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Certificate

This is to certify that the **annual progress report** entitled as "**Flow mixing using fluid-structure interaction.**" which is being submitted by **Gaurav Singh (18AT92R08)** for the annual session for the course in **Doctor of Philosophy** at **Advanced Technology & Development Centre, IIT Kharagpur, India**. This is a record of bonafide research work and is approved for submission.

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Course Works

Fluid Structure Interaction

- Introduction, Application areas;
- Structural Dynamics of continuous systems (plates and membranes);
- Basics of fluid dynamics and wave equations;
- Sloshing of fluids in rigid containers;
- Sloshing of fluids in rigid containers subjected to harmonic loads; Sloshing of fluids in elastic containers interacting with fluidCoupled and Partitioned systems (local and global partitions) simple examples;
- Hydroelasticity as coupled system approach; as partitioned approach;
- Lagrangian formulations: Total and Updated Lagrangian;
- Eulerian formulations; Arbitrary Lagrangian Eulerian formulation;
- Elastic interacting solid fully submerged in fluid;
- Elastic interacting fluid partially submerged in fluid;
- Shock waves interacting with solid.

High Performance Computing

- Sparse Matrices
- Discretization of differential equations,
- Storage Schemes for sparse matrices,
- Permutations and reorderings, Direct Solution Methods
- Iterative methods and Convergence
- SOR, Gradient search Methods: steepest descent, Conjugate gradient algorithm, Krylov subspaces methods: Arnoldis method, GMRES, Symmetric Lanczos algorithm,
- Convergence Analysis, Block Krylov Methods,
- Preconditioning techniques, ILU factorization preconditioners, Multigrid methods.
- Domain Decomposition,
- Schwarz algorithms and the Schur Complement, Graph partitioning: Geometric Approach, Spectral Techniques

- Parallel Computing
- Architectures for parallel computing, Shared and distributed memory
- Performance metrics, Parallelization of simple algorithms
- MPI and OpenMP,Parallelizing matrix solvers using domain decomposition
- CUDA, GPGPU architecture
- Thread algebra for Matrix operations

Computational Fluid Dynamics

- Spatial and temporal discretization
- Introduction to grid generation;
- structured, unstructured and hybrid grids, various grid generation techniques;
- finite difference method, convergence and stability,
- explicit and implicit methods;
- finite difference method applied to model equations;
- Numerical solutions Euler equations, incompressible and compressible
- Navier-Stokes equations, Concepts of upwinding, flux splitting and limiters, weighted integral and various finite element formulations.

Grades Achieved:

S. NO.	Course Works	Grade
1.	Fluid Structure Interaction	B
2.	High Performance Computing	A
3.	Computational Fluid Dynamics	C

Contents

1 Fluid mixing using fluid interaction interaction

1.1 Background and motivation

Fluid flow over structural bodies like elastic plates show very complex physics involving creation of wide range of flow scales, self-induced oscillations, and acoustic noise production. Few examples regarding fluid-structure interactions (FSI) are:

- human voice is an outcome of interaction between a pulsatile air flow passing over oscillating vocal-folds.
- blood flow through flexible arteries and veins.
- fluttering of plane wings, bio-sensing applications, etc

The flow-structure instabilities generated play a significant role in optimizing the mentioned engineering designs. Even in case of simple geometries like cylinder, sphere or plate which is not flexible flow instabilities may take place if the flow Reynolds number exceeds a critical value. If flow takes place on an elastic body instead of rigid, then complicated flow-structure instabilities take place. The factors that can affect the reported fluid-structure instabilities are mainly the fluid inertia, the bending rigidity of the plate, and the density ratio of the plate to the fluid. Recent technological advances are actively looking at adding one or more flexible plates on channel walls of Lab-on-a-chip device to take advantage of enhanced mixing rates, see [1, 2, 3, 4, 5, 6]. In these devices, the plates are either self-excited because of the flow currents or activated by external fields (such as magnetic fields).

