

UNIVERSITY

LEVEL's Degree in COURSE



LEVEL's Degree Thesis

Study and development of fault tolerant
operating systems for aerospace
applications

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Summary

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Acknowledgements

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*“HI”
Goofy, Google by Google*

Table of Contents

List of Tables	VI
List of Figures	VII
Acronyms	IX
1 Introduction	1
1.1 Thesis Motivation	3
2 Background	4
2.1 Hardware Technology	4
2.1.1 FPGA Architecture	4
2.1.2 FPGAs vs. ASICs	6
2.1.3 FPGA or ASIC in Aerospace Applications?	7
2.2 Radiations	9
2.2.1 Radiation sources	9
2.2.2 Radiation problems on Earth: the Nintendo's Super Mario 64 glitch	10
2.2.3 Types of radiation	10
2.2.4 Single Event Effects	11
3 Hello	15
3.1 Extremely long name with manual linebreak which otherwise would not fit the page	15
A Galileo	19
Bibliography	20

List of Tables

3.1	Examples of activation functions, operating either element-wise or vector-wise, depending on the function	16
3.2	y is the output of the network, N is the batch size multiplied by the number of outputs (e.g. pixels), C is the number of classes and \hat{y} is the correct output.	18

List of Figures

2.1	Simplified schematic of a FPGA cell	5
2.2	The intrinsic BJTs in the CMOS Technology that can cause a Latchup. Deepon, CC BY-SA 3.0, via Wikimedia Commons	12
2.3	Example of a Single Event Upset in a memory element.	13
2.4	Simple SRAM Cell layout. Inductiveload, Public domain, via Wiki- media Commons.	13
3.1	Hi	15
3.2	HI	16
3.3	SVG	16

Acronyms

SEU

Single Event Upset

COTS

Commercial Off-The-Shelf

FPA

Field Programmable Gate Array

ASIC

Application Specific Integrated Circuit

CLB

Configurable Logic Block

LAB

Logic Array Block

LUT

Look-up Table

HDL

Hardware Description Language

CPU

Central Processing Unit

DSP

Digital Signal Processing

CMOS

Complementary Metal-Oxide Semiconductor

TMR

Triple Module Redundancy

SEE

Single Event Effect

Chapter 1

Introduction

In the last past years, the number of missions devoted to the exploration of the universe has increased. Predictions shows that the number of missions in the current decade is expected to be almost three times the number of missions in the previous decade, without considering low-cost and low-weight missions like the one including cubesats.

Due to this increase in the number of missions, the number of electronic devices on board has increased, and the job complexity assigned to those devices has increased as well. Nowadays, electronic instruments are used not only for navigation purposes, but also for the analysis and manipulation of data. The most advanced spacecrafts are capable of decide autonomously the trajectory to follow, or to apply some complex algorithms to the data collected before sending them back to the ground.

Whatever is the purpose of a spacecraft, from the smallest one to a complete rover exploring another planet, electronic devices must be tailored to work in a realiable way, even in a complex environment like the space, where there are many disturbances like big temperature variations or radiations, one of the most common causes of failure in the spacecraft and greatest enemy of electronic instruments.

To understand better the problem, we can start from a real-world example, a piece of history. On September 22, 2021, the ESA's INTEGRAL spacecraft autonomously entered into emergency safe mode. INTEGRAL is a space telescope for observing gamma rays, and it was launched into Earth orbit in 2002. Something catastrophic was happening for the missions itself: one of the spacecraft's three reaction wheels had switched off without warnings. This caused a ripple effect that brought the satellite to begin to rotate incontrollably.

This episode created a lot of problems for the spacecraft, and the team of engineers responsible for the INTEGRAL spacecraft had to deal with it: due to the

fact that the spacecraft was spinning, data from the spacecraft were only reaching ground control in a difficult way, and the batteries were quickly discharging because of the missing orientation of the solar panels towards the Sun. ESA was going to lose a 19-years old space telescope.

With only a few hours of energy left to save the mission, the Integral Flight Control Team, together with Flight Dynamics and Ground Station Teams started working on a solution, and with quick thinking and ingenious ideas, they found the cause of the problem and rescued the spacecraft. The root of the problem was radiations. Charged, ionised particles, from the Van Allen belt, caused a SEU in the control system of the spacecraft, deciding erroneously to shut down the reaction wheel.

