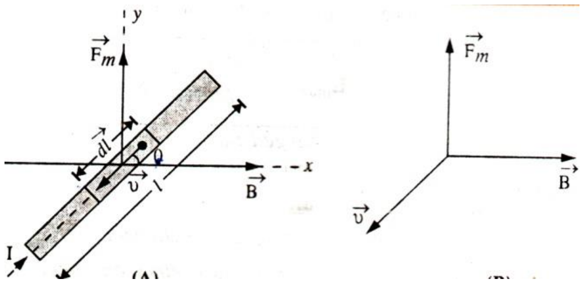
MAGNETIC FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN A MAGNETIC FIELD: -

A conductor contains large number of free electrons. These electrons move with drift velocity $\vec{v}$ in a direction opposite to the direction of conventional current flowing in the conductor. These electrons moving in a uniform magnetic field experience a deflecting force which is transmitted to the conductor.

Consider a conductor of length l carrying a current (I) placed in a uniform magnetic field $\vec{B}$.



Let n = Number of free electrons per unit volume of the conductor.

A = Area of cross-section of the conductor.

Magnetic Lorentz force acting on an electron

$$\vec{F}_m = -e\left(\vec{v} \times \vec{B}\right) \qquad \text{... (1)}$$

The direction of force $\vec{F}_m$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

Now consider a small element of length dl.

Number of electrons in the small element = n × volume of the element = nAdl

Magnetic Lorentz force experienced by the element,

$$d\vec{F}_m = nAdl\left[-e\left(\vec{v} \times \vec{B}\right)\right] = -nAedl\left(\vec{v} \times \vec{B}\right) \qquad \text{... (2)}$$

Drift velocity, $\vec{v} = -\dfrac{d\vec{l}}{dt}$ $\qquad$ ($\because$ $d\vec{l}$ is in a direction opposite to $\vec{v}$)

$$\therefore d\vec{F}_m = (nAdl)e\left[\frac{d\vec{l}}{dt} \times \vec{B}\right] = \frac{(nAdl)e}{dt}\left(d\vec{l} \times \vec{B}\right) \qquad \text{... (3)}$$

Now, $\dfrac{(nAdl)e}{dl} = \dfrac{dq}{dt} = I$ $\qquad$ (dq is the charge on the element)

$$\therefore d\vec{F}_m = I\left(d\vec{l} \times \vec{B}\right)$$

Since conductor is made of large number of such elements, therefore, total force experienced by the conductor is

$$\vec{F}_m = \int d\vec{F}_m = \int I\left(d\vec{l} \times \vec{B}\right)$$

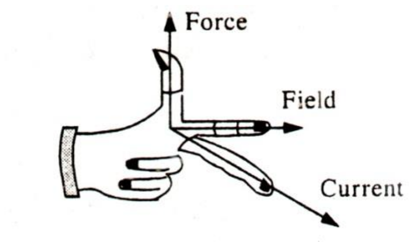
or $\qquad \vec{F}_m = I\left(\vec{l} \times \vec{B}\right) \qquad \text{... (4)}$

Magnitude of F_m is $BIl\sin\theta$

where, θ is the angle between $\vec{l}$ and $\vec{B}$.

Direction of $\vec{F}_m$ is perpendicular to the plane containing $\vec{l}$ and $\vec{B}$ and can be determined using Fleming's left hand rule.

Fleming's Left Hand Rule: - Stretch the left hand such that the fore-finger, the central finger and the thumb are mutually perpendicular to each other. When fore-finger points in the direction of magnetic field and the central-finger points in the direction of current, then the thumb gives the direction of the force acting on the conductor.