Mixing is an omnipresent phenomenon observed in a cup of coffee to a huge cement silo. The complex physics involved in mixing has a wide range of multiple scales involving, inertial and diffusive range of time and length scales. In any industry, for example, to make better pharma products, making paints, alloy making or in food processing, mixing plays a vital role in bringing the correct composition to improve the quality and establish a good degree of homogeneity. The process rates in these applications depend on mixer designs such as flow inlet rates, flow properties, and mainly mixer geometry. Industries are actively looking for new and repeatable integrated mixers (similar to the modular framework) instead of a single large scale design. One of the key attractions of a micro-scale mixer is its portability. However, mixing on the micro-scale is quite challenging due to the increase in pressure loss

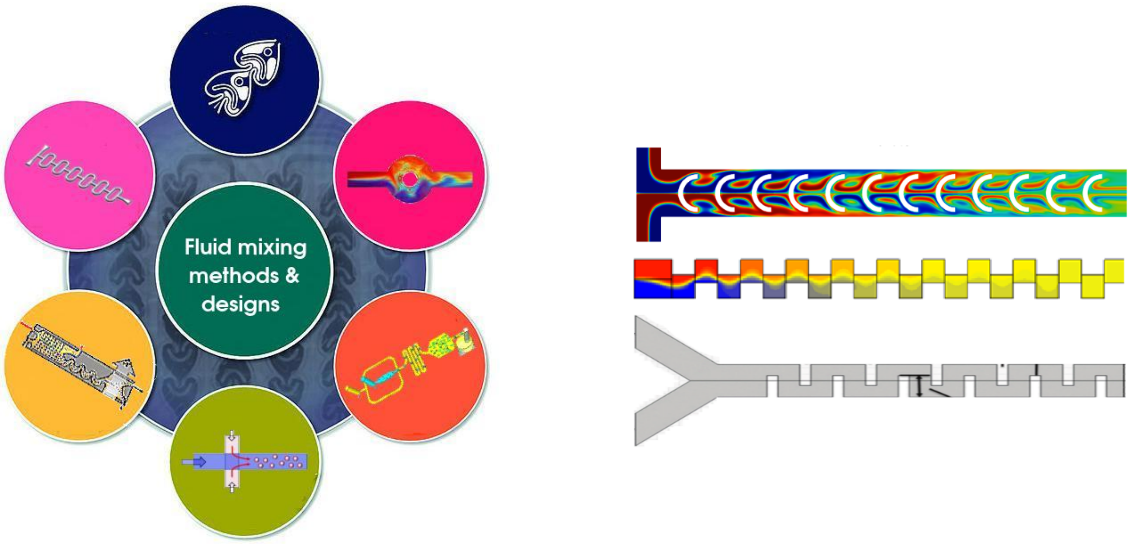

Figure 1: Presently available fluid mixing channel designs.

and molecular type diffusion. Is there a way the mixing rates can be increased by several orders of magnitude in these devices?

1.2 Research proposal

To achieve high mixing rates, a common method is to split geometry into several branches (passive method) or by externally forcing the flow (active method). In passive methods, the fluid naturally mixes, either by a change in local geometry or due to the presence of free surfaces. For active methods, external energy to move parts is required. Though the active methods are popular in achieving high transport rates when compared to passive methods, moving parts have reliability issues due to wear and tear. [7] were one of the first to demonstrate in developing a multi-stage multi-layer laminating mixer channel to high transport rates in passive micromixers, see some of the popular micromixing designs in figure 1. Is there a way one can achieve better transport rates than what has been already reported in the scientific community? Can we develop a passive design with moving parts, but not externally controlled, and achieve transport rates such as thousand times the ordinary molecular diffusion rates, especially in micro-scale channels? To probe these questions, I propose to work on different small scale channel flow designs with the addition of short flexible obstructions in the flow in the form of various configurations such as in figure 2. By inducing self-sustained oscillations to the flexible obstructions either by flow generated

