

# IMPERIAL

IMPERIAL COLLEGE LONDON

DEPARTMENT OF MATHEMATICS

MSCI RESEARCH PROJECT

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## Title

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## **Abstract**

Hi

## Acknowledgments

## **Plagiarism statement**

The work contained in this thesis is my own work unless otherwise stated.

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# **Chapter 1**

## **Introduction**

- 1.1 The Sphere Packing Problem**
- 1.2 The Work of Maryna Viazovska**
- 1.3 The Formalisation Movement**
- 1.4 Progress in Formalising Viazovska's Solution in Dimension 8**
- 1.5 The Scope of this Project**

## Chapter 2

# A Roadmap to Constructing the Magic Function

We mentioned, in the introduction, that the scope of this project is to construct Viazovska's Magic Function in Lean and prove that it satisfies certain specific properties, such as satisfying the hypotheses of the Cohn-Elkies Linear Programming Bound. In this chapter, we will outline the steps we will take to achieve this goal. In particular, we will list all the conditions we need to prove that the Magic Function satisfies. Our approach will be to construct the Magic Function in terms of two intermediary functions. Proving it satisfies the necessary conditions will then be a matter of proving that these intermediary functions satisfy certain properties. We will list these properties as well.

### 2.1 On Schwartz Functions

### 2.2 The Cohn-Elkies Conditions

### 2.3 The Desired Properties of the Magic Function

### 2.4 The Magician's Assistants: $a$ and $b$

# Bibliography