

# P451/551 Lab:

## Superconducting Quantum Interference Devices (SQUIDs)

### AN INTRODUCTION TO SUPERCONDUCTIVITY AND SQUIDS

Superconductivity was first discovered in 1911 in a sample of mercury metal whose resistance fell to zero at a temperature of four degrees above absolute zero. The phenomenon of superconductivity has been the subject of both scientific research and application development ever since. The ability to perform experiments at temperatures close to absolute zero was rare in the first half of this century and superconductivity research proceeded in relatively few laboratories. The first experiments only revealed the zero resistance property of superconductors, and more than twenty years passed before the ability of superconductors to expel magnetic flux (the Meissner Effect) was first observed. Magnetic flux quantization – the key to SQUID operation – was predicted theoretically only in 1950 and was finally observed in 1961. The Josephson effects were predicted and experimentally verified a few years after that.

SQUIDs were first studied in the mid-1960's, soon after the first Josephson junctions were made. Practical superconducting wire for use in moving machines and magnets also became available in the 1960's. For the next twenty years, the field of superconductivity slowly progressed toward practical applications and to a more profound understanding of the underlying phenomena. A great revolution in superconductivity came in 1986 when the era of high-temperature superconductivity began. The existence of superconductivity at liquid nitrogen temperatures has opened the door to applications that are simpler and more convenient than were ever possible before.

The superconducting ground state is one in which electrons pair up with one another such that each resultant pair has the same net momentum (which is zero if no current is flowing). In this ground state, all the electrons are described by the same wavefunction. In the usual or “normal” state, the wavefunctions describing the electrons in a material are unrelated to one another. In a superconductor, on the other hand, a single wavefunction describes the entire population of superconducting electron pairs. That wave function may differ in phase from one place to another within the superconductor, but knowing the function in one place determines it in another. As we will see, it is the existence of this coherent wavefunction that accounts for the phenomena associated with superconductivity.

## The Quantum of Flux

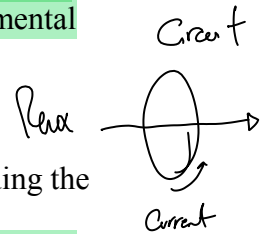
We are familiar with the concept of quanta as applied to charge, mass, and energy. In general, quantization is only readily apparent when we are dealing with microscopic objects. Macroscopic objects are composed of enormous numbers of elementary particles whose complicated interactions and chaotic thermal motion completely masks the discrete nature of the microscopic world.

Superconductivity offers a unique opportunity to observe the quantization of a physical quantity in a macroscopic, readily observable system. The reason for this is that closed superconducting circuits can only contain discrete units of magnetic flux known as fluxons. More precisely, the product of the magnetic field times the area of a closed superconducting loop must always be a multiple of  $h/2e$ , where  $h$  is Planck's constant and  $2e$  is the charge of an electron pair. In other words, a physically observable property of a macroscopic system must occur in units defined entirely by fundamental physical constants.

### Superconducting Circuits

The closed superconducting ring is a particularly convenient system to study for understanding the properties of superconductors. Consider the following experiment. We cool a ring of superconductor in a small magnetic field that corresponds to one flux quantum threading the ring. We now have a superconducting ring threaded by a single flux quantum. Suppose we now turn off the applied field. According to Faraday's Law of Induction, the moment that we change the field lines that thread the ring, a current flows in the ring. The induced current tries to oppose the change in magnetic field by generating a field to replace the field we removed. In an ordinary material, that current would rapidly decay away. In the superconductor, something entirely different happens. If the induced current decreased just a little bit in the ring, then the flux threading the ring would be a little less than a flux quantum. This is not allowed. The next allowable value of flux would be zero flux. Therefore, the current would have to abruptly cease rather than decay away. Because the superconducting state is composed of an enormous number of electrons that are paired up and occupying the same quantum state, a current reduction of the sort needed would require all the electrons to jump into another state simultaneously. This is an extraordinarily unlikely event. Practically speaking, it will never happen. As a result, the current induced in a superconducting ring will flow indefinitely. People have actually tried this experiment for years on end. As long as the ring is kept cold, the current flows without resistance.

Thus, the concept of flux quantization gives us an insight into why superconductors pass current without any resistance. How can we understand flux quantization itself? The answer lies in the long-range coherence of the superconducting wavefunction. As we said before, the value of the wavefunction in one place in a superconductor is related to the value at any other place by a simple phase change. The case of a superconducting ring places special restrictions on the superconducting wavefunction. The wavefunction  $\psi$  at the point marked by the black dot on the ring in Figure 1 must be the same wavefunction obtained by traveling around the ring one full circuit – it is the same spot. The phase change for this trip must be  $2\pi$  in order for the wavefunction to have a single value at a given point in space.



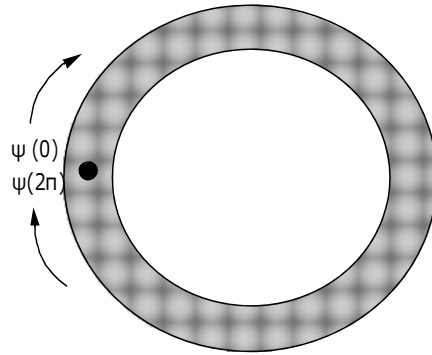


Figure 1 A superconducting ring.

According to electromagnetic theory, applying a magnetic field to a superconductor induces a change in the phase of the wavefunction. This results from the relationship between phase and canonical momentum ( $p - eA$ , where  $p$  is the mechanical momentum and  $A$  is the vector potential.) A given amount of magnetic field creates a specific phase change in the wavefunction. Since the phase change going completely around the ring must be some multiple of  $2\pi$  in order to maintain the single-valuedness of the wavefunction, the amount of flux contained within in the ring can only assume certain discrete values. This quantum mechanical property is the origin of flux quantization and, therefore the persistence of current, in a superconducting ring. Finally, there is one more aspect of superconductivity that we need to know a bit about: the Josephson effect.

### Josephson Junctions

The Josephson effect is yet another manifestation of the long-range quantum coherence of superconductors. The simple picture of this effect is as follows:

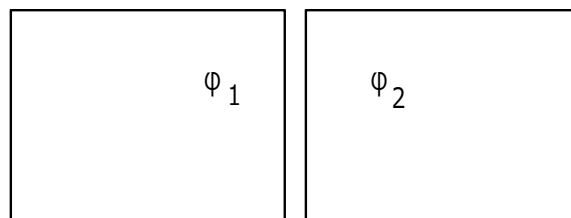


Figure 2 Schematic diagram of two superconducting regions separated by a thin gap.