This story is an example of the problems that can happen during space missions due to radiations affecting the on board electronics. From this example, we can understand how crucial is the fault tolerant analysis during all the stages of development of a new space component, in order to produce a dependable system. The concept of dependable system is a complex one, and in space missions there are mainly three factors that can affect the dependability of a system:

- *Reliability*: the probability of a system to work as expected, continuously, in a given period of time (usually it coincides with the period of time of the mission itself).
- *Availability*: the probability of a system to work as expected at a generic moment in time, in the future.
- *Safety*: the ability of a system to work in a given environment, without any risk of serious damage.

With the increasing need for protection against unwanted effects caused by radiations, since the first interplanetary mission in the 60s with the Mariner 2 mission, there have been an increasing number of studies and techniques developed to deal with the problem. At the hardware level, there are *hardware mitigation techniques*, where usage of radiation-tolerant hardware components are used and hardware created with those components is called *radiation-hard* or *rad-hard* for simplicity. In most of the cases, *COTS* (Commercial Off-The-Shelf) hardware is used, which is a hardware meant to be used in a generic environment, and on top of that logical mitigation techniques are used to protect the system from the effects of radiation. The latter solution is easier to implement, and it is more efficient than the former one.

1.1 Thesis Motivation

The main motivation for the development of this thesis is to develop some techniques to deal with the problem of radiation in the space. In particular, the main goal is investigating the outcome that can occur when a SEU faults affects the CPU of a system (like the navigation system of a spacecraft), and how to deal with them by applying some innovative ideas to enhance the system's robustness and so the global fault tolerance of the system.

The hardware model on which the techniques are developed is the FPGA. FPGAs are used on a lot more space missions nowadays than in the past, for all the reasons that make FPGAs better than ASICs, mainly due to their flexibility. Because of the complexity of space missions, flexibility is a key factor in the success of a mission, both during the development and during the operational phases.

For this thesis, the usage of FPGAs has one big advantage, among other things: randomly generated SEU faults can be injected easily without using any strange and sophisticated instruments, a PC is enough. This is crucial in the study of radiation effects: it's possible to develop a systematic way to inject faults, and they can be repeated over time in order to be able to study the effect of the same SEU with different solutions.

Chapter 2

Background

Before going further in the implemented solutions, it's better to introduce a few background concepts. In particular, concepts about how FPGAs works, what kind of radiations exists and how FPGAs are affected by them.

2.1 Hardware Technology

2.1.1 FPGA Architecture

FPGAs (Field Programmable Gate Arrays) are used in a wide range of applications, from signal processing to machine learning applications. In particular, it is an integrated circuit designed to be general purpose: after manufacturing, it has no functionalities. It is a hardware that can be programmed to perform specific tasks.

It differs from a CPU. A CPU is an already designed hardware that is designed to do only one thing in a very optimized way: execute code, from a pre-defined Instruction Set. In this case, the action of *programming* is referred to the process of writing a series of instructions that the CPU will eventually execute. This is done by exploiting Programming Languages. A FPGA, instead, is like LEGO bricks. Each LEGO brick alone does not have any function or purpose, but when assembled (so put together with other bricks), it can be used to perform a specific task. Here, the action of *programming* is referred to the process of writing a *description* on how all the bricks will be assembled to perform the specific task we want. The description is done exploiting Hardware Description Languages (HDL) like VHDL or Verilog.

The basic FPGA design consists of I/O pads (to connect with the outside world), a set of routing channels and a set of LEGO bricks. A LEGO brick in the FPGA is a logic block (and depending on the vendor, it can be called CLB or LAB) that

can be programmed to perform a very specific task that in the overall design helps in achieving the goal of the User's Application.

