Unit 4 Electromagnetic Induction and alternating current (Remaining Notes)

Alternating Current (A.C.)

An electric current, magnitude of which changes with time and polarity reverses periodically is called alternating current (A.C.).

The sinusoidal alternating current (a.c.) is expressed* as

$$I = I_0 \sin \omega \qquad \ldots(1)$$

where, I_0 is the ***maximum value*** or ***peak value*** or ***amplitude*** of a.c. and ω is the angular velocity where $\omega = \frac{2\pi}{T} = 2\pi\nu$. Here T is the ***time period*** and ν is called the ***driving frequency.*****

The variation of a.c. with time is shown in figure 1.

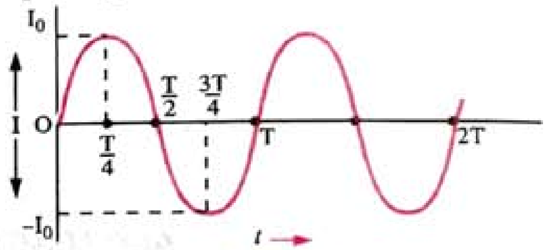


Figure 1

Sinusoidal voltage ***of an a.c. source*** is given by,

$$V = V_0 \sin \omega t \qquad \ldots(2)$$

Figure 2

where, V_0 is the *maximum value* or *peak value* or *amplitude* of *voltage* and ω is *driving angular velocity.*

The ***pictorial symbol*** used to represent the a.c. source is shown in figure 2.

MEAN OR AVERAGE VALUE OF ALTERNATING CURRENT

Derive an expression for average value of alternating current.

Mean or average value of alternating current is that value of steady current which sends the same amount of charge through a circuit in a certain time interval as is sent by an alternating current through the same circuit in half cycle.

Mean value of an a.c. over half cycle is 63·7% of its peak value.

Derivation of Expression for mean value of *a.c.*

Let an alternating current be represented by,

$$I = I_0 \sin \omega t$$

The charge sent by the alternating current I in time dt is given by,

$$dq = I\,dt = I_0 \sin \omega t\, dt \qquad \left(\because I = \frac{dq}{dt}\right)$$

Therefore, the charge sent by a.c. in the first half cycle (*i.e.* $t = 0$ to $t = T/2$) is given by,

$$\int dq = \int_0^{T/2} I_0 \sin \omega t\, dt$$

or

$$q = I_0 \int_0^{T/2} \sin \omega t\, dt = I_0 \left[\frac{-\cos \omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{I_0}{\omega}\left[\cos \omega t\right]_0^{T/2} = -\frac{I_0}{(2\pi/T)}\left[\cos \frac{2\pi}{T} t\right]_0^{T/2} \qquad (\because \omega = 2\pi/T)$$

or

$$q = -\frac{I_0 T}{2\pi}\left[\cos \frac{2\pi}{T} \times \frac{T}{2} - \cos 0\right] = -\frac{I_0 T}{2\pi}[\cos \pi - \cos 0]$$

$$= -\frac{I_0 T}{2\pi}[-1-1] \qquad (\because \cos \pi = -1 \text{ and } \cos 0 = 1)$$

$$q = \frac{I_0 T}{\pi} \qquad \ldots(i)$$

Let I_{av} be the mean or average value of a.c. over positive half cycle, then the charge sent by it in time T/2 is given by

$$q = I_{av} \times \frac{T}{2} \qquad \ldots(ii)$$

According to definitions, eqn. *(i)* = eqn. *(ii)*

$$\therefore \quad I_{av} \times \frac{T}{2} = \frac{I_0 T}{\pi}$$

or

$$I_{av} = \frac{2 I_0}{\pi} = 0{\cdot}637\, I_0 \qquad \ldots(13)$$

The mean or average value of a.c. over a complete cycle is zero.

(i) ***Mean value of a.c. over positive half of a.c. is 0·637 I_0. Similarly mean value of a.c. over negative half is − 0·637 I_0. Therefore, mean value of a.c. over a full cycle***

$$= 0\cdot637\, I_0 - 0\cdot637\, I_0 = 0.$$

(ii) ***Mean or average value of alternating voltage $V = V_0 \sin \omega t$ over a positive half cycle is, $V_{av} = 0\cdot637\, V_0$. Similarly mean or average value of alternating e.m.f. over a negative half cycle is, $V_{av} = -\,0\cdot637\, V_0$. Therefore, mean value of alternating e.m.f. or voltage over a complete cycle = $0\cdot637\, V_0 - 0\cdot637\, V_0 = 0$.***

(iii) ***Ordinary d.c. ammeter and d.c. voltmeter cannot measure alternating current or alternating voltage. They will give zero reading if connected to a.c. circuit because average value of a.c. or average value of alternating voltage ever a complete cycle is zero.***

ROOT MEAN SQUARE (R.M.S.) OR EFFECTIVE VALUE OF ALTERNATING CURRENT

Derive an expression for r.m.s. value of alternating current.

Root mean square value of a.c. is defined as that steady current which produces the same amount of heat in a conductor in a certain time interval as is produced by the a.c. in the same conductor during the time period T (i.e. full cycle).

It is represented by I_{rms} or simply I.

Root mean square value of a.c. is also known as **effective value (I_{eff})** ***or*** **virtual value (I_v).**

Derivation of Expression for r.m.s value of a.c.

Let an alternating current $I = I_0 \sin \omega t$ flow through a conductor of resistance R for time dt.

Then, heat produced in the conductor is given by,

$$dH = I^2 R\, dt = (I_0^2 \sin^2 \omega t)\, R\, dt$$

or

$$dH = I_0^2 R \sin^2 \omega t\, dt$$

Now heat produced in the conductor, when current flows for time period T (*i.e.* from $t = 0$ to $t = T$) is given by,

$$\int dH = \int_0^T I_0^2 R \sin^2 \omega t\, dt \text{ or } H = I_0^2 R \int_0^T \sin^2 \omega t\, dt$$

Since

$$\sin^2 \omega t = \left(\frac{1 - \cos 2\omega t}{2}\right)$$

$\therefore$

$$H = I_0^2 R \int_0^T \frac{(1 - \cos 2\omega t)}{2}\, dt = \frac{I_0^2 R}{2} \int_0^T (1 - \cos 2\omega t)\, dt$$

$$= \frac{I_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right] = \frac{I_0^2 R}{2}\left[[t]_0^T - \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T\right]$$

$$= \frac{I_0^2 R}{2}\left[(T - 0) - \frac{1}{2\omega}\left[\sin 2 \times \frac{2\pi}{T} t\right]_0^T\right] \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{1}{2\omega}\left(\sin 2 \times \frac{2\pi}{T} \times T - \sin 0\right)\right]$$

As

$$\sin 4\pi = \sin 0 = 0$$

$\therefore$

$$H = \frac{I_0^2 R}{2} T \qquad \ldots(i)$$

Let I_{rms} be the r.m.s. value of a.c. which flows through the conductor of resistance R for time T.