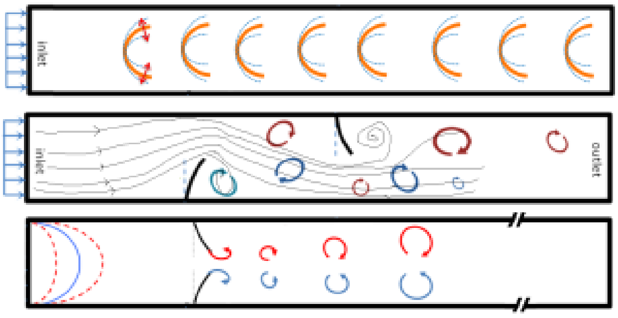

Figure 2: Rough sketch of improvements in the existing designs by either the introduction of flexibility in structure ir by varying flow consitions.

vortices or by imposing externally controlled oscillations we have observed an increase in the mixing level in the microchannels. Through this research plan, I would like to extend my preliminary studies on mixing in channel flows and develop a fluid-structure interaction based computational framework.

1.3 Literature review

Flow induced structural deformation has vital applications in several fields of engineering. For example, a soft structure in an oscillatory flow conditions resembles blood flow in veins and arteries [8, 9]. Also the elastic structures have potential of energy devices, and recent researches have shown thermal augmentation due to the deformable structure in a flow. Many biological systems are also composed of deformable membranes like heart valve, vocal folds etc, which causes the flow to undergo spatial variations normal to the flow. These variations lead to intricate flow dynamics in the downstream. Similar designs can also be seen in industries in the form of baffles and valvular designs. Such baffles are known to have larger control on fluid mixing phenomena, much required in food processing industry, chemical industries, bioreactors etc. However, industrial baffles in general are rigid. These baffles are an easy method of enhancing fluid mixing phenomena as it is a passive way requirng no external actuation. Such constrictions and obstructions in a channel flow develop vortical motions periodically. Vortical ejections have been under the purview of research in the recent few decades. In both two-dimensional and three-dimensional flows, coherent vortical structures are prevalent facets. The vortex structures in planar flows often come in pairs (or vortex dipoles) which is analogous to vortex rings in 3D flows. Studies have shown the process of such vortex generation and its growth formed by the wing tips of aircrafts, by the slender orifices, and by the locomotion of aquatic animals. Several works have been devoted to analyse the vortex generation due flow through orifices in terms of optimization of some integral quantities like vortex circulation, energy, thrust etc. Similar vortical motions are also known to be observed in the cardiovascular systems. The blood flow during the ventricular filling develop vortices which facilitate the blood ejections into primary circulation [10, 11]. [12] introduced optimization of vortex formation as a novel potential measure for cardiac health. This approach is based on the concept of universal formation time explained by [13].

Moreover, in the biological systems, essence of vortex structures can also seen in the mechanism of voice production. [14] has shown a strong correlation between intraglottal vorticity and acoustic energy i.e. the glottal airflow contains certain vortical structures that significantly contribute to voice quality. Further research in this line may lead to improved diagnosis and treatment of certain voice disorders. The biological systems mentioned contain movable parts such as in a cardiac system, blood flows from the atrium to ventricle through the mitral valve leaflets and glottal airflow passes past elastic vocal folds. Flow past such movable obstruction can be reduced to flow through orifice with variable opening due to the valve movements. A study on vortex formation process through a time-varying exit diameter is shown in [15] which shows that temporally increasing the opening results in higher-energy vortex ring structures with peak vorticity located further from the axis of symmetry relative to a static nozzle case. The analysis counts on predictive fluid-structure interactions models to explain coupled correlations between the fluid loading on the orifice exit and the generated global features. Similar ejections of vortex rings is also observed in "synthetic jets". These jets have wide scale engineering applications which includes control of flow separation over bluff bodies, enhancement of fluid mixing, electroning heat managment systems etc. These ejections also relate with the field of aquatic propulsion mechanisms such as squids, jellyfish motility and salps [16, 17, 18, 19]. Propulsion of jellyfish was experimentally stuied by Dabiri et al. 2005, and showed the development vortex rings within the wake.