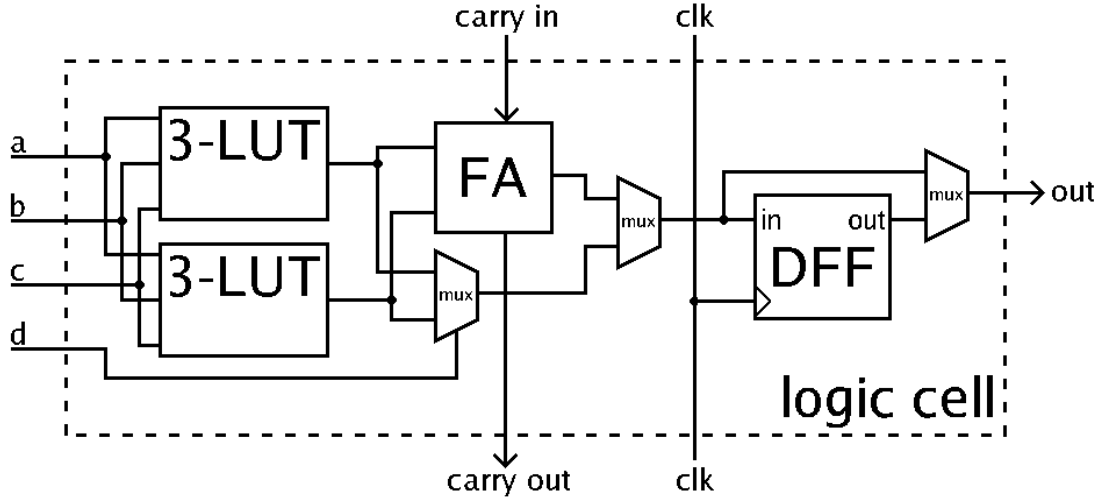


Figure 2.1: Simplified schematic of a FPGA cell

A basic logic block consists of a few Logic Elements. As shown in figure 2.1, a Logic Element is made of LUTs, a Full-Adder (FA), a D-Type Flip Flop and a bunch of multiplexers. This particular architecture can work in two modes: *normal* mode and *arithmetic* mode. Thanks to the Flip Flop, FPGAs can implement operations where some kind of memory is required.

Modern FPGAs are very complex and expand upon the above capabilities to include other functionalities in silicon. Having these common functions embedded in the circuit reduces the area required and gives those functions increased speed compared to building them from logical primitives (because they are implemented in-silicon, built out of transistor instead of LUTs, so they have ASICs-level performance). Examples of these include multipliers, generic DSP blocks, embedded processors, high speed I/O logic (like PCI/PCI-Express controllers, DRAM Controllers and so on and so forth) and embedded memories.

Once the User's Application is designed (i.e. the description of the FPGA is written), the design needs to be mapped onto the FPGA's hardware resources. This is done using the Vendor's specific software and it's in charge of deciding which FPGA's LE is assigned to which subpart of the description and how each LE is configured. Then, all the LEs need to be connected between themselves and the I/O pads, and this is done by routing algorithms that decide the best way to connect them. Once all the implementation steps are done, a configuration file is generated that will eventually be used to program the FPGA and is called *bitstream*.

All the programmable bits (like the content of the LUTs, some multiplexers selection signals or the routing details) are stored in the FPGA in memory elements that are outside the FPGA's functional blocks (i.e. the one that can be used by the user to implement the application). Those memory elements can be thought of as a big array of bits, or a *shift register*. It's the *configuration memory*: it stores the configuration bits of the entire FPGA and is loaded with the bitstream when the FPGA itself is programmed. Most FPGAs rely on an SRAM-based approach to be programmed: this allows to be in-system programmable (so the FPGA chip can be programmed without unmounting it from the board and from the system itself) and re-programmable (can be programmed as many times we want), but require external boot devices. Because the SRAM is a volatile memory, when the FPGA is powered off, the configuration memory content is lost. An external memory where the bitstream can be retrieved is required in order to re-program it. The SRAM approach is based on CMOS.

Consequently, FPGAs are alternatives to hard-core CPUs. This means that on a FPGA a CPU can be implemented out of logic primitives (called *soft-core*), alongside with the hardware that is used to implement the application like peripherals, memory and other components. Modern FPGAs supports *at runtime programming*, this lead to the idea of *reconfigurable systems*, where for example a CPU can be reconfigured in order to enable/disable some of its functionalities to suit the task at hand. The concept of *reconfigurable systems* is also used in another manner and will be explained further in the next chapters.

