious anatomical interpretation. He cites linguistic evidence for the existence of a pre-Sumerian base-5 system, which he conjectures became base 60 after contact with a base-12 culture.

16. This publication is a posthumous collection, Pascal having died in 1662. Adamson (1995, 280) gives the date of the relevant essay, *De numeris multiplicibus*, as 1654.

hence all represent the same power of 2). All that is left is to reduce the piles of tokens on the bottom margin to canonical form via replacements of the kind already mentioned—and then translate them into the final decimal result.[17]

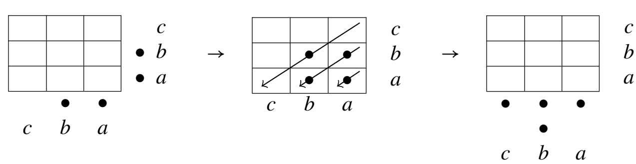
Napier went on to devise similar methods for division and even the extraction of square roots, a quite substantial labor. As a calculating aid, his assembly of rods and boards and tokens might best be compared to an abacus, a way that human calculators could substitute the movement of pieces for "the tedium of calculation," as he called it. It did not catch on; Napier died soon after this publication, and the work excited negligible interest. And perhaps that was as Napier intended and expected. He describes it as "more of a lark than a labor"; it was something of a game, a term he proudly used in an epigram at the beginning of the book to describe his ambitions for arithmetic more generally.[18]

Leibniz cited Napier's *Rabdologiæ* in his habilitation thesis of March 1666, written when he was nineteen years old (A VI 1, 203), and recalled reading Napier's book in his 1705 letter to César Caze ("I once saw Napier's book in Latin without considering then its relation to binary arithmetic, which is also noticeable in the divisions of weights among assayers"; see chapter 31). So it may well be that Leibniz's thinking about binary was influenced by Napier's devices, as well as by the use of weights in geometric progression and traditional arithmetic tricks involving duplation and mediation. But as Leibniz's first scribblings about binary were about binary notation, it is important to explore where, if anywhere, the idea might have come from to use positional notation with only two digit values.

At about the same time Napier was working on *Rabdologiæ*, proper binary notation and binary arithmetic made a brief and unheralded appearance in England and then in a flash disappeared. Thomas Hariot (1560–1621, sometimes "Harriot") was a mathematician and astronomer of great ability who published little; one of his few works is an account of his trip to the Virginia colony (Harriot 1590). In the mid-twentieth century, John Shirley (1951) discovered previously unpublished Hariot manuscripts in the British Museum, on the basis of which he and others (e.g., Glaser 1981, 11–14; Wolf 2000, 31; Ineichen 2008, 14) have claimed that Hariot developed the binary system first. There is no doubt that the manuscripts show calculations in binary—11 1011 + 111 0111 = 1011 0010 and another addition, two subtractions, two multiplications, and conversion back and forth to decimal, but no division. There is no indication of Hariot's motivation and no evidence that anyone except Hariot himself saw these manuscript pages in the 330 years or more that elapsed between their writing and their publication.[19] Certainly Leibniz never saw them.

Nor, as we shall see, was Leibniz aware of Juan Caramuel's *Mathesis biceps* of 1670, which contains a short article entitled "On Binary Arithmetic." In this article, Caramuel presents a table of binary numbers up to 32, using 0 and the letter *a* to form the sequence of binary numbers (for example, in his notation, 25 is *aa*00*a*). He then notes a convergence between binary arithmetic and music, "for each observes numbers, the former abstracted from material things, the latter discovered

17. Kolpas (2019) has a nicely illustrated explanation of Napier's "chessboard calculator." Here is a simple example: to compute 3×3, Napier would multiply $ab \times ab = abbc$ as shown below and then transform $abbc \rightarrow acc \rightarrow ad = 1 + 8 = 9$.



18. Arithmetic has now become a game.
Gone is the tedium of calculation.
And *Logarithms, Chess-board, Rod, and Strip,*
Confirm once more great the Napier's reputation.
(Napier 2017, 655)

19. Bauer (2010, 20) states that Napier published "a few years before Harriot," but as Hariot died four years after Napier's publication of *Rabdologiæ* and Shirley (1951) does not provide any information that could be used to date Hariot's manuscripts, no basis is evident for knowing which happened first.

in sounds," in particular the intervals in music, since a double geometric progression models the succession of octaves (Caramuel 1670, XXVII). Caramuel does not accord any further significance to the binary system, however, and in subsequent sections proceeds to identify and discuss a variety of different number systems, all of which are presented in positional notation. Aside from base 2, he discusses bases 3, 4, 5, 6, 7, 8, 9, 10, 12, and 60 (Caramuel 1670, XLV–LXI). For each of these, Caramuel provides a table of numbers, identifies nations or thinkers that had favored that system, and gives a series of examples of how each particular base number is embedded or reflected in the natural or supernatural world.[20] For example, when discussing "ternary arithmetic," Caramuel notes the theological importance of the doctrine "of the mystery of the most holy trinity," that angels are divided into three classes, and that the heavens are threefold, consisting of the Aëreum (in which the birds fly), the Aethereum (location of the stars), and the Empyreum (the throne of God). Similarly, in his discussion of "quaternary arithmetic," Caramuel (1670, L) points out that there are four elements and four cardinal directions, in "septenary arithmetic," seven principal planets, and so on (Caramuel 1670, LIV). His principal aim in treating the various bases is to show that they are all natural, being present in mundane and spiritual things without our thinking of them as bases. Accordingly, in his treatment of the different number systems, Caramuel often seems more concerned with a kind of number mysticism than he is with mathematics per se (in the words of Zacher 1973, 30, Caramuel "treats his systems without any mathematical or practical interest"). Even leaving this aside, his discussion of binary is disappointing because of what it doesn't offer: although the heading "binary arithmetic" would lead the reader to suppose that he will offer a full-blown binary arithmetic, Caramuel makes no attempt to show how arithmetic would work in that system, contenting himself with presenting a table of numbers up to 32.

Much more ambitious as an engagement with a nondecimal number system was the *Tetractys*, a short work published in 1673 by the mathematician and philosopher Erhard Weigel (1625–1699) to recommend conversion to the quaternary or base-4 system. Weigel favored the quaternary partly because it was easier to work with than the decimal, having fewer distinct digit values, and partly because he thought 4 to be the more natural unit of measure, being reflected everywhere in nature. To secure this latter point, Weigel (1673, 37–40) argues that the world, understood metaphysically, spiritually, and physically, consistently exemplifies the number 4; for example, there are 4 modes of being (necessary, contingent, possible, and impossible), 4 spirits (God, angel, devil, and rational soul), 4 types of parallelogram (square, rhombus, rhomboid, and rectangle), 4 islands of East Asia (Maluku, Philippines, Formosa, and Japan), and so on. To undercut potential resistance to his proposal, Weigel (1673, 9) notes that both the learned and unlearned already use quaternary measures routinely, such that "they make one digit from four barleycorns [i.e., from the breadth of four barleycorns], one palm from four digits, one foot from four palms, one pace from four feet, one perch from four paces ... and also separate day and night into four parts, the hour into four quarters," and so forth.[21] To further smooth the introduction of the new system, at least in Germany, Weigel (1673, 13) devises new German terms for key values: *Secht* for four fours and *Schock* for four *Secht* (thereby revaluing the term *Schock*, traditionally used to refer to the decimal number 60). More important from a mathematical perspective, Weigel (1673, 16–23) outlines methods of performing all four arithmetic operations in base 4, including the use of a dot to indicate carries when adding or multiplying, as well as methods for the extraction of square and cube roots.

20. The last of these features is likely derived from Schwenter (1636, 117–122), who provides a lengthy list of ways that the numbers 2–30 are embedded or reflected in the physical and spiritual realms. In a later work, Harsdörffer (1653, 80–128) examines the philosophical, theological, and scientific significance of an even wider range of numbers, namely, 1–24, 30, 40, 50, 60, 70, 80, 90, 100, and 1000. Caramuel uses many of Schwenter's examples.

21. Note that not all Weigel's measurements coincide with what Clavius (1570, 263) had earlier claimed was a system of measurements laid down by mathematicians as a universal standard, in an effort to avoid regional or national variations. For example, Clavius (1570, 264) describes a perch as equivalent to 10 feet (or 40 palms), not 16 feet (or 64 palms), as Weigel states.

Leibniz's Influences

The deep and varied interest in nondecimal number systems during the seventeenth century raises the question of who or what may have influenced Leibniz in his development of binary. This usually takes the form of trying to identify a single thinker from whom Leibniz may have drawn inspiration. Hence, Couturat (1901, 473)[22] suggested that Leibniz was influenced by Weigel, a view later endorsed by Ingaliso (2017, 111–112), while Tropfke (1980, 12) wondered whether Leibniz had been influenced by Weigel or Caramuel.[23]

Of these suggested influences or inspirations, that of Caramuel can be swiftly ruled out by the following remark Leibniz made in a letter to Friedrich Simon Löffler of 11 January 1711:

> Regarding Caramuel's *Mathesis biceps [Old] and New*, for which he asks ten thalers, I am unable to judge well because I have not yet seen it, and I fear it may contain vain subtleties, which is not unusual for Caramuel. (Dutens V, 418)

As Leibniz had not even seen a copy of Caramuel's book more than three decades after he had developed the binary system, clearly it could not have inspired or influenced him in this regard.[24]

As for the potential influence of Weigel's quaternary system on Leibniz's development of binary, this was first suggested in 1701 by Johann Bernoulli, who, upon learning of Leibniz's "new kind of arithmetic," informed him that "Weigel once devised something not unlike this . . . , the only difference being that in place of your binary sequence he adopts the quaternary" (A III 8, 614). In his response to Bernoulli, Leibniz flatly denied Weigel's influence, insisting that he had "reflected upon this [i.e., the binary system] for many years before any mention was made of the *Tetractys* being revived" (A III 8, 639). Unfortunately, Leibniz's implicit assertion that he developed the binary system before 1673 (the year Weigel's *Tetractys* was published) cannot be verified, as there is no evidence of his writings on binary going back that far. But the textual evidence we do have at least supports his claim that he had not been influenced by Weigel's book: the earliest evidence we have of his having read it is his brief reading notes on it,[25] and from the watermark, we know that they were written in the first half of 1683, by which point Leibniz had already written many manuscripts on binary. Prior to that, Leibniz appears to have had no awareness of the quaternary system or Weigel's advocacy of it, since there is no mention of either in any of his pre-1683 writings. In fact, in his pre-1683 writings on binary, Leibniz mentions only one other nondecimal number system, namely, the duodecimal, which he believed some had preferred to the decimal (see, for example, LH 35, 1, 25 Bl. 3v; LH 35,

22. "[The binary system] probably had been suggested to him by the *Tetractys* of his old master Weigel, published in 1673."

23. "Perhaps Leibniz was inspired to create the dyadic by Weigel's four-system or the work of Caramuel y Lobkowitz."

24. This fact has not stopped Ares et al. (2018) from claiming that Leibniz not only knew of Caramuel's work on binary but deliberately plagiarized it. Oddly, they cite the same passage from Leibniz's 1711 letter to Löffler in order to support this claim, apparently not realizing that it does the very opposite. The other textual "evidence" Ares et al. provide is no better. First, they point the reader to Leibniz's dissertation for his Master of Philosophy degree. While Leibniz does mention Caramuel in this text (see A VI 1, 88), he does not mention the *Mathesis biceps*, nor indeed could he, since Leibniz's dissertation was written in 1664, six years *before* Caramuel published the *Mathesis biceps*! The final piece of "evidence" of Leibniz's supposed plagiarism offered by Ares et al. is a letter to Michael Gottlieb Hansch of 13 December 1714, in which Leibniz encourages his correspondent to undertake further investigation of the number periods he has found in the binary system and the rules underlying them (Dutens V, 170). However, there is no mention of Caramuel or the *Mathesis biceps* in this letter. The charge of plagiarism leveled by Ares et al. is therefore not merely baseless but manufactured and need not be taken seriously. More sober is the assessment of Eberhard Knobloch (2018, 242) that "there is not the least evidence that this book [Caramuel's *Mathesis biceps*] influenced Leibniz's invention of the dyadic."

25. See A VI 4, 1162–1163. The notes are not particularly interesting in themselves, as Leibniz mainly restricts himself to summarizing some of Weigel's claims. However, at the end, Leibniz does briefly offer his own thoughts, writing, "I think the binary is best absolutely, the ternary in planes, and the octal in solids" (A VI 4, 1163). Around the same time, Leibniz also made notes on the *Tetractyn*, the companion work by the Pythagorean Society (1672); see A VI 4, 1163–1164.

