Define electromagnetic induction.

The phenomenon of producing induced e.m.f and hence induced current in a closed circuit due to the change in magneticfield or change in magnetic flux linked with the closed circuit is known as electromagnetic inductin.

MAGNETIC FLUX

Define magnetic flux. Give special cases when angles between normal vector and uniform magnetic field are 0° and 90°.

Magnetic flux (ϕ) through any surface is defined as the total number of magnetic lines passing through that surface.

Consider a small surface of area A. Let $\hat{n}$ be the unit vector which is drawn normal to the plane of the surface. If θ *is the angle between* $\hat{n}$ *and the uniform magnetic field* $\vec{B}$ (Figure 1), then the magnetic flux (ϕ) through the surface is given by,

$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta \quad \ldots(1)$$

or

$$\phi_B = (B\cos\theta)\, A \quad \ldots(2)$$

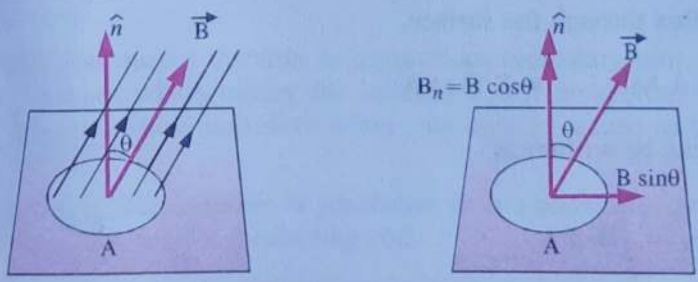


Figure 1 Figure 2

Now B cos θ is the component of the magnetic field normal to the plane of the surface and can be represented as B_n (Figure 2).

Then eqn. (2) can be written as, $\phi_B = B_n A$ ...*(ii)*

Thus, *magnetic flux over a given surface is defined as the product of the area of the surface and the component of the magnetic field* **(B_n)** *normal to the plane of the surface.*

Special Cases :

(i) When $\theta = 0°$, *i.e.* uniform magnetic field is acting perpendicular to the plane of the surface, then

$$\phi_B = BA\cos 0° \quad \text{or} \quad \phi_B = BA = \text{maximum} \quad (\because \cos 0° = 1)$$

Thus, magnetic flux through a given surface is maximum *i.e.* maximum number of magnetic field lines pass through the given surface, when $\theta = 0°$.

(ii) When $\theta = 90°$, *i.e.* uniform magnetic field is along the plane of the surface, then

$$\phi_B = BA\cos 90° = 0 \quad (\because \cos 90° = 0)$$

Thus, the magnetic flux through a given surface is zero, when $\theta = 90°$.

Definition of Magnetic flux density *i.e.*, Strength of Magnetic field or Magnetic Induction (B) :

Since $\phi_B = BA \quad \therefore \quad B = \dfrac{\phi_B}{A}$

Thus, *magnetic flux density (B) is defined as the magnetic flux (associated normally) per unit area.*

Units of magnetic flux

In SI, magnetic flux is measured in **weber** (Wb)

Since $\phi_B = B_n A$

$\therefore$ 1 weber = 1 tesla × 1 metre^2

i.e., 1 Wb = 1 T m^2

In c.g.s. system, unit of magnetic flux is **maxwell**

1 maxwell = 10^{-8} weber

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION (EMI)

State and explain Faraday's Laws of electromagnetic induction.

Faraday's First Law of EMI (Qualitative Law) :

Whenever magnetic flux linked with a conductor (or coil) changes, an e.m.f. is induced in it.

This induced e.m.f. lasts so long as the change in magnetic flux continues in the coil.

Faraday's Second Law of EMI (Quantitative Law) :

The magnitude of the induced e.m.f. is directly proportional to the rate of change of magnetic flux linked with the conductor (or coil).

Expression for Induced e.m.f.

Let ϕ_1 be magnetic flux linked with a closed circuit or coil at time t_1 and ϕ_2 be magnetic flux linked with a closed circuit or coil at time t_2

$\therefore$ Then change in magnetic flux in the time interval $(t_2 - t_1) = (\phi_2 - \phi_1)$

$$\therefore \quad \text{Induced } e.m.f., \ |\epsilon| \propto \frac{(\phi_2 - \phi_1)}{(t_2 - t_1)} \quad \text{or} \quad \epsilon = k\frac{(\phi_2 - \phi_1)}{(t_2 - t_1)}$$

where k is constant of proportionality. In SI, $k = 1$

$$\therefore \quad |\epsilon| = \frac{(\phi_2 - \phi_1)}{(t_2 - t_1)}$$

or

$$|\epsilon| = \frac{d\phi_B}{dt} \qquad ...(4)$$

If a coil has N number of turns, then eqn. (4) can be written as

$$|\epsilon| = N\frac{d\phi_B}{dt}$$

Induced current in the closed circuit is given by

$$I = \frac{e.m.f. \text{ (Induced)}}{\text{Resistance of the circuit}} = \frac{|\epsilon|}{R} = \frac{d\phi_B}{R\,dt}.$$

Limitation of Second Law of EMI

This law gives the magnitude of induced *e.m.f.* but does not give the polarity of induced *e.m.f.* or direction of induced current.