2.1.2 FPGAs vs. ASICs

An *ASIC* (application-specific integrated circuit) is an integrated circuit chip customized for a particular use. ASIC chips are typically fabricated using metal-oxide-semiconductor (MOS) technology. Thanks to the miniaturization of the MOS-based transistors and the improvement in the design tools, the maximum complexity (and hence functionality) possible in an ASIC has grown from 5000 logic gates to over 100 million.

They are designed using the same HDLs Languages as the FPGAs, but the similarities stop there. Once the description is complete, specific ASIC softwares are used to synthesize and implement onto a technology library. While the corresponding technology library in FPGAs is simpler (made of LEs and routing elements), on ASICs it's a lot more complex. A typical ASIC technology library consists of a set of basic logic gates (like 2 input NAND, 3 input OR, 2 input FA, etc.) provided by the manufacturer that will manufacture the chip. Once a HDL description is mapped on top of the ASIC library, the so called *gate-level netlist* is sent to the manufacturer. Here, ad-hoc technicians will start to work on this

netlist, doing the *route & place* of the netlist and as output of this process, a set of masks will be generated. The masks are used to *print* the circuit in the silicon. On top of all this process, tests engineers must prepare a set of tests that in order to test the correct functionalities of the circuit during the various stages of the manufacturing process, until the end of the process itself.

This allows to implement entire microprocessors, memories (including ROM, RAM, EEPROM and flash) and other large component in a single chip. Usually, for lower production volumes, FPGAs may be more cost-effective than an ASIC design. This is due to the non-recurring engineering (NRE) cost of an ASIC, that can run into millions of dollars.

To recap:

- ASICs circuits are faster, less power-hungry than FPGAs.
- ASICs are more complex to design and implement (hence more expensive) than FPGAs.
- FPGAs are more flexible than ASICs.

2.1.3 FPGA or ASIC in Aerospace Applications?

In the aerospace industry, we are witnessing a turnaround in the last years regarding the hardware technology. FPGAs are typically much less radiation hardened than ASICs, so they are more prone to SEUs as well as lower total ionizing dose tolerance, but there are techniques to reduce these deficiencies. However, FPGAs are used on a lot more missions nowadays than 15 years ago, for all the reasons that make FPGAs a better choice than ASICs.

As an example, Mars Exploration Rovers were something like 90% ASICs. The last JPL's Martian Rover, Perseverance, is a very complex system and it's a very challenging design from the engineering point of view: it has multiple sensors and cameras to collect as much data as possible and, due to the volume of live data being recorded and the long data transmission time from Mars to Earth, a powerful processing system is essential. Early Mars rovers were basing their workload mainly on CPUs and ASICs as the processing units, while nowadays FPGAs are taking on much of the workload, like in Perseverance.

There are different reasons behind this choice. The first one is the flexibility given by their re-programmability: because of the different stages a mission is made of, some parts of the system could be useful only in some of those stages (maybe intermediate ones) and they will never be used again. This is a waste of resources:

FPGAs can be a great help in this aspect and Perseverance rover is an example. It utilizes an almost decade-old FPGA technology (Xilinx Virtex-5, introduced in May 2006 on 65 nm technology) as one of the main processing units. This unit is responsible for rover entry, descent and landing on Mars. Once the rover is landed, this unit would be useless and would become a *dead hardware*. However, it's based on a FPGA hardware so it has been reprogrammed by NASA engineers from Earth to handle computer vision tasks.

Other units on Perseverance such as radars, cameras, UHF transceivers, radar, and X-ray (used to identify chemicals) are controlled using Xilinx's FPGAs. Another interesting point is that Perseverance uses machine learning algorithm running on FPGAs, and they are so well optimized that it's achieving higher performance levels (about 18 times) than Curiosity rover (landed on Mars in 2012 and still active).