$\therefore$ Heat produced in the conductor, $H = I_{rms}^2 RT$ ...*(ii)*

According to definition of r.m.s. value of a.c. ; eqn. *(i)* = eqn. *(ii)*

$\therefore$

$$I_{rms}^2 RT = \frac{I_0^2 RT}{2}$$

or

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0\cdot707\, I_0 \qquad \ldots(14)$$

$\sqrt{2}$

1. ***R.M.S. or virtual or effective value of alternating voltage is given as,***

$$V_{rms} = \frac{V_0}{\sqrt{2}} \text{ or } V_{rms} = 0{\cdot}707\ V_0$$

2. ***A.C. ammeter and voltmeter reading the r.m.s value i.e. effective value of alternating current and voltage respectively are known as hot wire meters.***

3. ***If an a.c. voltmeter plugged into a household electric outlet reads 150V, then 150 V is the r.m.s. value of voltage and the peak value of voltage is given by,***

$$V_0 = \sqrt{2}\ V_{rms} \text{ i.e. } 1{\cdot}414 \times 150 = 212{\cdot}1\ V$$

Numerical Illustration. **Calculate the r.m.s. value of a.c. if its peak value is 10 A.**

Solution. Here $I_0 = 10$ A

Using $I_{rms} = \frac{I_0}{\sqrt{2}} = 0{\cdot}707\ I_0$, we get

$$I_{rms} = 0{\cdot}707 \times 10\ A = \mathbf{7{\cdot}07\ A.}$$

PHASOR AND PHASOR DIAGRAM :

What do you understand by phasor and phasor diagram.

Phasor is a rotating vector which represents a quantity varying sinusoidally with time.

Rotation of a phasor is treated as anticlock with an angular speed (ω) equal to the angular frequency of *the quantity varying sinusoidally with time.*

For example, instantaneous voltage of *a.c.* source is given by,

$$V = V_0 \sin \omega t$$

So a phasor representing $V = V_0 \sin \omega t$ is shown in figure 11. Here, length of phasor $V \propto V_0$ and it describes an angle, $\theta = \omega t$ in time t, where ω is the angular frequency of the voltage of *a.c.* source. The vertical component of phasor $V = V_0 \sin \omega t$.

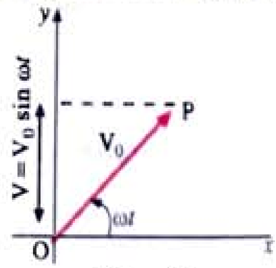


Fig. 11

Phasor diagram : In a phasor diagram, quantities varying sinusoidally with time are represented. The phase difference between two quantities is represented by the angle between their maximum values.

ALTERNATING VOLTAGE APPLIED TO A RESISTOR

An A.C. source connected to a resistor of resistance R is shown in figure 12. Such a circuit is known as a ***resistive circuit***.

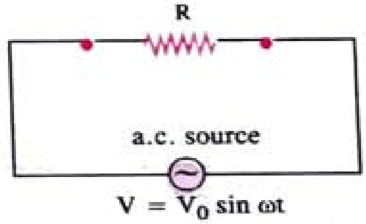


Figure 12

The applied alternating voltage is given by,

$$V = V_0 \sin \omega t \qquad \ldots (i)$$

Let I be the current in the circuit at any instant t.

As per *Ohm's Law*, potential difference across the resistor = IR.

So, $V = IR$ or $I = \frac{V}{R}$, using eqn. *(i)*, we get,

$$I = \frac{V_0 \sin \omega t}{R}$$

or

$$I = I_0 \sin \omega t \qquad \ldots (ii)$$

where, $I_0 = \frac{V_0}{R}$ is the *peak value* of alternating current.

If a graph for voltage across the resistor and current in the resistor is plotted with respect to time t, *it is observed that both* voltage and current have zero, maximum and minimum values at the same times respectively. Moreover, comparison of eqn. *(i)* and *(ii)* shows that the current and voltage across the resistor are **in *phase*** with each other (figure 13).

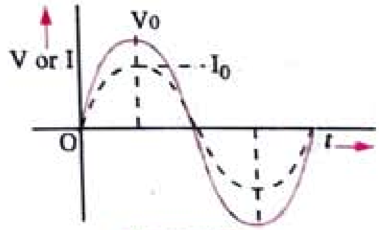


Figure 13

Phasor diagram for pure resistive circuit is shown in figure 14.

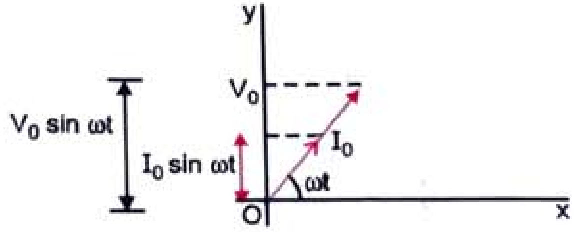


Figure 14

This diagram shows that the phase difference between I and V is zero

ALTERNATING VOLTAGE APPLIED TO AN INDUCTOR

Alternating supply is connected across an inductor. Find an expression for current passing throught the inductor. What is inductive reactance ? Give S.I. unit and dimensional formula of inductive reactance. Discuss behaviour of pure inductance in case of d.c. and a.c. supply.

An alternating source is shown connected to an ideal inductor of inductance L in figure 15. Such a circuit is known as purely ***inductive circuit***.

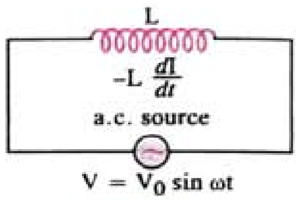


Figure 15

The alternating voltage across the inductor is given by,

$$V = V_0 \sin \omega t \qquad \ldots (i)$$

The induced *e.m.f.* across the inductor $= -L\frac{dI}{dt}$ which opposes the growth of current in the circuit.

As there is no potential drop across the circuit, so,

$$V + \left(-L\frac{dI}{dt}\right) = 0 \quad \text{or} \quad L\frac{dI}{dt} = V \quad \text{or} \quad \frac{dI}{dt} = \frac{V}{L}$$

Using eqn. *(i)*, we get,

$$\frac{dI}{dt} = \frac{V_0}{L}\sin \omega t \quad \text{or} \quad dI = \frac{V_0}{L}\sin \omega t\, dt \qquad \ldots (ii)$$

Integrating both sides, we get,

$$\int dI = \int \frac{V_0}{L}\sin \omega t\, dt = \frac{V_0}{L}\int \sin \omega t\, dt$$

or

$$I = \frac{V_0}{L}\left(-\frac{\cos \omega t}{\omega}\right) = \frac{V_0}{L\omega}(-\cos \omega t)$$

Since $(-\cos \omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$ and peak value, $I_0 = \frac{V_0}{L\omega}$

$$\therefore \quad I = I_0 \sin(\omega t - \pi/2) \qquad \ldots (iii)$$

Comparison of eqn. *(i)* and *(iii)* shows that in this case the ***current lags behind the voltage by an angle of π/2***. In other words, ***phase difference exists between voltage and current in an a.c. circuit having pure inductor.*** [Time diagram in this respect is shown in Figure 16(A)].

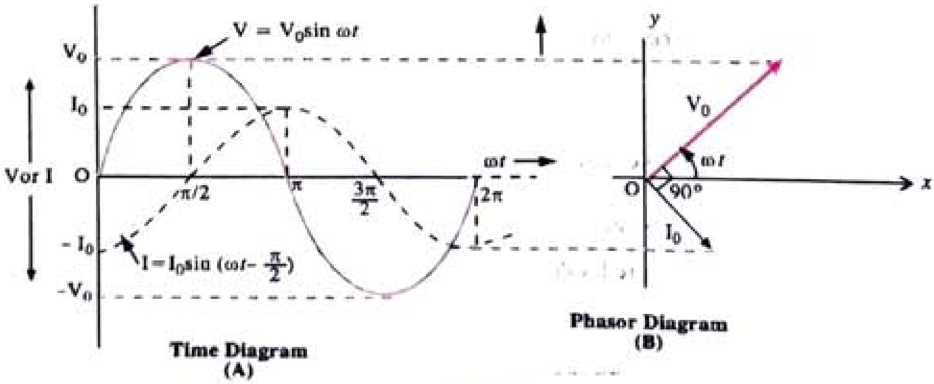


Figure 16

The phasor diagram for purely inductive circuit is shown in figure 16(B).

