LEIBNIZ'S "IMAGE OF CREATION."

Author(s): Florian Cajori

Source: *The Monist*, OCTOBER, 1916, Vol. 26, No. 4 (OCTOBER, 1916), pp. 557-565
Published by: Oxford University Press
Stable URL: https://www.jstor.org/stable/27900610

LEIBNIZ'S "IMAGE OF CREATION."

IN the achievements of great men the trivial and curious frequently loom higher than the solid and substantial. At one time Kepler's fame centered largely around the pseudo-discovery of fanciful relations between the regular solids and planetary distances. Placing the icosahedron, dodecahedron, octahedron, tetrahedron and cube, one within the other at such distances that each solid was inscribed in the same sphere about which the next outer solid was circumscribed, he found that the radii of the spheres were roughly in the ratio of successive planetary distances. Only in an uncritical age could such very crude numerical resemblances command any attention, especially as there seemed to exist no causal relation between said radii and the distances of planets from the sun.

Nowadays we smile at Kepler's early speculation. Nevertheless it is a fair question to ask why the regular solids should be less likely to play a part in the mathematical theory of planetary motion than does that conic section which was destined later in Kepler's career to lend itself to the establishment of his permanent world-fame as an astronomer—why the ellipse rather than the "Platonic figures"? In Kepler's time the law of gravitation could not be appealed to for arbitration; it was still hidden away in the realm of the unknown. No *a priori* decision was within reach at that time. The answer could come only after painstaking measurements, combined with mathematical deduction, which, in this case, confirmed one guess and exploded the other.

The famous mathematician Sylvester derived great satisfaction from lecturing in Baltimore on versification and displaying his skill in the making of rhymes. Rumor has it that he was fonder of his grotesque booklet, the *Laws of Verse*, than of any of his great mathematical discoveries.

Thus it was also with Leibniz. He was strangely partial to a discovery of very minor importance that he made relating to the so-called binary numbers, which are constructed on the scale of 2 instead of 10 and require only two symbols, namely 0 and 1. In his scale, 1 is written 1, 2 is written 10, 3 is written 11, 4 is written 100, 5 is written 101, and so on.

The charm of Kepler's regular-solid-theory of planetary distances lay in the unexpected relation thought to exist between magnitudes so foreign to each other that not the remotest cause for such intimate relation could be imagined. Sylvester's fantastic performances lay in the acrobatic groupings of words similar in their terminal sounds. The fascination in the dual arithmetic of Leibniz lay in the philosophical and religious mysticism associated with it. The 0 and 1, by which any number could be represented in that system, symbolized the creation of everything out of nothing; it afforded a phase of religious mystery which was thought to be helpful in the conversion of the heathen. The idea of Leibniz was based upon sound psychology. The mind of man delights in figures of speech, in analogies, in images. Here was an *"imago creationis,"* truly novel and simple. The fact that it rests upon a mathematical basis was no drawback. Had not number-theory figured prominently in ancient religious mysticism?

With Leibniz his dual arithmetic was more than a passing fancy. He had reflected on this subject for over twenty years, before he permitted an account of his meditations to appear in type. He made his first full statement of the binary scale and its symbolic interpretation in a letter written on January 2, 1697, to Duke Rudolph August of Brunswick. A little later, on May 17, 1698, Leibniz touched upon this subject in a letter to Johann Christian Schulenburg of Bremen, in which he states that his first thoughts on this matter antedate the year 1678. Accordingly his first ideas on binary numbers go back to the time when he was making his marvelous invention of the differential calculus. In April, 1701, Leibniz wrote enthusiastically on these numbers to John Bernoulli, then at Groningen in the Netherlands. Two years later, on July 12, 1703, he sent an account of his new arithmetic to Fontenelle, the secretary of the French Academy of Sciences, and it was published in 1703 under the title "Explication de l'arithmétique binaire," in the *Mémoires de l'Académie des Sciences de Paris*. This was the earliest appearance of this subject in print. The perusal of this article convinces the reader that Leibniz regarded it with parental pride.