In a reduced order approach [20] and [21] reported the behaviour of (single) flexible and stiff leaflets under leaflets under pulsatile flow conditions. Another analysis of the coupled interaction between bi-leaflets and pulsatile flows invesigated on possible prosthetic heart valve designs [22]. The vortex formation analysis in a reduced model (2D) sometimes shows a better approximation for such valves as these undergo shape tapering. In certain conditions because of available constraints which include gerometrical restrictions such as flows in thin soap films and stratified rotating ocean currents which generate vortex dipoles.

1.4 Problem Definition

The current work is a reduced two dimensional model flow emanating through a flexible flap pair mounted on opposite channel wall. However, in the present study, we intend to vary deformability and inlet pulsation so as to achieve better understanding of these systems. We present a reduced model as two dimensional channel of height, h, consisting elastic plates which block 85 percent of the channel cross-section, at a distance $2h$ from the inlet. The elastic plates conform transiently with the incoming flow which in our case is pulsatile in nature. Thus the opening caused by the two flexible plates also pulsates in magnitude. We hereby enquire the effect of the flow inlet conditions past the flexible plates and the flow ejection in the form of vortical structures in the downstream of the plate which extends upto $8h$ length beyond the flexible plates and is supported by sponge layer boundary conditon to avoid possible numerical reflections whatsoever.

The vortical ejections of an incompressible flow in a confined channel through a wall mounted flexible flaps, is investigated. A schematic of the model as two dimensional channel geometry of height h and length $14h$ as shown in 3. Two flexible plates, each of length $l = 0.425h$ are fixed at a distance $2h$ from the channel inlet. The thickness of each plate is $b = 0.005h$ and width $w = 0.125h$ (into the plane). A pulsatile flow enters from the left (in the figure) with a mean value of $Re = 500$, based on channel height h and mean free-stream speed u_{mean}. This time dependent inlet Reynolds Number, $\overline{Re}(t)$, has uniform cross

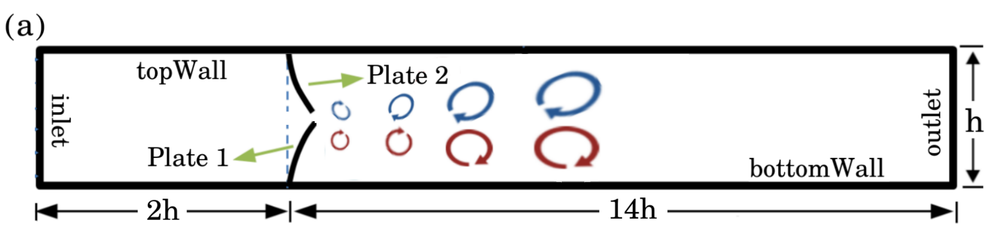

Figure 3: Schematic of the problem definition. Pulsatile flow inlet from the left which is objected by the two oppositely mounted plates.

sectional velocity and does not vary in wall-normal direction. The oscillating flow pulses with a sinusoidal funtional shape. The waveform applied in the pulsatile flow is:

$$\overline{Re}(t) = \overline{Re}_{mean}\{1 + 0.5sin(St_f t)\} \tag{1}$$

1.5 Mathematical formulation

We have considered a two-way coupling between fluid and structure (or plate). The governing equations for fluid and structure regions are formulated by using Arbitrary Lagrange Euler (ALE) framework, in which the change in volume of each control element between two adjacent time steps is always equal to the volume swept by the mesh cell boundary, see more details in [23, 24, 25]. The flow is considered incompressible and Newtonian in nature. The mass and momentum equations are given by,

$$\begin{aligned} \frac{\partial \rho_f}{\partial t} + \nabla \cdot [\rho_f(\mathbf{v}_f - \mathbf{v}^m)] &= 0 && (2) \\ \frac{\partial \mathbf{v}^f}{\partial t} + \left[(\mathbf{v}^f - \mathbf{v}^m) \cdot \nabla\right] \mathbf{v}_f &= -\frac{1}{\rho_f}\nabla p + \nu \nabla^2 \mathbf{v}_f, && (3) \end{aligned}$$