Two regions of superconductor are placed very close to one another as in Figure 2. The quantum mechanical phase on the left is  $\phi_1$  and the phase on the right is  $\phi_2$ . In an ordinary material, the phases at two different spots are unrelated. In a single piece of superconductor, the phases at two different places have a specific relationship to one another. In the situation represented in Figure 2 above, if the two regions of superconductor are close enough together, their phases will also be related. In other words, they will act like a single superconductor. Functionally speaking, electrical currents can flow between the two regions with zero resistance. Such currents are called *Josephson currents* and physical systems composed of two regions of superconductor that exhibit this property are called *Josephson junctions*. Strictly speaking, the resistanceless currents that flow in a Josephson junction are a manifestation of the *DC Josephson effect*; a second property of junctions by which the current oscillates at high frequencies is called the *AC Josephson effect*.

Any weak coupling between two regions of superconductor – such as tiny constrictions, microscopic point contacts, weakly conducting layers, or certain crystallographic grain boundaries exhibit the Josephson effect. Josephson junctions generally have a much lower critical current ( $I_c$ ), which is the maximum current that can be carried before resistance begins to appear, than that of the two superconducting regions that it connects.

Josephson junctions are the essential active devices of superconductive electronics, much as the transistor is the essential active device of semiconductor electronics. Junctions can be used in a variety of electronic circuits as switching devices, sensors, variable inductors, oscillators (because of the AC Josephson effect), and for other applications. People have built Josephson electronic circuits that contain up to a million junctions. At the opposite extreme, one of the most useful circuits made from Josephson junctions is the DC SQUID, which contains only two junctions, which we describe next.

### The DC SQUID

Voltage frequency  $\propto$  int flux ?

The DC SQUID is actually a rather simple device consisting of two Josephson junctions connected in parallel on a closed superconducting loop, as shown in Figure 3. As we have said, a fundamental property of superconducting rings is that they can enclose magnetic flux only in multiples of a universal constant called the flux quantum. Because the flux quantum is very small, this physical effect can be exploited to produce an extraordinarily sensitive magnetic detector known as the Superconducting QUantum Interference Device, or SQUID. SQUIDs actually function as magnetic flux-to-voltage transducers where the sensitivity is set by magnitude of the magnetic flux quantum ( $2 \times 10^{-15}$  Wb). To set the scale, the Earth's magnetic field produces a flux through a typical SQUID sensor corresponding to over 100 flux quanta. The small fraction of the Earth's field that is not attenuated by the magnetic shield surrounding the SQUID probe in this setup is sufficient to shift the  $V-\phi$  curve by several flux quanta. The output voltage of a SQUID is a periodic function of applied magnetic flux, going through one complete cycle for every flux quantum applied. While it would be possible to obtain quite a sensitive measure of a magnetic signal simply by counting flux quanta, practical SQUID systems involve control electronics that interpolates between whole numbers of flux quanta and greatly enhances their ultimate sensitivity. SQUID sensitivity is finally limited by the intrinsic noise in the device, which in 4.2 K niobium DC SQUIDs, for example, typically approaches a millionth of a flux quantum (corresponding to a few billionths of the Earth's field passing through a 100- $\mu$ m diameter SQUID).

The inherent periodicity of the SQUID implies that it cannot distinguish between zero applied field and any other field that generates an integral number of flux quanta. This allows the dynamic range of the SQUID to be extended almost indefinitely by re-zeroing the SQUID in a controlled way. It also means that in order to measure the absolute value of an applied field, it is necessary to

reset the SQUID in a known field, or to rotate the SQUID with respect to the field. Nevertheless, it is very often the case that only changes in field are of interest, in which case no special measures are necessary.

*Heh*

### Details of DC SQUID Operation

So far we have discussed what it is that SQUIDS do, namely: SQUIDS convert magnetic flux, which is hard to measure, into voltage, which is easy to measure. Now we will describe how SQUIDS work. As we have said, a DC SQUID is a superconducting loop with two Josephson junctions in it. Suppose we pass a constant current, known as a bias current, through the SQUID. If the SQUID is symmetrical and the junctions are identical, the bias current will split equally, half on each side.

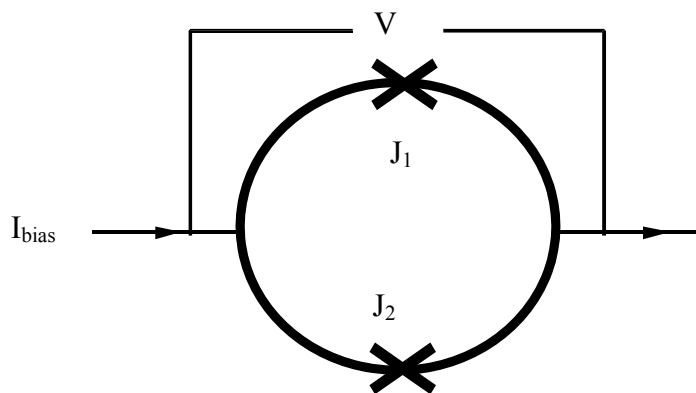
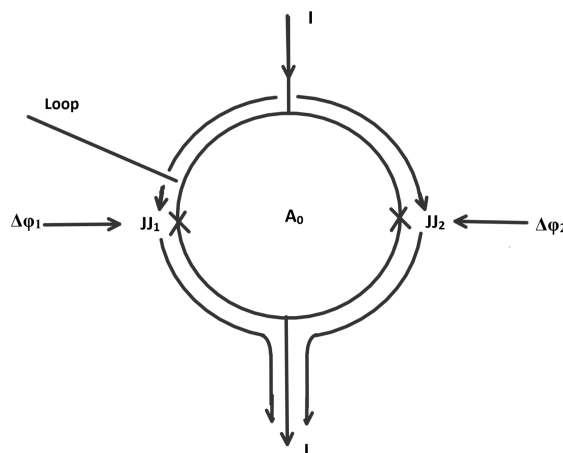


Figure 3 A schematic representation of the DC SQUID.

A supercurrent will flow through the SQUID, as long as the total current flowing through it does not exceed the critical current of the Josephson junctions, which as we discussed earlier, have a lower critical current than the rest of the superconducting ring. The critical current is the maximum zero-resistance current which the SQUID can carry, or the current at which a voltage across it develops. You could measure the critical current of a SQUID by ramping the bias current up slowly from zero until a voltage appears, then reading the current with an ammeter. The value of current determined in this way is the critical current of the SQUID.

When the two junctions in the SQUID are identical, the loop is symmetrical, and the applied field is zero, both junctions will develop a voltage at the same time. So the critical current of the SQUID is twice that of one of the junctions. The voltage-current characteristic, or  $V-I$  curve, of a SQUID, looks very much like the  $V-I$  curve of a bulk superconductor, except the value of the critical current is smaller. A typical  $V-I$  characteristic for the SQUID used in this apparatus is shown in Figure 4.



