MPI EXECUTION & ANALYSIS

LORENZ ATTRACTOR

High Performance Computing Project Report

By,

SAI KAUSHIK SUDHAKARAN

CED18I044

Department of Computer Science and Engineering,

Indian Institute of Information Technology Design and Manufacturing

Kancheepuram, Chennai

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I am ensuring that this project was done by me and not copied from anywhere.

ABSTRACT

This project is designed to implement parallel programs for the simulation of objects following **Lorenz attractor** satisfying the Lorenz system.

Lorenz System is a system of differential equations, noted to have chaotic solutions for given parameters and the initial conditions. Lorenz attractor is the set of chaotic solution to the above system.

Using multi-dimensional arrays and multiple threads to increase the speed-up of the program. Generating high performance programs for the same of multi-core processor (OpenMP), cluster computers (MPI), and general-purpose graphic processing unit (Cuda C/C++).

Drawback of serial programs is the inability to run multiple objects to visually see the chaos with slight variations in the initial conditions.

TABLE OF CONTENT

INTRODUCTION

The Lorenz System is a system of differential equations, noted to have chaotic solutions for given parameters and the initial conditions, first studied by Edward Lorenz in 1962 [1].

Lorenz attractor is the set of chaotic solutions of the Lorenz System. It is a part of an area of study, Chaos Theory.

The shape of the Lorenz attractor, when plotted graphically, represents that of a butterfly. The butterfly effect is a real-world implication of the Lorenz attractor.

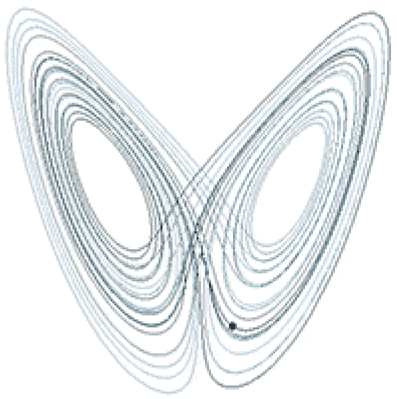


Figure 1: *Lorenz Attractor Animation*

This project is designed to implement parallel programs for the simulation of objects following Lorenz attractor satisfying the Lorenz system.

LITERATURE SURVEY

CHAOS THEORY

Chaos theory is a branch of mathematics focusing on the study of chaos—dynamical systems whose apparently random states of disorder and irregularities are actually governed by underlying patterns and deterministic laws that are highly sensitive to initial conditions [2].

A commonly used definition, to classify a dynamical system as chaotic, it must satisfy three properties:

- It must be sensitive to initial conditions.
- It must be topologically transitive.
- It must have dense periodic orbits.

BUTTERFLY EFFECT

In chaos theory, the butterfly effect is the sensitive dependence on initial conditions in which a small change in one state of a deterministic nonlinear system can result in large differences in a later state. [3]

It is metaphorically derived from the example of a tornado being influenced by minor perturbations such as a distant butterfly flapping its wings several weeks earlier.

ATMOSPHERIC CONVECTION

Atmospheric convection is the result of a temperature difference layer in the atmosphere. Different rate of temperature change with altitude within the air masses lead to instability. This instability with moist air masses causes thunderstorms, which are responsible for severe weather in the world. [4]

HISTORY

Lorenz had been studying the simplified models describing the motion of the atmosphere, in terms of ordinary differential equations depending on few variables. In his 1962 article, he analysed differential equations, for atmospheric convection, in a space of 12 dimensions, in which he numerically detects a sensitive dependence to initial conditions. [5]

In 1963, with the help of Ellen Fetter, Lorenz simplified the above mathematical model to a system of differential equations, now known as the Lorenz equations.