13, 3 Bl. 33).[26] If, as seems likely, the quaternary system appeared on Leibniz's radar only in 1683, when he read Weigel's book on the subject, the case for thinking Weigel influenced Leibniz in the development of binary must be considered a lost cause.[27]

On the question of Leibniz's influences, then, aside from ruling out Caramuel as a candidate, and all but doing the same for Weigel, nothing definitive can be said except that Leibniz's idea for binary did not arise in a vacuum. He intimates as much himself on the occasions he offers his own narrative regarding his invention of binary, though the story he tells is not consistent and likely incomplete. In his "Essay on a New Science of Numbers" from February 1701, he identifies his inspiration as the use other people had made of nondecimal systems:

> Everyone agrees that the decimal progression is arbitrary, so others have sometimes been used. That made me think of the binary, or double geometric progression, which is the simplest and most natural. (p. 138 in chapter 23)

A similar claim was made in his letter to Bouvet of 15 February 1701:

> As we are usually accustomed to make use of progression by ten, and as some have used other progressions, I wanted to consider what would be the simplest progression possible, which is the binary progression or the double geometric progression. (pp. 127–128 in chapter 22)

While these writings do not specify the bases of the number systems that others had used, in a letter to Grimaldi written in early 1697, Leibniz identifies them as the duodecimal and quaternary:

> It is apparent that some considered the duodecimal to be more useful while others took pleasure in the Pythagorean tetractys. At some point it occurred to me to consider what would ultimately be revealed if we used the simplest of all [progressions], namely the dyadic or binary. (p. 110 in chapter 19)

The story Leibniz would tell decades after his invention of the binary system, then, was that he had hit upon binary as the simplest number system in conscious reflection upon other systems, sometimes unspecified but occasionally identified as the duodecimal and quaternary. Scholars, of course, must be wary of taking such *ex post facto* narratives on trust and will want textual evidence that confirms them before considering them reliable. And in this case, the textual evidence only partly supports Leibniz's story, for while we know that he was aware of the duodecimal system as least as early

26. "It is well known that all fractions can be expressed by an infinite sequence of integers of a certain progression, for example, the decimal, or even the duodecimal, or the one I prefer overall, the binary" (LH 35, 13, 3 Bl. 33, from August 1680). The relevant passage from LH 35, 1, 25 Bl. 3v is quoted above. In a letter written in 1712, Leibniz (mistakenly) identifies the German mathematician Daniel Schwenter (1585–1636) as a proponent of the duodecimal system: "Some report in the German *Deliciae mathematicae* has given preference to the duodecimal progression, in which eleven digits will be needed, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, δ, ε, where δ is 10 and ε is 11" (LBr 705 Bl. 93r). The reference is to Schwenter (1636) or perhaps Harsdörffer (1651) or Harsdörffer (1653). However, in none of those works does Schwenter mention, let alone endorse, the duodecimal system. Leibniz had cited Schwenter (1636) in a dissertation written in 1666 (see A VI 1, 173; English translation: PPL 78); it is not surprising that almost fifty years later, he had only an imperfect recollection of a work he had read in his youth.

27. It is worth noting that Leibniz had studied under Weigel at the University of Jena for the summer term of 1663. Whether Weigel was advocating the quaternary system at this time—a decade before he published the *Tetractys*—is unknown. However, it is at least possible that he was. In the preface to the counterpart publication to Weigel's *Tetractys*—namely, the *Tetractyn*, written by "The Pythagorean Society" of the University of Jena—it is noted that when the Society was formed, it had scarcely started to discuss Pythagoras's famous tetrad when Weigel was appointed director (Pythagorean Society 1672, 4). That appointment was made in 1662, though how soon after that Weigel developed the quaternary system—which is quite different from the Pythagorean tetrad—is unknown. There is also no evidence that Leibniz was a member of the Pythagorean Society during his term in Jena, though of course this cannot be ruled out. As Zacher (1973, 33) notes, one could certainly form and entertain the hypothesis that Leibniz had learned about the quaternary system through Weigel in 1663, but as there is no evidence to support the hypothesis, it should be treated as nothing more than speculation.

as 1670,[28] there is, as noted above, no evidence in his writings that he had any awareness of the quaternary system prior to 1683. Moreover, as also noted above, Leibniz elsewhere explicitly denied that he knew of the quaternary system prior to his invention of binary.

In truth, one thing that beleaguers most attempts to determine influence, whether on oneself or others, is the natural impulse to restrict oneself to identifying a single influential figure or single influential idea. Given the complexity of the human mind and the range of different factors impinging thereon, this impulse is probably best resisted as a general rule. Certainly, in the case of Leibniz's invention of binary, yielding to it (as he himself did and as some of his commentators have done) is likely to make us miss much of what is important. For as we have seen, the idea of exploring nondecimal number bases was not uncommon in the seventeenth century, and Leibniz himself was aware of some of this work. He may not have been privy to Hariot's experiments with binary arithmetic, Caramuel's flirtation with binary notation, or perhaps even Weigel's work on the quaternary system, but he did know Napier's *Rabdologiæ* as well as Pascal's offhand remark about nondecimal bases being possible (alluding to it occasionally; see, for example, chapter 19). He was also aware—as indeed was everyone else!—of the wide use of nondecimal number bases for measurements and coinage, and he even explored some of these himself prior to his invention of binary. Most notably, in the middle of 1674, he drafted two manuscripts on the sexagesimal system. In these two manuscripts, Leibniz concerns himself with the mathematical foundations of the sexagesimal system, working out procedures for converting sexagesimal numbers to decimal and for multiplying a three-digit sexagesimal number by an improper fraction (see LH 35, 3 A 26 Bl. 17 and LH 35, 8, 30 Bl. 27). No doubt it could be claimed that Leibniz's investigation into the mathematical foundations of the long-known sexagesimal system directly or indirectly spurred him to consider other number bases, but such a claim would be undermotivated and in any case repeats the mistake of supposing that a single figure or idea was his sole inspiration. A better conclusion to draw from Leibniz's work on sexagesimal is that it would have made it easier for him to adapt the algorithms for multiplication and division to other bases, as he did with binary and sedecimal shortly thereafter. Other than that, perhaps all that can be said about who or what inspired Leibniz to invent binary is that no simple or easy answer presents itself, and all that can be done by way of trying to answer it is what we already have done, namely sketching what we might call the mathscape of his time, noting the cognate ideas with which he would been familiar and treating the complex of these as the catalyst for his invention of binary, while declining to privilege any one of them in particular.

Leibniz's Legacy: The Birth of Computer Arithmetic

The use of duplation and mediation in arithmetic, as well as the use of positional notation with nondecimal bases, runs deep in human history and mathematical experimentation. By the seventeenth century, the overlapping waves of influence on individual scholars had become so turbulent that it is impossible to give a simple answer to a big question such as, "Who gave us the binary system?" Narrower questions are more approachable. If the question is, "Who wrote the first binary sum?" the answer seems to be Thomas Hariot, from what we know today, but it would surprise no one if an earlier and unrelated example turned up in some manuscript collection in the future. If the question is, "Who first realized that any integer can be expressed as a sum of powers of 2?" the answer could be the scribe of the Rhind papyrus (though the record includes only worked examples) or some Egyptian arithmetician who figured out the trick a millennium earlier and passed it down through a now-lost chain of cultural transmission.[29] Or perhaps credit for binary decomposition should go to the forgotten grain merchant who first used a pan balance with weights in geometric progression.

But if the question is, "Who gave binary computer arithmetic to the modern world?" the answer is Leibniz. We will make the case for forward influence below, but we pause here to reflect on what change of direction in intellectual history is presented by the disorderly collection of manuscripts

28. Early in 1670, Leibniz wrote, "In arithmetic, and in other disciplines as well, truths remain the same even if notations are changed, and it does not matter whether a decimal or a duodecimal number system is used" (A VI 2, 429; English translation: PPL 128).

29. Separately, Eglash (2005, chap. 7) proposes that binary began in sub-Saharan Africa.

and publications presented in these pages. Leibniz's work combines three elements that had not been seen together before and have never been completely disjoined since. First, he (like Hariot) made the leap from positional notation in the abstract, and multiple specific examples of positional number systems, to the special case of base 2. Second, Leibniz developed a complete suite of paper-and-pen algorithms for binary arithmetic, including not just the four standard arithmetic operations, but also extraction of square roots, the use of two's complement arithmetic, and the use of (as we would call it today) boolean algebra to describe the results of binary operations in terms of their operands. And third, he developed all this with an agenda to mechanize thought and actually designed machines (not just calculating aids, like Napier's chessboard) that could, in principle, carry out binary arithmetic calculations with minimal human intervention.

To say a few more words about the third point: discrete mathematics has always had game-like qualities. When only whole numbers are allowed in the solution to a problem, what otherwise looks amenable to well-understood methods can turn into an intractable puzzle. The "Cattle Problem" of Archimedes (Bell 1895) resisted solution for two millennia. With its stipulations that there are $\frac{1}{3} + \frac{1}{4}$ as many white cows as black cattle and so on, finding the size of the herd looks like a straightforward exercise in solving linear equations—except that certain of the bulls are to be arranged in a perfect square and others in an equilateral triangle. (The smallest solution turns out to be on the order of 10^{206545}.) Such puzzles have always brought mathematicians joy, because they have the appearance of reality (we can all picture arranging bulls in rows and columns) without any of the true utility that motivated the development of linear algebra or the infinitesimal calculus, for example.

So when Napier said that his location arithmetic was a "lark" and he had reduced calculation to a "game," he meant it. But when Leibniz said that binary was for theory, he was only half serious. He surely recognized that no one would want to carry out arithmetic with numbers twenty or thirty bits long (though his manuscripts show his own attempts) and that mathematically, binary notation was a theoretical extremum on a spectrum where decimal stood at a more useful midpoint. But he immediately followed that thought with a sedecimal notation "for practice," recognizing that binary-sedecimal conversion was trivial. And he designed not one but two different calculating engines based on binary principles. Perhaps binary is for theory, and perhaps he knew he would have trouble building the binary machines he sketched (he had enough trouble with his decimal machine), but he also had supreme confidence that thought could be mechanized and that the day would come when human disputes would be settled by calculation. His binary explorations were, from the beginning, part of that large agenda for him and became part of the development of the decision-making computers that would be built two and a half centuries later.

In his ambivalence about the utility of his discrete mathematical explorations, Leibniz foreshadows the spirit of computer science and diverges from the spirit of mathematics. When G. H. Hardy famously characterized his favorite branch of mathematics, number theory, as "useless," he was speaking as a premier mathematician of his time. When Alan Turing a few years later, and Rivest, Shamir, and Adleman (1978) some decades after that, used that theory for powerful cryptography and cryptanalysis, they were birthing computer science. In proposing to mechanize binary arithmetic, Leibniz was their ultimate progenitor.

A line can be drawn between Leibniz's writings on binary and its adoption in the first modern computers designed by Konrad Zuse (1910–1995), the pioneering computer scientist who in 1934 made the decision to use binary numbers and code in his Z1 and subsequent machines. As we shall see, this line passes through the various books, articles, and encyclopedia and dictionary entries that promulgated Leibniz's work on binary from his own time to the 1930s. To trace this line, let us begin with published treatments during Leibniz's own lifetime.

Aside from Leibniz's own "Explanation of Binary Arithmetic," published in 1705, two further treatments of his binary system were published during his lifetime, both of which he had a hand in. One of these was the aforementioned essay by Dangicourt on column periodicity of binary numbers, which was written at Leibniz's behest and published in *Miscellanea Berolinensia*, a journal of which Leibniz was the editor. The other, an essay by Leibniz's friend and long-time correspondent, Wilhelm Tentzel (1659–1707), was published in 1705 under the title: "Explanation of Binary Arithmetic, Which Was Used by the Chinese 3000 Years Ago and Lost by Them but Recently Found Again by Us." Unlike Dangicourt's essay, which does not mention Leibniz's name at all, let alone credit him with the invention of the binary system (and this presumably with Leibniz's blessing, since he commissioned and published the essay), the essay by Tentzel (1705, 81) opens with an acknowledgment of his debt to Leibniz: "To curious people in general, especially mathematicians and arithmeticians, I offer something quite new for the New Year, which Privy Counselor Leibniz of Hanover discovered and recently communicated to me in accordance with his usual kindness." He then offers brief details of binary numeration, examples of the four standard arithmetic operations in binary, and the periodicity of columns of binary numbers, all within the space of four pages. The remaining twenty-eight pages are devoted to an outline and detailed exploration of Bouvet's hypothesis of a correlation between binary numeration and the hexagrams of the Yijing. Tentzel (1705, 112) concludes his study by describing the table of sixty-four hexagrams as a "labyrinth" from which not even Confucius could escape, but that "Leibniz alone has discovered the thread of Ariadne," echoing Leibniz's claim that an ancient Chinese enigma had been solved by a European. While Tentzel's essay, essentially an expanded version of—and apology for—Leibniz's "Explanation of Binary Arithmetic," went largely unnoticed, its nonmathematical focus would be mirrored in many of the treatments of the binary system during the eighteenth and (to a lesser extent) the nineteenth centuries, as we shall see.

In the decades after Leibniz's death, references to his work on binary inevitably drew upon the few of his writings on the subject that had been published at the time, in particular the 2/12 January 1697 letter to Duke Rudolph, with its focus on the binary-creation analogy and proposal for a commemorative medal, "Explanation of Binary Arithmetic," which proposed a parallel between binary and the hexagrams of the Yijing, and "Discourse on the Natural Theology of the Chinese," which likewise explored the link between binary and the hexagrams.[30] Leibniz's 1697 letter to Duke Rudolph appeared in six different books published between 1720 and 1768,[31] and was discussed in various others, such as Wideburg (1718). While interest in the binary-creation analogy waned over the course of the eighteenth century, it was occasionally resuscitated by those keen to deride it, such as Laplace (1825, 211), who wrote,

> Leibniz thought he saw the image of creation in his binary arithmetic, in which he used only the two characters, 0 and 1. He imagined that God could be represented by 1, and nothingness by 0; the Supreme Being had drawn all beings from nothing, like 1 with 0 expresses all numbers in this system of arithmetic. This idea pleased Leibniz so much that he communicated it to the Jesuit Grimaldi, president of the tribunal of Mathematics in China, in the hope that this symbol of creation would convert the emperor of the time, who was particularly fond of science, to Christianity. I report this feature only to show how far the prejudices of infancy can lead the greatest men astray.