*Better Schematic*

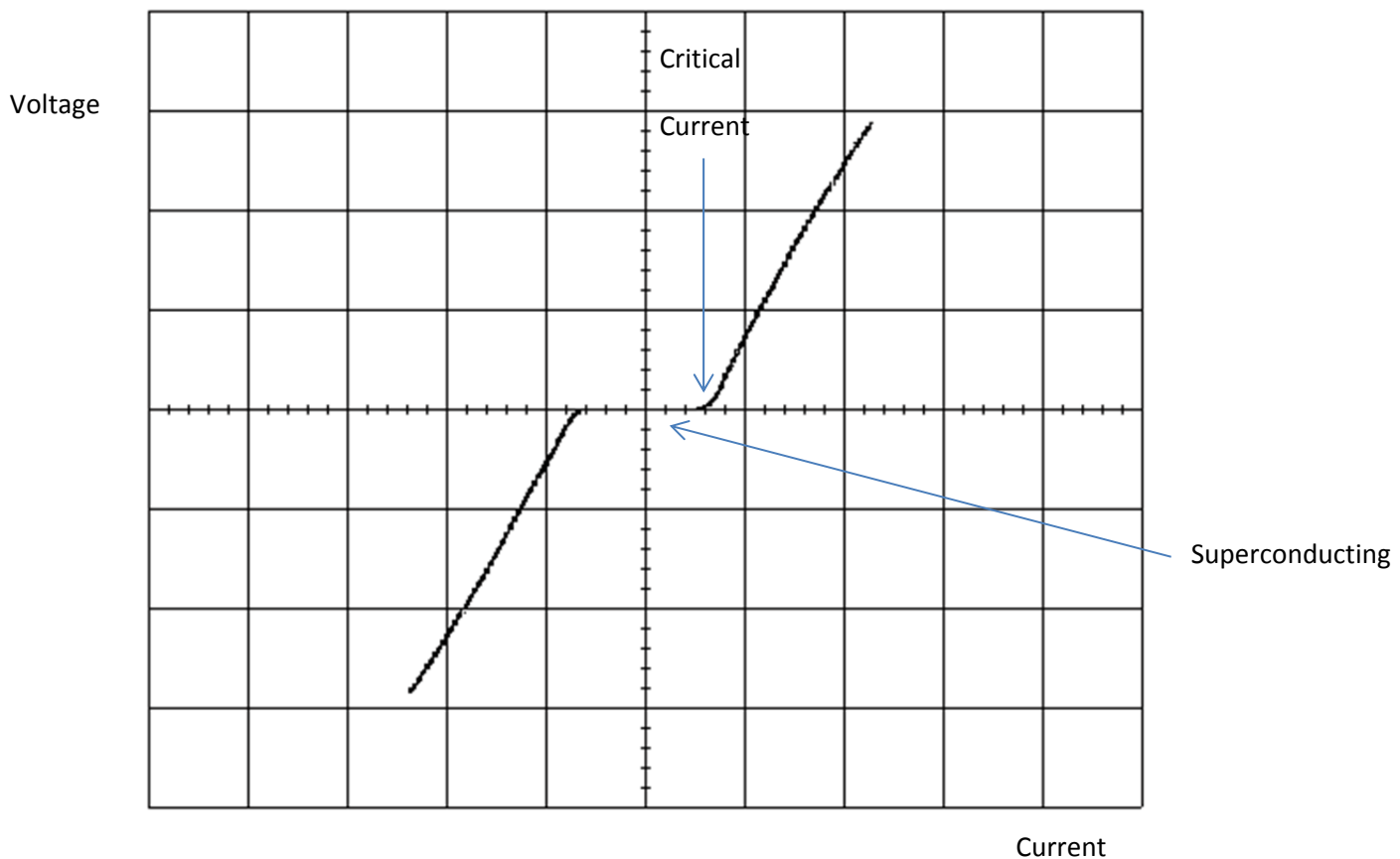


Figure 4 A typical V-I curve ( $50 \mu\text{A}/\Phi_0$  horizontal,  $200 \mu\text{V}/\text{div}$  vertical).

Now imagine what happens if a magnetic field is applied to the SQUID. First, let's bias the SQUID with a current well below its critical current. Then, if we apply a tiny magnetic field to the SQUID, the magnetic field wants to change the superconducting wave function. But the superconducting wavefunction doesn't want to change — as discussed earlier, it must maintain an integral number of wavefunction cycles around the loop. So the superconducting loop does what you would expect: it opposes the applied magnetic field by generating a screening current  $I_s$ , that flows around the loop. The screening current creates a magnetic field equal but opposite to the applied field, effectively canceling out the net flux in the ring. The applied magnetic field has lowered the critical current of the SQUID — in other words, it has reduced the amount of bias current we can pass through the SQUID without generating a voltage, since the total current flowing through SQUID is the sum of the screening current and bias current in one of the Josephson junctions. When the total exceeds the critical current, the SQUID becomes resistive, so a voltmeter will register a voltage across it.

As we increase the applied magnetic flux, the screening current increases. But when the applied magnetic flux reaches half a flux quantum, something interesting happens. The junctions momentarily go normal. The continuity of the superconducting loop is destroyed long enough for one quantum of magnetic flux to pop inside the loop. Then superconductivity around the loop is restored. (This is illustrated in Figure 5.) Thus, the junctions serve as gates that allow magnetic flux to enter (or leave) the loop.

One movement of the entire wavefunction for electrons

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If we consider the behavior of the screening current as more and more magnetic flux is applied, we would obtain the plot shown in Figure 5. As you see, the screening current changes sign (really, it changes direction), when the applied flux reaches half of a flux quantum. Then, as the applied flux goes from half a flux quantum toward one flux quantum, the screening current decreases. When the applied flux reaches exactly one flux quantum, the screening current goes to zero. At that point, the magnetic flux inside the loop and the magnetic flux applied to the loop are equal, so there is no need for a screening current. If you increase the applied magnetic flux a little more, a small screening current starts to flow in the positive direction, and the cycle begins again. The screening current is periodic in the applied flux, with a period equal to one flux quantum.

Amount

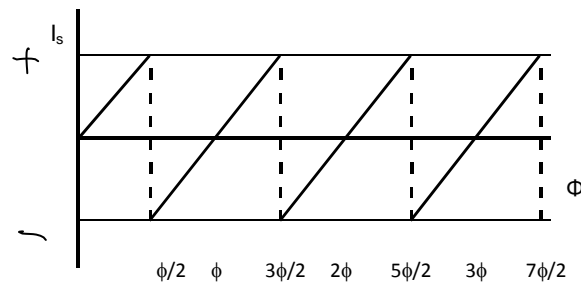


Figure 5: Relationship between screening current and applied magnetic flux.

Since you already know these two facts:

- The screening current of a SQUID is periodic in the applied flux, and
- The critical current of a SQUID depends on the screening current,

it makes sense that the SQUID critical current also is periodic in the applied magnetic flux. The critical current goes through maxima when the applied magnetic flux is an integer multiple of the flux quantum, because that's when the screening current is zero. It goes through minima when the applied magnetic flux is an integer of the flux quantum plus one half, because that is when the screening current is largest. Because the critical current of the SQUID is periodic in the applied field the  $V-I$  curve of a SQUID oscillates periodically between two extremes as shown in Figure 6 as the applied field is increased.

