

# Constraint Satisfaction

# Constraint Satisfaction

- Constraint satisfaction problems solve some problem state using set of constraints instead of finding optimal path to the solution.

Example:

1. Cryptography Problem : A number puzzle in which a group of arithmetical operations has some or all of its digits replaced by letters and the original digits must be found.
2. N-Queen problem : The condition is that no two queens are in same row, column or diagonal.
3. Map coloring problem : Given a map color the nodes such that no two neighbors have same colors.

# Constraint Satisfaction

- Definition:

- A set of variables  $\{x_1, x_2, x_3, \dots, x_n\}$ , with each  $x_i \in D_i$  with the values .
- A set of constraints i.e relations that are assumed to hold between the values of the variables.
- The problem is to find, for each  $i$ ,  $1 \leq i \leq n$ , a value of  $x_i \in D_i$ , so that all constraints are satisfied.
- A CS problem is represented as a graph called **constraint graph** which is undivided.
- Nodes are variables and edges are binary constraints.

# Constraint Graph

- Constraint Graph
  - ✓ Start state: The un assignment variables.
  - ✓ Goal state: all variables are assigned values satisfying constraints.
  - ✓ Operator: assigns value to unassigned variable provided that it does not conflict with previously assigned variables.
- Every solution must be a complete assignment, it appears at depth  $n$  if there are  $n$  Variables.
- Search tree extends to depth  $n$ , so DF Search is implemented.

# ALGORITHM

- Until a complete solution is found or all paths have lead to dead ends  
{
  - Select an unexpanded node of the search graph;
  - Apply the constraint inference rules to the selected node to generate All possible new constraints;
  - If the set of constraints contain a contradiction, then report that this path is a dead end;
  - If the set of constraints describes a complete solution, then report success;
  - If neither a contradiction nor a complete solution has been found, then apply the problem space rules to generate new partial solutions that are consistent with the current set of constraints. Insert these partial solutions into the search graph;}
- Stop

# Crypt-Arithmetic Puzzle

- Solve the given puzzle by assigning a numeral (0-9) in such a way that each letter is assigned unique digit which satisfy the given addition.

$$\begin{array}{r} \text{B A S E} \\ + \text{B A L L} \\ \hline \text{G A M E S} \\ \hline \end{array}$$

## Conditions /Requirements

1. Digit ranges from 0 to 9 only.
2. Each variable should have unique and distinct value.
3. Each letter represents only one digit throughout the problem.
4. You have to find the value of each letter in the crypt arithmetic.
5. There must be only one solution to the problem.
6. The numerical base unless specified is 10.
7. Numbers should have not begin with zero.
8. After replacing letters by their digits, the resulting arithmetic operations must be correct.

# Solution

- Constraints : No two letters have the same value
- Initial Problem state :  $G=?$ ,  $A=?$ ,  $M=?$ ,  $E=?$ ,  $S=?$ ,  $B=?$ .  $L=?$
- Apply constraint inference rules to perform all assignments required by the current set of constraints. Then choose another rule to generate an additional assignment, which will, in turn generate new constraints at the next cycle.
- At each cycle, there may be several choices of rules to apply.
- A useful heuristics can help to select the best rule to apply first

C4	C3	C2	C1	
	B	A	S	E
+	B	A	L	L
G	A	M	E	S

- Constraints equations are:

$$E + L = S \quad \rightarrow C_1$$

$$S + L + C_1 = E \quad \rightarrow C_2$$

$$2A + C_2 = M \quad \rightarrow C_3$$

$$2B + C_3 = A \quad \rightarrow C_4$$

$$G = C_4$$

# Exercise

Problem 1 :

C4	C3	C2	C1	
	B	A	S	E
+	B	A	L	L
G	A	M	E	S

## Crypt-Arithmetic Solution Trace

Constraints Equations	Initial State
$G = C_4$	$G = ?; A = ?; M = ?; E = ?;$
$2B + C_3 = A \rightarrow C_4$	$S = ?; B = ?; L = ?$
$2A + C_2 = M \rightarrow C_3$	
$S + L + C_1 = E \rightarrow C_2$	
$E + L = S \rightarrow C_1$	

- We can easily see that G has to be non-zero digit, so that the value of Carry  $C_4$  should be 1 and hence  $G=1$ .

$$1. \quad G = C_4 \Rightarrow \boxed{G = 1}$$

$$2. \quad 2B + C_3 = A \longrightarrow C_4$$

2.1 Since  $C_4 = 1$ , therefore,  $2B + C_3 > 9 \Rightarrow B$  can take values from 5 to 9.

2.2 Try the following steps for each value of  $B$  from 5 to 9 till we get a possible value of  $B$ .

- If  $B = 5$ 
  - if  $C_3 = 0 \Rightarrow A = 0 \Rightarrow M = 0$  for  $C_2 = 0$  or  $M = 1$  for  $C_2 = 1 \times$
  - if  $C_3 = 1 \Rightarrow A = 1 \times$  (as  $G = 1$  already)
- For  $B = 6$  we get similar contradiction while generating the search tree.
- If  $\boxed{B = 7}$ , then for  $C_3 = 0$ , we get  $\boxed{A = 4} \Rightarrow M = 8$  if  $C_2 = 0$  that leads to contradiction later, so this path is pruned. If  $C_2 = 1$ , then  $\boxed{M = 9}$

