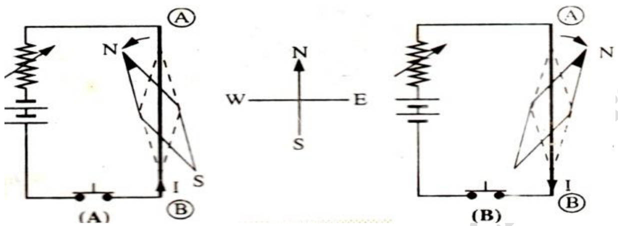
Unit III: Magnetic Effects of Current and Magnetism

(8 Marks)

Oersted's Experiment:-

Hans Christian Oersted performed a simple experiment which established the relationship between Electricity and Magnetism.

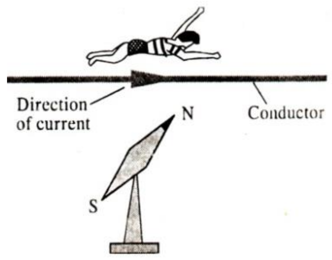
When a large current was allowed to flow through a wire AB placed parallel to the axis of a magnetic needle kept directly bellow and sufficiently close the wire, the needle was found to be deflected from its normal position as shown in figure. The deflection of the needle was found to be in the opposite direction on reversing the direction of the current by reversing the polarity of the battery as shown in figure.(If we stop the flow of current, the needle comes at rest.)



These observations led Oersted to interpret that there must be some magnetic effect around the wire carrying current which deflected the magnetic needle. Thus, an electric current (i.e. flow of electric charge) produces manetic effect in the space around the conductor. In other word, flow of electric charge is the source of magnetic field.

Direction of the deflection of the magnetic needle due to electric current can be found by applying

Ampere's Swimming Rule: - Imagining a man who swims along the conductor in the direction of current facing the needle such that current enters his feet then north of needle will deflect towards his left hand.



Ampere's Swimming Rule

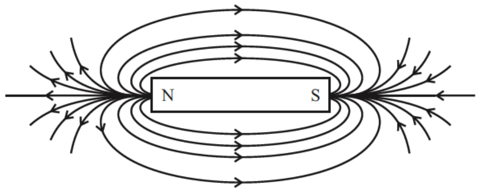
[**Important Notes: -** Ampere's swimming rule is also referred to as SNOW rule if current flows from south to north in a wire kept over a magnetic needle, the north of the needle will deflect towards west.]

Magnetic field: - It is the region surrounding a magnet, in which force of magnet can be detected. It is a vector quantity, having both direction and magnitude.

The SI unit of magnetic field is tesla (T) and its CGS unit is gauss (G). 1 tesla $= 10^4$ gauss

Compass Needle: - It is a small bar magnet, whose north end is pointing towards North Pole and south end is pointing towards South Pole of the earth.

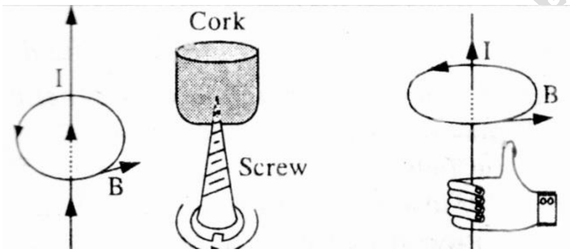
Magnetic field lines: - When a bar magnet is placed on a cardboard and iron filling are sprinkled, they will arrange themselves in a pattern as shown bellow



The lines along which the iron filling align themselves represent magnetic field lines.

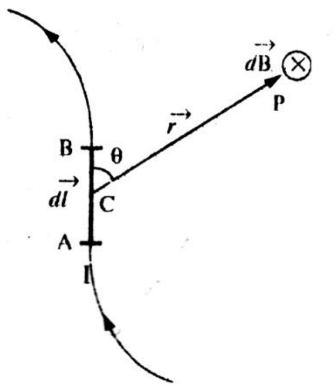
Hence, magnetic field line is a path along which a hypothetical free north pole tends to move towards South Pole.

Maxwell's Cork Screw Rule (Right Handed Screw Rule): - If the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.



Right Hand Thumb Rule: - If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the curled fingers encircling the conductor will give the direction of the magnetic lines of force.

Biot-Savart's Law: - Biot-Savart's law is used to determine the strength of the magnetic field at any point due to a current carrying conductor.



Consider a very small element AB of length dl of a conductor carrying current I. The strength of the magnetic field dB due to this small current element at a given P distant r from the element is found to be depending upon the quantities as under:

$i. dB \propto dl$

$ii. dB \propto I$

$iii. dB \propto \sin\theta$ where θ is the angle between $\vec{dl}$ & $\vec{r}$.

$iv. dB \propto \frac{1}{r^2}$

Combining (i) to (iv) we get

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

or $$dB = k\frac{Idl\sin\theta}{r^2} \quad \text{... (1)}$$

where k is the constant of proportionality.

In S.I. units, $k = \frac{\mu_o}{4\pi}$

where μ_o is called absolute permeability of free space i.e. vacuum.

Value of μ_o in S.I. units $= 4\pi \times 10^{-7}\, TmA^{-1} or Wbm^{-1}A^{-1}$

$$\therefore \frac{\mu_O}{4\pi} = 10^{-7}\, TmA^{-1}$$

Hence, eqn. (1) can be written as

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2} \quad \text{... (2)}$$

**** Current Element: -** is a vector quantity whose magnitude is equal to the product of current and length of small element having the direction of the flow of current.

**** Permeability: -** is the capability of a substance or a medium to have magnetism in magnetic fields. It indicates the degree or extent to which magnetic field can enter a substance. It is denoted by μ. Permeability $\mu = \mu_o \mu_r$, where μ_o is absolute permeability of free space and μ_r is the relative permeability.

Biot-Savart's Law Vector Form: - If we consider length element as $\vec{dl}$, distance of p as displacement vector $\vec{r}$ and unit vector along CP as $\hat{r}$ then

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I\vec{dl} \times \hat{r}}{r^2}$$

Now, $\vec{r} = |\vec{r}|\hat{r}$ $\quad \therefore \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$

$$\therefore d\vec{B} = \frac{\mu_o}{4\pi}\frac{I(\vec{dl} \times \vec{r})}{r^3} \quad \text{... (3)}$$

Magnitude of the strength of the magnetic field is given by

$$|d\vec{B}| = \frac{\mu_o}{4\pi}\frac{I|\vec{dl} \times \vec{r}|}{r^3}$$

Direction of $d\vec{B}$ is same as that of direction of $d\vec{l} \times \vec{r}$ which can be determined using Right Handed Screw Rule or Right Hand Thumb Rule for the vector product of vectors. Here at point P, direction of $d\vec{B}$ is perpendicular to the plane containing $d\vec{l}$ and $\vec{r}$ and is directed inside the plane of the paper represented by $\otimes$ in the figure.