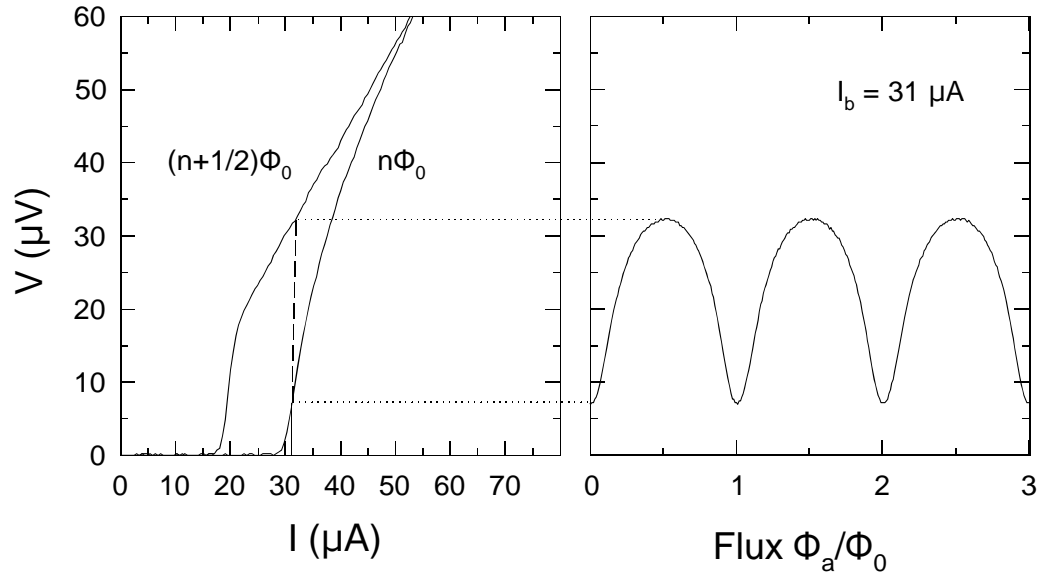


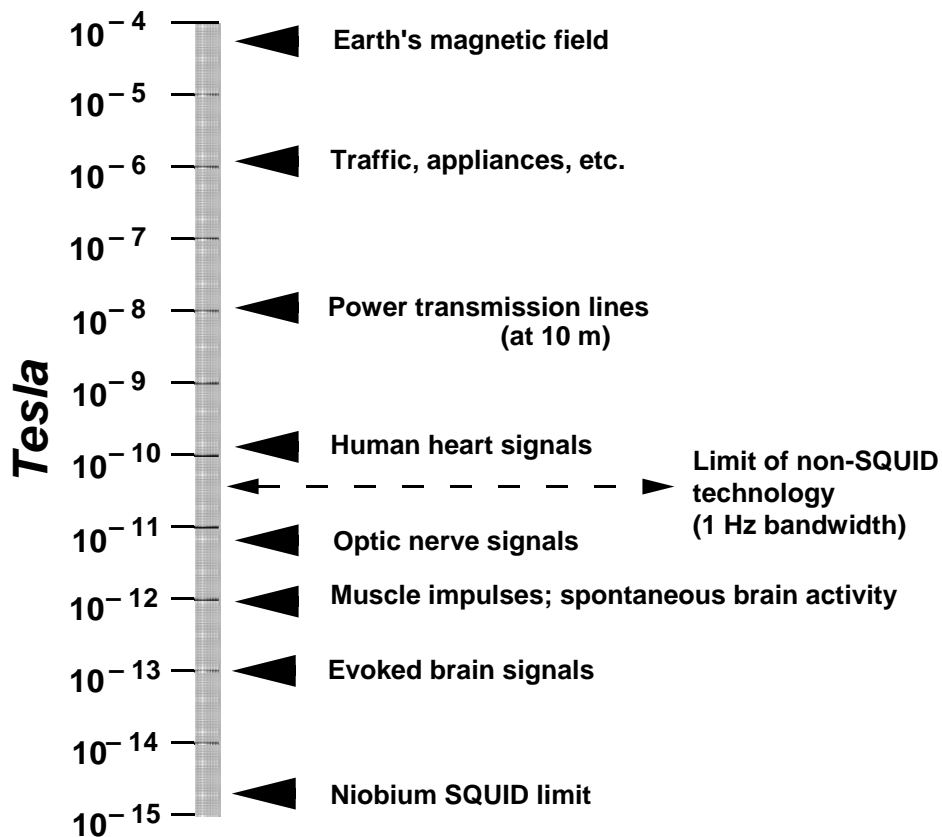
Figure 6: Voltage-Current ( $V-I$ ) and Voltage-Flux ( $V-\Phi$ ) characteristics of a dc SQUID showing the periodic dependence of the SQUID voltage on applied flux for fixed bias current  $I_b$ .



First, look at the left plot in Figure 6. The rightmost  $V$ - $I$  curve is what you see when the applied magnetic flux is a multiple of one flux quantum. The  $V$ - $I$  curve on the left is what you see when the applied magnetic flux is a multiple of one flux quantum plus one half. As you increase the magnetic flux continuously from zero, the SQUID  $V$ - $I$  curve oscillates continuously between these two extremes with a period of one flux quantum.

To make a magnetometer, or magnetic field detector, we operate the SQUID with a constant bias current slightly greater than the critical current, so the SQUID is always resistive. Under these conditions, there is a periodic relationship between the voltage across the SQUID and the applied magnetic flux, with a period of one flux quantum. Note that, at fixed bias current, the voltage across the SQUID is a maximum when the critical current is a minimum, and vice versa. The relationship between the input flux and the output voltage across the SQUID looks like the right side of the diagram on the previous page. This is the phenomenon scientists and engineers exploit to create the world's most sensitive magnetic field detectors.

## Magnetic Signal Levels



## **SQUID Applications**

Liquid-helium-cooled SQUIDs have been commercial products for two decades. In the early stages, the primary uses for these devices were in laboratory instrumentation. Generally speaking, technically trained users such as physicists and electrical engineers used commercial SQUIDs as highly sensitive magnetic field detectors, voltmeters, or null detectors for experiments that were already being conducted at cryogenic temperatures. During the past decade, however, complete instruments have become available that incorporate helium-temperature SQUIDs that require less expertise on the part of the user. The prime example of this type of application is the SQUID susceptometer that is widely used by many laboratory scientists. Ironically, this instrument has found its greatest application in the study of material properties of high-temperature superconductors. Other examples include rock magnetometers that analyze the magnetic properties of mineral samples. These SQUID-based instruments feature liquid helium dewars with hold times greater than one year! Clearly, for at least some fixed installations, liquid helium does not impose great hardships.

