CIS 301: Logical Foundations of Programming

Fall 2024

Exam 2 – 100 points

**This test is closed-notes and closed-computers.**

There are 7 questions worth 12-15 points each.

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Score: \_\_\_\_\_\_\_\_\_\_\_\_

1. (15 pts) Use natural deduction to prove the following sequent:

( a → c, b → f ∧ c) ⊢ ( a ∨ b → c )

Proof(

1 ( a → c ) by Premise,

2 ( b → f ∧ c ) by Premise,

//COMPLETE THE PROOF HERE

)

2. (15 pts) Use natural deduction to prove the following sequent:

( q ∨ a → b, p → ¬b ) ⊢ ( ¬( p ∧ q) )

Proof(

1 ( q ∨ a → b ) by Premise,

2 ( p → ¬b ) by Premise,

//COMPLETE THE PROOF HERE

)

3. (16 pts) Consider the following questions about sets.

1. (5 pts) List the elements in the set:

{x : x = n2+1, n ∈ ℤ, -1 ≤ n ≤ 4 }

1. (5 pts) Suppose A = {4,1,17,11,23} and B = {4,0,23,2,10}. List (i.e., write them all out) the elements in A - B.
2. (6 pts) Use set builder notation to describe a set with the elements below. Use only mathematical notation and not words to describe the set. (Hint: think about how each element compares to a multiple of 4.)

{-6, -2, 2, 6, 10, 14}

4. (14 pts) Consider the following questions about set operations.

1. (7 pts) Suppose we have the following results about the sets A, B, and C:

A ⋂ B = {2,3,5}

A ⋃ C = { x : x ∈ ℕ , x ≤ 6}

C = A – B

B ⋂ C = ∅ (the empty set)

Find the elements in each set (i.e., list out the elements in A, the elements in B, and the elements in C). Recall that ℕ starts with 1.

1. (7 pts) Suppose A and B are sets. Must A – B = A – (A ⋂ B)? Either explain why this must be true (using a Venn diagram, if it helps) or find a counterexample (with specific sets) where this property does not hold.

5. (13 pts) Consider the following statements over the domain of people, which use the predicates below:

Student(x): person x is a student

Adult(x) : person x is an adult

Job(x): person x has a job

Taller(x, y): person x is taller than person y

1. (4 pts) Assuming “Jill” is an individual in our domain, translate to predicate logic: “There is no adult student that is taller than Jill.”
2. (5 pts) Translate to predicate logic: “Every adult with a job is taller than some student.”
3. (4 pts) Translate to English: ¬(∀ x (Job(x) → Student(x)))

6. (12 pts) Consider the following questions about predicate logic and sets of numbers.

1. (6 pts) Is the statement ¬(∀ x (2x ≥ x2)) true or false in the domain of natural numbers numbers (ℕ)? Write “true” or “false”, and then explain your answer.
2. (6 pts) Is the statement ¬(∃ x ∃ y (x < 0 ∧ y < 0 ∧ x – y > 0)) true or false in the domain of integers (ℤ)? Write “true” or “false”, and then explain your answer.
3. (15 pts) Consider the following partially completed proof, where some of the claims, assumptions, and justifications are missing. Next to each “**???**”, write the appropriate claim, assumption, or justification to complete the proof.

(

¬(∃ ((x: T) => ( ¬P(x) ⋀ Q(x)) )

)

⊢

( ∀ ((x: T) => ( ¬P(x) → ¬Q(x) ) ) )

Proof(

//COMPLETE THE PROOF HERE

1 ( ¬ (∃((x: T) => (¬P(x) ⋀ Q(x)))) ) by *Premise*,  
 2 Let ((a: T) => SubProof(  
 3 SubProof(  
 4 Assume( **???** ),  
 5 SubProof(  
 6 Assume( **???** ),  
 7 ( **???** ) by *AndI*(4, 6),  
 8 ( **???** ) by *ExistsI*[T](7),  
 9 (*F*) by by **???**

),  
 10 ( **???** ) by *NegI*(5),  
 ),  
 11 (¬P(a) → ¬Q(a)) by **???**

)),  
 12 (∀((x: T) => (¬P(x) → ¬Q(x)))) by **???**

)