where ρ_f is the fluid density, $\mathbf{v}^f$ is the flow velocity, and p is the pressure. The operator $\frac{\partial}{\partial t}$ is the partial derivative with time, and ∇ is the spatial gradient. The elastic plate's motion (or the structural displacement, $\mathbf{u}$), is computed on an arbitrary Lagrangian frame on which the mesh velocity ($\mathbf{v}^m$) equals the material velocity, i.e., $\mathbf{v}^m \equiv \frac{\partial \mathbf{u}}{\partial t}$. The mass and momentum conservation equations for the plate are given by,

$$\begin{aligned} \frac{\partial \rho_s}{\partial t} &= 0, && (4) \\ \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} &= \nabla \cdot \boldsymbol{\sigma}_s + \rho \mathbf{b}, && (5) \end{aligned}$$

where $\boldsymbol{\sigma}_s$ is the Cauchy stress tensor and $\mathbf{b}$ is external force (fluid) loading on the structure. In our simulations, a linear elastic material is used for which the Cauchy stress tensor ($\boldsymbol{\sigma}_s$) and the Lagrangian Green's tensor ($\boldsymbol{\varepsilon}_s$) are connected as: $\boldsymbol{\sigma}_s = 2\mu\boldsymbol{\varepsilon}_s + \lambda \nabla \cdot \mathbf{u}\mathcal{I}$. Here, $\mu = E/2(1+\upsilon)$ and $\lambda = \upsilon E/[(1+\upsilon)(1-2\upsilon)]$ are Lame's constants; υ is the Poisson's ratio; E is the Youngs modulus; $\mathcal{I}$ is the identity tensor of rank two, and $\boldsymbol{\varepsilon} = \frac{1}{2}[(\nabla\mathbf{u} + \nabla\mathbf{u}^T) + (\nabla\mathbf{u} \cdot \nabla\mathbf{u}^T)]$. The superscript ^{T} is the transpose of a given tensor.

1.6 Computational Method

We have used a partitioned approach based solver to resolve fluid-structure interaction problem. This approach shall involve strong coupling between fluid and the structure. We shall employ a second-order based finite volume discretization is used for spatial terms to solve the fluid equations. The temporal terms are discretized using Euler implicit technique. In addition to these, the following inlet-outlet flow boundary conditions are used. At the inlet, a uniform velocity V_∞ and a zero pressure gradient are applied. Whereas, at the outlet, zero gradient in the velocity and a fixed (zero) pressure is maintained.

1.7 Research Objectives

The incoming flow, at different pulsatile frequency, bends the plates along the inlet flow direction i.e., downstream. We can see the development of vortical structures in the flow due to the presence of plates and also because of the inlet frequency in figure 4. The evolution of the vortex structures follow intriguing pattern which needs to be thoroughly studied and understood. So, we enlist some of the objectives of the research as:

- to study the structural kinematics.
- to analyse of different modes of the structure oscillations.
- to find frequency modes of the structures.
- to inspect the vortical ejection.
- to discover the frequency of the vortical ejections.

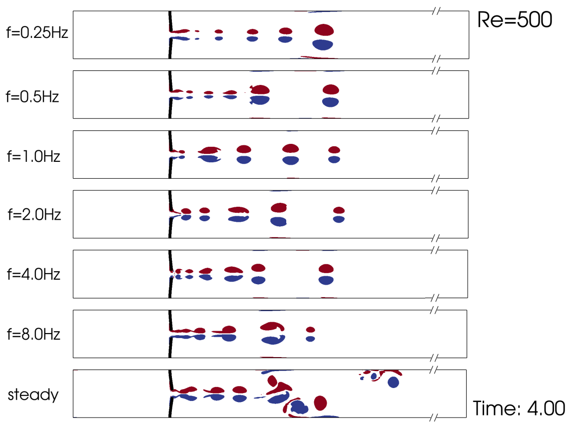

Figure 4: Vortical ejections at different inlet flow pulsation. These vortex structures accommodate the efficiency of mixing in the flow.

- to quantify of the special leap-frogging of vortex.
- to evaluate flow circulation and flow impulse in the downstream.
- to inspect of the flow energy dissipation.
- to discover any other emanating flow features.
- to estimate the efficiency of the flow mixing because of the presence of the flexible plates.