Special-purpose SQUID instruments for users other than laboratory scientists have been slower to reach the marketplace. The main example in this category is the multi-channel SQUID-based system for biomedical applications. Several companies worldwide market systems of this kind, which sell for as much as \$3 million. The two main applications are magnetoencephalography (MEG) and magnetocardiography (MCG), in which the magnetometers measure magnetic signals generated in the brain and heart, respectively. The SQUID system is contactless and may yield additional or complementary information to the conventional electroencephalogram (EEG) or electrocardiogram (EKG). There are a number of applications beyond MEG and MCG that make use of SQUIDs. The combination of SQUIDs and superconducting magnets can be used as a “magneto-ferritometer,” which is a specialized instrument used to monitor iron levels in the liver. This is an important diagnostic tool for identifying a condition known as hemochromatosis, which can be extremely serious if not detected.

In addition to the commercial marketplace, there has been considerable development effort on a variety of SQUID applications for the military. The U.S. Navy has sponsored research for many years on SQUIDs for submarine and mine detection in both airborne and ocean-going platforms. For this application, the remote fielding and demanding environment for the system are major barriers to the use of liquid-helium-cooled SQUIDs. Nitrogen cooling offers a far more viable alternative if HTS SQUID performance can meet the requirements of this application.

A number of applications have been investigated in the area of geophysics. These range from prospecting for oil and minerals to earthquake prediction. There are a variety of different techniques that have been explored that use of SQUIDs. Active systems introduce pulsed magnetic signals into the earth and then detect the response by means of the SQUID. One

version of this technique is essentially a type of NMR that determines the properties of the different strata that make up the ground in a test site. Currently, conventional (*i.e.* non-SQUID) magnetometers are used for “bore-hole logging,” an important technique for locating oil.

An important passive geophysical technique that utilizes SQUIDs is known as magnetotellurics. In this technique, the properties of an area of ground (actually the impedance tensor of the ground) are determined by comparing the signals at a reference SQUID to those at a test SQUID that is moved from point to point. During the early 1980's, several companies practiced this technique on a commercial basis. However, the subsequent drop in the cost of oil — and the removal of incentives to develop alternative energy sources, such as geothermal energy — ended the commercial viability of such surveying techniques. As in the case of the magnetic anomaly detection applications, remote magnetic field measurement is a critical issue for geophysical applications and, therefore, HTS SQUIDs represent a significant advance in practicality.

Another area of investigation has been non-destructive evaluation (NDE) using SQUIDs. Once again, there are a variety of proposed techniques and implementations, both active and passive. Of special interest is the testing of submerged or otherwise inaccessible pipes, the evaluation of structural members such as bridge components, the location of buried toxic waste drums, and the testing of welds in structures such as aircraft wings. Initial studies indicate that SQUIDs may be effective in all these areas, but considerable modeling and testing remains to be done to demonstrate the viability of the method.

In summary, there is a great variety of SQUID applications either under investigation or commercially available. What has kept the majority of them out of the marketplace is a combination of the difficulties imposed by cryogenic requirements and the lack of sufficient demonstration of their utility. As high-temperature superconductor-based SQUIDs alleviate the former problem, the opportunities to eliminate the latter problem will only increase. The Mr. SQUID<sup>®</sup> system before you represented the first step in bringing HTS SQUID technology into the marketplace.

# Summary of Studies

In this lab, you will observe the following fundamental properties of Josephson junctions and SQUIDS and calculate the values of these key parameters:

- Critical current of the SQUID,  $2I_c$ ,
- Normal-state resistance of the SQUID,  $R_N/2$ ,
- Characteristic voltage of the SQUID,  $I_c R_N$ ,
- Maximum voltage modulation depth,  $\Delta V$ ,
- Inductance of the SQUID,  $L$ , and
- Modulation parameter  $\beta_L$  determined using two methods.

You should use the Labview interface to the control box to acquire data for these measurements.

## Detailed Description of Initial Studies

The descriptions that follow assume that you have some familiarity with the basics of superconductivity including zero resistance, flux quantization, and the Josephson effect. Before starting, you should first read the short primer on these topics found at the start of this writeup above before going any further.

The DC SQUID you will be using is made from a thin film of YBCO superconductor. The DC SQUID is a simple circuit that can be represented schematically as shown in Figure 3.

The simplest experiment to perform with this circuit is to pass current “ $I$ ” from left to right through the ring and to measure the voltage “ $V$ ” that appears across the ring. If the two junctions in the SQUID are identical, in the absence of any magnetic field, the current will divide evenly and half of it will pass through each junction before recombining at the right side.

The Mr. SQUID<sup>®</sup> control box allows you to perform this experiment quite easily, as discussed in the next section.

## Setting Up the Mr. SQUID<sup>®</sup> System

In this section, we explain how to assemble the various parts of the Mr. SQUID<sup>®</sup> system. The set-up instructions refer to the various controls and connectors on the Mr. SQUID<sup>®</sup> electronics box shown in Figure 7 below:

- Configure your oscilloscope for x-y mode, and set the vertical sensitivity to 2 V/div and the horizontal sensitivity to 0.5 V/div. Use the vertical and horizontal position controls to move the oscilloscope dot to the center of the display.
- Using two BNC cables, connect the VOLTAGE output on the front panel of the electronics box to the vertical input on the oscilloscope and CURRENT output to the horizontal input on the oscilloscope.

- Confirm that the power switch on the front panel of the electronics box is in the OFF (down) position and that the mode switch is in the V-I (up) position.
- Plug the 5-pin power cable into the POWER connector on the rear panel of the electronics box, and plug the power supply into the power mains.
- Plug one end of the nine-pin DB-9 M/M cable into the PROBE connector on the rear panel of the electronics box, and plug the other end into the Mr. SQUID probe.
- Check that the FLUX OFFSET and CURRENT OFFSET controls are in the 12 o'clock position and that the SWEEP OUTPUT control is fully counter-clockwise. Turn on power to the electronics box by switching the POWER switch into the ON (up) position. Use the CURRENT OFFSET control on the front panel of the electronics box to position the dot near the center of the oscilloscope display. If you are unable to move the dot to the center of the display, check that the oscilloscope vertical and horizontal sensitivities are configured as described above and that the probe is connected to the electronics box using a nine-pin DB-9 M/M cable such as is provided with the system. If you are still unable to center the dot on the display see Section 6.



Figure 7: Front panel of the Mr. SQUID<sup>®</sup> electronic control box.

## Cooling the Mr. SQUID<sup>®</sup> Probe

### WARNING

**WEAR EYE PROTECTION AND GLOVES WHEN WORKING WITH LIQUID NITROGEN. UNDER ALL CIRCUMSTANCES, BE SURE TO FOLLOW THE SAFETY REGULATIONS OF YOUR LABORATORY. IF YOU ARE UNCERTAIN ABOUT HANDLING LIQUID NITROGEN, CHECK WITH RESPONSIBLE PEOPLE IN YOUR LABORATORY. THE DEWAR SUPPLIED WITH MR. SQUID<sup>®</sup> IS MANUFACTURED SPECIFICALLY TO CONTAIN LIQUID NITROGEN, BUT IT CONSISTS OF A GLASS VACUUM VESSEL THAT CAN SHATTER IF MISHANDLED.**

Fill the dewar about 3/4-full with liquid nitrogen. If you will be using the system for a long time (more than a few hours), the liquid nitrogen level in the dewar will decrease due to evaporation. If the liquid level drops below the SQUID sensor, simply refill the dewar to its original level with more liquid nitrogen.