Note: -

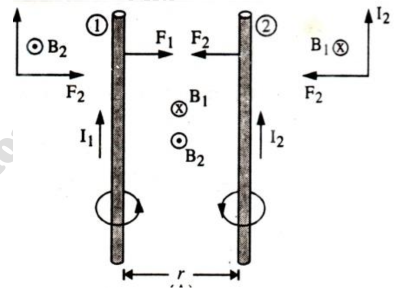
1. If $\theta = 0^o$ or 180^o, $F_m = 0$. It means current carrying conductor experiences no force when placed parallel or anti-parallel to the direction of the magnetic field.
2. If $\theta = 90^o$, $F_m = BIl$. It means, a current carrying conductor placed at right angle to uniform magnetic field experiences a maximum force.
3. If the conductor is not straight then it can be treated as combination of many small straight lengths. dl giving magnetic force.

$$F_m = \sum I(d\vec{l} \times \vec{B})$$

FORCES BETWEEN TWO INFINITELY LONG STRAIGHT PARALLEL CONDUCTORS CARRYING CURRENTS: -

Consider two infinitely long parallel conductors carrying currents I_1 and I_2 in the same direction.

Let r be the distance of separation between these two conductors.



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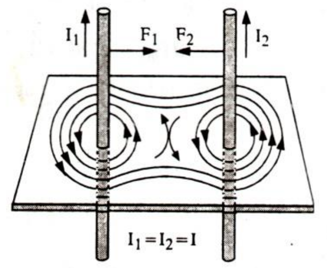


B

The current I_1 in the conductor (1) produces a magnetic field around it as shown in figure B.

The magnetic field at any point on the conductor (2) due to current I_1 in conductor (1) is given by

$$B_1 = \frac{\mu_o}{4\pi}\left(\frac{2I_1}{r}\right) \quad \text{... (1)}$$

The direction of B_1 with reference to conductor (2) is perpendicular to the plane of the conductor and is directed vertically downward (i.e. into the plane).

We know, a current carrying conductor placed at right angle to the magnetic field experiences a force,

$$F = BIl$$

Therefore, force experienced per unit length of conductor (2) in the magnetic field B_1 is given by

$$F_2 = B_1 I_2 \times 1 = B_1 I_2$$

Using equation (1), $F_2 = \frac{\mu_o}{4\pi}\left(\frac{2I_1}{r}\right)I_2 = \frac{\mu_o}{4\pi}\left(\frac{2I_1 I_2}{r}\right)$ ... (2)

Applying Fleming's left hand rule to conductor (2), the direction of F_2 is in the plane of the conductors directed towards conductor (1) (Refer Right hand side of figure A).

Similarly, the force experienced per unit length of conductor (1) in magnetic field (B_2) due to current carrying conductor (2) is given by

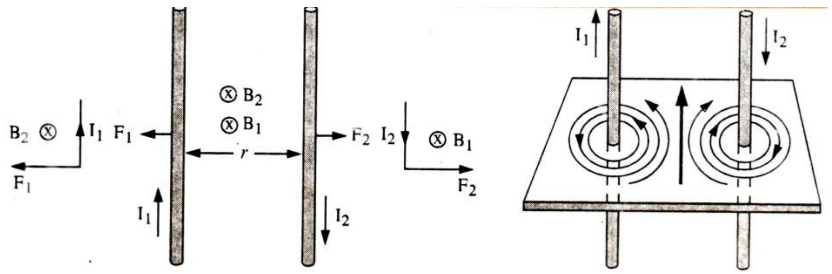
$$F_1 = \frac{\mu_o}{4\pi}\left(\frac{2I_1 I_2}{r}\right) \quad \text{... (3)}$$

Applying Fleming's left hand rule to conductor (1), the direction of F_1 lies in the plane of the conductors and is directed towards conductor (2) (Refer left hand side of figure A).

Since F_1 and F_2 are equal and opposite, so these forces pull the conductors towards each other.

Hence, we conclude that two long parallel conductors carrying currents in the same direction attracts each other.

Similarly it can be easily shown that two long parallel conductors carrying current in opposite direction repel each other.



Definition of ampere from force experienced by current carrying parallel conductors.