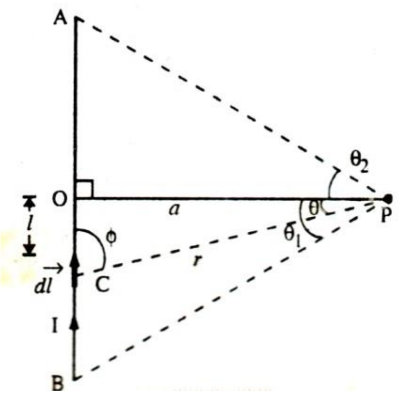
The resultant magnetic field at point P due to the whole conductor can be obtained by integrating eqn. (3) over the whole conductor.

i.e. $$\vec{B} = \int d\vec{B} = \frac{\mu_o}{4\pi}\int \frac{I\left(d\vec{l} \times \vec{r}\right)}{r^3}$$

APPLICATIONS OF BIOT-SAVART'S LAW

1. Magnetic field due to infinitely long straight wire carrying current: -

Consider a long straight wire AB carrying current I. Let P be the point at a distance 'a' from the wire where magnetic field is to be calculated.



Consider a small current element of length dl at a distance r from point P.

According to Biot-Savart's law, magnetic field at point P due to small element of the wire is given by

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\phi}{r^2} \qquad \text{... (1)}$$

In right angled triangle POC,

$$\sin\phi = \frac{a}{r} = \cos\theta \qquad \text{... (2)}$$

or, $$r = \frac{a}{\cos\theta} \qquad \text{... (3)}$$

Also $$\tan\theta = \frac{l}{a} \quad \text{or } l = a\tan\theta \Rightarrow \frac{dl}{d\theta} = \frac{d}{d\theta}(a\tan\theta)$$

i.e. $$dl = a\sec^2\theta d\theta \qquad \text{... (4)}$$

Substituting (3) and (4) in eqn. (1), we get

$$dB = \frac{\mu_o}{4\pi}\frac{I(a\sec^2\theta d\theta)\cos\theta}{\left(a^2/\cos^2\theta\right)}$$

or $$dB = \frac{\mu_O}{4\pi}\frac{I\cos\theta d\theta}{a}$$

Magnetic field due to the whole conductor AB can be calculated by integrating eqn. (5) within the limits θ_2 and - θ_1

$$\therefore B = \int_{-\theta_1}^{\theta_2} dB = \frac{\mu_o I}{4\pi a}\int_{-\theta_1}^{\theta_2}\cos\theta d\theta$$

or, $$B = \frac{\mu_o I}{4\pi a}[\sin\theta]_{-\theta_1}^{\theta_2} = \frac{\mu_o I}{4\pi a}[\sin\theta_2 - \sin(-\theta_1)]$$

or, $$B = \frac{\mu_o I}{4\pi a}\left[\sin\theta_1 + \sin\theta_2\right] \qquad \text{... (6)}$$

If the straight wire is infinitely long then θ_1 and θ_2 are taken as $\frac{\pi}{2}$. Then eqn. (6) becomes

$$B = \frac{\mu_o I}{4\pi a}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = \frac{\mu_o I}{4\pi a}(1+1)$$

$$\therefore B = \frac{\mu_o}{4\pi}\left(\frac{2I}{a}\right)$$

Here, the direction of magnetic field at point P will be perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$ and is directed inside the plane of the paper.

Important Note: - Expression $B = \frac{\mu_o}{4\pi}\left(\frac{2I}{a}\right)$ indicates cylindrical symmetry of magnetic field (It tells the magnetic field is same at all points on a circle of radius a).

Q. The magnitude of the magnetic field 80 cm from the axis of a long straight infinite wire is 70 μT. Calculate the current in the wire.

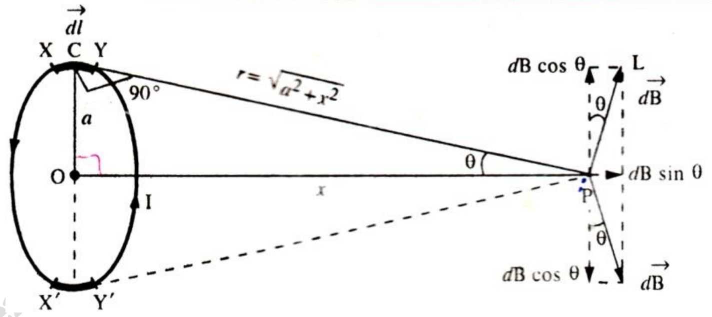
Soln.: - Here, B = 70 μT = 7 x 10^{-6}T; a = 80 cm = 0.8 m

Using $B = \frac{\mu_o}{4\pi}\left(\frac{2I}{a}\right)$, we have

$$I = \frac{Ba}{2\times\left(\frac{\mu}{4\pi}\right)} = \frac{7\times10^{-6}\times0.8}{2\times10^{-7}} = 28A$$

2. Magnetic field on the axis of a circular loop (or ring or a coil) carrying current: -

A circular loop is a plane circular ring of conducting wire.



Let P be the point on the axis of a circular loop or coil of radius a carrying current I. The distance of P from the centre of the loop is x.

Let XY be a small element of length 'dl' at a distance r from point P.

According to Biot-Savart's law, magnetic field due to a small element XY at point P is

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2} \qquad \text{... (1)}$$

Angle θ between $\vec{dl}$ and $\vec{r}$ can be taken as $\frac{\pi}{2}$ because the radius of the loop is small.

So eqn. (1) can be written as

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\frac{\pi}{2}}{r^2} = \frac{\mu_o}{4\pi}\frac{Idl}{r^2} \qquad \left[\because \sin\frac{\pi}{2} = 1\right]$$

The direction of $d\vec{B}$ is perpendicular to the plane formed by $d\vec{l}$ and $\vec{r}$ and is along PL, which is perpendicular to PC.

Resolving dB into two components

i. dBcosθ, which perpendicular to the axis of the coil.

ii. dBsinθ, which is along the axis of the coil and away from the centre of the coil.

dBcosθ components balance each other.