Confirm that the mu-metal shield is securely installed on the Mr. SQUID probe. The mu-metal shield helps screen the SQUID from external magnetic noise sources.

Wear eye protection and gloves during the following procedure as the liquid nitrogen may splash as the probe is introduced into the dewar. *Slowly* lower the sensor end of the probe into the dewar taking care to avoid excess boiling and splashing of the liquid nitrogen. The foam cap for the Mr. SQUID<sup>®</sup> dewar has a hole and a slot in it to center the Mr. SQUID<sup>®</sup> probe in the dewar, as shown in Figure 8 below. It will take several minutes for the SQUID sensor at the end of the probe to reach the boiling point temperature of liquid nitrogen, 77 K (at sea level). The critical temperature ( $T_c$ ) for the YBCO superconductor in Mr. SQUID<sup>®</sup> is approximately 90 K.

It is important to cool the SQUID into the superconducting state with a minimum of external magnetic fields present. This will reduce the effects of a phenomenon known as magnetic flux trapping, which will adversely affect the performance of the SQUID. We will discuss the consequences of flux-trapping later.



Figure 8 Mr. SQUID<sup>®</sup> probe inserted into the liquid nitrogen dewar.

### 3.3 Varying the Current Offset

The CURRENT OFFSET control is used to send a DC offset current through the SQUID. Slowly turn this knob in either direction. The spot on the oscilloscope screen should initially move horizontally in response to the changing current, then with a slope corresponding to the SQUID resistance as the critical current is exceeded. As you turn the knob back and forth, you will trace out a curve representing the relationship between the current fed through the SQUID and the voltage across the SQUID. This curve is called the  $V-I$  curve for the SQUID.

### 3.4 Varying the Amplitude of the Sweep Output

Return the CURRENT OFFSET control to the 12 o'clock position and now slowly turn the SWEEP OUTPUT control in the clockwise direction. This function sweeps the current through the SQUID back and forth, automating the procedure you performed by hand using the CURRENT OFFSET control. A solid curve should now appear on the oscilloscope screen.

If you now turn the CURRENT OFFSET control, the center point of the curve being traced on the screen will move. The CURRENT OFFSET control sends a DC offset current through the SQUID, whereas the SWEEP OUTPUT sweeps the current back and forth about the DC offset current value.

### 3.5 Calculating the Current and Voltage

Your output device acts like a voltmeter. The sensitivity settings on it determine how much voltage corresponds to a division on the screen. The CURRENT output on the front panel of the Mr. SQUID<sup>®</sup> electronics box is the voltage output of an operational amplifier inside the box that measures the voltage drop across a precision (0.1%) 10- $\Omega$  resistor through which the current to

the SQUID flows. According to Ohm's Law ( $I = V/R$ ), the current flowing through this resistor is equal to the voltage across it divided by the resistance. The operational amplifier is configured with a gain of 1,000 so a voltage of 1 V at the CURRENT monitor output corresponds to a current of  $(1/1,000)/10 = 1/10,000$  A or 100  $\mu$ A.

Typical voltage levels across SQUIDs are very small and amplification is required to measure these voltages using an external display device such as an oscilloscope. The Mr. SQUID<sup>®</sup> electronic control box includes an amplifier circuit that can be configured for a gain of 100, 1,000 or 10,000 using an internal switch. The default factory- configured gain setting is 10,000. Thus, to calculate the actual voltage across the SQUID, the measured value on the oscilloscope should be divided by 10,000. That is, 1 V at MONITOR output corresponds to 100  $\mu$ V at the SQUID.

### **Observing V-I Characteristics**

If the settings on the control box and your oscilloscope are configured properly, as the SWEEP OUTPUT is increased you will see a curve on the oscilloscope display that appears more-or-less like the curve shown in Figure 4. It is important that there is a flat region in the center of the curve as shown above, although the width may vary from device to device. If what you see looks like a straight line, then either you do not have enough liquid nitrogen in the dewar, the probe is not sufficiently cold, or there may be trapped magnetic flux in the SQUID. The latter is a very common occurrence because the SQUID is very sensitive to external magnetic fields. If you observe this behavior and have checked your LN2 levels, see the instructor.



At this point, you can adjust the FLUX OFFSET control on the electronics box and observe the effect on the  $V-I$  characteristic. This control feeds a DC current into a coil that produces a magnetic field that couples to the loop of the SQUID. As explained above, a changing magnetic flux through the SQUID loop will modulate the critical current of the SQUID in a very specific manner. As the FLUX OFFSET control is rotated, the maximum zero-voltage current through the SQUID will change visibly on the oscilloscope screen. Try to adjust the FLUX OFFSET current such that the current corresponding to the “knee” in the positive-going side of the  $V-I$  curve is maximized.

Note that the current at the “knee” in the negative-going part of the  $V-I$  curve will not necessarily be maximized at the same FLUX OFFSET setting. This is a consequence of the small magnetic flux coupled to the SQUID by the current flowing through the SQUID (the current through Mr. SQUID<sup>®</sup> does not flow symmetrically as shown in the simplified schematic model in Figure 3-1, which would produce negligible magnetic flux coupling to the SQUID).

The response to the flux offset current can be quite sensitive; increasing the horizontal sensitivity on the oscilloscope may make it easier to adjust the FLUX OFFSET control to maximize the critical current. With the flux offset current adjusted to maximize the zero-voltage SQUID current in this way, the flux through the SQUID is an integral multiple of the flux quantum; if the flux offset is adjusted to minimize the zero-voltage SQUID current, the flux through the SQUID is a half-integral multiple of the flux quantum.

### ***Measurement 1:***

Determine the critical current of the junctions in Mr. SQUID<sup>®</sup> by measuring the voltage at the “knee” in the  $V-I$  curve (after adjusting the FLUX OFFSET to maximize the zero-voltage current at the location of the “knee”) and dividing that number by 10,000 to convert your answer into Amperes of current. If the “knee” is at 0.5 Volt, for example, the corresponding current is 50  $\mu\text{A}$ .

Note that this is the current through both junctions in the SQUID, not just one. Therefore, assuming the junctions are identical, the current through one junction is half this value (in our example above,  $50/2 = 25 \mu\text{A}$ ).

### ***Measurement 2:***

In addition to the critical current of the junction (usually written as  $I_c$ ) which you measured above, there is also a parameter of the junction known as the normal-state resistance or  $R_N$ . You can determine this parameter by measuring the slope of the  $V-I$  curve when the SQUID is well into the normal resistive region. Measure  $R_N$ .

To measure the slope, ground the horizontal input on the oscilloscope and measure the voltage corresponding to the length of the vertical line displayed on the oscilloscope. This is the (amplified) voltage across the SQUID; to get the actual voltage you must divide this number by 10,000. Now reconfigure the horizontal input for DC coupling and ground the vertical input. Measure the voltage corresponding to the length of the horizontal line displayed on the oscilloscope. As described above, the actual current through the SQUID is equal to this voltage divided by 10,000. The normal resistance of the SQUID is then the voltage across the SQUID divided by the current through the SQUID.