The vast majority of eighteenth-century treatments of Leibniz and binary followed the thread of "Explanation of Binary Arithmetic," by outlining both binary notation and its apparent correlation with the hexagrams of the Yijing. Such an account was added to the biography of Leibniz by Lamprecht (1740, 74–75), as well as to some editions of Leibniz's works, such as the German edition of the *Theodicy* (Leibniz 1744, 824–828). An article on "binary arithmetic," written by Samuel Formey (1711–1797), was also included in the monumental enterprise of Diderot and D'Alembert (1751, 257–258), the *Encyclopédie*. Opening with "Binary arithmetic is a new kind of arithmetic that Mr. Leibniz based on the shortest and simplest progression," the article focuses on binary notation and conversion of decimal to binary, though a few lines are devoted to the claimed connection between binary and the hexagrams of the Yijing, about which no judgment is offered. Other encyclopedia editors opted not to include a dedicated entry on binary but rather discussed the possible binary–Yijing connection in other articles; Croker, Williams, and Clark (1766, n.p.), for example,

30. Other writings of Leibniz on binary that were published in the eighteenth century often had the same focus, for example, his letter to Ferdinand Orban of 27 August 1705, which was first published in Will (1778, 57–65); English translation: Leibniz (1705b).

31. Namely, Leibniz (1720, 103–112), Nolte (1734, 1–16), Ludovici (1737, 132–138), Leibniz (1739, I: 234–239), Leibniz (1740, 92–100), and Leibniz (1768, III, 346–348).

did so in an article on "Chinese philosophy," in which they asserted that "Leibnitz decyphered the aenigma, and demonstrated that Fohi's [Fuxi's] two lines were only the elements of binary arithmetic." Striking a more cautious note was Georg Bernhard Bilfinger (1693–1750), who dealt with the binary and Yijing question at the end of a lengthy book on Chinese moral and political philosophy. While accepting that Leibniz and Bouvet were probably right in thinking that binary was the key to deciphering the hexagrams of the Yijing, Bilfinger (1724, 359–360) pointed out that the Chinese usually arrange the hexagrams in a different way from the ordering sent to Leibniz by Bouvet (i.e., the so-called Fuxi ordering), which led him to insist that the Chinese, whose mystery it was, ought to have the final say on whether Bouvet and Leibniz were right to treat the hexagrams as a form of binary numeration.

While the possible link between binary and the hexagrams of the Yijing dominated the eighteenth-century treatments of Leibniz's binary system in the secondary and tertiary literature, there were some serious mathematical treatments also. Wenceslaus Josephus Pelicanus (1712) published a short book whose title translates as "The Perfect Arithmetic for Anyone Who Does Not Know How to Count to Three," in which he offered a detailed exposition and analysis of the binary number system, albeit under his own preferred term "dual arithmetic." After describing binary notation, Pelicanus describes how the four standard arithmetic operations work in binary before considering how to perform the same operations with binary fractions. He also investigates the extraction of square and cube roots and provides tables for easy conversion between binary and decimal. Curiously, Pelicanus says nothing about the inventor of the "dual arithmetic" and mentions Leibniz not once, which suggests that Pelicanus was either yet another independent inventor of binary or not very good at acknowledging intellectual debts. Although Leibniz was made aware that Pelicanus was working on binary, he appears not to have seen Pelicanus's book;[32] had he done so, he might have realized that it was precisely the sort of work that he could and should have written thirty years earlier. Twenty-five years after Leibniz's death, Buffon (1741, 219–221) published a short paper outlining a method for the conversion of numbers between different bases, including binary, using logarithms; the paper's summary, written by the perpetual secretary of the Académie Royale des Sciences, Bernard le Bovier de Fontenelle (1657–1757), reminds the reader that Leibniz had published on the binary system back at the turn of the eighteenth century. Three decades after Buffon's paper, Brander (1775, 3) published a short pamphlet on binary arithmetic (which "owes its invention . . . to the excellent Mr. Leibniz," he noted at the outset), detailing algorithms for the four base-2 arithmetic operations, which he appears to have worked out independently of both Leibniz and Pelicanus.

During the eighteenth century, outlines of the binary system also started to appear in mathematics textbooks, often followed by an account of Bouvet and Leibniz's hypothesis that binary was the key to deciphering the Yijing. In some cases this hypothesis was merely described without assessment (for example, see Vieth 1796, 13), while in others, it was evaluated, not always positively (for example, see Ozanam 1790, 5, in which the hypothesis is described as "more imaginative than sound").[33] But authors of textbooks, encyclopedias, and other reference works varied in how much space they were prepared to devote to the Yijing during their treatment of binary. Andrés (1790, 71–73) mentions the hypothesis of the Yijing almost in passing in his outline of binary notation, while Hutton (1795, I: 144, 206) mentions it not at all.

In this matter, little changed in the first two-thirds or so of the nineteenth century. Encyclopedias and other reference works from this time treated Leibniz's binary system—and it was almost always seen as Leibniz's[34]—as a curious, if impractical, form of numeration, often devoting as much space to binary notation as to the purported parallel with the hexagrams of the Yijing, with an occasional

32. See Michael Gottlieb Hansch's letter to Leibniz of 4 October 1714 (LBr 361 Bl. 49–50), in which he outlines Pelicanus's method of determining the decimal equivalent of a long binary string but does not mention Pelicanus's book.

33. The same verdict is also to be found, verbatim, in Montucla (1799, 457).

34. Even brief references to binary notation in encyclopedias and dictionaries were typically accompanied by an assertion that Leibniz was the author thereof; see, for example, Lieber, Wigglesworth, and Bradford (1854, 335) and Heyse (1859, 289).

mention of the binary-creation analogy thrown in for good measure (see, for example, Barlow 1814, n.p., art: "binary arithmetic"; Brewster 1832, II: 382–384; Anon 1857, III: 591; Schlömilch and Witzschel 1858, 338). Even serious mathematical works tended to follow this (by then rather tired) formula; for example, in an essay on the history of different number systems, Krist (1859, 49–50) wrote,

> For $g = 2$ one arrives at the so-called dyadic or binary system, which needs only the digits 1 and 0 for number figures, according to which our 2 is represented by 10, our 3 by 11, our 4 by 100, our 5 by 101, and so on. This system is of historical interest insofar as it was established around 1697 by the great German philosopher Leibniz, after it had already been found in 1670 by the bishop of Campagna and Satriano, Johann Caramuel. Leibniz, however, came up with it independently of the latter, and dealt with it merely in the hope of being able to examine the properties of numbers more easily with its help. Leibniz also showed, and after him Brander, how the dyadic system can be used for calculation, but without the intention of thereby displacing the already prevailing decimal system. Interesting is the fact that Leibniz used the dyadic system to symbolize the origin of the universe. In a letter to the Duke of Brunswick from 1697, he added a medallion on which he sketched light and darkness behind a table filled with dyadic numbers, and to which he wrote the following sentence: "In order to bring forth everything out of nothing, unity is sufficient" in Latin as a circumscription. At the same time, Leibniz also expressed the intention of sending his number system to the missionary Grimaldi in China, in the hope that the Chinese emperor might be converted to Christianity through its profound meaning. Although this hope was not realized, the missionary Bouvet was informed of the dyadic and was thereby able to unravel many ancient Chinese manuscripts, which at the same time revealed that the dyadic system was already known to the Chinese and that among them the invention of the same is attributed to their emperor Fuxi, the founder of the Chinese empire and Chinese scholarship. But Fuxi is said to have lived around 2200 BC.

It is a notable that, until deep into the nineteenth century, virtually all treatments and discussions of binary and the hexagrams of the Yijing merely repeated the claims made by Bouvet and Leibniz at the very start of the eighteenth century. Moreover, those who sought to assess these claims did so either with a partial understanding of the Yijing itself or in complete ignorance of it, as it was not available to European scholars in its entirety until the first complete translation into a European language—Latin—appeared in the 1830s (see Mohl 1834, 1839), with various others following thereafter (for example, McClatchie 1876 and Legge 1882). The availability of the full text of the Yijing engendered an explosion of scholarship, and eventually the deeper understanding of the Yijing by European scholars began to put strain on the hypothesis that its hexagrams were a form of binary notation. For example, Cantor (1863, 48) considered the evidence that the Chinese had used a base-2 system in the hexagrams of the Yijing and dismissed it, claiming that binary numeration was an original invention of Leibniz. Thanks to such efforts, as the nineteenth century wore on, it became increasingly common for binary to be treated more as a system of numeration and arithmetic than as an intercultural curiosity; see, for example, Privat-Deschanel and Focillon (1864, 264), Lübsen (1869, 232–234), Figuier (1870, 73), and Collignon (1897). And by the end of the nineteenth century, the idea of mechanizing calculations in binary arose for the first time since Leibniz first entertained it in 1679, with Lucas (1891, 148) stating that "this system [i.e., binary] would lend itself more naturally than any other to the manufacture of arithmetic machines." Inspired by this suggestion, Peano (1898, 10) designed and built a stenograph based on "binary writing," where groups of binary digits corresponded to the common sounds of various languages, making it possible to input shorthand transcriptions faster than in a conventional stenograph.

This decoupling of binary and the Yijing, along with the accompanying re-mathematization of Leibniz's binary system, was driven not just by a greater understanding of the Yijing but also by increasingly greater availability of Leibniz's writings on binary. In the mid-nineteenth century, the German mathematician Karl Immanuel Gerhardt (1816–1899) published a seven-volume series of Leibniz's mathematical writings, *Mathematische Schriften*, containing many previously unpublished pieces. These included important works on binary such as "Demonstration That Columns of Sequences Exhibiting Powers of Arithmetic Progressions, or Numbers Composed from These, Are Periodic" and "On Binary," neither of which had been published before (see GM VII 228–238, corresponding to chapters 27 and 32 of the present book). Gerhardt's volumes also brought together items from Leibniz's correspondence in which he discussed binary, including letters to Johann Christian Schulenburg of 1698 (GM VII 238–243) and Johann Bernoulli of 1701 (GM III 2, 656–657, 660–662, 669–670). All this enabled nineteenth-century readers to get a better—albeit still very incomplete—understanding of Leibniz's studies on the subject.

Tracing the references to Leibniz and binary across the eighteenth and nineteenth centuries is not merely an exercise in *Rezeptionsgeschichte* but also makes it possible to show that Leibniz's work on binary appeared in the sort of mathematics textbooks that would likely have been used by Konrad Zuse during school or at the Technical University of Berlin in the late 1920s and 1930s. Indeed, there is no shortage of German-language textbooks from the first quarter of the twentieth century that (a) mentioned the binary system and (b) indicated Leibniz's authorship thereof. For example:

- Tropfke (1902, 4n3): "in *Histoire de l'acad. d. Paris* 1703 (printed 1705), pp. 85–89 (*Gesammelte Werke*, ed. Gerhardt, third series, volume 7, Halle 1863, pp. 223–227) he [Leibniz] discusses the advantages and disadvantages of the binary system."
- Wieleitner and Braunmühl (1911, 74) notes Leibniz's development of binary and refers the reader to Leibniz's "Explanation of Binary Arithmetic" as well as letters to Schulenburg, Bernoulli and so on (all of which were available in Gerhardt's *Mathematische Schriften* by that point).
- Weber (1922, 30): "With the exception of 1, any number g can be taken as the base number of a number system with g digits 0, 1, 2, … $(g-1)$. The theoretically simplest is the binary system with the base number 2, which manages with only two digits, 0 and 1 (Leibniz, *Math. Schriften*, ed. C. J. Gerhardt 7, 223)."

The first two of these works were and still are held by the library at the Technical University of Berlin,[35] where Zuse studied between 1927 and 1935. The same library also holds original copies of Leibniz's *Mathematische Schriften*. Whether in the aforementioned works or others, it would not have been difficult for Zuse to come across references to Leibniz and binary, making it likely that he would have read about them firsthand prior to beginning work on his first computer in 1934. That computer, the Z1, was organized on the principle of binary representation of numbers—several years before John von Neumann (1993, written in 1945) adopted binary for the EDVAC and roughly simultaneously with the adoption by Howard Hathaway Aiken of decimal for the "Automatic Sequence Controlled Calculator" he was designing at Harvard (Aiken, Oettinger, and Bartee 1964, written in 1937). And indeed, Zuse emphasized that he adopted the binary system in the early 1930s and that the "birth of modern computer science" was with his realization that "all data could be represented through bit patterns" (Zuse 1987) and "data processing begins with the bit" (Zuse 1980, 621). Zuse singled out Leibniz as the forefather of the modern computer, in no small part because of his work on the binary system:

> The story [of the computer's prehistory] begins in the seventeenth century with Leibniz, who, together with Schickard and Pascal, was one of the pioneers of computing machine construction. He developed the mathematics of the binary number system and made one of the first formulations of symbolic logic—what we today call propositional calculus or boolean algebra. It was precisely this symbolic logic that later proved extraordinarily useful in the construction of computing machines and led to the generalization of the concept of computation. (Zuse 1993, 33)

In bestowing such an honor upon Leibniz, Zuse can be plausibly read as acknowledging an intellectual debt and, just as plausibly, influence. Petrocelli (2019) suggests that Zuse's use of the term

35. The search function for the catalog is available online at https://www.tu.berlin/en/ub/.

“Dyadik” is a direct borrowing from Leibniz.[36] While others such as Hariot and Caramuel may have claims to have entertained the idea of binary numeration before Leibniz, their work played no part in the birth of the computer age. Rather, it was Leibniz’s independent invention and development of binary that proved to have the only impact in this regard—an assessment shared by other scholars, for example, Garfinkel and Grunspan (2018, 32) (“All modern computers use binary notation and perform arithmetic using the same laws that Leibniz first devised”), Stein (2006, 57) (“Leibniz’s Machina Arithmeticae Dyadicae is the forefather of today’s computers”), and Wiener (1961, 19) (“If I were to choose a patron saint for cybernetics out of the history of science, I should have to choose Leibniz”).[37]

36. Leibniz seems to have been responsible for introducing *Dyadik* into the German vocabulary. This German word is still used exclusively to refer to “the binary system of arithmetic.” The German word *binär* was not used to refer to the binary system until the twentieth century and even now is not the usual term, which is *Dyadik* or sometimes *Zweiersystem*, or *Dualsystem*.