The magnetic field is given by

$$B = \sum dB\sin\theta$$

or
$$B = \int \frac{\mu_o Idl}{4\pi r^2}\sin\theta$$

or
$$B = \frac{\mu_o I\sin\theta}{4\pi r^2}\int dl = \frac{\mu_o I\sin\theta \times 2\pi r}{4\pi r^2}$$

[Here I, θ and r are fixed and $\int dl = 2\pi a$ = length of the circular coil = circumference of the coil]

Now from the figure

$$\sin\theta = \frac{a}{r}$$

Then,
$$B = \frac{\mu_o I}{4\pi r^2}\cdot\frac{a2\pi a}{r} = \frac{\mu_o I}{4\pi}\cdot\frac{2\pi a^2}{r^3}$$

But
$$r = \sqrt{a^2 + x^2}$$

$$\therefore B = \frac{\mu_o}{4\pi}\cdot\frac{2\pi I a^2}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

If the coil has n turns, then magnetic field due to the coil at point P becomes

$$B' = nB = n \times \frac{\mu_o}{4\pi} \times \frac{2\pi I a^2}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

$$B' = \frac{\mu_o}{4\pi}.2\pi n I.\frac{a^2}{\left(a^2 + x^2\right)^{\frac{3}{2}}} \qquad ..(2)$$

Magnetic field at the centre of the coil can be determine by putting

x = 0 in eqn. (2)

$$B' = \frac{\mu_o}{4\pi}\frac{2\pi I}{a}$$

Special Case: -

If point of observation (i.e. point P) is far away from the loop or coil (i.e. x >> a) then a^2 can be neglected as compared to x^2, hence $\left(a^2 + x^2\right)^{\frac{3}{2}} = x^3$

Then eqn. (2) becomes $B' = \frac{\mu_o}{4\pi} \cdot \frac{2\pi n I a^2}{x^3}$

Since, $\pi a^2 = A$, Area of coil or loop

$$B' = \frac{\mu_o}{4\pi} \frac{2nIA}{x^3}$$

Q. A circular coil of wire of 50 turns, each of radius 0.08m carries a current of 0.8 A. Find the magnetic flux density at the centre of the coil.

Soln.: - Magnetic flux density is given by

$$B = \frac{\mu_o}{4\pi} \frac{2\pi n I}{A}$$

$= 10^{-7} \times 2 \times \frac{22}{7} \times \frac{50 \times 0.8}{8 \times 10^{-2}}$

$= 3.1 \times 10^{-4} T$

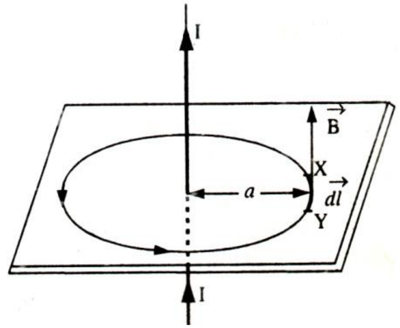
Ampere's Circuital Law: -

Ampere's Circuital law states that the line integral of the magnetic field around any closed path in free space is equal to absolute permeability (μ_o) times the net current enclosed by the path.

Mathematically $\oint \vec{B} . d\vec{l} = \mu_o I$

Where $\vec{B}$ is the magnetic field, $d\vec{l}$ is the small element, μ_o is the absolute permeability of free space and I is the current.

Proof: - Consider an infinitely long straight conductor carrying current I. The magnetic lines of forces are produced around the conductor as concentric circles.



The magnetic field due to this current carrying infinite conductor at a distance a is given by

$$B = \frac{\mu_o}{4\pi}\left(\frac{2I}{a}\right) \quad .... (1)$$

Consider a circle of radius a. Let XY be a small element of length dl.

$\vec{dl}$ and $\vec{B}$ are in the same direction because direction of $\vec{B}$ is along the tangent to the circle.

$\therefore \vec{B}.\vec{dl} = Ndl\cos\theta = Bdl\cos 0^o = Bdl$

Taking the line integral over the closed path

$$\oint \vec{B}.\vec{dl} = \oint Bdl$$

Substituting the value of B from equation (1) to the right side, we get

$$\oint \vec{B}.\vec{dl} = \oint \frac{\mu_o}{4\pi}\frac{2I}{a}dl = \frac{\mu_o I}{2\pi a}\oint dl$$

But $\oint dl =$ Circumference of the circle =2πa

$$\therefore \oint \vec{B}.\vec{dl} = \frac{\mu_o I}{2\pi a}\times 2\pi a$$

or $\oint \vec{B}.\vec{dl} = \mu_o I$

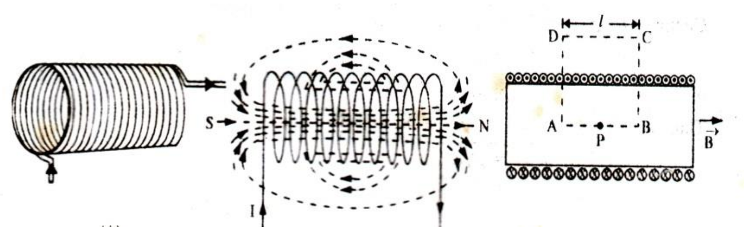
*Integral around the closed path is depicted as $\oint$

Applications of Ampere's Law

1. Magnetic Field Inside A Solenoid carrying Current: -

A cylindrical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a Solenoid.

Consider a very long solenoid having n turns per unit length of solenoid. Let current I be flowing through the solenoid. Figure shows that magnetic field inside the solenoid is uniform, strong and directed along the axis of the solenoid. The magnetic field outside a very long solenoid is very weak and can be neglected, i.e. the same may be taken as zero.



Let P be a point well within the solenoid. Consider any rectangular loop ABCD (known as Amperian Loop) passing through P as shown in figure.

then, $\oint \vec{B}.\vec{dl} =$ Line integral of magnetic field across the loop ABCD

$$= \int_A^B \vec{B}.d\vec{l} + \int_B^C \vec{B}.d\vec{l} + \int_C^D \vec{B}.d\vec{l} + \int_D^A \vec{B},d\vec{l} \qquad \text{... (1)}$$

$\vec{B}$ is perpendicular to paths BC and AD i.e. angle between $\vec{B}$ and $d\vec{l}$ is 90° for these paths.

$$\therefore \int_A^B \vec{B}.d\vec{l} = \int_D^A \vec{B}.d\vec{l} = \int Bdl\cos 90^o = 0$$

Since path CD is outside the solenoid where $\vec{B}$ is taken as 0, so $\int_C^D \vec{B}.d\vec{l} = 0.$

F or path AB, the direction of $d\vec{l}$ and $\vec{B}$ is same i.e θ = 0.

Hence equation (1) becomes

$$\oint \vec{B}.d\vec{l} = \int_A^B \vec{B}.d\vec{l} = \int_A^B \vec{B}.d\vec{l}\cos 0 = \int_A^B Bdl \qquad \text{[since B is uniform]}$$

or $$\oint \vec{B}.d\vec{l} = Bl \qquad \text{... (2)} \qquad \left[\because \int_A^B dl = l\right]$$

According to Ampere's circuital law

$\oint \vec{B}.d\vec{l} = \mu_o \times$ net current enclosed by loop ABCD.