Inductive Reactance (X_L)

The inductive reactance is the effective opposition offered by the inductor to the flow of current in the circuit.

Comparing $I_0 = \frac{V_0}{L\omega}$ with $I_0 = \frac{V_0}{R}$, we conclude that, ($L\omega$) has the dimensions of resistance. The term ($L\omega$) is known as ***inductive reactance*** represented by X_L. *i.e.*, $X_L = L\omega$.

Behaviour of pure inductor in case of d.c. and a.c.

Inductive reactance, $X_L = L\omega = L \times 2\pi\nu$

For d.c. $\quad \nu = 0$

$\therefore \quad X_L = 0$

Clearly, *pure inductor offers no opposition to the flow of d.c. Hence d.c. can flow easily through an inductor.*

For a.c., $\quad \nu = finite$

$\therefore \quad X_L =$ finite value

Thus, *inductor offers finite opposition to the flow of a.c.*

S.I. unit of inductive reactance

$$X_L = L \times 2\pi\nu$$

$$\therefore \quad \text{S.I. unit for } X_L = \frac{\text{henry}}{\text{second}} = \frac{\text{volt}}{\text{ampere / second}} \times \frac{1}{\text{second}} = \frac{\text{volt}}{\text{ampere}} = \text{ohm}$$

Dimensional formula of inductive reactance = dimensional formula of resistance
$= [ML^2T^{-3}A^{-2}]$

X_L versus ν diagram of a purely inductive circuit is shown in figure 17.

Figure 17 shows that X_L increases linearly with the increase in the frequency of the current.

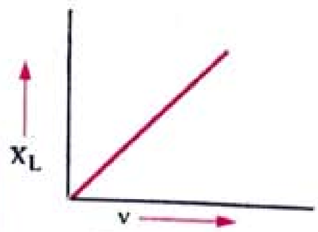


Figure 17

1. ***$X_L = L \times 2\pi\nu$, so X_L increases with the increase in the frequency. For very high frequency, $X_L \to \infty$. Thus, inductor behaves as good as an open circuit (i.e. $I = 0$) for very high frequency.***
2. ***Practically, an inductor always possesses some resistance because of the material of the wire used to make it. Therefore, it is only theoretical to consider an inductor to be pure.***

ALTERNATING VOLTAGE APPLIED TO A CAPACITOR

Find an expression for current in the case of a capacitor put across alternating voltage. What is capacitive reactance. Give S.I. unit and dimensional formula of capacitive reactance.

Alternating source connected to a capacitor is shown in Figure 20. Such a circuit is known as purely ***capacitive circuit***. The capacitor is periodically charged and discharged when alternating voltage is applied to it.

The alternating *voltage* applied across the capacitor is given by

$$V = V_0 \sin \omega t \qquad \ldots(i)$$

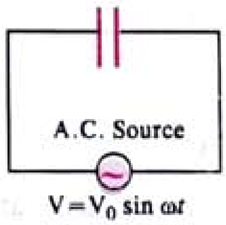


Figure 20

Let q be the charge on the capacitor at any instant.

$\therefore$ Then potential difference across the capacitor, $V_C = \dfrac{q}{C}$

But $\quad V_C = E$ or $\dfrac{q}{C} = V = V_0 \sin \omega t.$

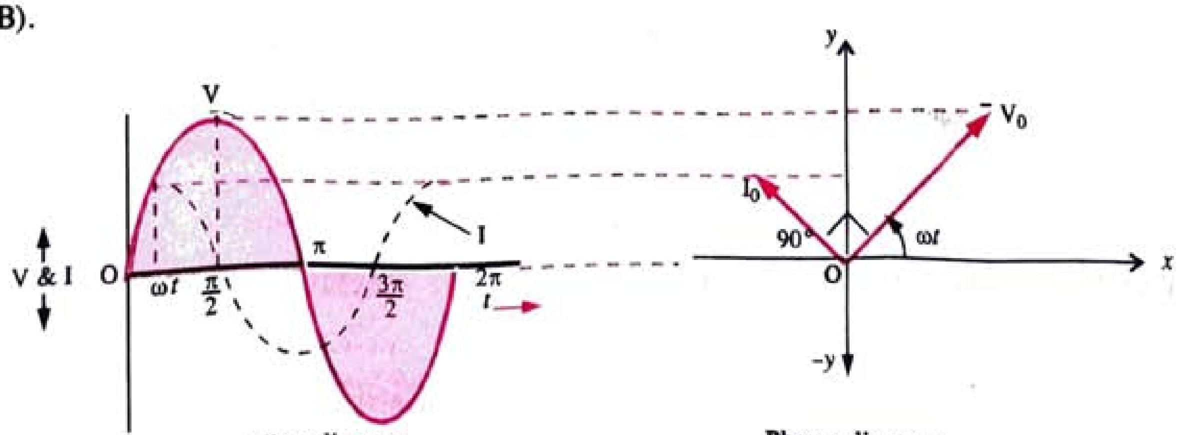
$\therefore \quad q = V_0 C \sin \omega t.$

Now, $\quad I = \dfrac{dq}{dt} = \dfrac{d}{dt}(V_0 C \sin \omega t) = V_0 C (\cos \omega t)\, \omega = \dfrac{V_0}{(1 / C\omega)} \cos \omega t.$

Since $\quad \cos \omega t = \sin\left(\omega t + \dfrac{\pi}{2}\right)$

$\therefore \quad I = I_0 \sin\left(\omega t + \dfrac{\pi}{2}\right)$ where $I_0 = \dfrac{V_0}{\left(\dfrac{1}{C\omega}\right)}$ is the peak value of a.c. $\qquad \ldots(ii)$

Comparison of equations *(i)* and *(ii)* shows that ***current leads the e.m.f. by an angle*** $\pi/2$ in a purely capacitive a.c. circuit. Time diagram for the same is shown in figure [Figure 21(A)]. The phasor diagram for capacitor is shown in Figure 21(B).



Time diagram
(A)

Phasor diagram
(B)

Figure 21

Capacitive Reactance (X_C)

The capacitive reactance is the effective opposition offered by a capacitor to the flow of current in the circuit.

Comparing $I_0 = \dfrac{V_0}{(1 / C\omega)}$ with $I_0 = \dfrac{V_0}{R}$, we conclude that $\left(\dfrac{1}{C\omega}\right)$ has the dimension of resistance. The term $(1/C\omega)$ is known as ***Capacitive reactance*** (X_C).

In ***S.I., unit*** of capacitive reactance is ***ohm***.

Behaviour of a capacitor in case of d.c. and a.c.

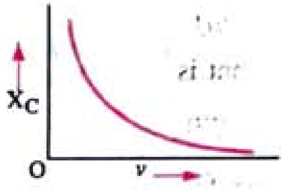


Figure 22

Capacitive reactance, $X_C = \dfrac{1}{C\omega} = \dfrac{1}{C \times 2\pi\nu}$

For D.C., $\quad \nu = 0 \;\therefore\; X_C = \dfrac{1}{0} = \infty.$

Thus, *capacitor offers infinite opposition to the flow of d.c. So direct current cannot pass through a capacitor, however small the capacitance of the capacitor may be.*

For A.C., $\quad \nu$ = finite

$$\therefore \qquad X_C = \frac{1}{\text{finite value}} = \text{smaller value.}$$

Thus, *capacitor offers small opposition to the flow of a.c. So a.c. can be considered to pass through a capacitor easily.*

Variation of X_c with ν is shown in figure 22.