LENZ' LAW

State and explain Lenz' Law.

According to Lenz'* law : ***The polarity of induced e.m.f. is such that it opposes the cause which produces it.***

i.e.,
$$\epsilon = -N\frac{d\phi_B}{dt}$$

Negative sign shows that induced e.m.f. opposes the change in magnetic flux. It was explained by Lenz. Consider a coil or closed loop and a bar magnet.

(i) When N pole of a bar magnet is brought *towards* a closed loop (figure 6A), *counter clockwise* current is produced in the loop. As per *Right handed screw rule,* this induced current produces a magnetic field in a direction which opposes the motion of the bar magnet *i.e.* nearer face of the loop as shown in figure 7 (A) behaves as N pole. ***Clock rule of fitting*** ***in the loop*** will give the direction of current induced in the loop. North pole of the loop will repel the N-pole of bar magnet.

(ii) When N pole of a magnet is taken away from the loop, the nearer face of the loop behaves as S pole (Figure 7B). ***Clock rule of fitting*** Ⓢ in the loop will give the direction of current induced in the loop. The induced current in the loop opposes the motion of the bar magnet away from the loop.

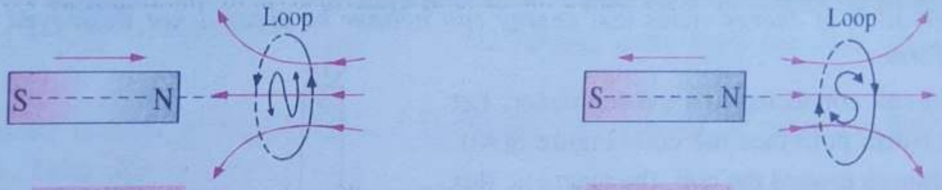


Figure 7 (A) **Figure 7 (B)**

It can be concluded that current due to induced e.m.f. produces magnetism such that it opposes the cause (i.e., motion of the magnet) of producing e.m.f.

NOTE

(i) If magnetic flux linked with a closed loop increases, current in the loop is anticlockwise.

(ii) If magnetic flux linked with a closed loop decreases, current in the loop is clockwise.

LENZ' LAW IS IN ACCORDANCE WITH THE LAW OF CONSERVATION OF ENERGY

Show that Lenz' law is in accordance with the law of conservation of energy.

The law of conservation of energy states that ***energy can neither be created nor destroyed but can change from one form to another form.***

Consider a closed coil connected to a galvanometer. Let a bar magnet with its North pole face the coil (Figure 8(A)). When the bar magnet moves toward the coil, the magnetic flux linked with it goes on increasing and hence induced *e.m.f.* is produced in the coil. Due to this induced *e.m.f.*, current flows through the coil and galvanometer shows deflection.