$= \mu_o \times$ number of turns in the loop ABCD × I = $\mu_o nlI$ ... (3)

Comparing eqn. (2) and (3), we get

$$Bl = \mu_o nlI$$

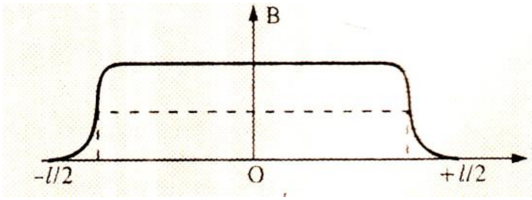
or $$B = \mu_o nI$$

Thus, magnetic field well within an infinitely long solenoid is given by $$B = \mu_o nI \qquad \text{... (4)}$$

Since $n = \frac{N}{l}$, where N = total number of turns of solenoid

$$\therefore B = \frac{\mu_o NI}{l}$$

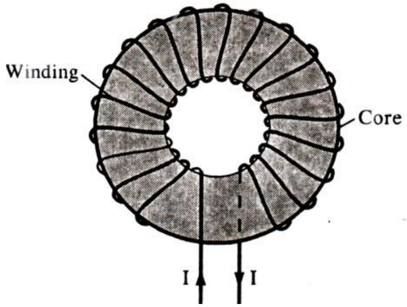
1. Magnetic field at each end of a long solenoid reduces to half of the value given by (4) i.e.
$$B_{end} = \frac{1}{2}\mu_o nI$$
2. Field and its variation with distance along the axis of solenoid is shown in the figure bellow.

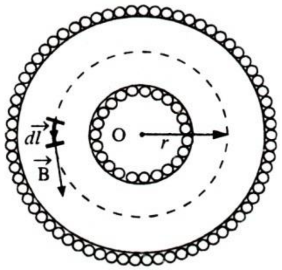


3. Magnetic field inside the solenoid is uniform magnetic field. On the other hand, magnetic field outside the solenoid is non-uniform.

2. Magnetic Field due to a Toroid carrying Current: -

A toroid can be considered as a ring shaped closed solenoid.
Consider a toroid having n turns per unit length.





Let I be the current flowing through the toroid. The magnetic lines of force mainly remain in the core of toroid and are in the form of concentric circles. Consider such a circle of mean radius r.

Now, $$\oint \vec{B}.d\vec{l} = \oint Bdl\cos\theta$$

By symmetry, magnetic field $\vec{B}$ in the coil is constant and is tangent to path $d\vec{l}$, therefore, angle θ between them is zero, hence

$$\oint \vec{B}.d\vec{l} = \oint Bdl\cos 0 = \oint Bdl = B\oint dl$$

$$= B\times \text{ Circumference of the circle of radius r.}$$

or $$\oint \vec{B}.d\vec{l} = B\times 2\pi r \qquad ...(1)$$

According to Ampere's circuital law

$$\oint \vec{B}.d\vec{l} = \mu_o \times \text{ net current enclosed by the circle of radius r}$$

$$= \mu_o \times \text{ total number of turns} \times I$$

$$= \mu_o (n\times 2\pi r)I \qquad ... (2)$$

Comparing eqn. (1) and (2), we get

$$B\times 2\pi r = \mu_o (n\times 2\pi r)I$$

or $$B = \mu_o nI \qquad ...(3)$$

Also, if N is the total number of turns of a toroid then

$$N = n\times 2\pi r \qquad \text{or} \qquad n = \frac{N}{2\pi r}$$

$\therefore$ Equation (3) can be written as

$$B = \frac{\mu_o NI}{2\pi r}$$

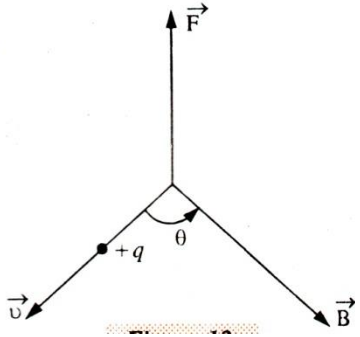
**For any point inside the empty space surrounded by toroid and outside the toroid, magnetic field B is zero because the net current enclosed in these spaces is zero.

<u>Lorentz Force</u>: -

Consider a charge +q moving with velocity $\vec{v}$ in magnetic field $\vec{B}$ such that $\vec{v}$ makes an angle θ with $\vec{B}$, then *force experienced by the charge in the magnetic field* is given by

$$\vec{F}_m = q(\vec{v}\times\vec{B}) \qquad ... (1)$$

This force is called ***Lorentz Magnetic force.***



Magnitude of force $\vec{F}_m$ is given by

$$F_m = qvB\sin\theta \qquad ... (2)$$

The direction of $\vec{F}_m$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. This can be found by right handed screw rule.

Special Cases for Lorentz magnetic force F_m.

1. If θ = 0° or θ = 180° i.e. charge q moves parallel or anti-parallel to the direction of magnetic field $\vec{B}$,

then $$F_m = 0$$

Thus, no force is experienced by a charged particle moving parallel or anti-parallel to the direction of magnetic field.

2. If θ = 90°, i.e. charge q moves at right angles to the magnetic field $\vec{B}$,

then $$F_m = qvB\sin 90^o$$

or $$F_m = qvB$$

3. If θ = 270°, then $F_m = qvB\sin 270^o$ or $F_m = -qvB$
4. If $\vec{v}$ = o, i.e., charged particle is at rest in the magnetic field. Hence force experienced by it is zero.

<u>Lorentz Force due to electric and magnetic fields</u>

When a charged particle having charge q moves in a region where both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, it experiences a net force called ***Lorentz force (F).***

Lorentz force $(\vec{F})$ = Force on charge due to electric field + Force on charge due to magnetic field $= \vec{F}_e + \vec{F}_m$

or $$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \text{ or } \vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] \qquad \text{... (30)}$$

Definition of Magnetic Field B from Lorentz Force

Lorentz magnetic force is given by $F_m = qvB\sin\theta$

or $$B = \frac{F_m}{qv\sin\theta}$$

where q is the charge on a particle, v is the velocity of the particle in magnetic field B.

If q = 1, v = 1 and θ = 90°, then

$$B = F_m$$

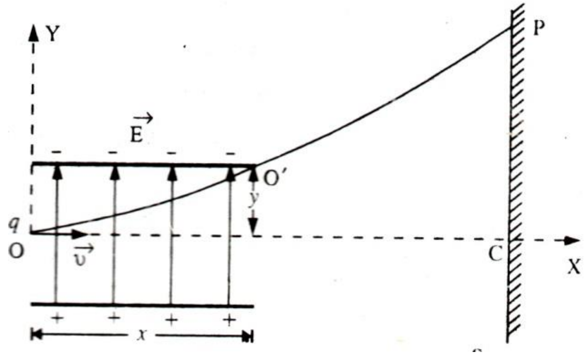
∴ ***Strength of magnetic field (B) at a point may be defined as the magnetic force experienced by a unit charge moving with unit velocity at right angles to the magnetic field.***

Moving Charged Particle in Electric and Magnetic Fields: -

Comparatively heavy charged particle can be accelerated to a sufficiently high energy with the help of electric field and strong magnetic field.

