THEORETICAL ANALYSIS ON LORENZ ATTRACTOR

SAI KAUSHIK S CED18I044

ABSTRACT

This project is designed to implement parallel programs for the simulation of objects following Lorenz attractor satisfying the Lorenz system.

Lorenz System is a system of differential equations, noted to have chaotic solutions for given parameters and the initial conditions. Lorenz attractor is the set of chaotic solution to the above system.

Using multi-dimensional arrays and multiple threads to increase the speed-up of the program. Generating high performance programs for the same of multi-core processor (OpenMP), cluster computers (MPI), and general-purpose graphic processing unit (Cuda C/C++).

Drawback of serial programs is the inability to run multiple objects to visually see the chaos with slight variations in the initial conditions.



INTRODUCTION

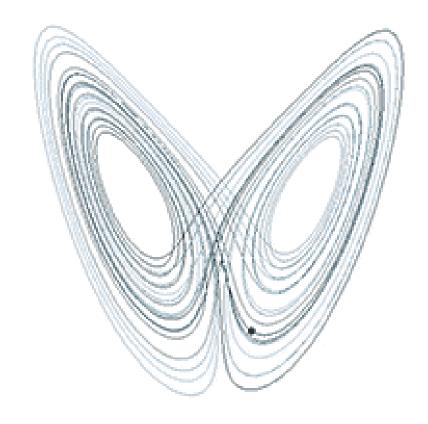


The Lorenz System is a system of differential equations, noted to have chaotic solutions for given parameters and the initial conditions, first studied by Edward Lorenz in 1962.

Lorenz attractor is the set of chaotic solutions of the Lorenz System. It is a part of an area of study, Chaos Theory.

The shape of the Lorenz attractor, when plotted graphically, represents that of a butterfly. The butterfly effect is a real-world implication of the Lorenz attractor.

This project is designed to implement parallel programs for the simulation of objects following Lorenz attractor satisfying the Lorenz system.



LITERATURE SURVEY

__

CHAOS THEORY



Chaos theory is a branch of mathematics focusing on the study of chaos—dynamical systems whose apparently random states of disorder and irregularities are actually governed by underlying patterns and deterministic laws that are highly sensitive to initial conditions.

A commonly used definition, to classify a dynamical system as chaotic, it must satisfy three properties:

- It must be sensitive to initial conditions.
 - It must be topologically transitive.
 - It must have dense periodic orbits.

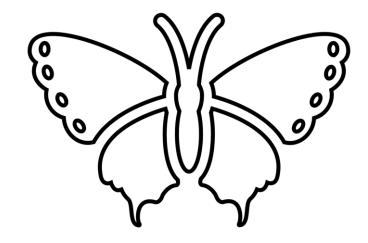
LITERATURE SURVEY

BUTTERFLY EFFECT

 \longrightarrow

In chaos theory, the butterfly effect is the sensitive dependence on initial conditions in which a small change in one state of a deterministic nonlinear system can result in large differences in a later state.

It is metaphorically derived from the example of a tornado being influenced by minor perturbations such as a distant butterfly flapping its wings several weeks earlier.



LITERATURE SURVEY

ATMOSPHERIC CONVECTION

Atmospheric convection is the result of a temperature difference layer in the atmosphere. Different rate of temperature change with altitude within the air masses lead to instability. This instability with moist air masses causes thunderstorms, which are responsible for severe weather in the world.

HISTORY

Lorenz had been studying the simplified models describing the motion of the atmosphere, in terms of ordinary differential equations depending on few variables. In his 1962 article, he analyzed differential equations, for atmospheric convection, in a space of 12 dimensions, in which he numerically detects a sensitive dependence to initial conditions.

In 1963, with the help of Ellen Fetter, Lorenz simplified the above mathematical model to a system of differential equations, now known as the Lorenz equations.

LORENZ EQUATIONS



$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

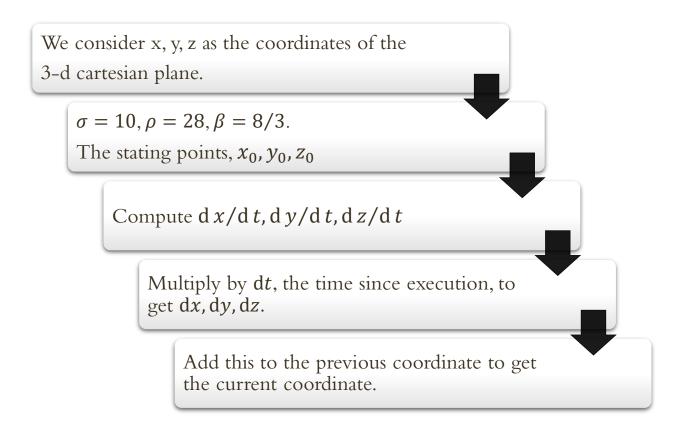
Here,

x is proportional to the rate on convection y is proportional to the horizontal temperature variation z is proportional to the vertical temperature variation σ is proportional to the Prandtl number, ratio of momentum diffusivity to thermal diffusivity

 ρ is proportional to the Rayleigh number, describes the behaviour of fluids when the mass density is non-uniform β is proportional to certain physical dimensions



IMPLEMENTATION



To observe the chaos in the system, we start multiple objects at the same time with the same start points, to observe their behavior.

```
for (j = 0; j < n; j++)
{
     xnew = rk4vec(t[j], m, x + j * m, dt, lorenz_rhs);
     for (i = 0; i < m; i++)
     {
          x[i + (j + 1) * m] = xnew[i];
     }
     free(xnew);
}</pre>
```

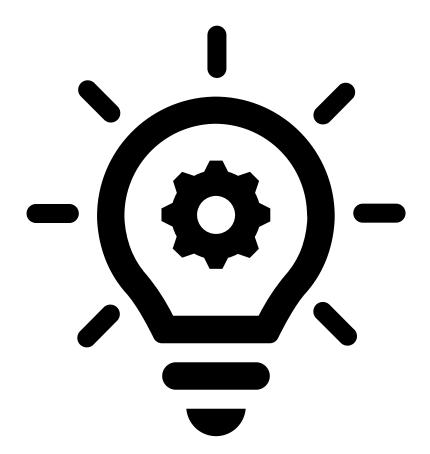
DRAWBACKS



In the critical part of the serial algorithm of Lorenz attractor, to generate the value of the point at (j + 1) * m, we need the value at j * m.

Since the current point depends on the position of the previous value, the main challenge is to remove the dependency to achieve high speed up when multiple threads are used.

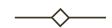
Multiple objects cannot be run at the same time in a serial code. So, to observe the chaos in the systems, a parallel program is necessary.



POSSIBLE SOLUTION

Multi-dimensional arrays to store the output coordinates, with multiple entries to the n-threads assigned for the program. Also, parallelize the execution of the different objects to get the synchronized output of the flow of the coordinates.

CONCLUSION



On perusal of the implementation and codes, the current parallelization factor is very low, on accounts on dependencies with previous values. This gives us a lot of room to apply parallelization techniques, to be learnt over the period of the course.