37. An alternative view, that Leibniz’s work was largely irrelevant to the development of the modern binary computer, has been asserted forcefully by Bernhard Dotzler:

- “Structurally, the back-projection of computer binarism onto the dyadic is almost the same story as the former identification of the system of binary numbers with the Yijing. Since its hexagrams are made up of only two elements—the whole and the broken line, they can formally be described as a binary system. However, this former interpretation is as wrong in terms of content as the updated one is in functional terms.” (Dotzler 2010, 29)
- “So one could say that with dyadics, esotericism was once set against esotericism: the esotericism of the dyadic penetration of creation against the esotericism of the Yijing interpretation, which had been declared false. To remember this, of course, cannot aim at bringing the associated metaphysics back into play. It is only a matter of keeping in focus this formerly different purpose of the binary number system: namely esoteric, and not cybernetics. Before it was seized by the binarism of information technology, the dyadic was an ontological instrument of understanding. Accordingly, it characterizes a functionality that may differ only minimally from the binary of the computer, but one which is fundamentally different.” (Dotzler 2010, 31)
- “With this, however, the dyadic stands for a paradox, which then counteracts the myth of its anticipatory conspiracy with the binary of the computer, with cybernetics and digital arithmetic. The formal does not correspond to a functional correspondence, and that means: The equation of the binarism of today with the dyadic of yore is actually—fiction.” (Dotzler 2010, 33)

However, it is clear from the timeline of Leibniz’s extensive algorithmic writings on binary arithmetic and his design sketches for two kinds of binary calculator that he saw the binary system as far more than an “ontological instrument” and that he did so well before he was made aware of the Yijing hexagrams. The assertion by Merzbach and Boyer (2011, 388) that Leibniz’s “noting of the binary system of numeration” was one of his “relatively minor contributions” can perhaps best be reconciled with its influence as an acknowledgment of the extraordinary breadth and range of Leibniz’s other work.

About the Texts, Translations, and Apparatus

The Texts

At the time of writing, almost 350 years have elapsed since Leibniz devised his binary system, and despite its importance, remarkably few of his writings on the subject have been published, and fewer still are available in English translation. In Leibniz's Nachlaß, there exist around 100 manuscripts devoted to binary and a similar number of letters in which binary is discussed (usually alongside other things). While a complete English edition of this corpus would be desirable, space constraints forced us to be selective. In narrowing our selection to thirty-two texts, we have focused on those writings which, taken together, tell the story of binary as it developed in Leibniz's hands, from first thoughts through to dissemination and eventual publication. It is, as we shall see, if not quite an epic then at least a protracted and sprawling story, featuring mathematical, philosophical, theological, and cultural twists and turns and such disparate artifacts as medals, wax seals, and the hexagrams of the Yijing.

Among the writings that we chose to omit were ones devoted to sequences of binary numbers along with examples of addition,[38] workings out of various binary sums,[39] investigations into the regularities in columns of binary numbers,[40] binary fractions,[41] the periodicity of binary fractions,[42] and binary algebra.[43] However, many of the ideas found in these omitted writings can also be found in the texts we have included. Twelve of our thirty-two texts have yet to be published in their original language (namely, chapters 2–4, 6–12, 20, 24, and 25) and all but three are appearing in English for the first time.[44]

Leibniz saved virtually everything he wrote, so the vast majority of his extant writings on binary were never intended for publication or dissemination in any form.[45] Many were simply rough notes or exploratory drafts in which he thinks as he writes, seeing where the ideas lead. Despite their rawness and sporadic clumsiness, these pieces afford a valuable insight into Leibniz's working methods, complete with their wrong turns, dead ends, and occasional calculation errors or lapses in reasoning as he charts new territory. Thus when reading this book, it is good to keep in mind that Leibniz was writing mostly for himself.

The Translations

As so many of Leibniz's writings on binary have yet to be published, including most of the important early ones that show Leibniz developing and experimenting with the system, we could not rely on published transcriptions and so made our own transcriptions from the original manuscripts, including of those writings for which a published transcription is available. Transcribing Leibniz's manuscript writings on binary proved to be a challenge, in no small part because many of them are hastily scribbled drafts often featuring dense tables, cramped scratchwork, and frequent deletions and revisions. We have sought to replicate the manuscript material as faithfully as the limitations of the printed page allow. We should note, however, that we have corrected Leibniz's errors (where we have spotted them!), indicating this in the footnotes, and have provided his deletions only where we consider

38. See LH 35, 3 B 5 Bl. 58.

39. See LH 35, 3 B 5 Bl. 28 and 62.

40. See LH 35, 3 B 5 Bl. 15–18, 25–26, 57, and 94–95.

41. See LH 35, 3 B 5 Bl. 47, 51, 52, and 89–90.

42. See LH 35, 3 B 5 Bl. 11–12.

43. See LH 35, 3 B 5 Bl. 78.

44. The three that have appeared in English before are "On the Organon or Great Art of Thinking" (chapter 10) in Leibniz (1973, 10–17); Leibniz's letter to Duke Rudolph August of 2/12 January 1697 (chapter 17) in Ching and Oxtoby (1992, 70–76) and partially in Cajori (1916, 560–563) and Glaser (1981, 31–36); and "Explanation of Binary Arithmetic" (chapter 30) in Glaser (1981, 39–43).

45. As Knobloch (2004, 75) observes, "Leibniz's texts reflect his thinking as thinking while writing. Every idea, every question, every doubt, every access and optimism, every provisional result, every plan or intention is written down."

them to have some significance or interest for contemporary scholars working in one or other of the fields of mathematics, computer science, philosophy, or theology (or the histories thereof).

Translating this material also proved quite challenging at times, usually due to Leibniz's terminology; the mathematics of his time had few agreed terms and little standardized notation. While we have tried to stay as close to the source language as possible, a literal translation of some of Leibniz's terms and symbols would have made for difficult reading today. We note below some of our more important translation choices.

Binary When referring to his notation of 0 and 1, Leibniz interchangeably used the borrowed Greek term *dyadica* and the Latin *binarius*, both of which mean "pertaining to two." Where possible, we have translated both terms as "binary," since this has greater currency in English than does the equivalent "dyadic," at least as regards the base-2 number system. Occasionally, Leibniz uses both *dyadica* and *binarius* separated by the "or" of equivalence, to indicate that they are alternative terms, and in such cases, we have translated the former as "dyadic" in order to avoid the obvious problem with "binary or binary." On the German word *Dyadik*, see note 36 above.

Carry Leibniz used the Latin term *residuus* (remainder, the remaining, left over) to refer to a quantity being transferred from one column to another. But "remainder" is a term now used in mathematics only to refer to an amount left over after a division operation; hence, when Leibniz uses *residuus* to refer to the transfer of quantities between columns, we translate it by the noun "carry," in line with modern usage. Leibniz also uses phrases such as the Latin *transmittit aliquid* (literally: transmits something) when the addition of two digits in one column causes a carry to the next; in such a case, we have translated the phrase as "causes a carry."

Digit In line with its etymology and usage at the time, Leibniz reserved the term "digit" (Latin: *digitus*) for the decimal numerals 0–9. When referring to binary digits, whether represented by 0 and 1 or by letters, he used the Latin *notae* (marks, signs) or the French *notes* (notes, marks). Since mathematicians and computer scientists now routinely refer to "binary digits," and since referring instead to marks, signs, or notes risks confusion, we have translated *notae* and *notes* as "digits" except for a few cases in which Leibniz used those terms to refer to the trigrams of the Yijing or their component lines, where we have opted for "trigrams" or "glyphs," respectively. (In our commentaries but not in the translation, we also use the conventional "bit" for "binary digit.")

Dot When indicating carries in a column of binary digits, Leibniz used the Latin *punctum*, the French *point*, and the German *Punct* (*Punkt* in modern German), all of which are most naturally translated as "point." However, in the context of a mark indicating a carry, we have opted to translate these terms as "dot," the term commonly used for a diacritic mark over a letter or numeral, as opposed to the decimal or binary "point" introducing a number's fractional part.

Progression Leibniz commonly referred to a sequence of binary digits by the Latin *progressio*, the French *progression*, and the German *Fortschreitung*, all of which are commonly rendered into English as "progression." Despite some reservations, we have decided to translate these terms by "progression" rather than "sequence," which is the English word we have used to translate the Latin *series* and French *série*. Others sometimes translate *progressio dyadica* as "binary system"; we have refrained from rendering *progressio* in this way to give readers the discretion to decide for themselves when Leibniz is referring simply to a geometric series and when to an associated system of notation and arithmetic.

Double geometric progression Leibniz uses the expression *progressio geometrica dupla*, literally "double geometric progression," to refer to the geometric progression in which the ratio of successive elements is 2, that is, 1, 2, 4, 8, 16, ...[46] (Or, sometimes, the progression in which the ratio is $\frac{1}{2}$, that is: 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ...) We use the literal English translation of this phrase since it is so much less

46. Leibniz is inconsistent as to whether $1 = 2^0$ is a member of the double geometric progression. In chapter 4 he explicitly includes it, but the deleted text in footnote 8 of chapter 23 requires that 2^1 be the first member of the progression, and the lower-left medal design near the end of chapter 17 affixes a star to 2, 4, 8, and 16, but not to 1.

wordy than a more precise phrasing, even though it may unintentionally suggest that the progression itself is “double” somehow.

The Apparatus

For each text, we have provided full details of the manuscript sources and available transcriptions, as well as a “headnote” in which we explain the context and content of the text as well as give reasons for our dating of the manuscript source in the many cases where Leibniz did not date the text himself. It should be noted that those texts written before March 1700 are given two dates: the first follows the Julian calendar, the second the Gregorian calendar, which was finally adopted in Protestant Germany on 1 March 1700. Until the switch, Leibniz dated his writings according to the Julian calendar, which was ten days behind the Gregorian. Although we have not amended any of the dates Leibniz supplied on his manuscripts, we have followed the convention of using a dual-dating system, consisting of old- and new-style dates, when referring to dates falling before the switch to the Gregorian calendar, for example, writing a date as 15/25 March, where the first date is that of the Julian calendar (old style) and the second that of the modern Gregorian calendar (new style).

In addition to the headnotes and footnotes, we have also supplied commentaries to explain Leibniz’s thinking and/or to identify his errors and how these affect his calculations. These commentaries are displayed against a gray background and are positioned alongside or underneath the relevant part of a text; the gray box indicates that the comments within are not part of the text proper but are an editorial comment thereon.

The layout of the manuscript material requires some comment. While it might seem most fitting to reproduce the layout of Leibniz’s manuscripts exactly on the printed page, this was not always possible or desirable in practice. Many of the manuscripts we have used are cluttered and untidy, with Leibniz adding new material wherever he could find space, sometimes indicating by a line where in the text it should fit, sometimes not. Some feature passages written in the margin or vertically down the side of the page. Many of the manuscripts contain lengthy tables and/or rows of sums, sometimes with no workings out, and many also contain material that Leibniz chose to delete. Representing all of this faithfully on the printed page would have made many of the texts unreadable. Hence, we have sought to order and arrange the material as best we can, seeking to remain faithful to Leibniz’s intentions where these could be discerned. When Leibniz indicates where a marginal addition should go, we put it there, likewise with tables and sums, so far as typesetting limitations permit. Material whose placement could not be discerned is recorded in footnotes. In a direct quotation from Leibniz, a period or comma positioned outside the closing double quotation mark, rather than inside as would be conventional, indicates that the text quoted does not include that punctuation mark.

We have also used the following conventions:

- [ ] In our editorial matter, square brackets are used to indicate English translations of titles or organizations given in their original language. In the translations (including translated material in the headnotes or footnotes), square brackets denote editorial interpolations, usually to indicate words omitted by the original author.
- *italics* In the translations, italics are used to emphasize words or passages that were underlined in the manuscript, as well as to indicate book titles and words or expressions left in other languages.
- grayed-out text In chapter 1, we have used grayed-out text to indicate manuscript material that was not in Leibniz’s hand.

Typographical Conventions Each chapter begins with a chapter number, title, and date, for example:

5 Binary Arithmetic Machine (before 15/25 March 1679)

If Leibniz supplied a title, we use it as the chapter title and also present it at the beginning of the text itself; if he did not include a title, we have devised one, but have not put this at the beginning of the text itself as it is not part of the manuscript.

Next comes information about the manuscript and available transcriptions, for example:

Manuscript:
M: LH 42, 5 Bl. 61. Draft. Latin.

Transcription:
LVM: Mackensen (1974, 256, 259).