$X_C = \frac{1}{C \times 2\pi\nu}$, i.e. $X_C \propto \frac{1}{\nu}$. For very high frequency. $X_C \longrightarrow 0$. Thus, capacitor behaves as a conductor for high frequency alternating current.

L, C AND R IN SERIES ACROSS AN ALTERNATING SUPPLY

Give phasor diagram and analytical solutions of series LCR circuit across a.c. supply.

A circuit containing inductor of pure inductance (L), capacitor of pure capacitance (C) and resistor of resistance (R), all joined in series across an a.c. supply, is shown in figure 25. Let V be the r.m.s. value of the applied alternating *e.m.f.* to the LCR circuit.

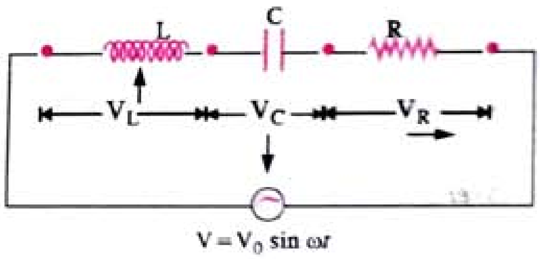


Figure 25

(A) Phasor-diagram Solution of Series LCR circuit

Let I be the r.m.s. value of current flowing through all the circuit elements.

The potential difference across inductor,

$$V_L = I X_L \qquad \ldots(i)$$

This potential difference or voltage leads current I by an angle of $\pi/2$.

The potential difference across C,

$$V_C = I X_C \qquad \ldots(ii)$$

This potential difference or voltage lags behind the current I by an angle of $\pi/2$.

The potential difference across R,

$$V_R = IR \qquad \ldots(iii)$$

This potential difference or voltage is in phase with the current.

Since V_R and I are in phase, so V_R is represented by OA in the direction of I (figure 26).

The current lags behind the potential difference V_L by angle of $\pi/2$, so V_L is represented by OB perpendicular to the direction of I.

The current leads the potential difference V_C by an angle of $\pi/2$, so V_C is represented by OF perpendicular to the direction of I.

Since V_L and V_C are in opposite phase, so their resultant $(V_L - V_C)$ is represented by OD (Here, $V_L > V_C$).

The resultant of V_R and $(V_L - V_C)$ is given by OH. The magnitude of OH is given by

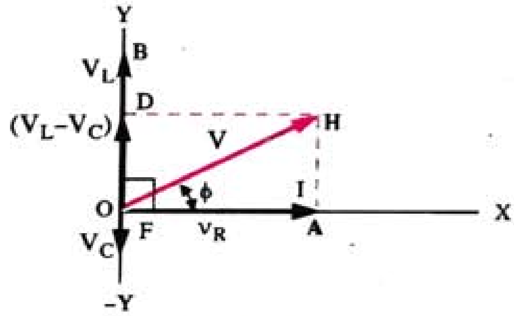


Figure 26

$$OH = \sqrt{(OA)^2 + (OD)^2} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

or
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Using eqns. *(i)*, *(ii)* and *(iii)*, we get,

$$V = \sqrt{I^2R^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

or
$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

But $\frac{V}{I} = Z$, where Z is the effective opposition of LCR circuit to alternating current called ***impedance*** of the circuit.

$\therefore$ ***Impedance (Z) of LCR circuit is given by,***

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \ldots(iv)$$

The relation *(iv)* is known to be the ***general relation for impedance***. Clearly, *(i)* if X_L and X_C both are equal *i.e.*, $X_L = X_C$, then $Z = R$ *i.e.* expression for pure *resistive circuit*, *(ii)* If $X_L = 0$, then $Z = \sqrt{R^2 + X_C^2}$ *i.e.* expression for ***series RC circuit***. *(iii)* If $X_C = 0$, then $Z = \sqrt{R^2 + X_L^2}$ *i.e.* expression for ***series RL circuit***.

Again,
$$I = \frac{V}{Z}$$

$\therefore$
$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$\left(\because\ X_L = L\omega \text{ and } X_C = \frac{1}{C\omega}\right)$$

Let ϕ be the phase angle between V and I, then from Figure 26,

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I X_L - I X_C}{IR} = \frac{X_L - X_C}{R}$$

Putting $X_L = L\omega$ and $X_C = \frac{1}{C\omega}$, we get,

$$\tan\phi = \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R} \qquad \text{...(8)}$$

Also
$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Special Cases

If $V = V_0 \sin\omega t$, then $I = I_0 \sin(\omega t \pm \phi)$ depending upon the values of X_C and X_L

(i) When $X_L = X_C$ or $L\omega = \frac{1}{C\omega}$, then $\tan\phi = 0$ or $\phi = 0°$

Thus, there is no phase difference between current and potential difference. Therefore, ***the given LCR circuit is equivalent to a pure resistive circuit***.

The impedance of such LCR circuit is given by $Z = R$. It is independent of the frequency of alternating current.

(ii) When $X_L > X_C$ or $L\omega > \frac{1}{C\omega}$, $\tan\phi = +\text{ve}$ or $\phi = +\text{ve}$.

This means, the potential difference leads the current by an angle of ϕ. Such LCR circuit is known as ***inductance dominated circuit***.

(iii) When $X_L < X_C$ or $L\omega < \frac{1}{C\omega}$, $\tan\phi = -\text{ve}$ or $\phi = -\text{ve}$.

Thus, the potential difference lags behind the current by an angle of ϕ. Such LCR circuit is called ***capacitance dominated circuit***.

LC-OSCILLATIONS

Electrical oscillations produced by the exchange of energy between a capacitor which stores electrical energy and an inductor which stores magnetic energy are called LC oscillations.

An electric circuit containing an inductor of inductance (L) and a capacitor of capacity (C) connected in parallel is called as ***tank circuit***. This circuit gives rise to oscillations called LC oscillations.

Consider that capacitor of the LC circuit is fully charged [Figure 32(A)]. A potential difference exists between the plates of the capacitor and the energy is stored in the electric field of the capacitor ($U_e = Q_0^2/2C$). Now the capacitor begins to discharge through the inductor and hence current starts flowing through the inductor. As a result of this, magnetic field is set up around the inductor (Figure 32(B)). When the capacitor is discharged completely, the energy is stored in the magnetic field around the inductor $\left(U_m = \frac{1}{2}LI_0^2\right)$. In order words, ***electric energy is completely converted into magnetic energy.***