Charged Particle Moving In Electric Field: -

Consider a uniform electric field $\vec{E}$ set up between two oppositely charged parallel plates. Let a positively charged particle having charge +q and mass m enters the region of electric field $\vec{E}$ at O with velocity $\vec{v}$ along X-direction.



Step 1. Force acting on the charge +q due to electric field $\vec{E}$ is given by

$$\vec{F} = q\vec{E}$$

The direction of the force is along the direction of $\vec{E}$ and hence the charged particle is deflected accordingly.

Acceleration produced in the charged particle is given by

$$\vec{a} = \frac{\vec{F}}{m} \qquad \text{or} \qquad \vec{a} = \frac{q\vec{E}}{m} \qquad ...(1)$$

The magnitude of $\vec{a}$ is $\qquad a = \frac{qE}{m}$

Step 2. The charged particle will accelerate in the direction of $\vec{E}$. As soon as the particle leaves the region of electric field, it travels in a straight line due to inertia of motion and hits the screen at point P.

Let t be the time taken by the charged particle to traverse the region of electric field of length x. Let y be the distance travelled by the particle along y-direction (i.e. direction of electric field).

Using a standard equation of motion,

$$S = ut + \frac{1}{2}at^2 \qquad ... (2)$$

For horizontal motion, S = x, u = v and a = 0

($\because$ no force acts on the particle along x-direction)

From eqn. (2), we have

$$x = vt \qquad \text{or} \qquad t = \frac{x}{v}$$

For vertical motion, S = y, u = 0 ($\because$ initially the particle was moving along x-direction)

$\therefore$ From eqn. (2) we have $\qquad y = \frac{1}{2}at^2$

Substituting the value of ‘a’ from eqn. (1), we get

$$\therefore y = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

Using eqn. (3), we get $$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2$$

or $$y = \frac{qEx^2}{2mv^2} = Kx^2$$

where $$K = \frac{qE}{2mv^2}, \quad \text{a constant}$$

Eqn. (1) is the equation of **parabola**.

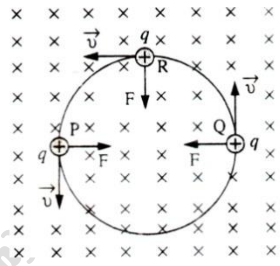
Hence a charged particle moving in a uniform electric field follows a parabolic path.

Charged Particle Moving in a Uniform Magnetic Field: -

Case I. A moving charged particle does not experience any force if its motion is parallel or antiparallel to the magnetic field.

Case II. When charged particle moves at right angle to the magnetic field.

Consider a uniform magnetic field $\left(\vec{B}\right)$ acting perpendicular to the plane of the paper and directed into the paper (shown by ×) in figure. Let a charged particle having charge +q enters the region of the magnetic field with velocity $\vec{v}$ and perpendicular to the direction of magnetic field $\left(\vec{B}\right)$.



Force acting on the charge +q due to the magnetic field is

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

or $$F = qvB\sin 90^o$$

or $$F = qvB \qquad \textbf{... (1)}$$

This force is always perpendicular to the direction of motion as well as the magnetic field. (Three different positions P, Q and R are shown in figure)Thus, the path of the charged particle is *circular*.

If m is the mass of the charged particle and r be the radius of the circular path, then the necessary centripetal force required by the particle to move in a circular path $= \dfrac{mv^2}{r}$.

This centripetal force is provided by the magnetic Lorentz force qvB.

$$\therefore \frac{mv^2}{r} = qvB \qquad \text{or} \qquad \frac{mv^2}{qvB} = r$$

i.e. Radius of circular path,

$$r = \frac{mv}{qB} \qquad \ldots (2)$$

Thus, radius of the circular path is directly proportional to speed of the particle and mass of the particle.

From eqn. (2), it is clear that slow charged particles move in small circles and fast charged particles move in large circles.

Time period of the charged particle is

$$T = \frac{Distance}{Speed} = \frac{Circumference-of-the-circle}{Speed} = \frac{2\pi r}{v}$$

From eqn. (2), we have

$$T = \frac{2\pi}{v} \times \frac{mv}{qB}$$

or

$$T = \frac{2\pi m}{qB} \qquad \ldots (3)$$

From eqn. (3), it is clear that time period (or frequency) is independent of speed of particle and radius of the orbit. It depends only on the field B and the nature of the particle i.e. specific charge q/m. This fact is used to accelerate charged particles in ***Cyclotron.***

Frequency, $\nu = \frac{1}{T} = \frac{qB}{2\pi m}$ ***..(4)***

Angular frequency, $\omega = 2\pi\nu$

or $$\omega = 2\pi \times \frac{qB}{2\pi m}$$

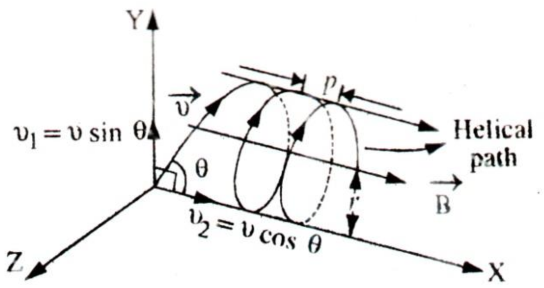
or $$\omega = \frac{qB}{m} \qquad \ldots (5)$$

This angular frequency is called ***Cyclotron Frequency or Gyro Frequency.***

From eqn. (5), it is clear that angular frequency does not depend upon the speed of the particle (provided speed of the particle is very small as compared to the sped of the light). Thus, all the charged particles take the same time to complete the small or large circles, provided their specific charge (q/m) is same.

CaseIII: - When the charged particle moves at an angle to the magnetic field (other than 0º, 90º, 180º).

Consider a charged particle of mass m moving with velocity $\vec{v}$ at an angle θ with the direction of magnetic field $\vec{B}$. (along X-axis in figure).



Now $v_2 = v\cos\theta$ is the component of the velocity $\vec{v}$ along the direction of $\vec{B}$ and $v_1 = v\sin\theta$ is the component of $\vec{v}$ perpendicular to the direction of $\vec{B}$.

The charged particle moves with constant velocity vcosθ along the magnetic field, as no force acts on the charged particle when it moves parallel to the magnetic field.