Where there exists only a single manuscript source, it is designated M, but where there are multiple manuscript sources, they are labeled M1, M2, and so on, and in such cases, we indicate at the start of the translation which source(s) we have used. We have likewise provided an abbreviation for available transcriptions. Where there are discrepancies between our reading of a manuscript source and that found in a published transcription, we record it in a footnote, referring to each by the abbreviations indicated at the start of the chapter. Similarly, where there are variations between different manuscript sources, this is recorded in a footnote. Abbreviations such as "LH" are explained in the "Abbreviations" section. "Bl" stands for "Blatt," German for "sheet." "Mackensen (1974)" can be identified in the bibliography at the end of this book.

Our headnote follows and is ended by the following ornament:

Leibniz's text follows and continues to the end of the chapter.

Material that appears in a box on a gray background, like this, is our explanatory gloss. Sometimes it appears alongside Leibniz's text; sometimes, as here, as a paragraph interpolated between sections of Leibniz's text.

To improve readability, long binary numerals are sometimes presented as blocks of four bits separated by narrow spaces, for example, "10100001" as "1010 0001."

A numeral with slashes through digits, "$\not{1}\not{0}\not{1}\not{0}\ \not{0}001$", for example, transcribes a numeral that Leibniz has crossed out digit by digit, in whole or in part (in this case, Leibniz wrote "1010 0001" and crossed out each of the first five digits). This is not an instance of Leibniz correcting a mistake; rather, he is checking off digits in the course of carrying out an arithmetic algorithm, for long division perhaps. Depending on typographical convenience, deleted passages are noted either in footnotes or by ~~crossing them out~~.

When captured in a footnote, deleted material is presented between ▷ and ◁ (the former symbol indicating where a deleted passage begins, the latter where it ends). Variants between different manuscripts of the same text are recorded in a similar way. After ◁ we indicate whether the material is a deletion or a variant in a different source from that used for the translation of a chapter. In cases where a deleted passage includes a restart which is also deleted, we indicate this using numbers in the following way:

{1} the deleted passage starts; {2} Leibniz strikes out everything in the whole of {1} and starts again.

Bibliography

Adamson, Donald. 1995. *Blaise Pascal. Mathematician, Physicist and Thinker about God.* Basingstoke: Palgrave.

Aiken, Howard, A. G. Oettinger, and T. C. Bartee. 1964. "Proposed Automatic Calculating Machine." *IEEE Spectrum* 1 (8): 62–69.

Aiton, E. J. 1981. "An Unpublished Letter of Leibniz to Sloane." *Annals of Science* 38 (1): 103–107.

Aiton, E. J. 1985. *Leibniz: A Biography.* Bristol: Adam Hilger.

Alexandre, Noel. 1700. *Conformite des ceremonies chinoises avec l'idolatrie grecque et romaine. Pour servir de confirmation à l'Apologie des Dominicains Missionnaires de la Chine.* Cologne: Corneille d'Egmond.

Andrés, Giovanni. 1790. *Dell'origine, progressi e stato attuale d'ogni letteratura.* Parma: Dalla Stamperia reale.

Anon. 1682. "Machines rares et surprenantes." *Journal des sçavans:* January 19, 16–18.

Anon. 1700. *Historia cultus Sinensium: seu varia scripta de cultibus Sinarum, inter vicarios apostolicos gallos aliosque missionarios, & patres societatis Jesu controversis, oblata Innocentio XII.* Cologne: n.p.

Anon. 1857. *Nuova enciclopedia popolare italiana, ovvero Dizionario Generale di Scienze, Lettere, Arti, Storia, Geografia, Ecc. Ecc.* 25 vols. Torino: Dalla Societa l'Unione Tipografico-Editrice.

Antognazza, Maria Rosa. 2009. *Leibniz: An Intellectual Biography.* Cambridge: Cambridge University Press.

Antognazza, Maria Rosa. 2018. "Leibniz as Historian." In *The Oxford Handbook of Leibniz,* edited by Maria Rosa Antognazza, 591–608. Oxford: Oxford University Press.

Ares, J., J. Lara, D. Lizcano, and M. A. Martinez. 2018. "Who Discovered the Binary System and Arithmetic? Did Leibniz Plagiarize Caramuel?" *Science and Engineering Ethics* 24:173–188.

Aristotle. 1984. *The Complete Works of Aristotle.* Edited by Jonathan Barnes. 2 vols. Princeton, NJ: Princeton University Press.

Astin, J. 1984. "Binary Expansion of Reciprocals." *International Journal of Mathematical Education in Science and Technology* 15 (4): 423–428.

Augustine, Saint. 1952. *The City of God, Books VIII–XVI.* Translated by Gerald G. Walsh and Grace Monahan. Washington, DC: Catholic University of America Press.

Bacon, Francis. 1620. *Novum Organum, sive indicia vera de interpretatione naturae.* London: Billium.

Bacon, Francis. 2000. *The New Organon.* Edited by Lisa Jardine and Michael Silverthorne. Cambridge: Cambridge University Press.

Barlow, Peter. 1814. *A New Mathematical and Philosophical Dictionary: Comprising an Explanation of the Terms and Principles of Pure and Mixed Mathematics, and Such Branches of Natural Philosophy as Are Susceptible of Mathematical Investigation.* London: Whittingham/Rowland.

Bauer, Friedrich L. 2010. *Origins and Foundations of Computing.* Dordrecht: Springer.

Bayer, Gottlieb Siegfried. 1730. *Museum sinicum in quo sinicae linguae et literaturae ratio explicatur. Tomus primus.* St. Petersburg: Academia imperatoria.

Beeley, Philip. 2020. "Leibniz and the Royal Society Revisited." In *Leibniz's Legacy and Impact,* edited by Julia Weckend and Lloyd Strickland, 23–52. New York: Routledge.

Belaval, Yvon. 2005. *Leibniz: Initiation à sa philosophie.* Paris: Vrin.

Bell, A. H. 1895. "The 'Cattle Problem' by Archimedes 251 B.C." *The American Mathematical Monthly* 2:140–141.

Bilfinger, Georg Bernhard. 1724. *Specimen doctrinae veterum sinarum moralis et politicae.* Frankfurt: Andreas and Hort.

Boerhaave, Herman. 1715. *Sermo academicus de comparando certo in physici.* Leiden: Vander.

Boole, George. 1854. *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities.* London: Walton/Maberly.

Bouvet, Joachim. 1697. *Portrait historique de l'Empereur de la Chine.* Paris: Michallet.

Bouvet, Joachim. 1704. "Extrait d'une lettre écrite à Mr. Leibnits par le P. Bouvet de la Compagnie de Jesus. A Pekin le 4. Novembre 1701." *Memoires pour l'histoire des sciences et des beaux arts:* January, 128–165.

Brancato, Mattia. 2021. "Leibniz's Binary Algebra and its Role in the Expression and Classification of Numbers." *Philosophia Scientiae* 25:71–94.

Brander, Georg Friedrich. 1775. *Arithmetica Binaria Sive Dyadica das ist Die Kunst nur mit zwey Zahlen in allen vorkommenden Fällen sicher und leicht zu rechnen.* Augsburg: Klett.

Brewster, David. 1832. *The Edinburgh Encyclopaedia.* 18 vols. Philadelphia: Joseph and Edward Parker.

Buffon, Comte de, Georges-Louis Leclerc. 1741. "Formule sur les échelles arithmetiques." *Histoire de l'Académie Royale des Sciences. Année MDCCXLI* (Paris): 219–221.

Cajori, Florian. 1916. "Leibniz's 'Image of Creation'." *The Monist* 26 (4): 557–565.

Cantor, Moritz. 1863. *Mathematische Beiträge zum Kulturleben der Volker.* Halle: Schmidt.

Caramuel, Juan. 1670. *Mathesis biceps, vetus et nova.* Campagna: Annison.

Carhart, Michael C. 2016. "Leibniz between Paris, Grand Tartary, and the Far East: Gerbillon's Intercepted Letter." In *China in the German Enlightenment,* edited by Bettina Brandt and Daniel Leonhard Purdy, 80–96. Toronto: University of Toronto Press.

Caze, César. 1705. *Letter to Gottfried Wilhelm Leibniz.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/caze1705.htm.

Ceulen, Ludolph van. 1615. *Vanden circkel. Daer in gheleert werdt te vinden de naeste proportie des circkels-diameter tegen synen omloop daer door alle circkels (met alle figueren ofte landen met cromme linien besloten) recht ghemeten kunnen werde.* Delph: Andriesz.

Ching, Julia, and Willard G. Oxtoby. 1992. *Moral Enlightenment: Leibniz and Wolff on China.* New York: Routledge.

Clavius, Christopher. 1570. *In sphaeram Ioannis de Sacro Bosco commentarius.* Rome: Victorium Helianum.

Clüver, Philipp. 1663. *Germania antiqua, cum vindelicia et norico auctoris methodo, verbis & tabulis geographicis retentis contracta.* Wolfenbüttel: Buno.

Collani, Claudia von. 1988. "Tianxue benyi—Joachim Bouvets Forschungen zum Monotheismus." *China-Mission Studies (1550–1800)* Bulletin X:9–33.

Collani, Claudia von. 2007. "The First Encounter of the West with the Yijing: Introduction to and Edition of Letters and Latin Translations by French Jesuits from the 18th Century." *Momumenta Serica* 55:227–387.

Collignon, Eduard. 1897. "Note sur l'arithmétique binaire." *Journal de mathématiques élémentaires* 21:101–105.

Couturat, Louis. 1901. *La logique de Leibniz.* Paris: Félix Alcan.

Crippa, David. 2019. *The Impossibility of Squaring the Circle in the 17th Century. A Debate among Gregory, Huygens and Leibniz.* Cham: Birkhäuser.

Croker, Henry Temple, Thomas Williams, and Samuel Clark, eds. 1766. *The Complete Dictionary of Arts and Sciences. In Which the Whole Circle of Human Learning Is Explained, and the Difficulties Attending the Acquisition of Every Art, Whether Liberal or Mechanical, Are Removed, in the Most Easy and Familiar Manner.* London: J. Wilson & J. Fell.

Curione, Celio Secondo. 1617. *De amplitudine regni coelestis.* Frankfurt: Palthenius.

Dangicourt, Pierre. 1710. "De periodis columnarum in serie numerorum progressionis arithmeticae dyadice expressorum." *Miscellanea Berolinensia* 1:336–376.

Dascal, Marcelo. 2006. *Gottfried Wilhelm Leibniz: The Art of Controversies.* Dordrecht: Springer.

De Risi, Vincenzo. 2007. *Geometry and Monadology: Leibniz's Analysis Situs and Philosophy of Space.* Basel: Birkhäuser.

De Volder, Burcher. 1695. *Exercitationes academicae quibus Ren. Cartesii philosophia defenditur adversus Petri Danielis Huetii Episcopi Suessionensis Censuram philosophiae Cartesianae.* Amsterdam: Ravestein.

Descartes, René. 1898. *Oeuvres de Descartes: Correspondance II.* Edited by Charles Adam and Paul Tannery. Paris: Cerf.

Diaz, Don Francisco. 1640–1643. *Vocabulario de letra China con la explicacion Castellana hecho con gran propriedad y abundancia de palabras por el Padre Don Francisco Diaz de la Orden de Predicadores ministro incansable en esto Reyno de China.* Berlin. https://digital.staatsbibliothek-berlin.de/werkansicht/?PPN=PPN3358707539.

Diderot, Denis, and Jean le Rond D'Alembert, eds. 1751. *Encyclopédie: ou dictionnaire raisonné des sciences, des arts et des métiers. Tome second. B–CEZ.* Paris: Briasson, David, Le Breton, and Durand.

Dotzler, Bernhard J. 2010. "Erschaffung aller Dinge aus Nichts: Leibniz und die Muster der Binarität." In *Die Enzyklopädik der Esoterik: Allwissenheitsmythen und universalwissenschaftliche Modelle in der Esoterik der Neuzeit,* edited by Andreas B. Kilcher and Philipp Theisohn, 21–34. Paderborn: Wilhelm Fink.

Eglash, Ron. 2005. *African Fractals: Modern Computing and Indigenous Design.* New Brunswick, NJ: Rutgers University Press.

Figuier, Louis. 1870. *Vies des savants illustrés du XVIIIe siècle: avec l'appréciation sommaire de leurs travaux.* Brussels: Lacroix, Verboeckhoven & co.

Fontenelle, Bernard le Bovier de. 1705a. *New Binary Arithmetic.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/newbinary.htm.

Fontenelle, Bernard le Bovier de. 1705b. "Nouvelle arithmetique binaire." In *Histoire de l'Académie Royale des Sciences. Année MDCCIII. Avec les memoires de mathematique & de physique, pour la même année,* 58–63. Paris: Boudot.

Fontenelle, Bernard le Bovier de. 1705c. "Sur les tangentes et les secantes des angles." In *Histoire de l'Académie Royale des Sciences. Année MDCCIII. Avec les memoires de mathematique & de physique, pour la même année,* 64–65. Paris: Boudot.

Fontenelle, Bernard le Bovier de. 1718. "Eloge de M. Leibnitz." In *Histoire de l'Academie Royale des Sciences. Année M.DCCXVI. Avec les memoires de mathematique & de phisique, pour la même année,* 94–128. Paris: L'imprimerie Royale.

Gage, Thomas. 1696. *Histoire de l'Empire mexicain représentée par figures. Relation du Mexique ou de la Nouvelle Espagne, par Thomas Gages, traduite par Melchisédec Thévenot.* Paris: Moette.

Galilei, Galileo. 1638. *Discorsi E Dimostrazioni Matematiche.* Leiden: Elzevier.

Galilei, Galileo. 1914. *Dialogue Concerning Two New Sciences.* Translated by Henry Crew and Alfonso de Salvio. New York: Macmillan.