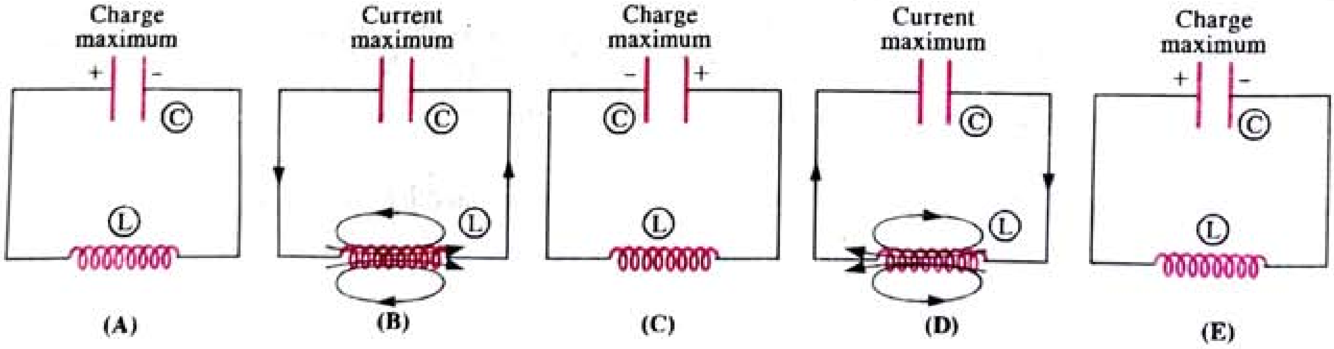


Figure 32

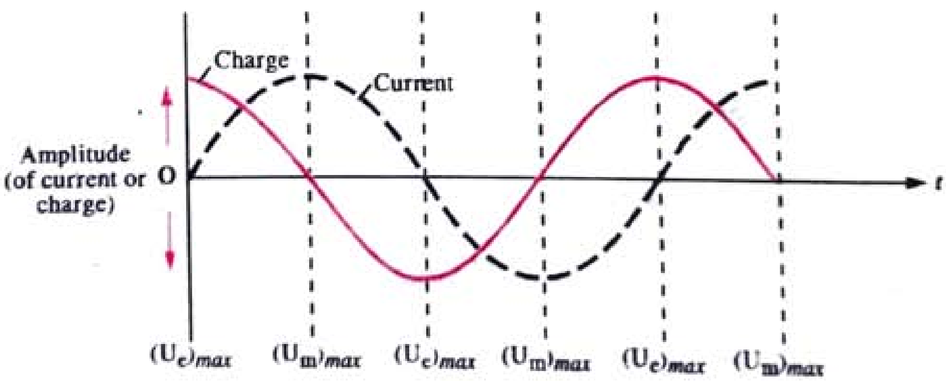
When the magnetic field energy becomes maximum, the capacitor begins to re-charge itself in the opposite direction. Now the energy stored in the magnetic field is converted into the energy stored in the electric field of the capacitor [Figure 32(C)].

When the capacitor is fully re-charged, whole of the energy is stored in the electric field of the capacitor. At this instant, the capacitor is again discharged through the inductor. The current flows through the inductor and energy is stored in the magnetic field around it [Figure 32(D)].

Now again the capacitor is re-charged in the opposite direction [Figure 32(E)] and the energy is stored in the electric field of the capacitor.

In the whole process, there is exchange of electric energy and magnetic energy. *This exchange of energy from electric form to magnetic form gives rise to oscillations called LC oscillations.*

The variation of current with time (gives rise to magnetic energy) and variation of charge with time (gives rise to electrical energy) are shown in Figure 31.



$(U_e)_{max}$ = maximum electrical energy

$(U_m)_{max}$ = maximum magnetic energy

Figure 33

The oscillations having constant amplitude are called ***undamped oscillations.***

Frequency of LC oscillations

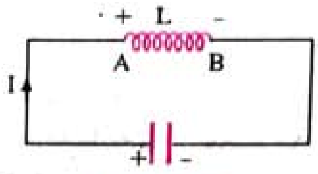


Figure 34

Let a capacitor C be charged and connected to an inductor L at $t = 0$. After some time t, let charge on capacitor be q and I be the current in the parallel LC circuit.

Then

$$\frac{q}{C} = L\frac{dI}{dt}, \quad \text{Since} \quad I = \frac{dq}{dt}$$

so,
$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0 \quad \text{or} \quad \frac{d^2q}{dt^2} + \frac{q}{LC} = 0$$

This equation is comparable to equation of S.H.M* *i.e.*, $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Thus,
$$\omega = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \nu = \frac{1}{2\pi\sqrt{LC}}$$

This is the natural frequency of charge which varies sinusoidally as $q = q_0 \cos(\omega t + \phi)$, where ϕ is a phase constant.

Realistic LC oscillations–damped oscillations.

Damped oscillations are those oscillations whose amplitude decreases with time and ultimately dies off.

In actual LC circuit, the oscillations are not undamped.

An actual LC circuit has finite resistance. Due to this resistance (R), energy is dissipated in the form of heat (I^2Rt), when current (I) flows through the circuit for some time (t). So amplitude of LC oscillations goes on decreasing. There is radiation energy loss also. The damped oscillations are shown in Figure 35.

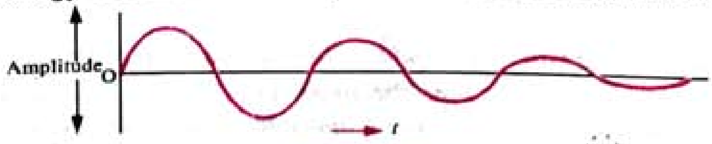


Figure 35

ELECTRICAL RESONANCE

What is meant by series circuit electrical resonance ? Derive an expression for resonant frequency.

Electrical resonance is said to take place in a series LCR circuit, when the circuit allows maximum current for a given frequency of the source of alternating supply for which capacitive reactance becomes equal to the inductive reactance.

Resonance occurs at a particular frequency called **resonant frequency.** At this frequency, the amplitude of the oscillations becomes very large.

The current (I) in a series LCR circuit is given by,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \quad \ldots(i)$$

From eqn. *(i)*, it is clear that *current* I *will be maximum if the impedance (Z) of the circuit is minimum.*

If $X_L = X_C$ for a particular frequency ν_0, then the impedance of LCR circuit is given by Z = R (minimum). Therefore, ***at this particular frequency (ν_0), the current amplitude in LCR circuit becomes maximum.***

The frequency ν_0 is known as the ***resonant frequency*** and the phenomenon is called ***electrical resonance***.

Expression for resonant frequency

For electrical resonance, $X_L = X_C$

or
$$L\omega_0 = \frac{1}{C\omega_0} \quad \text{or} \quad \omega_0^2 = \frac{1}{LC}$$

or
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad (2\pi\nu_0) = \frac{1}{\sqrt{LC}}$$

or
$$\nu_0 = \frac{1}{2\pi\sqrt{LC}} \quad \ldots(ii)$$

Eqn. *(ii)* is the expression for resonance frequency.

The resonance frequency is independent of the resistance (R) of the circuit. The variation of current (I) with the angular frequency (ω) for a series LCR circuit is shown in figure 36. The sharpness of the resonance curve increases with the decrease in the resistance (R). At a resonant frequency, the current in the circuit has maximum value.

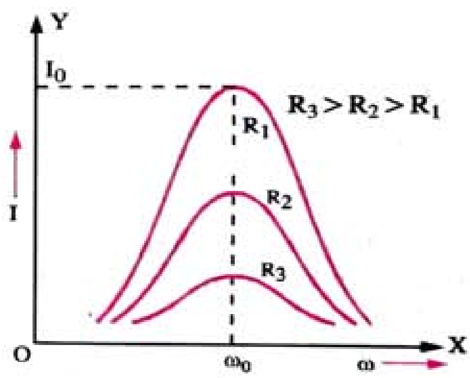


Figure 36

Q-FACTOR

What is Q-factor ? Derive an expression for Q-factor.

Q-factor or quality factor of LCR circuit is defined as the ratio of the voltage developed across inductor (or voltage developed across capicted) at the resonance to the voltage applied (i.e., voltage across resistance) to of the circuit.

The sharpness of a resonance curve is determined by quality factor of the circuit.

i.e.,
$$Q = \frac{V_L \text{ or } V_C}{V} = \frac{IX_L}{IR} \text{ or } \frac{IX_C}{R} = \frac{XL}{R} \text{ or } \frac{X_C}{R}$$

or
$$Q = \frac{L\omega_0}{R} \text{ or } Q = \frac{1}{C\omega_0 R} \quad \ldots(i)$$

Since
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$\therefore$
$$Q = \frac{L}{R} \times \frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} \text{ or } Q = \frac{1}{CR \times \frac{1}{\sqrt{LC}}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Thus,
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q-factor of series resonance circuit is also known as **voltage multiplier** of the circuit.