Force experienced per metre length of conductor by parallel infinitely long straight conductors carrying currents I_1 and I_2 is given by

$$F = \frac{\mu_O}{4\pi}\left(\frac{2I_1 I_2}{r}\right) Nm^{-1}$$

Where, r is distance of separation between two conductors

If $I_1 = I_2 = 1 ampere$ and r = 1m, then

$$F = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 1 \times 1}{1} = 2 \times 10^{-7}\, Nm^{-1}$$

Thus, ampere is that current which if maintained if two infinitely long parallel conductors of negligible cross-sectional area separated by 1 metre in vacuum causes a force of 2×10^{-7} on each metre of the other wire.

Note: -

1. If current in both parallel wires is equal and in same direction then magnetic field at a point is exactly half way between the wires is

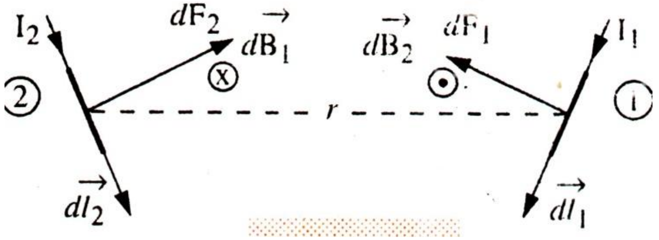
$$B_P = B_1 - B_2 = \frac{\mu_o 2I}{4\pi r} - \left(\frac{\mu_o 2I}{4\pi r}\right) = 0$$

But if the wire carry currents in opposite direction then

$B_P = B_1 + B_2 = 2\left[\frac{\mu_O 2I}{4\pi r}\right]$ because the fields will add up as shown in figure.

Force between Current Carrying Short Wires not parallel to each other: -

Consider two current elements 1 and 2 with length $d\vec{l}_1$, $d\vec{l}_2$ and currents I_1, I_2 respectively.



Let $d\vec{B}_1$ be the field produced due to element 1 directing towards the plane of the paper. Force experienced by element 2 is,

$$d\vec{F}_2 = I_2\left(d\vec{l}_2 \times d\vec{B}_1\right)$$

But $\quad d\vec{B}_1 = \frac{\mu_o}{4\pi}\frac{I_1\left(d\vec{l}_1 \times \vec{r}_{12}\right)}{r_{12}^3}$ (Biot-Savart's law)

$$\therefore d\vec{F}_2 = I_2\left(d\vec{l}_2 \times \frac{\mu_o}{4\pi}\frac{I_1\left(d\vec{l}_1 \times \vec{r}_{12}\right)}{r_{12}^3}\right) = \frac{\mu_o}{4\pi}\frac{I_1 I_2}{r_{12}} d\vec{l}_2 \times \left(d\vec{l}_1 \times \vec{r}_{12}\right)$$

Similarly force experienced by the element 1 is

$$d\vec{F}_1 = \frac{\mu_o}{4\pi}\frac{I_1 I_2}{r_{12}^3} d\vec{l}_1 \times \left(d\vec{l}_2 \times \vec{r}_{12}\right)$$

Using triple-product of vectors i.e. $\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \left(\vec{C}.\vec{A}\right)\vec{B} - \left(\vec{B}.\vec{A}\right)\vec{C}$ we get,

$$d\vec{l}_1 \times \left(d\vec{l}_2 \times \vec{r}_{12}\right) = \left(\vec{r}_{12}.d\vec{l}_1\right)d\vec{l}_2 - \left(d\vec{l}_2.d\vec{l}_1\right)\vec{r}_{12} = \left(r_{12} dl_1 \cos\theta_1 d\vec{l}_2 - dl_1 dl_2 \cos\theta_2 \vec{r}_{21}\right)$$