Garfinkel, Simson L., and Rachel H. Grunspan. 2018. *The Computer Book: From the Abacus to Artificial Intelligence, 250 Milestones in the History of Computer Science.* New York: Sterling.

Gimmestad, Beverly J. 1991. "The Russian Peasant Multiplication Algorithm: A Generalization." *The Mathematical Gazette* 75 (472): 169–171.

Glaser, Anton. 1981. *History of Binary and Other Nondecimal Numeration.* 2nd ed. Los Angeles: Tomash.

Golius, Jacob. 1655. "De regno catayo additamentum." In *Novus atlas sinensis,* edited by Martino Martini. Amsterdam: J. Blaeu.

Golius, Jacob. 1696. *Catalogus Insignium in omni facultate, linguisque, Arabica, Persica, Turcica, Chinensi &c. Librorum M.Ss.* Leiden: Vivie.

Grillet de Roven, René. 1673. *Curiositez mathematiques de l'invention du Sr Grillet horlogeur à Paris.* Paris: Coignard.

Guericke, Otto von. 1672. *Experimenta nova (ut vocantur) Magdeburgica de spatio vacuo.* Amsterdam: Waesberge.

Harriot, Thomas. 1590. *A briefe and true report of the new found land of Virginia: of the commodities and of the nature and manners of the naturall inhabitants. Discouered by the English colony there seated by Sir Richard Greinuile Knight in the yeere 1585. Which remained vnder the gouernement of twelue monethes; at the speciall charge and direction of the Honourable Sir Walter Raleigh Knight lord Warden of the stanneries who therein hath beene fauoured and authorised by her Maiestie and her letters patents.* Frankfurt: Johann Wechel, Theodor de Bry, and Sigmund Feyerabend.

Harsdörffer, Georg Philipp. 1651. *Delitiae mathematicae et physicae der Mathematischen und Philosophischen Erquickstunden. Zweiter Theil.* Nuremberg: Dümler.

Harsdörffer, Georg Philipp. 1653. *Delitiae mathematicae et physicae der Mathematischen und Philosophischen Erquickstunden. Dritter Theil.* Nuremberg: Endters.

Heinekamp, Albert. 1978. "Kochański als Leibniz-Korrespondent." *Organon* 14:73–106.

Heyse, Johann-Christian-August. 1859. *Allgemeines verdeutschendes und erklärendes Fremdwörterbuch.* 12th ed. Hanover: Hofbuchhandlung.

Hill, Thomas Wright. 1860. *Selections from the Papers of the Late Thomas Wright Hill, Esq. F.R.A.S.* London: John W. Parker / Son.

Hochstetter, Erich. 1966. *Herrn von Leibniz' Rechnung mit Null und Eins.* Berlin: Siemens.

Hoffmann, Friedrich. 1700. *Demonstrationes physicae curiosae, experimentis et observationibus mechanicis ac chymicis illustratae.* Halle: Zeitlerum.

Huet, Pierre-Daniel. 1690. *Censura philosophiae Cartesianae.* Frankfurt and Leipzig: Grenz.

Huet, Pierre-Daniel. 2003. *Against Cartesian Philosophy.* Edited and translated by Thomas M. Lennon. Amherst, NY: Humanity Books.

Hutton, Charles. 1795. *A Mathematical and Philosophical Dictionary: Containing an Explanation of the Terms, and an Account of the Several Subjects, Comprized under the Heads Mathematics, Astronomy, and Philosophy both Natural and Experimental.* 2 vols. London: J. Johnson and G. G. and J. Robinson.

Huygens, Christiaan. 1693. "De problemate Bernouilliano." *Acta eruditorum:* October, 475–476.

Huygens, Christiaan. 1698. Κοσμοθεωρός*, sive de terris coelestibus earumque ornatu conjecturae.* The Hague: Adriaen Moetjens.

Ifrah, Georges. 1998. *The Universal History of Numbers.* Translated by David Bellos, E. F. Harding, Sophie Wood, and Ian Monk. New York: John Wiley & Sons.

Ineichen, R. 2008. "Leibniz, Caramuel, Harriot und das Dualsystem." *Mitteilungen der deutschen Mathematiker-Vereinigung* 16 (1): 12–15.

Ingaliso, Luigi. 2017. "Leibniz e la lettura binaria dell'I Ching." In *Alterità e Cosmopolitismo nel Pensiero Moderno e Contemporaneo,* edited by Giancarlo Magnano San Lio and Luigi Ingaliso, 109–132. Soveria Mannelli: Rubbettino.

Intorcetta, Prospero, Christian Wolfgang Herdtrick, François de Rougemont, and Philippe Couplet, eds. and trans. 1687. *Confucius Sinarum philosophus.* Paris: Daniel Horthemels.

Irson, Claude. 1692. *Arithmetique pratique et raisonnée.* Paris: Chardon.

Jeake, Samuel. 1696. Λογιστολογία*, or Arithmetick Surveighed and Reviewed.* London: J. R / J. D.

Jones, Matthew L. 2018. "Calculating Machine." In *The Oxford Handbook of Leibniz,* edited by Maria Rosa Antognazza, 509–525. Oxford: Oxford University Press.

Jordan, G. J. 1927. *The Reunion of the Churches: A Study of G. W. Leibnitz and His Great Attempt.* London: Constable & Co.

Justinian. 1998. *The Digest of Justinian.* Translated by Alan Watson. 4 vols. Philadelphia, PA: University of Pennsylvania Press.

Kircher, Athanasius. 1650. *Obeliscus Pamphilius.* Rome: Grignani.

Kircher, Athanasius. 1652–1654. *Oedipus Aegyptiacus.* 3 vols. Rome: Mascardi.

Kircher, Athanasius. 1667. *China monumentis, qua sacris qua profanis, nec non variis naturæ & artis spectaculis, aliarumque rerum memorabilium argumentis illsutrata.* Amsterdam: Jansson.

Knobloch, Eberhard. 2004. "Leibniz and the Use of Manuscripts: Text as Process." In *History of Science, History of Text,* edited by K. Chemla, 51–79. Dordrecht: Springer.

Knobloch, Eberhard. 2015. "Analyticité, équipollence et théorie des courbes chez Leibniz." In *G. W. Leibniz, Interrelations between Mathematics and Philosophy,* edited by Norma B. Goethe, Philip Beeley, and David Rabouin, 89–110. New Studies in the History and Philosophy of Science and Technology 41. Dordrecht: Springer Netherlands.

Knobloch, Eberhard. 2018. "Determinant Theory, Symmetric Functions, and Dyadic." In *The Oxford Handbook of Leibniz,* edited by Maria Rosa Antognazza, 225–246. Oxford: Oxford University Press.

Knuth, Donald E. 1997. *The Art of Computer Programming, Volume 2: Seminumerical Algorithms.* 3rd ed. Upper Saddle River, NJ: Addison-Wesley Professional.

Kolpas, Sid. 2019. "Napier's Binary Chessboard Calculator." *Math Horizons* 27:25–27.

Kopp, Franz Otto, and Erwin Stein. 2008. “Es funktioniert doch—mit zwei Korrekturen: Zehnerüberträge der Leibniz-Maschine.” *Unimagazain Hannover* 3/4:56–59.

Krist, Joseph. 1859. “Über Zahlensysteme und deren Geschichte.” In *Vierter Jahres-Bericht der Kaiserlich-Königlichen Ober-Realschule der königlich freien Haupstadt Ofen,* 32–73. Ofen: Bagoschen.

L’Hospital, Guillaume-François-Antoine. 1696. *Analyse des infiniment petits, pour l’intelligence des lignes courbes.* Paris: L’imprimerie royale.

La Hire, Philippe de. 1695. *Traité de mécanique, ou l’on explique tout ce qui est nécessaire dans la pratique des Arts, & les propriétés des corps pesants lesquelles ont un plus grand usage dans la Physique.* Paris: Anisson.

Lagny, Thomas Fantet de. 1725. “Second memoire sur la goniometrie purement analytique, ou methode nouvelle & generale, pour déterminer exactement, lorsqu’il est possible, ou indéfiniment près lorsque l’exactitude est impossible, la valeur des trois angles de tout triangle rectiligne, soit rectangle, soit obliquangue, dont les trois côtés sont donnés en nombre; & cela par le seul calcul analytique sans tables des sinus, tangentes & secantes.” In *Histoire de l’Académie Royale des Sciences. Année M.DCCXXV. Avec les memoires de mathematique & de physique, pour la même année,* 282–323. Paris: L’imprimerie royale.

Lamprecht, Jacob Friedrich. 1740. *Leben des Freyherrn Gottfried Wilhelm von Leibniz.* Berlin: Haude.

Laplace, Pierre-Simon de. 1825. *Essai philosophique sur les probilités.* 5th ed. Paris: Courcier.

Legge, James. 1871. *The Chinese Classics.* 7 vols. London: Trübner & Co.

Legge, James. 1882. *The Sacred Books of The East. Vol. XXVII: The Texts of Confucianism.* Translated by James Legge. Oxford: Clarendon Press.

Leibniz, Gottfried Wilhelm. 1682. “De vera proportione circuli ad quadratum circumscriptum in numeris rationalibus expressa.” *Acta Eruditorum:* January, 41–46.

Leibniz, Gottfried Wilhelm. 1683. *Essay on the Number of Men.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/numbermen.htm.

Leibniz, Gottfried Wilhelm. 1684. “Nova methodus pro maximis et minimis.” *Acta eruditorum:* October, 457–473.

Leibniz, Gottfried Wilhelm. 1686a. “Brevis demonstratio erroris memorabilis Cartesii.” *Acta eruditorum:* March, 161–163.

Leibniz, Gottfried Wilhelm. 1686b. “De geometria recondita et analysi indivisibilium atque infinitorum, Addenda his quae dicta sunt in Actis a. 1684, Maji p. 233; Octob. p. 264; Decemb. p. 586.” *Acta Eruditorum:* June, 292–300.

Leibniz, Gottfried Wilhelm. 1689. “Tentamen de motuum coelestium causis.” *Acta eruditorum:* February, 82–96.

Leibniz, Gottfried Wilhelm. 1691a. “De legibus naturae et vera aestimatione virium motricium contra Cartesianos.” *Acta eruditorum:* September, 439–447.

Leibniz, Gottfried Wilhelm. 1691b. “De linea in quam exile se pondere proprio curvat.” *Acta eruditorum:* June, 277–281.

Leibniz, Gottfried Wilhelm. 1692a. “De la chainette, ou solution d’un Problème fameux proposé par Galilei.” *Journal des sçavans:* March, 147–153.

Leibniz, Gottfried Wilhelm. 1692b. “Generalia de natura linearum, anguloque contactus & osculi, provolutionibis aliisque cognatis, & eorum usibus nonnullis.” *Acta eruditorum:* September, 440–446.

Leibniz, Gottfried Wilhelm, ed. 1693. *Codex juris gentium diplomaticus.* Hanover: n.p.

Leibniz, Gottfried Wilhelm. 1694. *Binary Division.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/division.htm.

Leibniz, Gottfried Wilhelm. 1695a. *My Arithmetic Machine.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/machine.htm.

Leibniz, Gottfried Wilhelm. 1695b. “Specimen dynamicum.” *Acta eruditorum:* April, 145–157.

Leibniz, Gottfried Wilhelm. 1696. *Sketch for a Medal Commemorating His Formula for the Quadrature of the Circle.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/medal.htm.

Leibniz, Gottfried Wilhelm. 1697. *Novissima sinica.* n.p.: n.p.

Leibniz, Gottfried Wilhelm. 1699. *Reflections on Locke’s Second Reply.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/locke.htm.

Leibniz, Gottfried Wilhelm, ed. 1700. *Mantissa codicis juris gentium diplomatici.* Hanover: Freytag.

Leibniz, Gottfried Wilhelm. 1701. *Letter to Jean de Fontaney.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/fontaney1701.htm.

Leibniz, Gottfried Wilhelm. 1703a. *Binary and the Hexagrams of Fuxi.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/fuxi.htm.

Leibniz, Gottfried Wilhelm. 1703b. *Letter to Jean Fontaney.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/fontaney1703.htm.

Leibniz, Gottfried Wilhelm. 1705a. "Explication de l'aritmètique binaire, qui se sert des seuls caracteres 0 & 1; avec des Remarques sur son utilité, & sur ce qu'elle donne le sens des anciennes figures Chinoises de Fohy." In *Memoires de l'Académie Royale des Sciences. Année MDCIII. Avec les Memoires de Mathematique & de Physique, pour la même Année,* 85–89. Paris: Boudot.

Leibniz, Gottfried Wilhelm. 1705b. *Letter to Ferdinand Orban.* Translated by Lloyd Strickland. http://www.leibniz-translations.com/orban.htm.

Leibniz, Gottfried Wilhelm. 1718. *Otium Hanoverarum, sive miscellanea.* Edited by Joachim Friedrich Feller. Leipzig: J. C. Martini.

Leibniz, Gottfried Wilhelm. 1720. *Lehr-Sätze über die Monadologie: ingleichen von Gott und seiner Existentz, seinen Eigenschaften und von der Seele des Menschen.* Edited by Heinrich Köhler. Frankfurt: Meyers.

Leibniz, Gottfried Wilhelm. 1739. *Leibnitii Tentamina Theodicaeae.* Edited by Johann Ulrich Steinhofer. 2 vols. Frankfurt: Berger.

Leibniz, Gottfried Wilhelm. 1740. *Kleinere Philosophische Schriften.* Edited by Caspar Jacob Huth. Jena: Mayer.