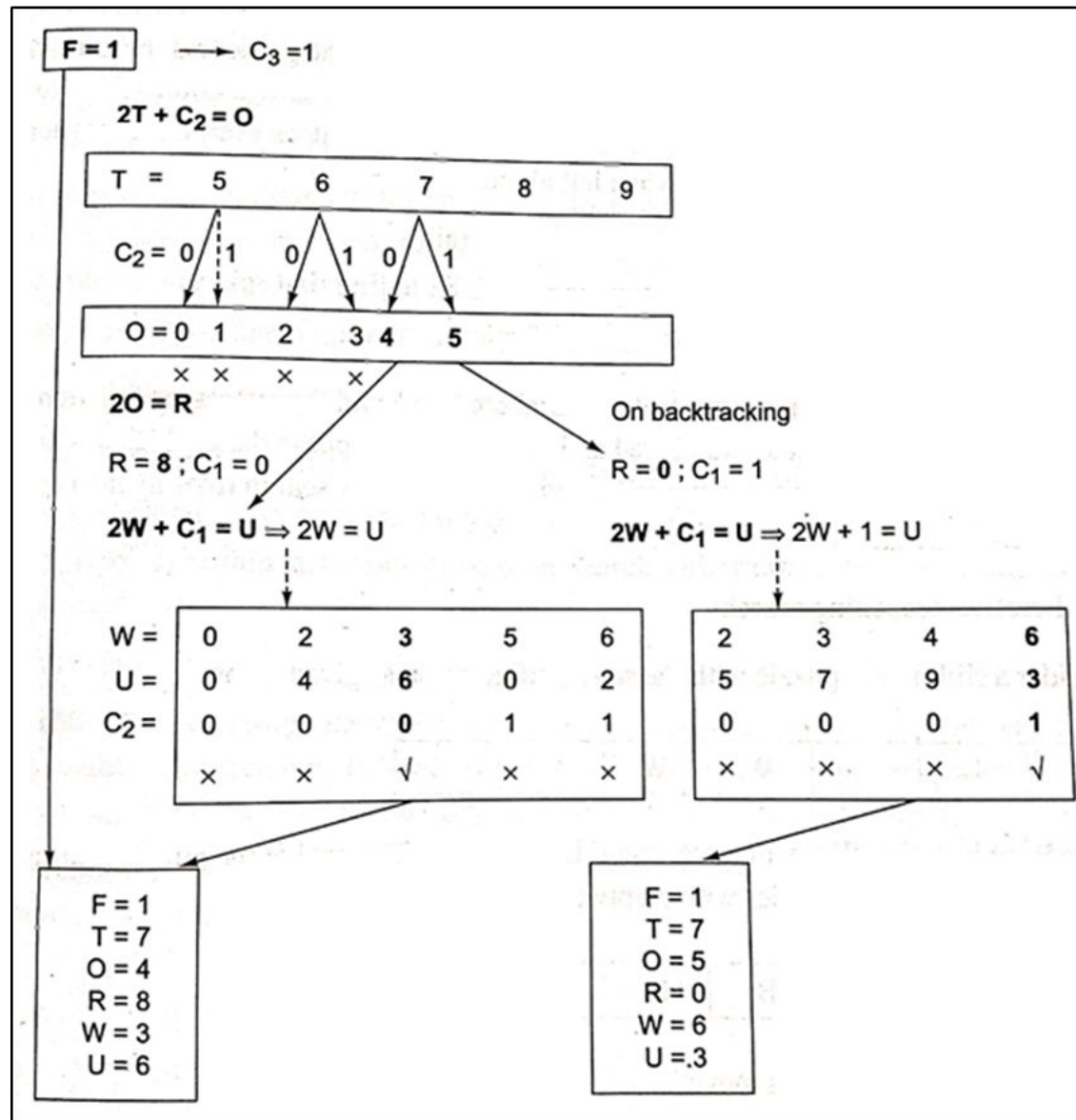
$$3. \quad \text{Let us solve } S + L + C_1 = E \text{ and } E + L = S$$

- Using both equations, we get  $2L + C_1 = 0 \Rightarrow \boxed{L = 5}$  and  $C_1 = 0$
- Using  $L = 5$ , we get  $S + 5 = E$  that should generate carry  $C_2 = 1$  as shown above
- So  $S + 5 > 9 \Rightarrow$  Possible values for  $E$  are  $\{2, 3, 6, 8\}$  (with carry bit  $C_2 = 1$ )
- If  $E = 2$  then  $S + 5 = 12 \Rightarrow S = 7$  (as  $B = 7$  already)
- If  $E = 3$  then  $S + 5 = 13 \Rightarrow S = 8$ .
- Therefore  $\boxed{E = 3}$  and  $\boxed{S = 8}$  are fixed up

4. Hence we get the final solution as given below and on backtracking, we may find more solutions. In this case we get only one solution.

$$\boxed{G = 1 ; A = 4 ; M = 9 ; E = 3 ; S = 8 ; B = 7 ; L = 5}$$

## The search tree using DFS



Search Tree for Crypt – Arithmetic Puzzle

# Exercise

Problem 2 :

C3	C2	C1	
	T	W	O
+	T	W	O
F	O	U	R

Possible  
Solutions :

F	T	O	R	W	U
1	8	6	2	3	7
1	8	6	2	4	9
1	8	7	4	6	3
1	9	8	6	2	5