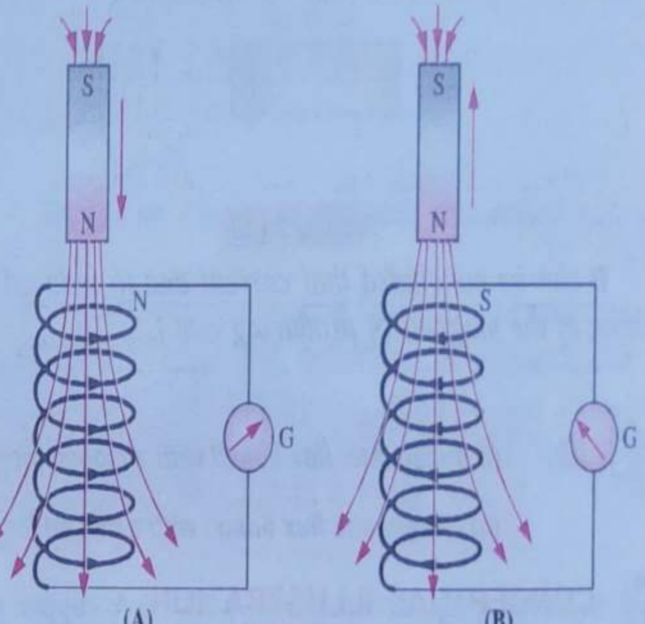


Figure 8

According to **Lenz' law**, *the direction of induced e.m.f. and hence induced current is such that it opposes the cause (say motion of the magnet) which produces it*. To oppose the cause (motion of the magnet), the upper face of the coil acquires ***North*** polarity. The north pole of the magnet and the north pole of the coil repel each other. To move the magnet towards the coil, mechanical work has to be done to overcome the *force of repulsion* between the north poles of the bar magnet and the coil. *This mechanical work done is converted into electrical energy* (indicated by the deflection in the galvanometer). This electrical energy is converted into heat energy due to Joule's effect.

Similarly, when the magnet moves away from the coil (Figure 8(B)), the upper face of the coil acquires ***South*** polarity. In this case, the induced e.m.f. will oppose the outward motion (*cause*) of the magnet. Again mechanical work has to be done to overcome the *force of attraction* between North pole of the magnet and South pole of the coil. This work done is converted into electrical energy.

If the magnet is not moved, no mechanical work is done, then no *e.m.f.* (thus no current) is induced in the coil *i.e.*, no electrical energy is produced.

Thus, ***Lenz' law is in accordance with the law of conservation of energy.***

METHODS OF PRODUCING INDUCED *E.M.F.*

Describe various methods of producing induced e.m.f.

From Faraday's Law, induced e.m.f. is given by, $\varepsilon = -N\frac{d\phi}{dt}$, where magnetic flux, $\phi = BA\cos\theta$.

This shows that e.m.f. (ε) depends upon *(i)* number of turns N and *(ii)* rate of change of magnetic flux *(ϕ)*

Magnetic flux can be changed by

(i) *changing the strength of the magnetic field* (B).

(ii) *changing the orientation* (θ) *of the coil with respect to the magnetic field.*

(iii) *changing the area* (A) *of the coil.*

(A) Induced e.m.f. by changing the magnetic field.

In Faraday's experiment 1 (Figure 4), when magnet is moved towards the coil, the strength of magnetic field at any point of the coil increases as more and more magnetic field lines pass through the coil. The galvanometer deflection indicates the presence of the current in the coil. On the other hand, when magnet is moved away from the coil, strength of the magnetic field linking with the coil decreases. Again, galvanometer shows deflection. It indicates the presence of current in the coil. Thus, e.m.f./ current is induced in the coil by changing the strength of the magnetic field.

(B) Induced e.m.f. by changing the area (Motional e.m.f.) : Induced e.m.f. produced by changing the area of a closed circuit by the movement of the circuit or part of it through a uniform magnetic field is known as *motional e.m.f.*

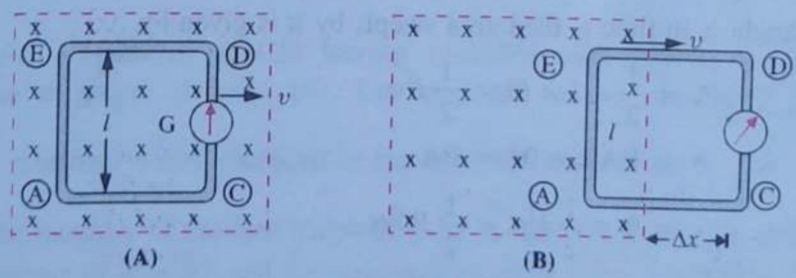


Figure 11

Consider a closed loop or circuit ACDE placed in a uniform magnetic field $\vec{B}$ directed into the page. Let the loop moves with a uniform velocity υ as shown in the figure 11.

Change in magnetic flux linked with the closed circuit in the time interval Δt, when coil moves out of magnetic field through a small distance Δx is given by

$$\Delta\phi_B = -B \times \text{change in area of the circuit} = -B \times l \times \Delta x$$

or $$\Delta\phi_B = -Bl\Delta x$$

Negative sign shows that area of the closed circuit inside the magnetic field decreases.