<u>**Special Case**</u>

If $\theta_2 = 0^o$ and $\theta_1 = 90^o$ (i.e. dl_1 and dl_2 are parallel with r as perpendicular distance between them) then

$$d\vec{l}_1 \times \left(d\vec{l}_2 \times \vec{r}_{21}\right) = \vec{r}_{21} dl_1 d\vec{l}_2 \cos 90^o - dl_1 dl_2 \vec{r}_{21} \cos 0^o = -dl_1 dl_2 \vec{r}_{21}$$

$$\therefore d\vec{F}_1 = -\frac{\mu_o}{4\pi} I_1 I_2 \frac{dl_1 dl_2}{r_{21}^3}\vec{r}_{21} \qquad \text{... (1)}$$

Similarly $\quad d\vec{F}_2 = -\frac{\mu_O}{4\pi} I_1 I_2 \frac{dl_1 dl_2}{r_{12}^3}\vec{r}_{12}$

But $\quad \vec{r}_{12} = -\vec{r}_{21}$

$$\therefore d\vec{F}_2 = \frac{\mu_O}{4\pi} I_1 I_2 \frac{dl_1 dl_2}{r_{21}^3}\vec{r}_{21} \qquad \text{... (2)}$$

From eqns. (1) and (2), we have

$$d\vec{F}_2 = -d\vec{F}_1$$

Magnitude of these forces $\quad F = \frac{\mu_O}{4\pi}\frac{I_1 I_2 dl_1 dl_2}{r^2}$

Clearly two parallel current elements are exert equal and attractive forces on each other when they carry currents in the same direction.

Ratio of Electrical and Magnetic Forces: -

Electrostatic force between two stationary charge q_1 and q_2 separated by a distance r in vacuum is given by Coulomb's law

i.e. $$F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \qquad \text{... (1)}$$

The magnetic force between two parallel current elements I_1dl_1 and $I_2\,dl_2$ separated by a distance r,

$$F_m = \frac{\mu_o}{4\pi}\left(\frac{I_1 I_2}{r^2}\right)dl_1 dl_2 \qquad \text{... (2)}$$

But $$I_1 dl_1 = \frac{q_1}{t}.dl_1 = q_1\left(\frac{dl_1}{t}\right) = q_1 v_1$$

Similarly, $I_2 dl_2 = q_2 v_2$

Here, v_1 and v_2 are the drift velocities of electrons in two current elements respectively.

Hence eqn. (ii) becomes $F_m = \frac{\mu_o}{4\pi}\frac{q_1 q_2}{r^2}v_1 v_2$ ... (3)

Dividing (1) by (3), we get $\frac{F_e}{F_m} = \frac{1}{(\mu_o\varepsilon_o)v_1 v_2}$ ... (4)

Now $c = \frac{1}{\sqrt{\mu_o\varepsilon_o}}$ or $\mu_o\varepsilon_o = \frac{1}{c^2}$

where c is the speed of light in vacuum = 3×10^8 ms^{-1}

$$\therefore \frac{F_e}{F_m} = \frac{c^2}{v_1 v_2} \qquad \text{... (5)}$$

Drift velocity of electron is of the order of 10^{-5} ms^{-1}.

i.e. $v_1 = v_2 = 10^{-5}$ ms^{-1}

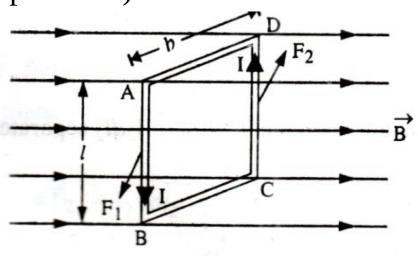
$$\therefore \frac{F_e}{F_m} = \frac{(3\times10^8)^2}{(10^{-5})^2}$$

or $$\frac{F_e}{F_m} \approx 10^{27}$$

Thus, electric force is much larger than the magnetic force.

Torque on Current Loop in a Magnetic Field: -

Consider a rectangular conducting loop (ABCD) of length l and breadth b placed in a uniform magnetic field $\vec{B}$. Let I be the current flowing in the loop in anticlockwise direction. Let θ be the angle between the normal (n) of the plane of the loop and magnetic field $\vec{B}$.(Refer figure B which shows magnified top view of arm AD of the loop ABCD).