Another advantage of using FPGAs is the faster time-to-space. There are different points that help in achieving this advantage. Not only the development on FPGA is faster than on ASICs (cost of design, development and fabrication of an ASIC are not present), but the most important thing is that there are many and many changes in the processing unit's architecture during project's development phase. There is usually a very strict launch window for the mission that can be missed, and FPGAs help in two ways mainly:

- Physically changing or adding more to a space system is a real challenge. The installation itself is not that difficult, but the system has to be recertified, proving that it is still dependable. Furthermore, FPGAs simplify this greatly: the only thing to prove is that the FPGA chip is safe to fly with. Once this is done, the overall number of different parts to be certified is reduced. Second, a change of the bitstream or of the software running on a *soft-core* take a lot less time to certify.
- Software and Hardware development can be done in parallel. This is a great advantage for the software development team, because a first iteration of the hardware can be prepared and ready to use by the software team faster and the software team can start to work on the software itself.

FPGAs are not only helpful during the development phase, but even during the operational phase. Missions are prepared to last a relatively long time, but usually the quality of the work is so high that they last much longer. Examples are Mars rovers: Opportunity landed on the Red Planet in 2003 and it was ended by a martian dust storm in 2018, so it lasted for 15 years. Curiosity in 2012 and in 2022 is still active. This is a so long period that, speaking again about

re-programmability, the processing system architecture may require changes to let the mission continue working. In fact, different things can go wrong in a decade and having a full reconfigurable system (from remote in particular) is a must, giving ground engineers a lot more possibilities to fix the system or to add/remove components.

On the radiation tolerant side, vendors offer radiation-tolerant FPGAs. On top of that, it's possible to apply some logic changes to the design like TMR (Triple Module Redundancy) to a portion of the design or even to the entire design. Basically, it consists in triplicating the design and add a voter at the outputs. If a radiation error occurs, it will theoretically affect only one module so there will be two different results from the three modules (two correct and one wrong caused by the radiation). The voter will select the correct result (that is the majority). This is an example of making a design more robust to radiation.

2.2 Radiations

We are going to understand better why radiation effects regarding electronic devices are one of the primary concerns for the aerospace industry.

2.2.1 Radiation sources

Where does the radiation originate from? Unfortunately, the Universe and in particular the Solar System are full of radiations. The natural space radiation environment can damage electronic devices in different ways, ranging from a degradation in performances to a complete functional failure. More and more a space system goes deeper in the space, less and less it is protected by the Earth's atmosphere.

Close to the Earth, there are two three sources of radiation: the Van Allen Belts, the Sun and the Cosmos itself. Van Allen Belts are zones of energetic charged particles, that are generated for example by the Sun, and captured by the Earth's magnetosphere. By trapping those charged particles, the magnetic field deflects them and protects the atmosphere from destruction. The two Earth's main belts extends from an altitude of 640 km to 58.000 km, in which radiation levels vary. Between the two belts, the *inner* and the *outer* there is a zone called *safe zone* where the level of radiation is pretty low. Spacecrafts travelling beyond the LEO (Low Earth Orbit) go through the two belts, and beyond the belts they face additional hazards from cosmic rays and solar particle events (coronal mass ejections and solar flares).

2.2.2 Radiation problems on Earth: the Nintendo's Super Mario 64 glitch

Here on Earth, electronic devices are often not shielded or design to tolerate radiations. Usually, only safety-critical systems undergo the same kind of radiation-tolerant techniques as the ones used in the space system, like Aviation and Nuclear Power Plants, for instance.

Even if there is a big magnetosphere protecting the planet's surface, some charged particles still escape and travel until they reach the ground and some everyday device. In 2013, a player was challenging another player in Nintendo's Super Mario 64 game. Suddenly, Mario was teleported into the air, saving crucial time and providing an incredible advantage in the game. The glitch caused the attention of a lot of players, and a \$1000 reward was offered to anyone who could replicate the glitch. Users tried in vain to recreate the scenario, but no-one was able to emulate that giant leap. In the end, after eight years, user concluded that the glitch was not replicable because it was caused by a charged particle coming from the outer space that caused a bit-flip in the value that defines the player's height.

Another curious case was the one related to the electronic voting machine in Belgium in 2003. A bit-flip here caused an adding of 4096 extra votes to a candidate. The error was only detected because there were more preferential votes than the candidate's own list, which is impossible in the voting system. The official explanation was "the spontaneous creation of a bit at the position 13 in the memory of the computer". It's not a coincidence that the value added was exactly 4096, in hexadecimal $0x1000$, that is 2^{12} .