Leibniz, Gottfried Wilhelm. 1742. "The True Proportion of the Circle to the Circumscribed Square, in Rational Numbers, Trans. A Society of Gentlemen." *Acta Germanica* (London) 1:59–62.

Leibniz, Gottfried Wilhelm. 1744. *Theodicee, das ist, Versuch von der Güte Gottes, Freyheit des Menschen, und vom Ursprunge des Bösen.* Translated by Johann Christian Gottsched. Hanover: Försters/Erben.

Leibniz, Gottfried Wilhelm. 1768. *G. G. Leibnitii Opera Omnia.* Edited by Ludovic Dutens. 6 vols. Geneva: Fratres de tournes.

Leibniz, Gottfried Wilhelm. 1845. *Godofredi Guilielmi L. B. de Leibnitz Lipsiensis epistolae XLVI at Teuberum concionatorem aulae cizensis.* Leipzig: Staritz.

Leibniz, Gottfried Wilhelm. 1849–1863. *Leibniziens Mathematische Schriften.* Edited by C. I. Gerhardt. 7 vols. Halle: H. W. Schmidt.

Leibniz, Gottfried Wilhelm. 1857. *Nouvelles lettres et opuscules inédits de Leibniz.* Edited by A. Foucher de Careil. Paris: Durand.

Leibniz, Gottfried Wilhelm. 1875–1890. *Die Philosophischen Schriften von Leibniz, Gottfried Wilhelm.* Edited by C. I. Gerhardt. 7 vols. Berlin: Weidmann.

Leibniz, Gottfried Wilhelm. 1903. *Opuscules et fragments inédits de Leibniz.* Edited by Louis Couturat. Paris: Félix Alcan.

Leibniz, Gottfried Wilhelm. 1923–. *Sämtliche Schriften und Briefe.* Edited by Berlin-Brandenburgische Akademie der Wissenschaften. 8 series, each divided into multiple volumes. Berlin: Akademie Verlag.

Leibniz, Gottfried Wilhelm. 1969. *Leibniz, Gottfried Wilhelm: Philosophical Papers and Letters.* Edited and translated by Leroy E. Loemker. 2nd ed. Dordrecht: D. Reidel.

Leibniz, Gottfried Wilhelm. 1971. *Calcolo con zero e uno.* Translated by Lia Ruffino. Milan: Etas Kompass.

Leibniz, Gottfried Wilhelm. 1973. *Leibniz: Philosophical Writings.* Edited and translated by Mary Morris and G. H. R. Parkinson. London: Everyman.

Leibniz, Gottfried Wilhelm. 1977. *Leibniziens Mathematische Schriften.* Edited by C. J. Gerhardt. Hildesheim: Georg Olms.

Leibniz, Gottfried Wilhelm. 1992. *De Summa Serum: Metaphysical Papers 1675–1676.* Edited and translated by G. H. R. Parkinson. New Haven, CT: Yale University Press.

Leibniz, Gottfried Wilhelm. 1993. "An Essay on the Causes of Celestial Motions." In *Equivalence and Priority: Newton versus Leibniz,* edited by Domenico Bertoloni Meli, 126–142. Oxford: Clarendon Press.

Leibniz, Gottfried Wilhelm. 1994. *Writings on China.* Edited and translated by Daniel J. Cook and Henry Rosemont Jr. Chicago: Open Court.

Leibniz, Gottfried Wilhelm. 2001. "Two Papers on the Catenary Curve and Logarithmic Curve." Translated by Pierre Beaudry. *Fidelio* 10 (1): 55–58.

Leibniz, Gottfried Wilhelm. 2006a. *Der Briefwechsel mit den Jesuiten in China (1689–1714).* Edited by Rita Widmaier. Hamburg: Felix Meiner Verlag.

Leibniz, Gottfried Wilhelm. 2006b. *Shorter Leibniz Texts.* Edited and translated by Lloyd Strickland. London: Continuum.

Leibniz, Gottfried Wilhelm. 2011a. *Gottfried Wilhelm Leibniz: Die mathematischen Zeitschriftenartikel.* Edited by H.-J. Hess and Malte-L. Babin. Hildesheim: Georg Olms Verlag.

Leibniz, Gottfried Wilhelm. 2011b. *Leibniz and the Two Sophies.* Edited and translated by Lloyd Strickland. Toronto: CRRS.

Leibniz, Gottfried Wilhelm. 2013. *The Leibniz–De Volder Correspondence.* Edited and translated by Paul Lodge. New Haven, CT: Yale University Press.

Leibniz, Gottfried Wilhelm. 2016a. *Leibniz on God and Religion.* Edited and translated by Lloyd Strickland. London: Bloomsbury.

Leibniz, Gottfried Wilhelm. 2016b. *The Leibniz-Arnauld Correspondence.* Edited and translated by Stephen Voss. New Haven, CT: Yale University Press.

Leibniz, Gottfried Wilhelm. 2020. *Dissertation on Combinatorial Art.* Edited by Massimo Mugnai, Han van Ruler, and Martin Wilson. Oxford: Oxford University Press.

Lewis, Harry R., ed. 2021. *Ideas That Created the Future.* Cambridge: MIT Press.

Leybourn, William. 1700. *Arithmetick, Vulgar, Decimal, Instrumental, Algebraical. In Four Parts.* 7th ed. London: J. Matthews.

Lieber, Francis, E. Wigglesworth, and T. G. Bradford, eds. 1854. *Encyclopædia Americana: A Popular Dictionary of Arts, Sciences, Literature, History, Politics and Biography.* Vol. IX. Boston: B. B. Mussey & Co.

Loosen, Renate, and Franz Vonessen. 1968. *Zwei Briefe über das binäre Zahlensystem und die chinesische Philosophie.* Stuttgart: Belser.

Lübsen, Heinrich B. 1869. *Ausführliches Lehrbuch der Arithmetik und Algebra: zum Selbstunterricht und mit Rïcksicht auf die Zwecke des practischen Lebens.* Leipzig: Brandstetter.

Lucas, Édouard. 1891. *Récréations mathématiques.* Paris: Gauthier-Villars.

Ludovici, Carl Günther. 1737. *Ausführlicher Entwurff einer vollständigen Historie der Leibnitzischen Philosophie zum Gebrauch seiner Zuhörer.* 2 vols. Leipzig: Johann Georg Löwe.

Mackensen, Ludolf von. 1974. "Leibniz als Ahnherr der Kybernetik—ein bisher unbekannter Leibnizscher Vorschlag einer 'Machina arithmeticae dyadicae'." In *Akten des II Internationalen Leibniz-Kongresses,* II:255–268. Wiesbaden: Steiner Verlag.

Madrolle, Claudius. 1901. *Les premiers voyages français à la Chine: la Compagnie de la Chine, 1698–1719.* Paris: Challamel.

Malebranche, Nicolas. 1692. *Des loix de la communication des mouvements.* Paris: Pralard.

Marcellinus, Ammianus. 1935. *History.* Translated by John C. Rolfe. 3 vols. London: Heinemann.

Mariotte, Edme. 1673. *Traité de la percussion ou choc des corps, dans lequel des principales regles du mouvement, contraires à celles que Mr Descartes, & quelques autres modernes ont voulu establir, sont demonstrées par leur veritable causes.* Paris: Michallet.

Martin, Bruce A. 1968. "Letters to the Editor: On Binary Notation." *Communications of the ACM* 11 (10): 658.

Martini, Martino. 1655. "Novus Atlas Sinensis." In *Theatrum orbis terrarum, sive Atlas Novus, Pars VI,* edited by Willem Blaeu and Joan Blaeu. Amsterdam: Blaeu.

Martini, Martino. 1658. *Sinicae historiae decas prima.* Munich: Lucas Straub.

McClatchie, Thomas. 1876. *A Translation of the Confucian Yi-king.* Shanghai: American Presbyterian Mission Press.

Merzbach, Uta C., and Carl B. Boyer. 2011. *A History of Mathematics.* 3rd ed. Hoboken, NJ: Wiley.

Mohl, Julius, ed. 1834, 1839. *Y-King antiquissimus Sinarum liber quem ex latina interpretatione P. Regis aliorumque ex Soc Jesu P.P.* 2 vols. Stuttgart: Cotta.

Montucla, Jean Étienne. 1799. *Histoire des mathématiques. Tome premier.* Paris: Agasse.

Morar, Florin-Stefan. 2015. "Reinventing Machines: The Transmission History of the Leibniz Calculator." *British Journal for the History of Science* 48 (1): 123–146.

Morland, Samuel. 1673. *The Description and Use of Two Arithmetick Instruments.* London: Moses Pitt.

Morrison, Theron V. 1952. "A Short Method of Long Division." *Journal of Applied Social Psychology* 52 (9): 684–686.

Müller, Andreas. 1671. *Disquisitio geographica & historica de Chataja.* Berlin: Rungian.

Müller, Andreas. 1674. *Inventum Brandenburgicum sive Andreae Mulleri Greiffenhagii, Praepositi Berlinensis, Proposito super Clave sua Sinica.* n.p.: n.p.

Mungello, David E. 1977. *Leibniz and Confucianism: The Search for Accord.* Honolulu: The University Press of Hawaii.

Napier, John. 1617. *Rabdologiæ.* Edinburgh: Andreas Hart.

Napier, John. 1990. *Rabdology.* Translated by William F. Richardson. Cambridge: MIT Press.

Napier, John. 2017. *The Life and Works of John Napier.* Edited and translated by Brian Rice, Enrique González-Velasco, and Alexander Corrigan. Cham: Springer.

Neumann, J. von. 1993. "First Draft of a Report on the EDVAC." *IEEE Annals of the History of Computing* 15 (4): 27–75.

Newman, James R., ed. 1956. *The World of Mathematics.* New York: Simon and Schuster.

Newton, Isaac. 1671/2. "A letter of Mr. Isaac Newton, Professor of the Mathematicks in the University of Cambridge; containing his new theory about light and colors: sent by the author to the publisher from Cambridge, Febr. 6. 1671/72; in order to be communicated to the R. Society." *Philosophical Transactions* 80 (February): 3075–3087.

Nilous, Horapollo. 1840. *The Hieroglyphics of Horapollo Nilous.* Translated by Alexander Turner Cory. London: William Pickering.

Nolte, Rudolph August. 1734. *Gottfried Wilhelms Baron von Leibnitz Mathematischer Beweis der Erschaffung und Ordnung der Welt.* Leipzig: Langenheim.

Nystrom, John W. 1862. *Project of a New System of Arithmetic, Weight, Measure and Coins, Proposed to be Called the Tonal System, with Sixteen to the Base.* Philadelphia, PA: J. B. Lippincott & Co.

Nystrom, John W. 1863. "On a New System of Arithmetic and Metrology, Called the Tonal System." *Journal of the Franklin Institute* 76:263–275, 337–348, 402–407.

Ozanam, Jacques. 1790. *Récréations mathématiques et physiques. Nouvelle édition, totalement refondue et considerablement augmentée par M. de M***. Tome premier.* Paris: Firmin Didot.

Papin, Denis. 1695. *Fasciculus dissertationum de novis quibusdam machinis atque aliis argumentis philosophicis.* Marburg: Jacob Estienne.

Pappas, Theoni. 1991. *More Joy of Mathematics.* San Carlos, CA: Wide World Publishing.

Parent, Antoine. 1700. *Élémens de méchanique et de physique.* Paris: Florentin/Laulne.

Pascal, Blaise. 1645. *Lettre dédicatoire à Monseigneur le Chancelier sur le sujet de la machine nouvellement inventée par le Sieur B.P. pour faire toutes sortes d'opérations d'arithmétique, par un mouvement reglé, sans plume ny jettons. Avec un advis necessaire à ceux qui auront curiosité de voir ladite machine, & de s'en servir.* n.p.: n.p.

Pascal, Blaise. 1665. *Traité du triangle arithmetique, avec quelques autres petits traitez sur la mesme materiere.* Paris: Guillaume Desprez.

Pasini, Enrico. 2016. "Leibniz's Concept of Unity and Its Aristotelian Origins." In *Beiträge zu Leibniz' Rezeption der Aristotelischen Logik und Metaphysik,* edited by J. A. Nicolas and N. Öffenberger, 325–332. Hildesheim: Olms.

Peano, Giuseppe. 1898. "La numerazione binaria applicata alla stenografia." *Accademia reale delle scienze di Torino* 34:4–11.

Peet, E. Eric, ed. 1923. *The Rhind Mathematical Papyrus, British Museum 10057 and 10058.* Translated by T. Eric Peet. London: Hodder/Stoughton.

Pelicanus, Wenceslaus Josephus. 1712. *Arithmeticus Perfectus Qui tria numerare nescit. Seu Arithmetica dualis, In qua numerando non proceditur, nisi ad duo, & tamen omnes quaestiones arithmeticae negotio facili enodari possunt.* Prague: Rosenmüller.

Petrocelli, Carla. 2019. "Konrad Zuse and His Plankalkül." *International Journal of Humanities and Arts Computing* 13 (1–2): 249–265.

Petty, William. 1682. *An Essay Concerning the Multiplication of Mankind: Together With Another Essay in Political Arithmetick, Concerning the Growth of the City of London.* London: Mark Pardoe.

Pignoria, Lorenzo. 1669. *Mensa Isiaca, qua sacrorum apud Aegyptios ratio & simulacra subjectis tabulis aenis simul exhibentur & explicantur.* Amsterdam: Frisi.

Privat-Deschanel, Augustin, and Adolphe-Jean Focillon. 1864. *Dictionnaire général des sciences théoriques et appliquées. I^{re} Partie.* Paris: Tandou and Garnier Brothers.