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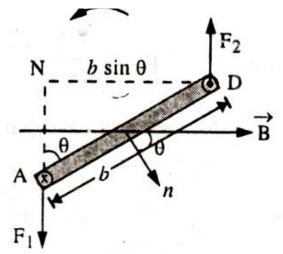


B

We know, force acting on a conductor of length l carrying current I in the magnetic field is given by

$$\vec{F} = I\left(\vec{l}\times\vec{B}\right)$$

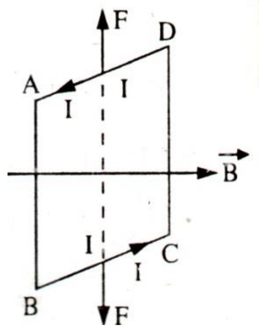
$\therefore$ Force acting on the arm AB of the loop,

$$\vec{F}_1 = I\left(\vec{l}\times\vec{B}\right) \qquad \text{... (1)}$$

Direction of $\vec{F}_1$ is perpendicular to the length of arm AB and directed outward of the sheet of paper (Fleming's left hand rule).
Similarly, force acting on the arm CD of the loop,

$$\vec{F}_2 = I\left(\vec{l} \times \vec{B}\right) \qquad \text{... (2)}$$

$\vec{F}_2$ is perpendicular to the length of the arm CD and is directed inside the sheet of the paper (Fleming's left hand rule).
Force acting on the arm BC and force acting on the arm DA of the loop are equal, opposite and act along the same line, hence they cancel each other.



$\vec{F}_1$ and $\vec{F}_2$ form a couple and try to rotate the loop anticlockwise.
The magnitude of the torque (τ) due to forces $\vec{F}_1$ and $\left(\vec{F}_2\right)$ is given by
τ = Magnitude of the either force × lever arm = F_1 × DN

$$= \left|I\left(\vec{l} \times \vec{B}\right)\right| \times DN$$

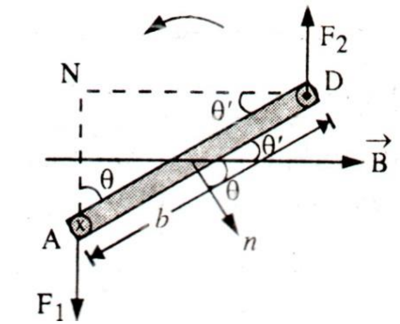
$$= I\left(lb \sin 90^o\right) \times b \sin \theta$$

or $\quad \tau = I(lb)B \sin \theta$
Since $\quad (lb) = A$, area of the loop

$$\therefore \tau = IAB \sin \theta \qquad \text{... (3)}$$

If the loop has N turns, then net torque acting on the loop is
$\tau_N = N_\tau = NIAB \sin \theta$

Another consideration



If the plane of loop makes an angle θ' with the magnetic field $\vec{B}$ then $\theta + \theta' = 90^o$ or $\theta = \left(90^o - \theta'\right)$
Hence eqn. (3) becomes

$$\tau_N = NIAB \sin\left(90^o - \theta'\right)$$

or

$$\tau_N = NIAB \cos \theta'$$

**A current carrying loop behaves as a magnetic dipole having dipole moment (M). When it is placed in a uniform magnetic field, it experiences a torque given by

$$\tau_N = NIAB \sin \theta$$

But M = NIA = magnetic dipole moment of a current carrying loop having N turns.