2.2.3 Types of radiation

The most common way to classify radiations is based on their effects on electronic devices. If the effect is the result of a cumulative damage (i.e. passage of many charged particles in different moments in time, and each particle has a relative low energy) then it can be a *total ionizing dose* or a *displacement ionizing dose*. If the effect is the result of a single charged particle (with a high energy) then it can be *destructive* or *non-destructive*, and they are usually referred as SEE (Single Event Effects).

Total ionizing dose

Most electronic devices are based on MOS transistors, forming the basis for digital logic. The common way to use those transistors is as *electronic switches*: there are

two isolated contacts, the source and the drain (i.e. the switch is off, no current). When a positive charge is applied to the gate (in the case of a NMOS transistor), electrons (that are negative charges) are allowed to pass from the two isolated contacts (i.e. the switch is on).

When ionizing radiations passes through the device, electrons are moved away from the material leaving “holes” of missing charge, acting as positive charge carriers. These holes can find their way to the gate oxide and become trapped: this phenomenon is called *total ionizing dose*. The effect of this phenomenon is the same as applying some positive voltage to the gate. With enough accumulated charges, the effect is to have the transistor always on, or better, in the *stuck-on state*.

Displacement ionizing dose

Another form of cumulative damage is the *displacement ionizing dose*. This is the effect of a single charged particle passing through the device. What happens is that an atom is displaced from the material, modifying the crystal structure of the material itself. These microscopic effects create traps and recombination centers, eventually leading to the modification of the free flow of the current. This will ultimately impact the device’s performance.

2.2.4 Single Event Effects

When a single high-energy charged particle passes through the device, it can cause a *destructive* or *non-destructive* effect. The particle creates a momentary change of charge in the device, creating an unexpected current that can affect the device in various ways. Some effects may be completely destructive, while others may degrade performance to the point that the device doesn’t work anymore in the limits required by the circuit or the system itself. Other effects cause the device to momentarily work in a wrong way, causing a functional failure (so it’s not destructive from the point of view of the device but can cause an functional error, for example a wrong value in the memory from *0xe* to *0xf*).

Within the destructive effect, the most common are Single Event Latchup (SEL), Single Event Burnout (SEB) and Single Event Gate Rupture (SEGR).

Single Event Latchup

In CMOS technology, there are a lot of intrinsic BJT (Bipolar Junction Transistor). When a special arrangement of PMOS and NMOS transistors is used, resulting in

a n-p-n-p structures (corresponding to a NPN and a PNP transistor stucked next to each other), a CMOS Latchup structure is created. If one of these two transistor is activated (accidentally by a high-energy charged particle), the other one will be activated too, creating a feedback loop. They will both keep each other activated for a long as some current flows through them. This phenomenon will increase the current draw and can bring to the destrupution of the device. Usually, the only way to correct this situation is to make a *power cycle*, so completely shutting down the device and then restarting it. However, latent damage may exists that may not appear until later.

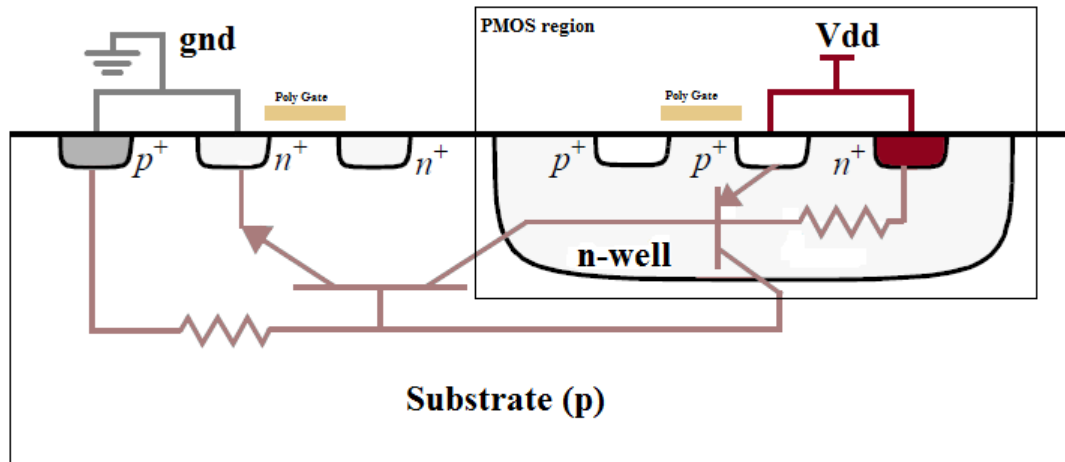


Figure 2.2: The intrinsic BJTs in the CMOS Technology that can cause a Latchup. Deepoon, CC BY-SA 3.0, via Wikimedia Commons