Probst, Siegmund. 2018. "The Calculus." In *The Oxford Handbook of Leibniz,* edited by Maria Rosa Antognazza, 211–224. Oxford: Oxford University Press.

Pythagorean Society. 1672. *Tetractyn tetracty Pythagoreae correspondentem, ut primum disceptationum suarum specimen, ulteriori curiosorum industriae exponit Societas Pythagorea in Alma Salana.* Jena: Meyer.

Qian, Sima. 1993. *Records of the Grand Historian: Han Dynasty II.* Rev. ed. Translated by Burton Watson. New York: Columbia University Press.

Rabouin, David. 2015. "The Difficulty of Being Simple: On Some Interactions Between Mathematics and Philosophy in Leibniz's Analysis of Notions." In *G. W. Leibniz, Interrelations between Mathematics and Philosophy,* edited by Norma B. Goethe, Philip Beeley, and David Rabouin, 49–74. New Studies in the History and Philosophy of Science and Technology 41. Dordrecht: Springer Netherlands.

Reyneau, Charles René. 1708. *Analyse démontrée, ou la méthode de résoudre les problèmes des mathématiques, et d'apprendre facilement ces sciences.* Paris: Quillau.

Riccioli, Giovanni Battista. 1672. *Geographiae et hydrographiae reformatae, nuper recognitae & auctae, libri duodecim.* Venice: LaNoù.

Rivest, R., A. Shamir, and L. Adleman. 1978. "A Method for Obtaining Digital Signatures and Public-Key Cryptosystems." *Communications of the ACM* 21 (2): 120–126.

Rudolph, Hartmut. 2018. "Scientific Organization and Learned Societies." In *The Oxford Handbook of Leibniz,* edited by Maria Rosa Antognazza, 543–562. Oxford: Oxford University Press.

Ryan, Michael T. 1991. "The Diffusion of Science and the Conversion of the Gentiles in the Seventeenth Century." In *In the Presence of the Past,* edited by Richard T. Bienvenu and Mordechai Feingold, 9–40. Dordrecht: Springer.

Schlömilch, Oskar, and B. Witzschel. 1858. *Zeitschrift für Mathematik und Physik.* Vol. 3. Leipzig: Teubner.

Schott, Gaspar. 1658. *Magia universalis naturae & artis operis quadripartiti. Tomus tertius & quartus.* Würzburg: Schönwetter.

Schwenter, Daniel. 1636. *Deliciae physico-mathematicae oder Mathematische und philosophische Erquickstunden.* Nuremberg: Dümmler.

Serra, Yves. 2010a. *Le manuscrit «De Progressione Dyadica» de Leibniz.* http://www.bibnum.education.fr/sites/default/files/leibniz-analyse.pdf.

Serra, Yves. 2010b. *Traduction du fac-similé du manuscrit de Leibniz du 15 mars 1679, «De Progressione Dyadica».* http://www.bibnum.education.fr/sites/default/files/leibniz-numeration-texte.pdf.

Shirley, John W. 1951. "Binary Numeration before Leibniz." *American Journal of Physics* 19:452–454.

Silva y Figueroa, García de. 1667. *L'ambassade de D. Garcia de Silva Figueroa en Perse, contenant la politique de ce grand Empire, les moeurs du Roy Schach Abbas, & une relation exacte de tous les lieux de Perse & des Indes, où cet ambassadeur a esté l'espace de huit années qu'il y a demeuré.* Translated by Abraham de Wicquefort. Paris: Billaine.

Stein, E., F. O. Kopp, K. Wiechmann, and G. Webe. 2006. "Calculemus! New Research Results and Functional Models Regarding Leibniz' Four Function Calculating Machine and Binary Calculating Machine." *Foundations of Civil and Environmental Engineering* 7:319–332.

Stein, Erwin. 2006. "Neue Forschungsergebnisse und Funktionsmodelle zur dezimalen Vier-Spezies-Rechenmaschine von und zur dyadischen Rechenmaschine nach Leibniz." In *Jahrbuch 2006 der Braunschweigischen Wissenschaftlichen Gesellschaf,* 56–57. Brunswick: J. Cramer Verlag.

Strabo. 1928. *The Geography of Strabo.* Translated by Horace Leonard Jones. 8 vols. London: Heinemann.

Strickland, Lloyd. 2005. "How Modern Was Leibniz's Biology?" *Studia Leibnitiana* 37 (2): 186–207.

Strickland, Lloyd. 2014. *Leibniz's Monadology: A New Translation and Guide.* Edinburgh: Edinburgh University Press.

Strickland, Lloyd. 2023. "Consilium Aegyptiacum." In *Companion to Leibniz's Political Thought and Activities,* edited by Luca Basso, Wenchao Li, Jaime de Salas, and Hartmut Rudolph. Berlin: De Gruyter.

Struik, Dirk Jan, ed. 1986. *A Source Book in Mathematics, 1200–1800.* Princeton, NJ: Princeton University Press.

Suetonius. 1914. *The Lives of the Caesars.* Translated by J. C. Rolfe. Vol. I. Cambridge, MA: Harvard University Press.

Swedenborg, Emanuel. 1754. "A Curious Memoir of M. Emanuel Swedenborg Concerning Charles XII of Sweden." *The Gentleman's Magazine, and Historical Chronicle* (London) XXIIII:423–424.

Swetz, Frank J. 2003. "Leibniz, the Yijing, and the Religious Conversion of the Chinese." *Mathematics Magazine* 76 (4): 276–291.

Swinton, John. 1789. *A Proposal for Uniformity of Weights and Measures in Scotland, By Execution of the Laws now in Force. With Tables of the English and Scotch Standards, and of the Customary Weights and Measures of the several Counties and Boroughs of Scotland;-Comparisons of the Standards with Each Other, and with the County-Measures;—Tables and Rules for their Reciprocal Conversion;-And some Tables of the Weight and Produce of Corn, &c.* 2nd ed. Edinburgh: Peter Hill.

Sypniewski, Bernard Paul. 2005. "China and Universals: Leibniz, Binary Mathematics, and the Yijing Hexagrams." *Monumenta Serica* 53:287–314.

Tachard, Guy. 1686. *Voyage de Siam, des peres Jesuites, envoyez par le Roy aux Indes & à la Chine.* Paris: Seneuze and Horthemels.

Tentzel, Wilhelm Ernst. 1696. *Epistola de sceleto elephantino Tonnae nuper effosso.* Gotha: Reyher.

Tentzel, Wilhelm Ernst. 1705. "Erklärung der Arithmeticae binariae, welche vor 3000 Jahren bey den Chinesern in Gebrauch gewesen und bißher bey ihnen selbst verlohren, neulich aber bey uns wieder funden worden." In *Curieuse Bibliothec oder Fortsetzung der Monatlichen Unterredungen,* edited by Wilhelm Ernst Tentzel, 81–112. Frankfurt: Stock.

Tentzel, Wilhelm Ernst. 2006. *Correspondence between Leibniz and Tentzel.* Edited and translated by Lloyd Strickland. http://www.leibniz-translations.com/tentzel.htm.

Terrentius, Joannes, and Johannes Kepler. 1630. *R.P.J. Terrentii e Societate Jesu Epistolium ex regno Sinarum ad mathematicos Europæos missum: cum Commentatiuncula Joannis Keppleri mathematici.* Żagań-Silesia: Cobius and Wiske.

Thévenot, Melchisédech. 1663. "Description des antiquitez de Persepolis, appellées maintenant Chimilnar, traduite de l'Anglois." In *Relations de divers voyages curieux, qui n'ont point esté publiées, ou qui ont esté traduites d'Hacluyt, de Purchas, & d'autres voyageurs Anglois, Hollandois, Portugais, Alemands, Espagnols; et de quelques Persans, Arabes, & autres autheurs orientaux. Premiere partie.* Paris: Langlois.

Thomas, Antoine, Filippo Grimaldi, Thomas Pereira, Jean-Francois Gerbillon, Joseph Suares, Joachim Bouvet, Kilian Stumpf, Jean-Baptiste Regis, Ludovicus Pernoti, and Dominique Parrenin. 1701. *Brevis relatio eorum, quae spectant ad declarationem Sinarum Imperatoris Kam Hi circa caeli, Cumfucii, et avorum cultum, datam anno 1700.* Peking: n.p.

Tropfke, Johannes. 1902. *Geschichte der Elementar mathematik in systematischer Darstellung. Erster Band: rechnen und Algebra.* Leipzig: Veit & Co.

Tropfke, Johannes. 1980. *Geschichte der Elementarmathematik. Band 1: Arithmetik und Algebra.* 4th ed. Berlin: De Gruyter.

Valeriano, Pierio. 1556. *Hieroglyphica, sive de sacris Aegyptiorum literis commentarii.* Basel: [Michael Isengrin].

Vieth, Gerhard Ulrich Anton. 1796. *Anfangsgründe der Mathematik: Erster Theil, Arithmetik und Geometrie.* Leipzig: Johann Ambrosius Barth.

Wakefield, Andre. 2010. "Leibniz and the Wind Machines." *Osiris* 25 (1): 171–188.

Weber, Heinric. 1922. *Enzyklopadie der Elementarmathematik: Ein Handbuch für Lehrer und Studierende. Erster Band. Arithmetik, Algebra und Analysis.* 4th ed. Leipzig: Teubner.

Weigel, Erhard. 1673. *Tetractys, Summum tum Arithmeticae tum Philosophiae discursivae compendium, artis magnae sciendi.* Jena: Meyer.

Weigel, Erhard. 1687a. "Aretologistica, die Tugendübende Rechen-Kunst." In *Wienerischer Tugend-Spiegel.* Nuremberg: Endtern.

Weigel, Erhard. 1687b. "Der Grund aller Tugenden, Nemlich eine Mathematische Demonstration wider alle Atheisten, Daraus man Rechenschafftlich erkennen kan dass ein Gott sey." In *Wienerischer Tugend-Spiegel.* Nuremberg: Endtern.

Weigel, Erhard. 1687c. "Kurzer Bericht auf eingenommenen Augenschein eines guten Grundes J. M. P. P. von dem Muster einer auf die Aretologistic gegründeten Tugend-Schul zu Jena." In *Wienerischer Tugend-Spiegel,* 100–112. Nuremberg: Endtern.

Weigel, Erhard. 1687d. “Logisticae virtutum genitricis, Das ist: Tugendübender Zahlen-Rechen-Kunst.” In *Wienerischer Tugend-Spiegel,* own pagination. Nuremberg: Endtern.

Weigel, Erhard. 1687e. *Wienerischer Tugend-Spiegel.* Nuremberg: Endtern.

Whitney, William Dwight. 1895. *The Century Dictionary. An Encyclopedic Lexicon of the English Language. Volume VII.* New York: The Century Co.

Wideburg, Johann Bernhard. 1718. *Dissertatio mathematica, de praestantia arithmeticae binariae prae decimali.* Jena: Krebsian.

Wieleitner, Heinrich, and Anton von Braunmühl. 1911. *Geschichte der mathematik. T.2, Von Cartesius bis zur Wende des 18. Jahrhunderts; von Heinrich Wieleitner; bearbeitet unter Benutzung des Nachlasses von Anton von Braunmühl. Hälfte 1, Arithmetik, Algebra, Analysis.* Leipzig: G.J. Göschen.

Wiener, Norbert. 1961. *Cybernetics, or Control and Communication in the Animal and the Machine.* 2nd ed. Cambridge, MA: MIT Press.

Wilhelm, Richard, and Cary F. Baynes. 1950. *The I Ching or Book of Changes.* Princeton, NJ: Princeton U. Press.

Wilkins, John. 1668. *An Essay Towards a Real Character, and a Philosophical Language.* London: Gellibrand.

Will, Georg Andreas. 1778. *Bemerkungen über einige Gegenden des katholischen Deutschlands auf einer kleinen gelehrten Reise gemachet Nebst sechs noch ungedruckten Leibnitzischen Briefen.* Nuremberg: Lochnerischen.

Witsen, Nicolaas. 1687. *Nieuwe Lantkaarte van het Noorder en Oosterdeel van Asia en Europa, strekkende van Nova Zemla tot China.* [Amsterdam]: n.p.

Wolf, Mark J. P. 2000. *Abstracting Reality: Art, Communication, and Cognition in the Digital Age.* Lanham, NY: Oxford University Press of America.

Zacher, Hans J., ed. 1973. *Die Hauptschriften zur Dyadik von G. W. Leibniz.* Frankfurt: Vittorio Klostermann.

Zhang, Dainian. 2002. *Key Concepts in Chinese Philosophy.* Translated by Edmund Ryden. New Haven, CT: Yale University Press.

Zhonglian, Shi. 2000. “Leibniz’s Binary System and Shao Yong’s Xiantiantu.” In *Das Neueste über China: G.W. Leibnizens Novissima Sinica von 1697,* edited by Wenzhao Li and Hans Poser, 165–169. Stuttgart: Franz Steiner Verlag.

Zuse, Konrad. 1980. “Some Remarks on the History of Computing in Germany.” In *A History of Computing in the Twentieth Century,* edited by N. Metropolis, J. Howlett, and G. C. Rota, 611–627. Saint Louis: Elsevier Science & Technology.

Zuse, Konrad. 1987. *Computer Design: Past, Present, Future: Talk in Lund, Sweden.* https://history.computer.org/pioneers/zuse.html.

Zuse, Konrad. 1993. *The Computer—My Life.* Translated by Patricia McKenna and J. Andrew Ross. Berlin: Springer.