Remember that the DC SQUID contains two junctions in parallel, so the resistance measured above corresponds to half the resistance of a single junction (assuming they are identical.) This

means that the normal resistance  $R_N$  of each Josephson junction is twice the normal resistance of the SQUID.

The product of the critical current and the normal state resistance ( $I_c R_N$ ) is a voltage that is an important parameter for the SQUID. Make a note of it now for use later. Note that  $I_c R_N$  for one of the junctions has the same value for the SQUID, which has two junctions in parallel. For the junctions in Mr. SQUID<sup>®</sup> operating in liquid nitrogen, you will probably obtain a value between 100 and 200  $\mu\text{V}$ . This value sets the maximum voltage change in the SQUID by an individual magnetic flux quantum, and is discussed later in this section.

### ***Measurement 3: Observing $V$ - $\Phi$ Characteristics***

Up to this point, we have been looking at the properties of Josephson junctions. Now we will turn our attention to the properties of the DC SQUID itself. The DC SQUID has the remarkable property that there is a periodic relationship between the output voltage of the SQUID and applied magnetic flux. This relationship comes from the flux quantization property of superconducting rings that is discussed in detail above.

As we saw before, in the  $V$ - $I$  operating mode one can apply a magnetic field to the SQUID using the FLUX OFFSET control. This control sends a DC current through the “internal” feedback coil that produces a magnetic field that couples to the SQUID; a second coil or “external” feedback coil allows you to couple a magnetic field to the SQUID produced by an external current source. Electrical connections to the “external” coil are made using the BNC connector on the back of the control box. The “internal” coil is not directly accessible to the user.

If you slowly turn the FLUX OFFSET control, you will see the change in the critical current and the changing  $V$ - $I$  curve that occurs as the magnetic flux threading the loop of the SQUID is varied. Another way to see the sensitivity of the SQUID to external fields is to rotate a small horseshoe magnet slowly in the vicinity of the dewar. If you experiment carefully with the FLUX OFFSET control, you will see that the critical current of the SQUID oscillates between a maximum value and a minimum value.

First adjust the FLUX OFFSET control so that the zero-voltage current is at its largest value, then adjust the SWEEP OUTPUT to zero, and set the CURRENT OFFSET to place the SQUID at the knee of the  $V$ - $I$  curve. Now turn the FLUX OFFSET control, which controls the amount of magnetic flux through the hole in the SQUID loop. You should see the point oscillate vertically on the screen as the FLUX OFFSET control is varied. This periodic motion arises because the screening current in the SQUID body depends on the applied magnetic flux in a periodic manner as described above. To automate this procedure, configure the electronic control box for the  $V$ - $\phi$  mode (MODE switch in the down position). In this mode, the sweep signal is sent through the feedback coil, which is linearly related to the magnetic field applied to the SQUID. Adjust the SWEEP OUTPUT control. Depending on the particular SQUID and coil in your probe, you should be able to see at least four or five periods of oscillation as shown in Figure 3-9.

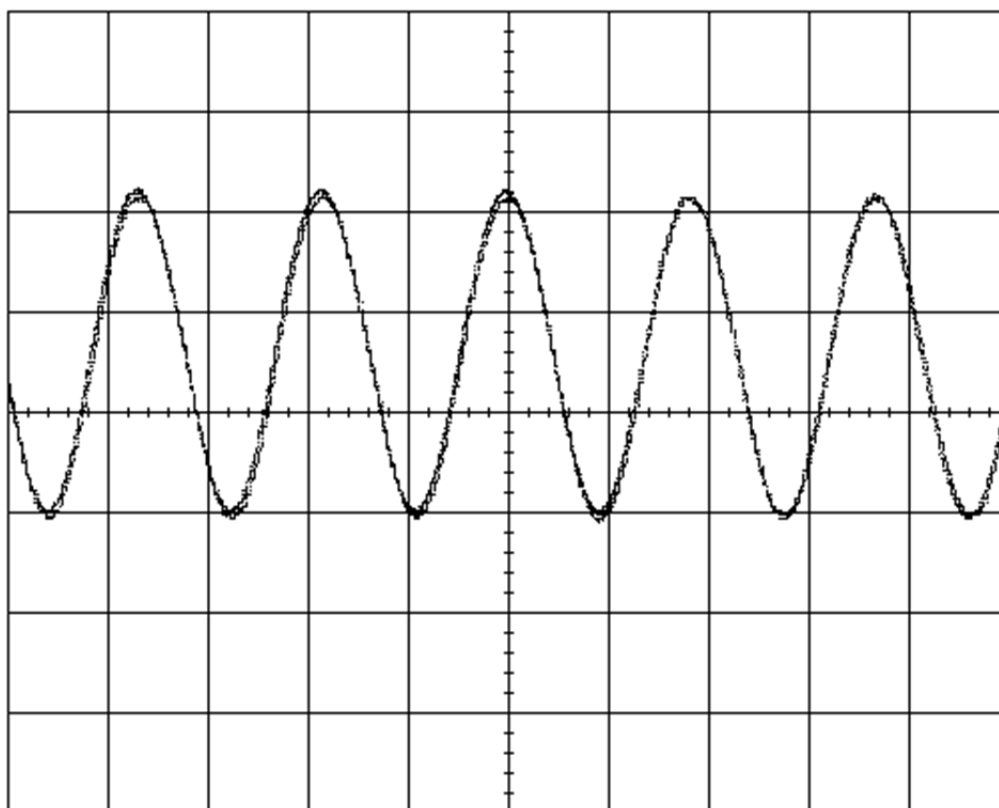


Figure 9 A typical  $V-\Phi$  characteristic ( $50 \mu\text{A}/\Phi_0$  horizontal,  $10 \mu\text{V}/\text{div}$  vertical). The maximum  $\Delta V$  may range from 10 to 30  $\mu\text{V}$  or higher.

The voltage change that occurs due to the influence of the magnetic field now appears on the vertical axis of your oscilloscope. You can maximize the signal by fine-tuning the CURRENT OFFSET control (just keep your eye on the  $V-\Phi$  curve as you “tweak” the CURRENT OFFSET control until you get the maximum modulation or voltage swing  $\Delta V$ ). Since the voltage amplification is unchanged from before, to determine the actual voltage swing across the SQUID, you must divide the voltage swing displayed on the oscilloscope by 10,000.

Notice the effect that the FLUX OFFSET control has on the  $V-\Phi$  curve. It allows you to set the central value of applied flux about which the SWEEP OUTPUT control varies the sweep signal. In other words, this control allows you to apply a static magnetic field to the SQUID on top of the oscillating field applied using the SWEEP OUTPUT control. Turning the FLUX OFFSET control will therefore allow you to move left and right along the  $V-\Phi$  curve and thereby explore points along this curve. You can accomplish the same thing in a less controlled manner by waving a small permanent magnet near the probe. Another way to see the effect of an additional applied field is to carefully rotate the Mr. SQUID probe. This rotates the SQUID with respect to the Earth's magnetic field and causes a shift in the total magnetic field coupled to the SQUID.

