Homework #2

Gajan Nagaraj
904937784
Dis 1D

3) a. If there is an optimal solution with $\delta = 0$ then:

$$y_i(\vec{w}^T\vec{x}_i + \theta) \geq 1$$

if $y=1$, then:

$$(\vec{w}^T\vec{x}_i + \theta) \geq 1 \geq 0$$

if $y=-1$ then:

$$(\vec{w}^T\vec{x}_i + \theta) \leq -1 < 0$$

We know that the two domains and ranges are disjoint, so we can reverse the condition and consequence and keep the same mapping.

D is linearly separable because it satisfies equation (1).

b. If there is a $\delta > 0$, then we can say that the data is linearly separable so long as $\delta \leq 1$. This will still satisfy equation (1), thus D will be still be linearly separable. If $\delta > 1$, we cannot conclude anything about the separability.

c. The optimal solution would be to set $\delta = 0$, $\vec{w} = 0$, $\theta = 0$. This would give us:

$$y_i(\vec{w}^T\vec{x}_i + \theta) \geq -\delta \rightarrow y_i(0 + 0) \geq 0 \rightarrow y_i(0) \geq 0 \rightarrow 0 \geq 0$$

This is the optimal solution because δ is minimized and the constraints are satisfied. This issue is that we form a bad hyperplane. In fact, we result in no hyperplane at all.

d. $x_1^T = [1 \; 1 \; 1]$ $\quad y_1 = 1$

$x_2^T = [-1 \; -1 \; -1]$ $\quad y_2 = -1$

Let $\delta = 0$:

$$\left(\vec{w}^T \begin{bmatrix}1\\1\\1\end{bmatrix} + \theta\right) \geq 1 \quad \text{for } y=1$$

$$-\left(\vec{w}^T \begin{bmatrix}-1\\-1\\-1\end{bmatrix} + \theta\right) \geq 1 \quad \text{for } y=-1$$

$$w_1 + w_2 + w_3 + \theta \geq 1 \quad \text{for } y=1$$

$$w_1 + w_2 + w_3 - \theta \geq 1 \quad \text{for } y=-1$$

$$\Rightarrow w_1 + w_2 + w_3 \geq 1 + |\theta|$$

Thus the optimal solution is:

$(\vec{w}, \theta, \delta)$ where $\delta = 0$ and $w_1 + w_2 + w_3 \geq 1 + |\theta|$

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