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

Here, x is proportional to the rate on convection

y is proportional to the horizontal temperature variation

z is proportional to the vertical temperature variation

σ is proportional to the Prandtl number, ratio of momentum diffusivity to thermal diffusivity

ρ is proportional to the Rayleigh number, describes the behaviour of fluids when the mass density is non-uniform

β is proportional to certain physical dimensions

CODE (LORENZ ATTRACTOR)

SERIAL CODE

```c
#include <stdio.h>
#include <stdlib.h>
#include <unistd.h>
#include <GL/freeglut.h>
#include <GL/glut.h>
#include <GL/glu.h>
#include <GL/gl.h>

typedef struct point
{
    double x;
    double y;
    double z;
    float t;
} Point;

double sigma = 10.0;
double rho = 28.0;
double beta = 8.0 / 3.0;

Point initial = {1.0, 1.0, 1.0, 0.0};
double dt = 0.01;

unsigned long int start = 0;
unsigned long int end = 1000000000000;

static GLfloat theta[] = {0.0, 0.0, 0.0};
GLint axis = 1;

unsigned long int op = 0;

Point differential(Point curr)
{
    Point diff;
    // 4 artih ops, 5 reads, 1 write
    diff.x = (sigma * (curr.y - curr.x)) * dt + curr.x;
    op += 4;
    // 5 arith ops, 6 reads, 1 write
    diff.y = (curr.x * (rho - curr.z) - curr.y) * dt + curr.y;
    op += 5;
    // 5 artih ops, 6 reads, 1 write
    diff.z = (curr.x * curr.y - beta * curr.z) * dt + curr.z;
    op += 5;
    // 1 arith op, 2 reads, 1 write
    diff.t = curr.t + dt;
    op += 1;
    return diff;
}

void lorenzGenerator()
{
    Point coord = initial;
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
    glLoadIdentity();
    glRotatef(theta[0], 1.0, 0.0, 0.0);
    glRotatef(theta[1], 0.0, 1.0, 0.0);
    glRotatef(theta[2], 0.0, 0.0, 1.0);
    glPointSize(1.0);
    FILE *fp = fopen("new.txt", "w");
    for (unsigned long int i = start; i < end; i++)
    {
        coord = differential(coord);
        fprintf(fp, "%f, %f, %f %f\n", coord.t, coord.x, coord.y, coord.z);
        glBegin(GL_POINTS);
        glColor3f(1, 1, 1);
        glVertex3f(coord.x, coord.y, coord.z);
        glEnd();
        glFlush();
        glutSwapBuffers();
    }
    fclose(fp);
    printf("Ops = %ld\n", op);
    glutLeaveMainLoop();
}

void spinCube()
{
    if (theta[axis] > 360.0)
        theta[axis] -= 360.0;
    else if (theta[axis] < 0)
        theta[axis] += 360.0;
    glutPostRedisplay();
}

void myReshape(int w, int h)
{
    glViewport(0, 0, w, h);
    glMatrixMode(GL_PROJECTION);
    glLoadIdentity();
    glOrtho(-50.0, 50.0, -50.0, 50.0, -50.0, 50.0);
    glMatrixMode(GL_MODELVIEW);
}

int main(int argc, char **argv)
{
    glutInit(&argc, argv);
    glutInitDisplayMode(GLUT_DOUBLE | GLUT_RGBA | GLUT_DEPTH);
    glutInitWindowSize(1280, 720);

    glutCreateWindow("Lorenz");
    glutReshapeFunc(myReshape);
    glutDisplayFunc(lorenzGenerator);
    glutIdleFunc(spinCube);
    glEnable(GL_DEPTH_TEST);
    glutMainLoop();
}
```