Single Event Burnout

Can happen when an incident particle initiates an avalanche charge multiplication effect. This leads to an increasing current, leading to a thermal runaway of the device, causing local melting or ejection of molten material in a small-scale explosion. Obviously, the result is a complete destruction of the device.

Single Event Gate Rupture

SEGR is the destructive rupture of a gate oxide (or any dielectric layer in a transistor). The effects can be observed in power MOSFETs with an increase of current flow when turned on, or in digital circuits with stuck bits.

Single Event Upset

This is the most common non-destructive effect. As known as *bit-flip*, it's caused by a particle that forces a digital signal to an opposite value momentarily. It can lean in a temporary modification of the digital output in a combinatory circuit, and the modified value can be memorized in a flip-flop or any other memory element if sampled at the same time a radiation arrives. In more complex circuits, it can cause other malfunctions like resets and memory values modifications.

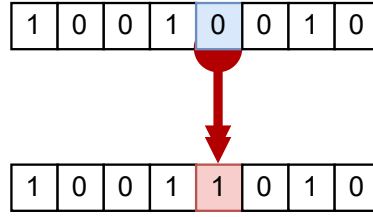


Figure 2.3: Example of a Single Event Upset in a memory element.

What is shown in Figure 2.3 can for example happen in a SRAM memory. Each cell is made of a cross-coupled transistors. Each side couple are connected forming an inverter (NOT logic function), and the output of the inverter is connected to the gates of the second couple.

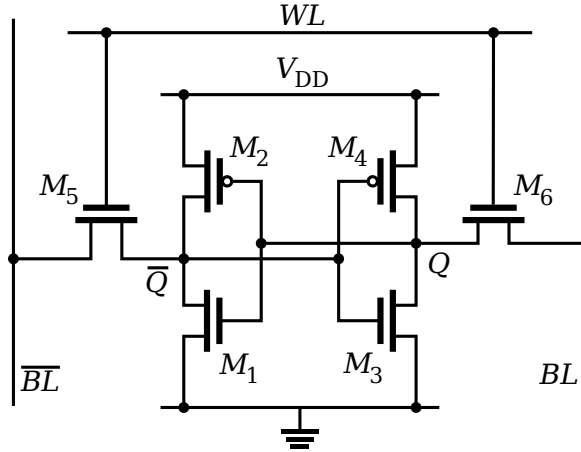


Figure 2.4: Simple SRAM Cell layout. Inductiveload, Public domain, via Wikimedia Commons.

In Figure 2.4, a simple layout is proposed. In order to have a logic 0 as output ($BL = 0$), M3 is active (thus M4 is not active). So M2 is active (thus M3 is not active). If a radiation strikes one of those transistor, can happen that the M3's

gate voltage goes low, causing a flip of the configuration thus a flip of the stored bit.

As explained in Section 2.1.1, most FPGAs' memory configuration are based on SRAM technology. If a bitflip occurs, the FPGA configuration itself is modified, leading to a malfunction of a module or to a routing modification.