Since vsinθ is perpendicular to the direction $\vec{B}$, so the particle moves in a circular path in the plane perpendicular to the magnetic field.

Centripetal force required to move the particle in a circle is provided by magnetic Lorentz force.

i.e. $$\frac{m(v\sin\theta)^2}{r} = q(v\sin\theta)B\sin 90^o = qvB\sin\theta$$

i.e. $$\frac{m(v\sin\theta)^2}{qvB\sin\theta} = r$$

i.e. ***Radius of the circular path***

$$r = \frac{mv\sin\theta}{qB} \quad \textbf{... (6)}$$

Time period of the particle is

$$T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi}{v\sin\theta} \times \frac{mv\sin\theta}{qB}$$

or $$T = \frac{2\pi m}{qB} \quad \textbf{. ..(7)}$$

Time period is independent of the velocity of the charged paticle.

In this case, charged particle moves along the direction of the magnetic field due to the horizontal component (vcosθ) of the velocity and at the same time moves in a circular path due to (vsinθ) component of the velocity. Thus the resultant path of the particle is helix with its axis parallel to the magnetic field $\vec{B}$.

The linear distance travelled by the charged particle in one rotation is called pitch of the helix

i.e. Pitch of the helix, $p = (v\cos\theta) \times T$

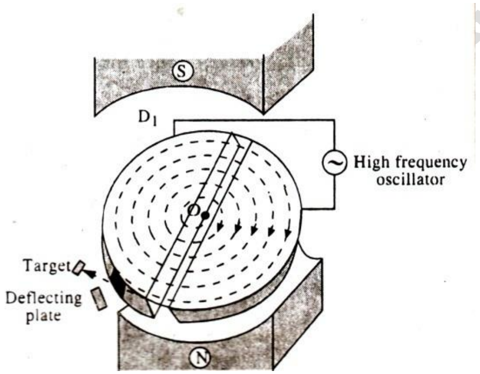
or $p = v\cos\theta \times \dfrac{2\pi m}{qB}$ [using eqn. (7)] or $p = \dfrac{2\pi mv\cos\theta}{qB}$

Cyclotron

Cyclotron is a device used to accelerate positively charged particles (like protons, α-particles, deuteron, ions etc.) to acquire enough energy to carry out nuclear disintegrations.

Principle: - Cyclotron is the application of cross fields i.e. electric field E and magnetic field B are at 90^o to each other. When a positively charged particle is made to move time and again in a high frequency electric field and using strong magnetic field, it gets accelerated and acquires sufficiently large amount of energy.

Construction: - It consists of two hollow D-shaped metallic chambers D1 and D2 called dees. These dees are separated by a small gap where a source of positively charged particles is placed. Dees are connected to high frequency oscillator, which provides high frequency electric field across the gap of the dees. This arrangement is placed between two poles of a strong electromagnet. The magnetic field due to this electromagnet is perpendicular to the palne of the dees.



Working: - If a positively charged particle (say proton) is emitted from O, when dee D_2 is negatively charged and dee D_1 is positively charged, it will accelerate towards D_2. As soon as it enters D_2, it is shielded from the electric field by metallic chamber. Inside D_2, it moves at right angle to the magnetic field and hence describes a semi-circle inside it. After completing the semi-circle, it enters the gap between the dees at the time when polarities of dees have been reversed. Now the proton is further accelerated towards D_1. Then it enters D_1 and again describes the semi-circle due to the magnetic field which is perpendicular to the motion of the proton. This process continues till the proton reaches the periphery of the dee system. At this stage, the proton (or a heavy charged particle) is deflected by the deflecting plate which then comes out through the window (W) and hits the target.

Theory: - When a proton (or other positively charged particle) moves at right angles to the magnetic field ($\vec{B}$) inside the dee, magnetic Lorentz force acting on it is

$$F = qvB\sin 90^o = qvB$$

This force provides the centripetal force $\frac{mv^2}{r}$ to the charged particle to move in a circular path of radius r.

$$\therefore qvB = \frac{mv^2}{r} \qquad \text{or } r = \frac{mv}{qB} \qquad ...(1)$$

Time taken by the particle to complete the semi-circle inside the dee,

$$t = \frac{Distance}{Speed} = \frac{\pi r}{v} \qquad \text{or } t = \frac{\pi}{v} \times \frac{mv}{qB} \quad \text{[Using eqn. (1)]}$$

or $$t = \frac{\pi m}{qB} \qquad \text{... (2)}$$

This shows that time taken by the positively charged particle to complete any semi-circle (irrespective of its radius) is same.

i. **Time Period: -** Let T be the time period of the high frequency electric field, then the polarities of dees will change after time $\frac{T}{2}$.

The particle will be accelerated if time taken by it to describe semi-circle is equal to $\frac{T}{2}$.

i.e. $$\frac{T}{2} = t = \frac{\pi m}{qB}$$

or $$T = \frac{2\pi m}{qB} \qquad \text{... (3)}$$

ii. **Cyclotron frequency: -** $\nu = \frac{1}{T} = \frac{qB}{2\pi m}$ **... (4)**

$\therefore$ **Cyclotron angular frequency,** $\omega = 2\pi\upsilon = \frac{qB}{m}$ **... (5)**

iii. **Energy gained: -** Energy gained by the positively charged particle is given by $E = \frac{1}{2}mv^2$

From eq. (1), we have $\upsilon = \frac{qBr}{m}, then$

$$E = \frac{1}{2}m \times \left(\frac{qBr}{m}\right)^2 \qquad \text{or} \qquad E = \frac{q^2B^2r^2}{2m} \qquad \text{... (6)}$$

Maximum energy gained by the positively charged particle,

$$E_{max} = \left(\frac{q^2B^2}{2m}\right)r_{max}^2 \qquad \text{... (7)}$$

Thus, the positively charged particle will acquire maximum energy when it is at the periphery of the dees (where r is maximum).

<u>Limitations of Cyclotron</u>

i. Cyclotron cannot accelerate uncharged particles like neutron.

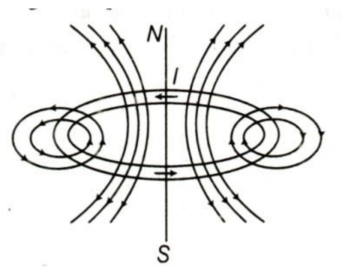
ii. Cyclotron cannot accelerate electrons because they have very small mass. Electrons start moving at a very high speed when they gain small energy in the cyclotron. Oscillating electric field makes them to go quickly out of step because of their very high speed.

Previous Years' Examination Questions

1 mark

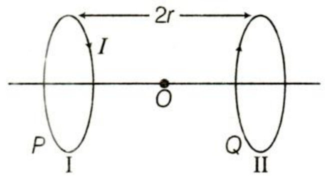
1. Draw the magnetic field lines due to a current carrying loop.