POWER CONSUMED IN A SERIES LCR CIRCUIT (PREDOMINANTLY INDUCTIVE)

Derive a formula for power consumed in a series LCR circuit which is dominated by an inductor. What is p.f. of an a.c. circuit ?

Power dissipated in an a.c. circuit is the product of r.m.s. value of voltage and component of current in phase with r.m.s. voltage.

Let in a series LCR circuit, the phase difference between current and voltage be ϕ.

The instantaneous values of voltage and current in LCR circuit are given by

$$V = V_0 \sin \omega t \quad \text{and} \quad I = I_0 \sin(\omega t + \phi).$$

$\therefore$ Instantaneous power input to LCR circuit is given by

$$P_i = VI = V_0 I_0 \sin \omega t \sin(\omega t + \phi)$$
$$= V_0 I_0 \sin \omega t [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$
$$[\because \sin(A + B) = \sin A \cos B + \cos A \sin B]$$
$$= V_0 I_0 [\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi]$$
$$= V_0 I_0 \left[\sin^2 \omega t \cos \phi + \frac{2 \sin \omega t \cos \omega t}{2} \sin \phi\right]$$

or
$$P_i = V_0 I_0 \left[\sin^2 \omega t \cos \phi + \frac{\sin 2\omega t \sin \phi}{2}\right] \quad \ldots(i)$$
$$[\because 2 \sin A \cos A = \sin 2A]$$

The ***average power*** over a complete cycle of a.c. through LCR circuit is given by,

$$P = \frac{\int_0^T P_i\, dt}{T} \quad \ldots(ii)$$

Using eqn. *(i)* in eqn. *(ii)*, we get,

$$P = \frac{1}{T}\int_0^T V_0 I_0 \left[\sin^2 \omega t \cos \phi + \frac{\sin 2\omega t \sin \phi}{2}\right] dt$$
$$= \frac{V_0 I_0}{T}\left\{\int_0^T \sin^2 \omega t \cos \phi\, dt + \int_0^T \frac{\sin 2\omega t \sin \phi}{2} dt\right\}$$
$$= \frac{V_0 I_0}{T}\left\{\cos \phi \int_0^T \sin^2 \omega t\, dt + \frac{\sin \phi}{2}\int_0^T \sin 2\omega t\, dt\right\} \quad \ldots(iii)$$

Now
$$\int_0^T \sin^2 \omega t\, dt = \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt = \frac{1}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right]$$
$$= \frac{1}{2}[T - 0] = \frac{T}{2} \quad \ldots(iv)$$

and $$\int_0^T \sin 2\omega t \, dt = 0 \quad \ldots(v)$$

Using eqns. *(iv)* and *(v)* in eqn. *(iii)*, we get

$$P = \frac{V_0 I_0}{T}\left\{\cos\phi \times \frac{T}{2} + \frac{\sin\phi}{2}\times 0\right\} = \frac{V_0 I_0 T}{2T}\cos\phi = \frac{V_0 I_0}{2}\cos\phi.$$

or $$P = \frac{V_0}{\sqrt{2}}\cdot\frac{I_0}{\sqrt{2}}\cos\phi \quad \text{or} \quad P = V_{rms} I_{rms}\cos\phi \quad \ldots(vi)$$

Here $\cos\phi$ is known as ***power factor***.

Here P is called ***True power*** (P) and $I_{rms} V_{rms}$ is called the ***apparent power*** or ***virtual power.***

Special Cases :

1. *A.C. circuit having resistor only*

In such a circuit, phase angle $\phi = 0$.

True power dissipated

$\therefore$ $$P = V_{rms} I_{rms}\cos 0 = V_{rms} I_{rms} \quad (\because \cos 0 = 1)$$

or True power = Apparent power

$\therefore$ Power loss = product of r.m.s. values of voltage and current

2. *A.C. Circuit having pure inductor only*

In such a circuit, the angle between voltage and current is $\pi/2$

i.e. $\phi = \pi/2$.

$\therefore$ Power dissipated, $P = V_{rms} I_{rms}\cos\left(\frac{\pi}{2}\right) = 0$ $\quad \left(\because \cos\frac{\pi}{2} = 0\right)$

Thus, *no power loss takes place in a circuit having pure inductor only.*

3. *A.C. circuit having pure capacitor only*

In such a circuit, the angle between voltage and current is $\pi/2$

i.e. $\phi = \pi/2$

$\therefore$ Power dissipated, $P = V_{rms} I_{rms}\cos(\pi/2) = 0$.

Thus, *no power loss takes place in a circuit having pure capacitor only.*

Power Factor of an A.C. Circuit

Ratio of true power and apparent power (virtual power) in an a.c. circuit is called as power factor of the circuit.

i.e. Power factor, $\cos\phi = \dfrac{P}{V_{rms} I_{rms}} = \dfrac{P}{P_{rms}}$

Power factor ($\cos\phi$) is always positive and not more than 1

(i) For circuit having pure resistor, $\cos\phi = 1$ $\quad (\because \phi = 0)$

(ii) For circuit having pure inductor or pure capacitor, $\cos\phi = 0$ $\quad \left(\because \phi = \frac{\pi}{2}\right)$

(iii) For RC circuit, $$\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad \ldots(i)$$

(iv) For LR circuit, $$\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \quad \ldots(ii)$$

(v) For LCR circuit, $\cos\phi = \dfrac{R}{Z}$ *i.e.*, $$\text{P.F.} = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \quad \ldots(iii)$$

WATTLESS CURRENT AND WATTFUL CURRENT

Compare wattless and wattful currents.

Wattless current *is that component of the circuit current due to which the power consumed in the circuit is zero.*

Wattful current *is that component of current due to which power is consumed in the circuit.*

In an a.c. circuit, the average power consumed over a complete cycle is given by

$$P = V_{rsms} I_{rms} \cos \phi$$

where ϕ is the phase angle between V_{rms} and I_{rms} (Figure 40).

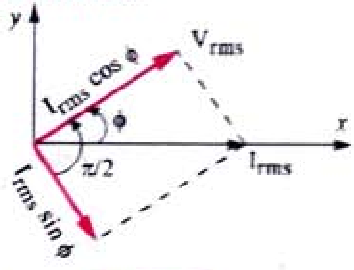


Figure 40

Resolving I_{rms}, we get two components

(i) $I_{rms} \cos \phi$ along V_{rms}.

(ii) $I_{rms} \sin \phi$ perpendicular to V_{rms}.

Now, the average power consumed in the circuit due to component ($I_{rms} \cos \phi$) of current (I_{rms}) is given by

$$P' = V_{rms} (I_{rms} \cos \phi) \cos 0°$$

($\because$ *phase angle between V_{rms} and $I_{rms} \cos \phi$ is zero*)

$$= V_{rms} (I_{rms} \cos \phi)$$

This component of current is called ***wattful component*** or ***wattful current***.

The average power consumed in the circuit due to component $I_{rms} \sin \phi$ of current (I_{rms}) is given by

$$P'' = V_{rms} (I_{rms} \sin \phi) \cos \pi/2$$

($\because$ *phase angle between V_{rms} and ($I_{rms} \sin \phi$) is $\pi/2$*)

or $$P'' = 0$$

Thus, the average power consumed in the circuit due to ($I_{rms} \sin \phi$) component of current is zero.

This component of current is known as ***wattless current***. Power is consumed due to $I_{rms} \cos \phi$ component so it is known as ***wattful component.***

TRANSFORMERS

What is a transformer ? Give main types of transformer. Discuss principle, theory, construction, working and uses of transformer.

Transformer is an electrical device used to convert low alternating voltage at higher current into high alternating voltage at lower current and vice-versa. In other words, a transformer is an electrical device used to increase or decrease alternating voltage.