1.8 Significance of research

Mixing is at the heart of most production systems in the chemical industry, pharmaceutical industry, food industry, and allied industries. It is of vital importance in processing solids, liquids, and gases in the polymer, glass, ceramics, building materials, pulp and paper, petroleum, and power industries and industrial waste treatment systems. The wide variety and ever-increasing complexity of mixing processes encountered in industrial applications require careful selection, design, and scale-up to ensure effective and efficient mixing. Improved mixing efficiency leads to shorter batch cycle times and operational costs. Today's competitive production systems necessitate robust equipment that is capable of faster blend times, lower power consumption, and adaptability of equipment for use with multiple products. A mixer is no longer a generic production tool, but a critical and decisive business tool. This is because profitability and competitive advantage are dependent on subtle improvements in product quality through gains in mixing performance and efficiency. A recently published handbook on industrial mixing estimates the cost of poor mixing to be as high as INR 7.5 billion per year [26]. So, good mixing is imperative for minimizing investment and operating costs providing high yields, and thereby enhancing profitability.

2 Future works

2.1 Fluid structure interaction designs in channel flows.

In the immersive field of fluid structure interaction, there could be a diffraction of applications and intricate physics to be discovered. As in the literature review, an extensive study on some possible enigineering designs for the purposes of achieving high mixing levels, is objected. We

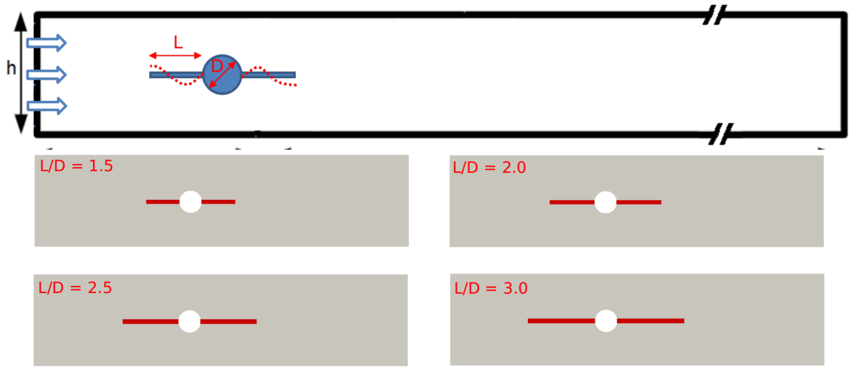

Figure 5: Schematic configuration of flexible structures in a uniform channel flow. The proposed configuration consist of splitter plate both in front and back of a cylinder.

plan to enhance the mixing levels in the channels to multiple order of magnitudes. Figure 5 shows a schematic configuration of flexible structures in a uniform channel flow. The proposed configuration consist of splitter plate both in front and back of a cylinder. We also intend to investigate:

1. impact of structure's flexural rigidity on a flow mixing phenomena.
2. comparative scaling for such interactive flow for different channel cross section such as circular or polygonal.
3. different spatial configurations of the obstructive plates.
4. some other drag reduction models or methods.

As of now, we are investigating reduced order models for the aforementioned cases. With an elementary understanding of such models, we shall introduce another spatial dimension to study the three dimensionality and its implications. The gradual upsacling of the basic model shall give us more realistic results. Simultaneously, we shall define our flow as under a higher Reynolds number to predict its effect.

2.2 Fluid structure interaction and multiphase flows

After a healthy and firm grip on the above standard cases, we shall try to compare the cases by introducing another phase into the picture. Interaction of multiple kinds of flows with flexible structures shall exhibit the mixing of flows in much more realistic sense. One such proposed designed configuration of the fluid structure interaction in a channel flow with two different fluids is shown in figure 6. This design is inspired out of already present flow

Figure 6: Schematic configuration for using flexible structure in a two-phase channel flow for efficient mixing.

mixing designs but with flexible structures. Presence of more than one phase shall introduce multiple sources for the breaking of symmetric phenomenon, if any, and shall contribute towards the physical understanding of the problems in the industrial applications such as food industries, chemical industries, etc.

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