Ans.: - Magnetic field lines due to a current carrying loop are given by



2 marks

2. Two identical circular loops P and Q, each of radius r and carrying equal currents are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and Q is anticlockwise as seen from O which is equidistant from the loop P and Q. Find the magnitude of the net magnetic field at O.



Ans.: - Magnetic field at O due to two rings will be in same direction (Q to P, along the axis) and of equal magnitude.

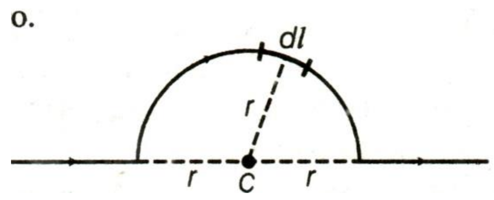
$$B = B_1 + B_2 \quad \text{but} \quad B_2 = B_1$$

$$\Rightarrow B = 2B_1 = 2\left[\frac{\mu_o I r^2}{2\left(r^2 + r^2\right)^{\frac{3}{2}}}\right]$$

$$\Rightarrow B = \frac{\mu_o I r^2}{\left(2r^2\right)^{\frac{3}{2}}} = \frac{\mu_o I r^2}{2^{\frac{3}{2}} r^3} = \frac{\mu_o I}{2^{\frac{3}{2}} r}$$

3. A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart's law to deduce an expression for the magnetic field at its centre due to the current I passing through it.

Ans.: - When a straight wire is bent into semi-circular loop, then there are two parts which can produce the magnetic field at the centre, one is circular part and other is straight part due to which filed is zero.



Since length L is bent into semi-circular loop. Length of wire = Circumference of semi-circular wire.

$$\Rightarrow L = \pi r \Rightarrow r = \frac{L}{\pi}$$

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C. Using Biot-Svart's law, we have

$$dB = \frac{\mu_O}{4\pi} \cdot \frac{Idl \sin 90^o}{r^2} \quad [\because Idl \perp r, \therefore \theta = 90^o]$$

$$dB = \frac{\mu_o}{4\pi} \cdot \frac{Idl}{r^2}$$

$\therefore$ Net magnetic field at C due to semi-circular loop,

$$\int_{semicircle} \frac{\mu_o}{4\pi} \frac{Idl}{r^2} \Rightarrow B = \frac{\mu_o}{4\pi} \frac{I}{r^2} \int_{semi-circle} dl$$

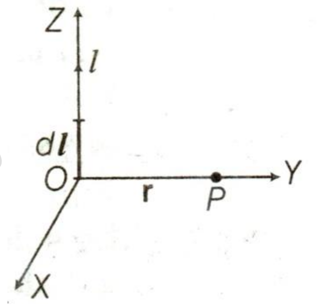
$$B = \frac{\mu_o}{4\pi} \cdot \frac{I}{r^2} L$$

But, $r = \frac{L}{\pi}$

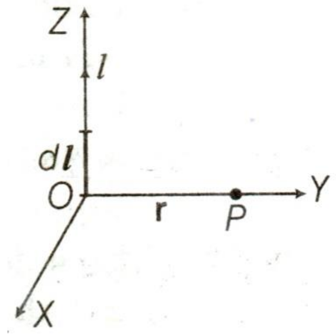
$$B = \frac{\mu_o}{4\pi} \cdot \frac{IL}{\left(L/\pi\right)^2} = \frac{\mu_O}{4\pi} \times \frac{IL}{L^2} \times \pi^2 \Rightarrow B = \frac{\mu_O I \pi}{4L}$$

This is the required expression.

4. State Biot-Svart's law. A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element dl at a point P situated at a distance r from the element as shown in the figure.



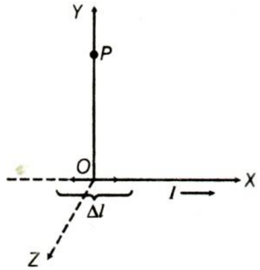
Ans.: - The direction of magnetic field can be obtained using right hand thumb rule.



$\therefore$ The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

5. An element $\Delta l = \Delta x I$ is placed at the origin (as shown in figure) and carries a current I = 2 A. Find out the magnetic field at a point P on Y-axis at a distance 1.0 m due to the

element $\Delta x = w$ cm. Also, give the direction of the field produced.



Ans.: - Biot-Savart's law states that

$$dB = \frac{\mu_O}{4\pi} \cdot \frac{Idl \times \hat{r}}{|r|^2}$$

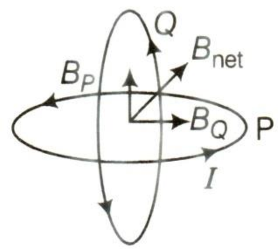
Given, $dl = \Delta x I\hat{i} = w \times 10^{-2} \times 2\hat{i}$ [∵ Δx = w cm]

$$dB = \frac{\mu_o}{4\pi} \cdot \frac{Idl \times \hat{r}}{r^2} = \frac{10^{-7} \times \left(2 \times 2 \times 10^{-2}\, w\hat{i}\right) \times \hat{j}}{12} = 4w \times 10^{-9}\,\hat{k}$$

and directed along +Z-axis.

6. Two identical circulars coils, P and Q each of radius R, carrying currents 1A and $\sqrt{3}A$ respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ planes. Find the magnitude and direction of the net magnetic field at the centre of the coils.

Ans.: -



Magnitude of magnetic field due to circular wire p,

$$B_P = \frac{\mu_o}{4\pi} \times \frac{2\pi I_1}{R} \quad \text{(along vertically upwards)}$$

$$= \frac{\mu_o I}{2R}$$

Magnitude of magnetic field due to a circular wire Q,

$$B_Q = \frac{\mu_o}{4\pi} \times \frac{2\pi I_2}{R} \quad \text{(along horizontal towards left)}$$

$$= \frac{\mu_o I_2}{2R}$$

Net magnitude of magnetic field at the common centre of the two coils,

$$B = \sqrt{B_P^2 + B_Q^2} \Rightarrow B = \sqrt{\left(\frac{\mu_O I_1}{2R}\right)^2 + \left(\frac{\mu_O I_2}{2R}\right)}^2$$

$$B = \sqrt{\left(\frac{\mu_O}{2R}\right)^2 \left(I_1^{\ 2} + I_2^{\ 2}\right)} \Rightarrow B = \frac{\mu_O}{2R}\sqrt{I_1^2 + I_2^2}$$

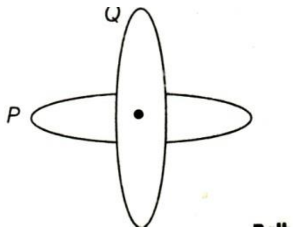
$$B = \frac{4\pi \times 10^{-7}}{2 \times R}\sqrt{(1)^2 + \left(\sqrt{3}\right)^2}$$

$$B = \frac{4\pi \times 10^{-7}}{R} T$$

Resultant magnetic field makes an angle θ with direction of B_Q, which is given by

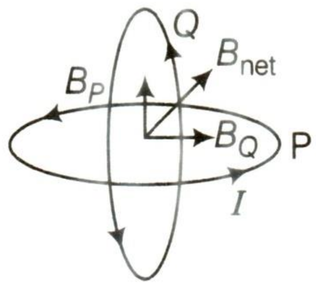
$$\tan\theta = \frac{B_P}{B_Q} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^O$$

7. Two identical loops P and Q each of radius 5cm are lying in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils, if they carry currents equal to 3A and 4A, respectively.