Chapter 3

Hello

[Hi 1, Goofy]
 kg s^{-1}



Figure 3.1: Hi

3.1 Extremely long name with manual linebreak which otherwise would not fit the page

1. A
2. B
3. C



**POLITECNICO
DI TORINO**

Figure 3.2: HI

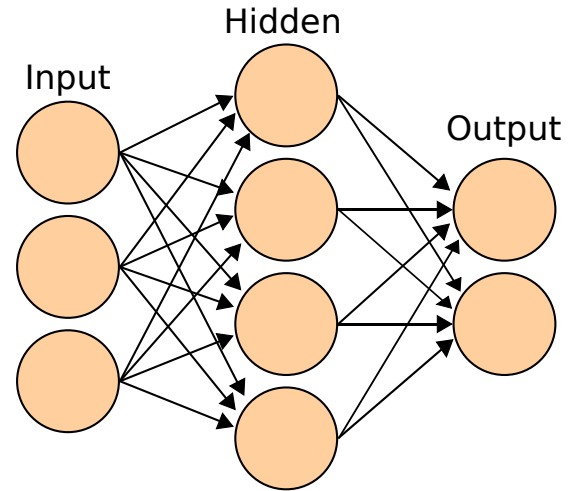


Figure 3.3: SVG

ReLU	$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x & \text{for } x > 0 \end{cases}$
Softmax	$f_i(\vec{x}) = \frac{e^{x_i}}{\sum_{j=1}^J e^{x_j}} i = 1, \dots, J$
tanh	$f(x) = \tanh(x) = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$

Table 3.1: Examples of activation functions, operating either element-wise or vector-wise, depending on the function

$$output = f_{activation} \left(\sum_{\#neurons} input_i + bias \right) \quad (3.1)$$

- A
- B
- C

Algorithm 1 Adam optimizer algorithm. All operations are element-wise, even powers. Good values for the constants are $\alpha = 0.001, \beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}$. ϵ is needed to guarantee numerical stability.

```
1: procedure ADAM( $\alpha, \beta_1, \beta_2, f, \theta_0$ )
2:    $\triangleright \alpha$  is the stepsize
3:    $\triangleright \beta_1, \beta_2 \in [0, 1)$  are the exponential decay rates for the moment estimates
4:    $\triangleright f(\theta)$  is the objective function to optimize
5:    $\triangleright \theta_0$  is the initial vector of parameters which will be optimized
6:    $\triangleright$  Initialization
7:    $m_0 \leftarrow 0$   $\triangleright$  First moment estimate vector set to 0
8:    $v_0 \leftarrow 0$   $\triangleright$  Second moment estimate vector set to 0
9:    $t \leftarrow 0$   $\triangleright$  Timestep set to 0
10:   $\triangleright$  Execution
11:  while  $\theta_t$  not converged do
12:     $t \leftarrow t + 1$   $\triangleright$  Update timestep
13:     $\triangleright$  Gradients are computed w.r.t the parameters to optimize
14:     $\triangleright$  using the value of the objective function
15:     $\triangleright$  at the previous timestep
16:     $g_t \leftarrow \nabla_{\theta} f(\theta_{t-1})$ 
17:     $\triangleright$  Update of first-moment and second-moment estimates using
18:     $\triangleright$  previous value and new gradients, biased
19:     $m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$ 
20:     $v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$ 
21:     $\triangleright$  Bias-correction of estimates
22:     $\hat{m}_t \leftarrow \frac{m_t}{1 - \beta_1^t}$ 
23:     $\hat{v}_t \leftarrow \frac{v_t}{1 - \beta_2^t}$ 
24:     $\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \frac{\hat{m}_t}{\sqrt{\hat{v}_t} + \epsilon}$   $\triangleright$  Update parameters
25:  end while
26:  return  $\theta_t$   $\triangleright$  Optimized parameters are returned
27: end procedure
```

MSE / L2 Loss / Quadratic Loss	$\frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{N}$
(Binary) Cross Entropy (average reduction on higher dimensions)	$\frac{\sum_{i=1}^N \sum_{j=1}^C \hat{y}_i \log(y_{i,j})}{N}$
Categorical Cross Entropy (sum reduction on higher dimensions)	$-\sum_{i=1}^N \hat{y}_i + \log\left(\sum_{i=1}^N \sum_{j=1}^C y_{i,j}\right)$

Table 3.2: y is the output of the network, N is the batch size multiplied by the number of outputs (e.g. pixels), C is the number of classes and \hat{y} is the correct output.

Appendix A

Galileo

```
1 import os
2 os.system("echo 1")
```

$\mathcal{O}(n \log n)$

numpy

Bibliography

- [1] S. Zhang, C. Zhu, J. K. O. Sin, and P. K. T. Mok. «A Novel Ultrathin Elevated Channel Low-temperature Poly-Si TFT». In: 20 (Nov. 1999), pp. 569–571 (cit. on p. 15).