**Measurement 4:  $\beta_L$** 

Now that you can measure a variety of properties of the SQUID, you can determine a key parameter of the device, namely the  $\beta_L$  parameter. This is defined by

Eqn 1 
$$\beta_L = \frac{2I_c L}{\Phi_0}$$

where  $L$  is the inductance of the SQUID loop.

Earlier we mentioned that the maximum modulation voltage depth  $\Delta V$  as measured from the  $V$ - $\Phi$  curve is related to the  $I_c R_N$  product that can be determined from the  $V$ - $I$  curve expressed simply in terms of the  $\beta_L$  parameter.

Eqn.2

$$\beta_L = \frac{4I_c R_N}{\pi \Delta V} - 1$$

The above expression provides an approximate way to determine the  $\beta_L$  parameter empirically (*i.e.*, without knowing the inductance  $L$  of the SQUID). This expression is strictly correct only if the critical currents of the two junctions are equal and only if thermal noise effects are negligible. Both of these are approximations.

The inductance of the SQUID may be written as the sum of three terms,

Eqn. 3 
$$L = L_{sl} + L_k + L_j,$$

where  $L_{sl}$  is the inductance of the long slit in the SQUID body,  $L_k$  is the small kinetic inductance of the SQUID body (arising from the “inertia” of the electrons), and  $L_j$  is the inductance of the Josephson junction bridges, which also includes a small kinetic inductance contribution. The slit inductance per unit length is about 0.46 pH/μm, and the slit length (excluding the length of the junction bridges) is 125 μm. Then,  $L_{sl} = 58$  pH. The kinetic inductance of the SQUID body is more difficult to determine precisely but is estimated to be about 7 pH. The inductance of the Josephson junction bridges is estimated to be about 8 pH, which includes the kinetic inductance contribution (the inductance per unit length of the bridges is much higher than 0.46 pH/μm because of the narrow width of the bridges).

The current gain in the electronics box is 10,000 V/A (*i.e.*, 1 Volt =  $10^{-4}$  Amperes). The period of the modulation of the magnetic flux in the Mr. SQUID<sup>®</sup> loop is the flux quantum,  $\phi_0$ . By measuring the amount of current ( $\Delta I$ ) in the coil that is required to produce a change of one fluxon through the SQUID, you can determine the mutual inductance ( $M$ ) of the internal feedback coil with respect to the SQUID, using the following formula:

Eqn. 4 
$$M = \phi_0 / \Delta I$$

### **Determine $\beta_L$ in two way, using Eqns 1 and 2.**

The fact that the values calculated using Eqn. 1 and Eqn. 2 do not agree was a mystery for a number of years after the 1986 discovery of high- $T_c$  superconductors, as these equations worked quite well for SQUIDs made using traditional low- $T_c$  superconductors. This lack of agreement was resolved in 1993 with the recognition that thermal effects play a large role in the behavior of high- $T_c$  SQUIDs. The lack of agreement between Eqn. 1 and Eqn. 2 is in large part due to the fact that the SQUID you’re studying is at a relatively high temperature superconductor. Including thermal effects, Eqn 2 becomes:

Eqn. 5 
$$\beta_L = \frac{4I_C R_N}{\pi \Delta V} \left( 1 - 3.57 \frac{\sqrt{kTL}}{\phi_0} \right) - 1$$

Figure 10 illustrates the differences between the  $\beta_L$  values calculated using Eqn. 1, Eqn. 2, and Eqn. 5. Compute  $\beta_L$  from Eqn. 5 and compare with your earlier results.

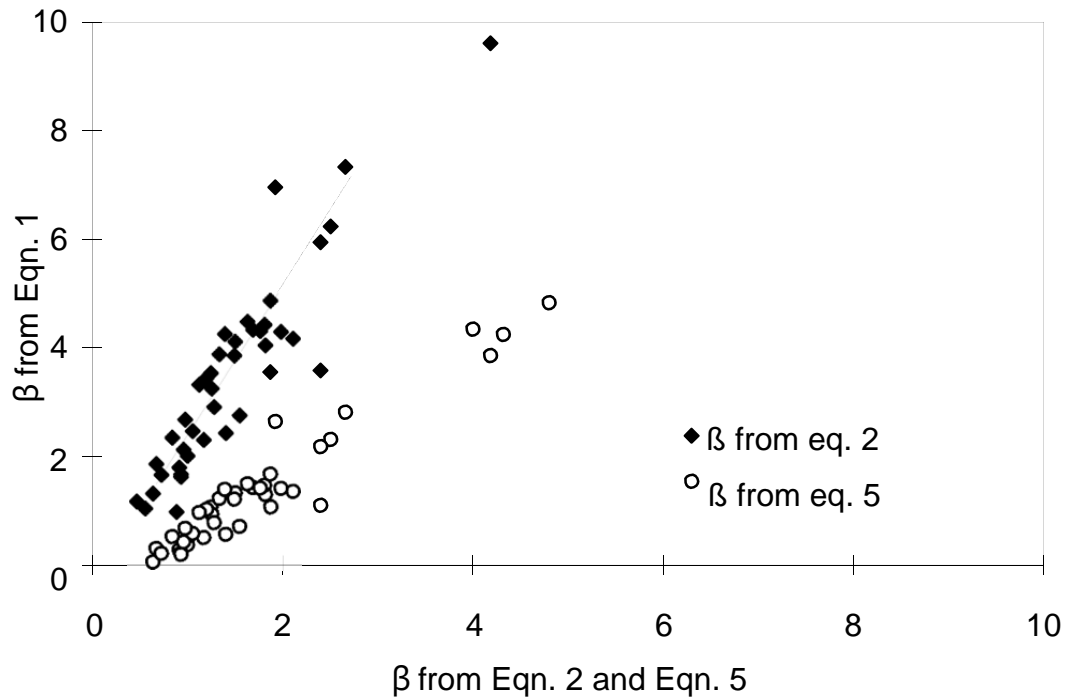


Figure 10 Values of  $\beta = 2I_c L / \Phi_0$  versus measurements of  $\beta$  based upon  $I_c R_N / \Delta V$  for 44 Mr. SQUID<sup>®</sup> probes.

As one can see from Figure 3-10, the method of calculating  $\beta_L$  that takes thermal effects into account agrees quite well with the inductive  $\beta_L$  measurements.

## Advanced Studies

Time permitting, refer to the Mr. SQUID manual, and carry out at least one of the advanced experiments, the descriptions for which begin on page 55 of the manual.

These include:

1. Measuring Resistance vs Temperature for the SQUID
2. Use Flux-Locked Loop to make a micro-voltmeter
3. Determine  $e/h$  using the AC Josephson Effect.
4. SQUID properties in pumped liquid nitrogen