$$\therefore \tau_N = MB \sin \theta$$

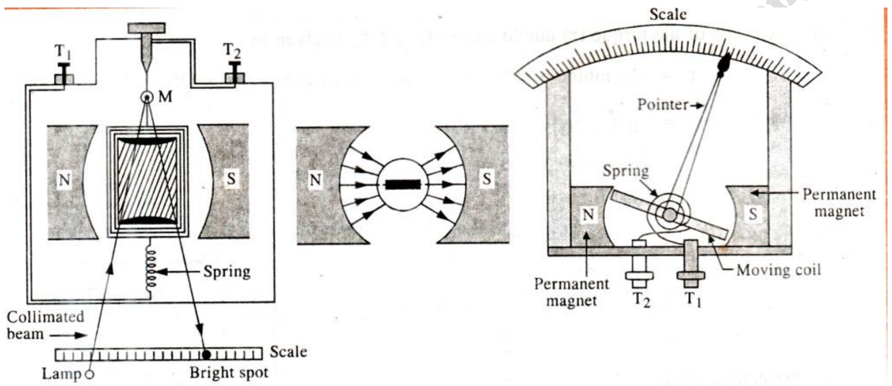
MOVING COIL GALVANOMETER

Moving Coil Galvanometer *is a device used to detect/measure small electric current flowing in the electric circuit.*

Principle: Moving coil galvanometer is based on the fact that when a current carrying loop or coil is placed in the uniform magnetic field, it experiences a torque.

Construction: It consists of a coil wound on a non-metallic frame. The coil is suspended by between two poles of a permanent magnet which is cylindrical in shape. The coil is suspended by a phosphor-bronze strip which acts as path for the current to the coil. The strip is finally connected to the terminal T_2 of the galvanometer. The other end of the coil is connected to a light spring which is finally connected to the terminal T_1. The spring exerts a small restoring couple on the coil.

A piece of soft iron is placed within the frame of the coil. A plane circular mirror M is attached to the suspension to note the deflection of the coil using lamp and scale arrangement.



Theory: Let B = Intensity of magnetic field

I = Current flowing through the coil

l = length of the coil

b = Breadth of the coil

$(l \times b)$ = A = Area of the coil

N = Number of turns in the coil

When current flows through the coil, it experiences a torque, which is given by

$$\tau = NIB\sin\theta$$

where, θ is the angle made by the normal to the plane of the coil with the direction of the magnetic field. If this angle is 90° then $\sin\theta = \sin 90^\circ = 1$. [It is possible when cylindrical poles of permanent magnet are used which produce radial magnetic field shown in figure].

Then $$\tau_N = NIAB \qquad \text{... (1)}$$

The torque is known as the **deflecting torque**.

As the coil gets deflected, the suspension wire is twisted and a restoring torque is developed in it. If k is the restoring torque per unit twist of the suspension wire, then the restoring torque for the deflection α is given by

$$\tau'_N = k\alpha \qquad \text{... (2)}$$

For equilibrium of the coil, Deflecting torque = Restoring torque

i.e. $$NIAB = k\alpha \qquad \text{... (3)}$$

or $$I = \left(\frac{k\alpha}{NAB}\right)$$

or $$I = G\alpha \qquad \text{... (4)}$$

where $G = \dfrac{k}{NAB}$ and is called galvanometer constant.

or $I \propto \alpha$ ... (5)

Thus, deflection of the coil is directly proportional to the current flowing through it. Hence we can use a linear scale in the galvanometer to detect the current in the circuit.

Sensitivity of a galvanometer:

A galvanometer is said to be sensitive if a small current flowing through the coil of galvanometer produces a large deflection in it.

i. **Current Sensitivity:** The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

i.e. Current sensitivity = $\dfrac{\alpha}{I} = \dfrac{\alpha(NAB)}{k\alpha} = \dfrac{NAB}{k}$ $\left(\because I = \dfrac{K\alpha}{NAB}\right)$

Current sensitivity of a galvanometer can be increased either by

a) Increasing the magnetic field B by using a strong permanent horse-shoe shaped magnet.
b) Increasing the number of turns N. (But, number of turns of the coil cannot be increased beyond a certain limit. This is because the resistance of the galvanometer will increase subsequently and hence the galvanometer becomes less sensitive).
c) Increasing the area of the coil A. (But this will make the galvanometer bulky and ultimately less sensitive).
d) decreasing the value of restoring force constant k by using a flat strip of phosphor-bronze instead of a circular wore phosphor-bronze because the value of k is small in case of a flat strip than a round phosphor bronze.