Ans.: - Magnetic field due to a circular loop P,

$$B_P = \frac{\mu_O I_P}{2r}$$



Magnetic field due to circular loop Q,

$$B_Q = \frac{\mu_O I_Q}{2r}$$

So, the net magnetic field at the common centre of the loop is

$$B_{net} = \sqrt{B_P^2 + B_Q^2} = \sqrt{\left(\frac{\mu_o I_P}{2r}\right)^2 + \left(\frac{\mu_o I_Q}{2r}\right)^2}$$

$$= \frac{\mu_o}{2r}\sqrt{I_P^2 + I_Q^2} = \frac{4\pi \times 10^{-7}}{2 \times 5 \times 10^{-2}} \times 5 = 2\pi \times 10^{-5} T$$

Resultant magnetic field makes an angle θ with BQ which is given by,

$$\tan\theta = \frac{B_P}{B_Q} = \frac{I_P}{I_Q} = \frac{3}{4}$$

8. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the radii of the circular path described by them?

Ans.: - For the given momentum of charge particle, radius of circular paths depends on charge and magnetic field as

$$r = \frac{mv}{qB}$$

For constant momentum, $r \propto \frac{1}{qB}$

$$\therefore r_{proton} : r_{deutron} = 1:1 = q_{deutron} : q_{proton}$$

9. Two particles A and B of masses m and 2m have charges q and 2q respectively. They are moving with velocities v_1 and v_2 respectively in the same direction, enters the same magnetic field B acting normally to their direction of motion. If the two forces F_A and F_B acting on them are in the ratio of 1:2, find the ratio of their velocities.

Ans.: -Ratio of forces acting on the two particles,

$$\frac{F_A}{F_B} = \frac{qv_1 B \sin 90^o}{(2q)v_2 B \sin 90^o} = \frac{1}{2}$$

$$\Rightarrow \frac{v_1}{v_2} = 1 \Rightarrow v_1 : v_2 = 1:1$$

10. A beam of α-particles projected along +X-axis, experiences a force due to which a magnetic field along the +Y-axis. What is the direction of the magnetic field?

Ans.: - Velocity of α-particles

$$v = v\hat{i}$$

[Projected along X-axis]

Magnetic force on α-particles,

$$F_m = q(v \times B) = q(v\hat{i} \times B)$$

As, $F_m = F_m \hat{j}$ [Oriented along Y-axis]

$$\Rightarrow F_m \hat{j} = q(v\hat{i} \times B) \Rightarrow B = -B\hat{k} = B(-\hat{k})$$

The direction of magnetic field must be along –Z-axis.

11. Use the expression $F = q(v \times B)$ to define the SI unit of magnetic field.

Ans.: -Given, $F = q(v \times B) \Rightarrow F = qvB\sin\theta$ where θ is the angle between v and B.

If q= 1C, v = 1 ms^{-1}, θ = 90°

The magnetic field at any point can be given by

$$B = \frac{1N}{\left[(1C)(1ms^{-1})\sin 90^o\right]} = 1NA^{-1}m^{-1} = 1T$$

Thus, the magnetic field induction at a point is said to be one tesla if a charge of one coulomb while moving at right angle to a magnetic field with velocity of 1ms^{-1} experiences a force of 1N at that point.

12. An electron does not suffer any deflection while passing through a region of a uniform magnetic field. What is the direction of magnetic field?

Ans.:- An electron does not suffer any deflection in the magnetic field, it means that the electron is moving parallel or antiparallel to the magnetic field, i.e. along or opposite to the direction of magnetic field.

13. Write the expression for Lorentz magnetic force on a particle of charge q moving with velocity v in a magnetic field B. Show that no work is done by this force on the charged particle.

Ans.: - 2nd part: -

Force is perpendicular to displacement made by charged particle.

$$\therefore W = Fd\cos 90^o = 0$$

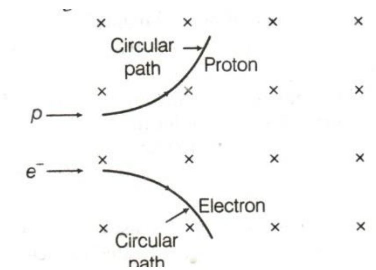
[Since Force F and displacement d are perpendicular to each other]

$$\Rightarrow W = 0$$

No work is done by magnetic Lorentz force on the charged particle.

14. An electron and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. Show the trajectory followed by the two particles in the magnetic field. Find the ratio of the radii of the circular paths which the particles may describe.

Ans.: - When charged particle enters in the magnetic field at right angle, then the particle follows a circular path. The trajectory of the two particles in the magnetic field is shown below.



Radius of the circular path, $r = \dfrac{mv}{qB}$

For same speed v, magnitude of charge and magnetic field

$$r \propto m \Rightarrow \frac{r_e}{r_p} = \frac{m_e}{m_p}$$

where, m_e and m_p are masses of electron and proton respectively.

$$\because m_e < m_p$$

(proton is much heavier than electron)

$$\Rightarrow r_e < r_p$$

The curvature of path of electron is much more than curvature of path of proton.

15. A cyclotron when being used to accelerate positive ions (Mass = 6.7 × 10 $^{-27}$ kg, charge = 3.2.×10 $^{-19}$ C) has a magnetic field of (π/2) T. What must be the value of the frequency of the applied alternating electric field to be used in it?

Ans.: - Frequency of alternating electric field is given by

$$\nu = \frac{qB}{2\pi m}$$

Here, q = 3.2.×10 $^{-19}$ C, m= 6.7 × 10 $^{-27}$ kg & B = (π/2) T

$$\therefore \nu = \frac{\left(3.2\times10^{-19}\right)\times\dfrac{\pi}{2}}{2\times\pi\times6.7\times10^{-27}} = 1.2\times10^{7}\,cycle/s$$