Types of Transformers

(i) Step-up transformer : *The transformer which converts low alternating voltage at higher current into a high alternating voltage at lower current is called step-up transformer.* In other words, a step up transformer gives increased alternating voltage output. Symbol of step-up transformer is shown in figure 41(A).

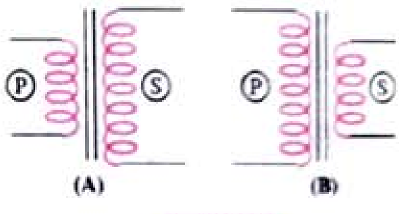


Figure 41

(ii) Step-down transformer : *The transformer which converts high alternating voltage at lower current into a low alternating voltage at higher current is called step-down transformer.* In other words, a step down transformer gives decreased alternating voltage output. Symbol of step down transformer is shown in figure 41(B).

Principle : A transformer is based on the principle of **mutual induction.** *An e.m.f. is induced in a coil, when a changing current flows through its nearby coil.*

Construction : It consists of two separate *coils* of insulated wire wound on same iron *core*. One of the coils connected to a.c. input is called ***primary*** (P) and the other winding giving output is called ***secondary*** (S) winding or coil (Figure 42(A)).

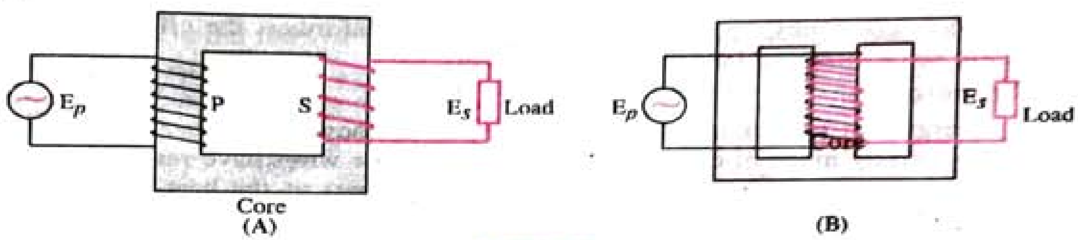


Figure 42

The primary coil is connected to a source of alternating voltage (E_p). The primary coil alongwith a source of alternating voltage is called ***primary circuit***. The output alternating voltage (E_s) is taken across the secondary coil and the *load* is connected to this winding. The secondary coil alongwith load is called ***secondary circuit.***

Figure 42(B) shows an arrangement in which primary and secondary windings are on single limb such that one winding is on the top of other.

Theory : When an alternating source of *e.m.f.* E_p is connected to the primary coil, an alternating current flows through it. Due to the flow of alternating current in the primary coil, an alternating magnetic field is produced and hence changing magnetic flux is linked with the coils. This changing magnetic flux induces an alternating *e.m.f.* in the secondary coil (E_s). Let N_p and N_s be the number of turns in the primary and secondary coils respectively. The iron core is capable of coupling the whole of the magnetic flux ϕ produced by the turns of the primary coil with the secondary coil.

According to Faraday's law of electromagnetic induction, the induced *e.m.f.* in the primary coil,

$$E_p = -N_p \frac{d\phi}{dt} \qquad \ldots(i)$$

The induced *e.m.f.* in the secondary coil,

$$E_s = -N_s \frac{d\phi}{dt} \qquad \ldots(ii)$$

Dividing *(ii)* by *(i)*, we get $\dfrac{E_s}{E_p} = \dfrac{N_s}{N_p}$

where $\dfrac{N_s}{N_p}$ = K, the ***transformation ratio or turns ratio***.

Then

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = K \qquad \ldots(iii)$$

K < 1 for *step down transformer*. In this case $N_s < N_p$ and $E_s < E_p$ *i.e.* output alternating voltage < Input alternating voltage.

K > 1 for *step up transformer*. In this case $N_s > N_p$ and $E_s > E_p$ *i.e.* output alternating voltage is greater than the input alternating voltage.

Relation is valid while following assumptions are made *(i)* Flux leakage is least *(ii)* Primary losses are small *(iii)* secondary losses are small. *(ii)* and *(iii)* is possible if currents are small and winding resistances are small.

For an ideal transformer (in which there are no energy losses), Output power = Input power ...*(iv)*

Let I_p and I_s be the current in the primary and secondary coils respectively.

Then Output power = E_sI_s ; Input power = E_pI_p

From equation *(iv)*, $E_sI_s = E_pI_p$ or $\dfrac{E_s}{E_p} = \dfrac{I_p}{I_s}$ ...*(v)*

In general, $E \propto \dfrac{1}{I}$

Eqn. *(v)* shows that for same power transfer, voltage increases with the decrease in current and vice-versa.

Thus, *whatever is gained in voltage ratio is lost in the current ratio and vice-versa.*

So, a ***step-up transformer*** increases the alternating voltage by decreasing the alternating current and ***step-down transformer*** decreases the alternating voltage by increasing the alternating current.

For a transformer, ***efficiency***, $\eta = \dfrac{\text{Output power}}{\text{Input power}} = \dfrac{E_sI_s}{E_pI_p}$

For an ***ideal transformer***, ***efficiency***, η is 100%. But in a real transformer, the efficiency varies from 90-99%. This indicates that there are some *energy losses* in the transformer.

Energy Losses in a Transformer

(i) Copper losses. Energy lost in windings of the transformer is known as *copper loss*. Primary and secondary coils of a transformer are generally made of copper wires*. These copper wires have resistance (R). When current (I) flows through these wires, power loss (I^2R) takes place. This loss appears as the heat produced in the primary and secondary coils. Copper losses can be reduced by using thick wires for the windings.

(ii) Flux leakage losses. In actual transformer, the coupling between primary and secondary coils is not perfect. It means the magnetic flux linked with the primary coil is not equal to the magnetic flux linked with the secondary coil. So a certain amount of electrical energy supplied to the primary coil is wasted.

(iii) Iron losses. These are grouped as below :

(a) Eddy Currents losses. When a changing magnetic flux links with the iron core of the transformer, *eddy currents* are set up. These eddy currents in the iron core produce heat which leads to the wastage of energy. This energy loss is reduced by using ***laminated iron cores.*** (Figure 43).

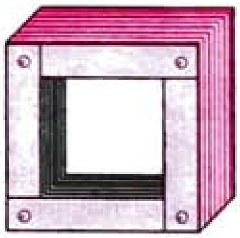


Figure 43

Eddy currents are reduced in a laminated core because their paths are broken as compared to solid core as shown in figure 44.

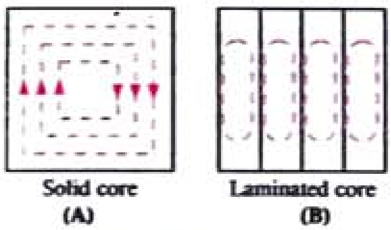


Figure 44

(b) Hysteresis Losses. When alternating current passes through the primary coil of the transformer, the iron core of the transformer is magnetised and demagnetised over a complete cycle. Some energy is lost in magnetising and de-magnetising the iron core. The energy loss in a complete cycle is equal to area of the hysteresis loop.

This energy loss can be minimized by using suitable material having narrow hysteresis loop for the core of a transformer.

(iv) Losses due to Vibration of Core. A transformer produces humming noise due to ***magnetostriction effect.*** Some electrical energy is lost in the form of mechanical energy to produce vibration in the core.