Quartz fibres can also be used for suspension of the coil because they have large tensile strength and very low value of k.

ii. **Voltage Sensitivity:**

Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

i.e. Voltage sensitivity = $\dfrac{\alpha}{V} = \dfrac{\alpha}{IR} = \dfrac{NBA}{kR}$

(R is the resistance of the coil)

Voltage sensitivity can be increased by

(a) increasing N (b) increasing B (c) increasing (A) (d) decreasing k and (e) decreasing R.

Advantages of a moving coil galvanometer:

i. It can be made extremely sensitive, so even a minute current in the electric circuit can be detected.
ii. Since deflection of the coil of the galvanometer is directly proportional to the current, so linear scale can be used.
iii. As the magnetic field (B) is very strong, so the external magnetic fields (say horizontal component of earth's magnetic field) cannot change the deflection of the coil

of the galvanometer. Thus, the galvanometer can be set in any position.

iv. It is a dead beat type galvanometer. (Dead beat type galvanometer comes to the rest position quickly after disturbance, hence the name of this type of galvanometer is given as dead beat galvanometer). When the coil rotates in the magnetic field, eddy currents are produced in the metallic frame on which the coil is wound. These eddy currents produce a dampening effect and hence the coil comes to the position of rest quickly.

Ammeter:

An ammeter is an instrument used to measure electric current in an electric circuit.

Conversion of Galvanometer into Ammeter with the help of Shunt:

A galvanometer can be converted into an ammeter by connecting a low resistance called Shunt parallel to the galvanometer.

Let G and S be the resistance of a galvanometer and shunt respectively. Let I be the total current to be measured by an ammeter in the circuit.

Let I_g be the current flowing through the galvanometer corresponding to which galvanometer gives the full scale deflection. The remaining current $(I - I_g)$ is to flow through the shunt.

Since G and S are parallel, the potential difference across them is same.

i.e. $$I_g G = (I - I_g)S \quad \text{or} \quad S = \left(\frac{I_g}{I - I_g}\right)G$$

This is the required value of shunt resistance to convert a galvanometer into an ammeter of range 0 - I ampere.

Effective resistance of ammeter

Total effective resistance R_{eff} of an ammeter is given by

$$\frac{1}{R_{eff}} = \frac{1}{G} + \frac{1}{S} = \frac{G+S}{GS} \quad \text{or} \quad R_{eff} = \frac{GS}{G+S}$$

Since G >> S, so (G+S) ≈ G

Hence $$R_{eff} = \frac{GS}{G} = S$$

Thus, an ammeter is a low resistance device.

Resistance of an ideal ammeter is zero.

- An ammeter is always connected in series in the circuit in which current is to be measured.
- If the ammeter is connected in parallel to the circuit, it will draw large amount of current because of its small resistance. Hence it may get damaged.

Voltmeter: A voltmeter is an instrument used to measure the potential difference across the two ends of a circuit element.

Conversion of galvanometer into Voltmeter

A galvanometer can be converted into a voltmeter by connecting a large resistance in series to the galvanometer.

Let G and R be the resistance of a galvanometer and a conductor connected in series with it respectively.

Let V volt be the potential difference to be measured by the voltmeter.

Let I_g be the current flowing in the circuit corresponding to which the voltmeter gives the full scale deflection.

Now potential difference between points A and B is given by

$$V = I_g R + I_g G = I_g (R+G)$$

$$\therefore R + G = \frac{V}{I_g}$$

or
$$R = \frac{V}{I_g} - G$$

This is the required value of resistance which must be connected in series to the galvanometer to convert it into a voltmeter of range

0 – V volt.

Effective Resistance of the Voltmeter:

Effective value of resistance of the voltmeter is given by

$R' = (R+G)$, which is very high.

Thus, voltmeter is a high resistance device.

Resistance of an ideal voltmeter is Infinite.