PARALLEL CODE

```
/*
mpicc -o lorenz_mpi lorenz_mpi.c -lGL -lGLU -lglut -lGLEW -lm
mpirun -n 5 -f machinefile ./lorenz_mpi
*/

#include <stdio.h>
#include <stdlib.h>
#include <unistd.h>
#include <GL/freeglut.h>
#include <GL/glut.h>
#include <GL/glu.h>
#include <GL/gl.h>
#include <mpi.h>
#include <stddef.h>

typedef struct point
{
    double x;
    double y;
    double z;
    double t;
} Point;

double sigma = 10.0;
double rho = 28.0;
double beta = 8.0 / 3.0;

Point initial = {1.0, 1.0, 1.0, 0.0};
double dt = 0.01;

unsigned long int start = 0;
unsigned long int end = 4096;

static GLfloat theta[] = {0.0, 0.0, 0.0};
GLint axis = 1;

Point differential(Point curr)
{
    int myid, numprocs;
    MPI_Comm_rank(MPI_COMM_WORLD, &myid);
    MPI_Comm_size(MPI_COMM_WORLD, &numprocs);
    MPI_Status status;

    MPI_Bcast(&curr, sizeof(Point), MPI_CHAR, 0, MPI_COMM_WORLD);

    Point diff;
    double temp0, temp1, temp2, temp3;
    switch (myid)
    {
    case 1:
        temp0 = sigma * (curr.y - curr.x);
        MPI_Send(&temp0, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    case 2:
        temp1 = curr.x * (rho - curr.z) - curr.y;
        MPI_Send(&temp1, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    case 3:
        temp2 = curr.x * curr.y;
        MPI_Send(&temp2, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    case 4:
        temp3 = beta * curr.z;
        MPI_Send(&temp3, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    }

    MPI_Barrier(MPI_COMM_WORLD);
    if (myid == 0)
    {
        MPI_Recv(&temp0, 1, MPI_DOUBLE, 1, 0, MPI_COMM_WORLD, &status);
        MPI_Recv(&temp1, 1, MPI_DOUBLE, 2, 0, MPI_COMM_WORLD, &status);
        MPI_Recv(&temp2, 1, MPI_DOUBLE, 3, 0, MPI_COMM_WORLD, &status);
        MPI_Recv(&temp3, 1, MPI_DOUBLE, 4, 0, MPI_COMM_WORLD, &status);

        diff.x = temp0 * dt + curr.x;
        diff.y = temp1 * dt + curr.y;
        diff.z = (temp2 - temp3) * dt + curr.z;
        diff.t = curr.t + dt;
    }
    MPI_Barrier(MPI_COMM_WORLD);

    MPI_Bcast(&diff, sizeof(Point), MPI_CHAR, 0, MPI_COMM_WORLD);
    return diff;
}

void draw(Point coord)
{
    glBegin(GL_POINTS);
    glColor3f(1, 1, 1);
    glVertex3f(coord.x, coord.y, coord.z);
    glEnd();
    glFlush();
    glutSwapBuffers();
```

```
}

void lorenzGenerator()
{
    Point coord = initial;
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
    glLoadIdentity();
    glPointSize(1.0);

    double startTime = MPI_Wtime();
    for (unsigned long int i = start; i < end; i++)
    {
        int myid, numprocs;
        MPI_Comm_rank(MPI_COMM_WORLD, &myid);
        MPI_Comm_size(MPI_COMM_WORLD, &numprocs);
        coord = differential(coord);
        if (myid == 0)
            draw(coord);
    }
    double endTime = MPI_Wtime();
    printf("%f\n", endTime - startTime);

    MPI_Finalize();
    glutLeaveMainLoop();
}

void myReshape(int w, int h)
{
    glRotatef(theta[0], 1.0, 0.0, 0.0);
    glRotatef(theta[1], 0.0, 1.0, 0.0);
    glRotatef(theta[2], 0.0, 0.0, 1.0);
    glViewport(0, 0, w, h);
    glMatrixMode(GL_PROJECTION);
    glLoadIdentity();
    glOrtho(-50.0, 50.0, -50.0, 50.0, -50.0, 50.0);
    glMatrixMode(GL_MODELVIEW);
}

void spinCube()
{
    if (theta[axis] > 360.0)
        theta[axis] -= 360.0;
    else if (theta[axis] < 0)
        theta[axis] += 360.0;
    glutPostRedisplay();
}

int main(int argc, char **argv)
```

```
{
    MPI_Init(NULL, NULL);

    glutInit(&argc, argv);
    glutInitDisplayMode(GLUT_DOUBLE | GLUT_RGBA | GLUT_DEPTH);
    glutInitWindowSize(1280, 720);
    glutCreateWindow("Lorenz");

    glutReshapeFunc(myReshape);
    glutIdleFunc(spinCube);
    glutDisplayFunc(lorenzGenerator);
    glEnable(GL_DEPTH_TEST);
    glutMainLoop();
}
```