A letter of Leibniz, written some years before, contains a statement which, we believe, has reference to the binary scale. It is a letter of September 8, 1690, sent to Placcius, who was professor of philosophy in the gymnasium in Hamburg. We may state parenthetically that this letter is of general interest, aside from its probable allusion to the binary scale. It reveals his ideas on the most profitable course of mathematical study and discloses information regarding the Hamburg mathematical club which ranks as the earliest organization of that sort known in mathematical history. We translate as follows:

"Recently I saw a book which deals in the German language with numerical problems, from which I gather that in Hamburg a few prominent arithmetical experts have combined and formed a society with which others in that vicinity have become affiliated, and that Meissner, one of your countrymen and a teacher of arithmetic, is the leader of this movement. I am much pleased with this organization and I expect from it excellent things if they can make up their minds to expend their efforts upon matters which will enlarge the boundary of science; for to spend the time on special problems is not quite worthy of this undertaking, unless these problems are of particular elegance and usefulness or help to enlarge the field of the general method itself. Nothing is simpler than to collect problems which are easy for us who know the mode of procedure, but which cause others unnecessary labor. One should endeavor to perfect analysis itself, and I do not believe that there is any one in Germany who has acted in this matter with more zeal—not to say with greater success—than have I myself....

"I am also in possession of an invention for the construction of algebraic tables which, if once made available, would simplify computation and would afford to analysis almost as much aid as do the sine tables and logarithmic tables in ordinary arithmetic."

Does the last paragraph in this quotation refer to the binary system? In the letter written some years later (Jan. 2, 1697) to Duke Rudolph August, he says of this system:

"At the bottom of this there are so many wonderful things to see, useful also in the advancement of science, that some members of the Hamburg Arithmetical Society, whose diligence and aims are praiseworthy, could enjoyably direct their thoughts upon this and, as I can assure them, find things therein which would bring no little renown to them, and also to the German nation for having been first brought forth in Germany. For I see that from this mode of writing numbers there can be derived wonderful advantages profitably applicable also in ordinary arithmetic."

And what are the advantages which can be claimed for the binary system? In the first place it has no multiplication table beyond $1 \times 1 = 1$. Practically all operations can be performed by mere addition and subtraction. Consider for example the multiplication of 2 by 3. In the binary system $10 \times 11 = 110$, $11 \times 11 = (10 + 1)11 = 110 + 11 = 1001$. To be sure nearly four times as many figures must be written down in the binary scale as in the decimal scale, but the absence of a multiplication table is a vital gain. "Calculation as an effort of mathematical thought," says a recent writer, "might be said to be entirely dispensed with, and the labor of the brain to be all transferred to the eye and hand."

In his letter of Jan. 2, 1697, Leibniz accompanies his New Year's greetings to Duke Rudolph August by the remark: "That I shall not come this time altogether empty, I send you a symbol of what I recently had the honor to mention to you. It is in the form of a thought-penny or medal; and while my design is trifling and to be improved according to one's taste, yet the thing itself is of such a nature that it would seem worthy to be exhibited to posterity in silver, if such were to be stamped by the command of your gracious Highness. For one of the chief tenets of Christian faith, one of those which have met with the least acceptance on the part of the worldly wise and are not easily imparted to the heathen, relates to the creation of all things out of nothing by the all-power of God. It can be rightly claimed that nothing in the world better represents this, indeed almost proves it, than the origin of number in the manner represented here, where, by the use simply of unity and zero or nothing, all numbers originate. In nature and philosophy it will hardly be possible to find a better symbol of this mystery, for which reason there is placed upon the design of the medal, *Imago Creationis*."

According to Leibniz, this image shows that God created all things well: "For while in the ordinary mode of writing numbers there can be recognized no order or definite sequence of characters or relations, there appears now, since one can see the innermost recesses and the primitive states, a wonderfully beautiful order and harmony which cannot be improved upon, and is exhibited, first of all, in a

LEIBNIZ'S IMAGO CREATIONIS.

fixed rule of alternation by which we can write down all members without computation and without aid of memory as far as we please, if we put in the first column on the right, or in the last position, alternately underneath each other: 0, 1, 0, 1, 0, 1, 0, 1, etc.; and put in the next column (proceeding from right to left): 0, 0, 1, 1, 0, 0, 1, 1, etc.; and in the third column: 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, etc.; in the fourth: 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, and so on,.... This continuous order and beauty can be seen in the small table on the medal, as far as 16 or 17....