Uses of a Transformer for Long Distance A.C. Supply

Electrical energy generated at a power station is transmitted to the consumer (*i.e.* a house, a shop or a factory) through wires. These wires have a resistance (R). So when current I flows through these wires for time t, the electric energy is wasted in the form of heat energy (I^2Rt) in the wires. Thus power loss (I^2R) during the long distance supply of alternating current depends upon :

(i) Resistance (R) of the wires *(ii)* square of current (I^2)

The power loss can, therefore, be reduced either by reducing the resistance of the transmission wire or by decreasing the magnitude of the current flowing through it.

(i) Resistance of a wire or a conductor is given by,

$$R = \rho \frac{l}{A}$$

where, l is the length of the wire, ρ is resistivity of the material used and A is the area of cross-section of the wire.

Thus for a selected conductor, resistance can be decreased either by decreasing length, l or by increasing area, A. Since the distance between the power station and the consumer's site is fixed (*i.e.* l = constant), so *R can be decreased by increasing the value of A.*

This can be done by using very thick wires. But it is not economical as the cost of manufacturing and installing such long and thick wires will be very high.

(ii) To decrease the power loss, the another option is to decrease the *current* flowing through the wires. This can be achieved by transmitting A.C. at very high voltage using a *step-up transformer.*

The electric power generated at the power station is fed to the primary coil of a step-up transformer. The output (across secondary coil) of the step-up transformer supplies very high voltage at low alternating current.

This low current at high voltage is carried by the transmission wire to the sub-station near to the consumer's site. Thus the power loss (I^2R) in the form of heat produced in the wire decreases considerably.

After transmission to the sub-station, a *step down transformer* is used to step down the alternating voltage to a value safe for distribution as shown in figure 45.

Figure 45

Thus, *transformer is an electrical device used for economical transmission of the electric energy to very long distances.*

A.C. GENERATOR (ALTERNATOR/A.C. DYNAMO)

An electrical machine used to convert mechanical energy into electrical energy is known as A.C. generator/alternator.

Principle : It works on the principle of ***electromagnetic induction*** *i.e., when a coil is rotated in uniform magnetic field, an induced e.m.f. is produced in it.*

Construction : The main components of a.c. generator (figure 33) are given below :

(i) ***Armature*** **:** Armature coil (ABCD) consists of large number of turns of insulated copper wire wound over a soft iron core.

(ii) ***Strong field magnet :*** A strong permanent magnet or an electro magnet whose poles (N and S) are cylindrical in shape used as a field magnet. The armature coil rotates between the pole pieces of the field magnet. The uniform magnetic field provided by the field magnet is perpendicular to the axis of rotation of the coil.

(iii) ***Slip Rings :*** The two ends of the armature coil are connected to two brass slip rings R_1 and R_2. These rings rotate alongwith the armature coil.

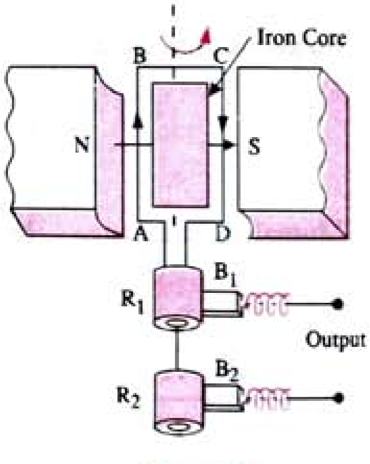


Figure 33

(iv) ***Brushes :*** Two carbon brushes (B_1 and B_2), are pressed against the slip rings. The brushes remain fixed while slip rings rotate alongwith the armature. These brushes are connected to the load through which the output is obtained.

Working : When the armature coil ABCD rotates in the magnetic field provided by the strong field magnet, it cuts the magnetic lines of force. The magnetic flux linked with the coil changes due to the rotation of the armature and hence induced e.m.f. is set up in the coil. The direction of the induced e.m.f. or the current in the coil is determined by the ***Fleming's right hand rule.***

The current flows out through the brush B_1 in one direction of half of the revolution and through the brush B_2 in the next half revolution in the reverse direction. This process is repeated. Therefore, *e.m.f.* produced is of alternating nature.

Theory : Consider the plane of the coil to be perpendicular to the magnetic field $\vec{B}$.

Let the coil be rotated anti-clockwise with a constant angular velocity ω (Figure 34). Then the angle between the normal to the coil and $\vec{B}$ at any instant t is given by

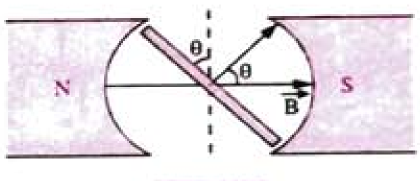


Figure 34

$$\theta = \omega t \qquad \ldots(i)$$

$\therefore$ The component of magnetic field normal to the plane of the coil $= B\cos\theta = B\cos\omega t$.

Magnetic flux linked with a single turn of the coil $= (B\cos\omega t)\,A$, where A is the area of the coil.

If the coil has n turns, then the total magnetic flux linked with the coil is given by

$$\phi_B = n(B\cos\omega t)\,A = nBA\cos\omega t \qquad \ldots(ii)$$

According to Faraday's laws of electromagnetic induction, the induced e.m.f. produced in the coil is given by

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(nBA\cos\omega t) = -nBA(-\omega\sin\omega t)$$

or

$$\varepsilon = nBA\,\omega\sin\omega t \qquad \ldots(28)$$

This is the expression for the induced *e.m.f.* produced in the coil at any instant t.

Induced *e.m.f.* will be maximum (*i.e.* $\varepsilon = \varepsilon_0$) if $\sin\omega t = 1$.

$\therefore$ From eqn. (28), we get maximum value of *e.m.f.*

$$\varepsilon_0 = nBA\omega \qquad \ldots(29)$$

Substituting the value of eqn. (29) in eqn. (28), we get,

$$\varepsilon = \varepsilon_0\sin\omega t$$

or

$$\varepsilon = \varepsilon_0\sin\theta \qquad \ldots(30)$$

Instantaneous current in the circuit is given by,

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R}\sin\omega t \qquad \ldots(31)$$

$$\varepsilon = \varepsilon_0 \sin \omega t$$

or $$\varepsilon = \varepsilon_0 \sin \theta \qquad ...(30)$$

Instantaneous current in the circuit is given by,

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \sin \omega t \qquad ...(31)$$

where R is the resistance of the circuit.

Variation of induced e.m.f. with different positions of the coil w.r.t. the magnetic field

(i) When $\theta = 0°$ *i.e.* the plane of the coil is perpendicular to the magnetic field, then

$$\varepsilon = \varepsilon_0 \sin 0° = 0 \qquad (\because \sin 0° = 0).$$

(ii) When $\theta = 90°$ *i.e.* the plane of the coil is along the direction of the magnetic field, then

$$\varepsilon = \varepsilon_0 \sin 90° = \varepsilon_0 \qquad (\because \sin 90° = 1)$$

(iii) When $\theta = 180°$ *i.e.* the plane of the coil is perpendicular to the magnetic field, then

$$\varepsilon = \varepsilon_0 \sin 180° = 0 \qquad (\because \sin 180° = 0)$$

(iv) When $\theta = 270°$ *i.e.* the plane of the coil is along the direction of magnetic field, then

$$\varepsilon = \varepsilon_0 \sin 270° = -\varepsilon_0 \qquad (\because \sin 270° = -1)$$

(v) When $\theta = 360°$ *i.e.* the plane of the coil is perpendicular to the direction of the magnetic field, then

$$\varepsilon = \varepsilon_0 \sin 360° = 0 \qquad (\because \sin 360° = 0)$$

Thus, when the coil is rotated from its position at right angle to the magnetic field through 180°, the induced e.m.f. and the current increases from zero to maximum (ε_0) and then decreases from maximum to zero in the same direction.

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