CRITICAL SECTION OF THE CODE

In the “differential” function,

```
switch (myid)
    {
    case 1:
        temp0 = sigma * (curr.y - curr.x);
        MPI_Send(&temp0, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    case 2:
        temp1 = curr.x * (rho - curr.z) - curr.y;
        MPI_Send(&temp1, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    case 3:
        temp2 = curr.x * curr.y;
        MPI_Send(&temp2, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    case 4:
        temp3 = beta * curr.z;
        MPI_Send(&temp3, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD);
        break;
    }

    MPI_Barrier(MPI_COMM_WORLD);
    if (myid == 0)
    {
        MPI_Recv(&temp0, 1, MPI_DOUBLE, 1, 0, MPI_COMM_WORLD, &status);
        MPI_Recv(&temp1, 1, MPI_DOUBLE, 2, 0, MPI_COMM_WORLD, &status);
        MPI_Recv(&temp2, 1, MPI_DOUBLE, 3, 0, MPI_COMM_WORLD, &status);
        MPI_Recv(&temp3, 1, MPI_DOUBLE, 4, 0, MPI_COMM_WORLD, &status);
        diff.x = temp0 * dt + curr.x;
        diff.y = temp1 * dt + curr.y;
        diff.z = (temp2 - temp3) * dt + curr.z;
        diff.t = curr.t + dt;
    }
```

The above snippet is the critical section of the program, i.e., the most frequently run section in the program. This section calculates the new point with the help of the Lorenz equations mentioned above [link].

Due to the dependency of the current point with the previous point, gives us a small room for parallelization.

Here, each processor has a copy of the current point which will be broadcasted in the start of the function execution.

We use a switch case with the id of the processor as the expression. As we have 4 critical expressions to be solved, we use the first four workers.

Each worker computes the value and sends it to the master, who then generates the next point based on the generated values.

We add a barrier before the master works because we need all the threads to reach the point. This way the master has all the values to compute.

Here, we use 4 workers and a master to complete the execution. Theoretically, the performance should remain the same for thread count of 5 or above. But practically, the performance will degrade due to the communication overhead and thread scheduling.

OBSERVATION

COMPILATION AND EXECUTION

Compiling the code with mpi compiler with all essential flags for OpenGL.

```
mpicc lorenz_mpi.c -o lorenz_mpi -lm -lGL -lGLU -lglut -lGLEW
```