"To explain the other parts of the medal I have marked the principal places with an asterisk, namely 10 or 2, 100 or 4, 1000 or 8, 10000 or 16; for if one examines just these, one derives therefrom the structure of the other numbers. Why, for instance 1101 stands for 13 is as follows:

1	1
00	0
100	4
1000	8
1101	13

and similarly for all others. On the sides of the table on the medal I have placed an example in addition and one in multiplication, that we may understand the operations and notice that the ordinary rules of computation hold here also—even though there is no intention on our part to use these modes of computation in any other way than to discover and exhibit the mysteries of numbers....

"If, as in perspective, one examines things from the proper point of view, one can see their symmetry. And this stimulates us more and more to praise and love the wisdom, goodness and beauty of the Highest Goodness, from whom all goodness and beauty flows. Hence, as I now write to Pater Grimaldi in China, a Jesuit and the president of the mathematical tribunal there, with whom I became acquainted in Rome,....to whom I thought it well to communicate this representation of numbers, with the hope—since, as he himself stated, the monarch of this extensive empire is a lover of arithmetic who learned from Pater Verbiest, Grimaldi's predecessor, European methods of computation—that this image of the mystery of creation might serve to bring more and more before his eyes the excellencies of the Christian faith."

To render this medal, designed as an image of creation, still more attractive and artistic, Leibniz suggested that it should also represent light and darkness, the spirit of God moving upon the face of the waters. As a motto he chose the following:

> "2, 3, 4, 5, etc. 0. *Omnibus ex nihilo ducendis sufficit unum.*" (To make all things from nothing, unity suffices.)

The binary arithmetic of Leibniz captured the attention of many mathematical writers. The mystic element put it in the class of mathematical recreations. Even Laplace, the heterodox, in his famous *Essai philosophique sur les probabilités,* speaks of it and its use in Chinese missions.

A curious blunder in mathematical history grew out of the binary arithmetic of Leibniz. The French Jesuit Bouvet, a missionary at Pekin and a zealous student of Chinese antiquities, learned of Leibniz's binary arithmetic and its theological interpretation. By the exercise of ingenious powers of coordination he found therein a key to the explanation of the *Cova,* or lineations of Fohi, the founder of the empire. They consisted of eight sets of three lines, either entire or broken lines, arranged in a circle. These *Cova* were held in great veneration in China, being suspended in all temples and, though not understood, were supposed to conceal great mysteries, embracing all true philosophy, both human and divine. Now Bouvet thought he had penetrated to the very depths of these mysteries when he announced triumphantly the discovery that in the *Cova* figures, the short lines meant 0 and the long lines meant 1, that Fohi possessed the principles of the binary arithmetic and that the *Cova* bore testimony to the unity of the Deity. Bouvet explained all this in a letter to Leibniz, dated Nov. 14, 1701. Leibniz, in turn, reported these findings in the paper to the Paris Academy which, as already related, was published in its *Mémoires* of 1703.

This application of the binary arithmetic to the interpretation of ancient oriental symbols afforded Leibniz profound pleasure. To the mathematician it meant the discovery that the Chinese had been in very early times in possession of binary arithmetic with its great principle of local value and the use of the zero. For the next 250 years the Chinese origin of this principle and of the zero appeared to be an established fact in mathematical history and was accepted as true even by the great mathematical historian of the nineteenth century, Moritz Cantor of Heidelberg, in his earlier publications. However, in 1863 Cantor became convinced that the traditional interpretation was incorrect, that the *Covas* of Fohi are not numbers at all, but have a physical significance, representing, respectively, air, rain, water, mountain, earth, thunder, fire, wind.

Thus it is seen that Leibniz's very minor invention of dual arithmetic was to him an object of contemplation for over a quarter of a century; it afforded him a satisfaction out of all proportion to its importance. He corresponded on the subject with mathematicians and religious teachers. It gave rise to an interesting chapter in modern religious mysticism and in the annals of foreign missions; it led to a blunder in the history of numeral notations which persisted for two centuries and a half, until the time of a great mathematical historian who is still living. It was the point of departure of interesting speculations as to the relative advantages of numeral notations whose bases are powers of 2, that is, the bases 2, 4 and 8.

FLORIAN CAJORI.

COLORADO COLLEGE.