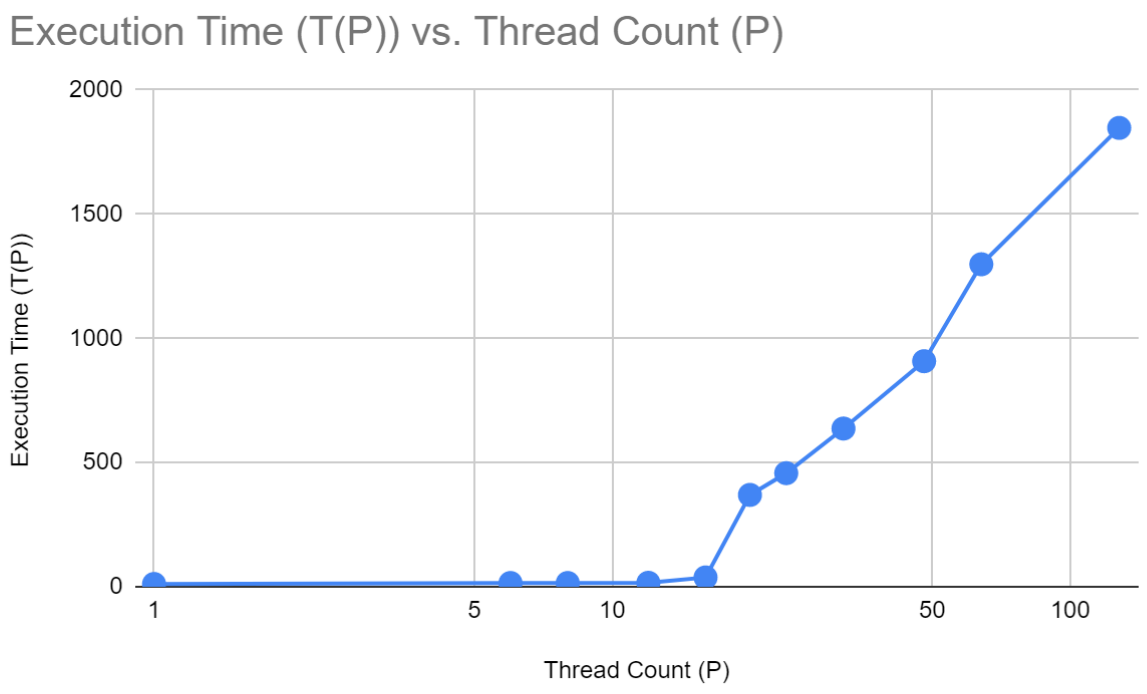
For execution, use

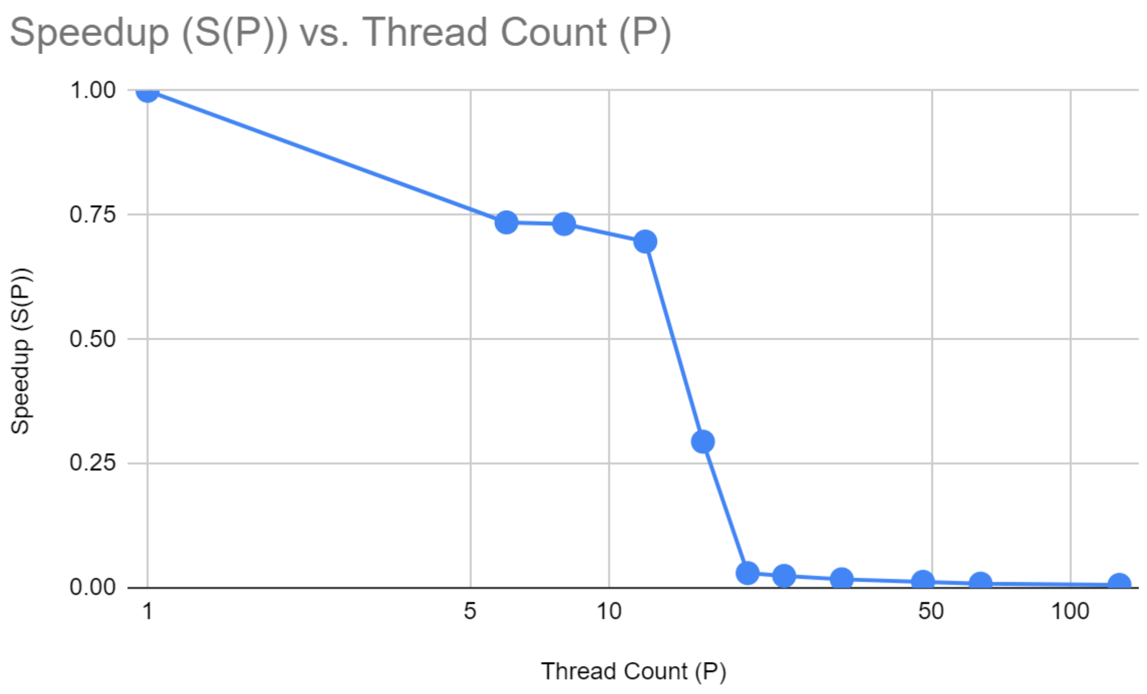
```
mpirun -n NUMBER_OF_PROCESSORS -f machinefile ./lorenz_mpi
```

TABLE

Lorenz Attractor (n = 4096)			
Thread Count (P)	Execution Time (T(P))	Speedup (S(P))	Parallelization Fraction (f(P))
1	10.963762	1	N/A
6	14.912688	0.7351969008	-43.22158033
8	14.978625	0.7319605104	-41.85073387
12	15.737066	0.6966839943	-47.49501793
16	37.273723	0.2941418543	-255.9701533
20	369.544086	0.02966834653	-3442.732272
24	457.15123	0.02398279012	-4246.598231
32	636.746772	0.01721840217	-5891.860465
48	907.864457	0.01207643048	-8354.647689
64	1298.230606	0.008445157547	-11927.47208
128	1847.237682	0.005935219981	-16880.45371

GRAPHS





CONCLUSION

The performance of the program worsens with the cluster execution as the same resources are divided among different virtual machines.

At any given instance of the program's runtime, there are 9 OS (1 Windows 11 and 8 Ubuntu 20.10) running, using the common resources from the laptop. This causes a bottleneck in the performance of the program.

Along with the OS, there are multiple background and foreground processes running, which use the system resources.

Due to the dependencies among the next and current point, we need multiple sends and receive to get the required output while using the concepts of MPI. Therefore, there is a very high communication overhead for each iteration of point generation.

The communication overhead crosses the computation overhead after 32 processors, causing the drastic reduction in the performance of the program.

REFERENCES

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