According to Faraday's law, induced e.m.f. in the circuit is given by

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = \frac{Bl\Delta x}{\Delta t}. \quad \text{Since } \frac{\Delta x}{\Delta t} = \upsilon$$

$$\therefore \quad \varepsilon = Bl\upsilon \qquad \ldots(5)$$

Eqn. (5) gives the expression for motional *e.m.f.*

If R be the resistance of the closed circuit, then the *induced current* flowing in the circuit is given by

$$I = \frac{\varepsilon}{R} = \frac{Bl\upsilon}{R} \qquad \ldots(6)$$

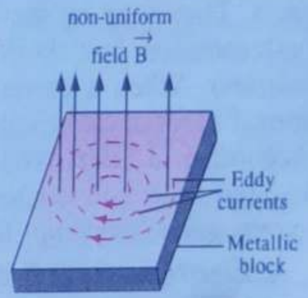
EDDY CURRENTS (FOUCAULT CURRENTS)

What is eddy current ? Give applications of eddy current. What are the disadvantages of eddy current and how to minimise them.

*The induced circulating (or whirling) currents produced in a conductor itself due to change in magnetic flux linked with the conductor are called eddy currents.**

These currents were discovered by Foucault, so they are also known as ***Foucault Currents.*** The direction of eddy currents is given by Lenz' law.

Eddy currents produced in a metallic block placed in a *non-uniform magnetic field* are shown in Figure 16.



Advantages/Applications of Eddy currents

1. ***Induction Furnace.*** *Induction furnace is based on the heating effect of eddy currents.* The metallic block to be melted is placed in a high frequency changing magnetic field. Strong eddy currents are induced in the block. Due to the high resistance of the metal, a large amount of heat is produced in it. This heat ultimately melts the metallic block. The induction furnace is used to separate metals from their ores and to make alloys.

2. ***Speedometer.*** *Speedometer is a device used to measure the instantaneous speed of a vehicle.*

In a speedometer, a small magnet is attached to the axle of the wheel. This magnet is surrounded by an aluminium drum and it rotates according to the speed of the vehicle inside the aluminium drum. Due to the rotation of the magnet, magnetic flux linked with the aluminium drum changes and hence eddy currents are produced in it. A pointer attached to the drum is deflected in the direction of the rotation of the drum. This speed is measured which corresponds to the deflection of the pointer.

3. ***Dead beat galvanometer.*** When current is passed through the coil of a galvanometer, the coil and the pointing needle (*i.e.*, pointer attached to the coil) get deflected in the magnetic field (*provided by the two pole pieces of a magnet*). They keep on oscillating for a long time before they come to rest. In order to stop the motion of the coil of the galvanometer in a shorter interval of time, the coil is wound on a non ferrous metallic frame (*made of copper or aluminium*). When a current is passed through the coil, the coil along with the metallic frame oscillates. Due to rotational motion, the magnetic flux linked with the metallic frame changes and hence eddy currents are produced in it. According to Lenz' law, these eddy currents oppose the motion of the frame and hence dampen the motion of the coil. Thus, the coil comes to rest in a shorter interval of time. This type of stopping or damping is called *Electromagnetic damping*. Such a galvanometer is called dead beat galvanometer.

4. ***Electromagnetic Brakes in electric trains.*** An electromagnet is attached to the axle of the electric train. The relative motion between the magnet and rails induces eddy currents in the rails. When the electric current passing through the solenoid of electromagnet is increased, the direction of eddy currents opposes the drag force on the moving train. Hence, the train slows down and ultimately comes to halt.

5. ***Diathermy.*** Eddy currents are used for the localised heating of tissues in human body. This type of treatment is called diathermy.

6. ***Analogue Energy meters.*** Concept of eddy currents is used in energy meters to record the consumption of electricity. Induced eddy currents are produced in aluminium disc used in these meters due to varying magnetic field. It rotates due to eddy currents produced in it.

7. ***Induction motor.*** Eddy currents are used to rotate the short circuited rotor of an induction motor. Ceiling fans have also induction motors which run on single phase alternating current. Eddy currents in the rotor interact with the magnetic field of the rotor producing rotating force on the rotor.

8. ***Induction cooking.*** Eddy currents produced in copper pots are used for cooking food. This process of cooking is known as induction cooking.

9. ***Inductothermy.*** The process of producing heating effect due to eddy current for localised heating of tissues is called inductothermy. In this process, a coil of many turns is placed around the affected part of human body (without touching the body part). An alternating current of high frequency is passed through the coil. It may be noted that inductothermy is far better treatment than the electrode diathermy.

Disadvantages/Undesirable Effects of Eddy Currents

(i) The production of eddy currents in a metallic block leads to the loss of electric energy in the form of heat.
(ii) The heat produced due to eddy currents breaks the insulation used in the electrical machines or appliances.
(iii) Eddy currents may cause unwanted *damping effect*.

Minimisation of losses due to eddy currents

The metallic cores are used in electrical devices like transformer, dynamo, choke etc. Due to changing magnetic field, large eddy currents are produced in the core which cause large amount of heat in the core. It results into loss of useful energy.

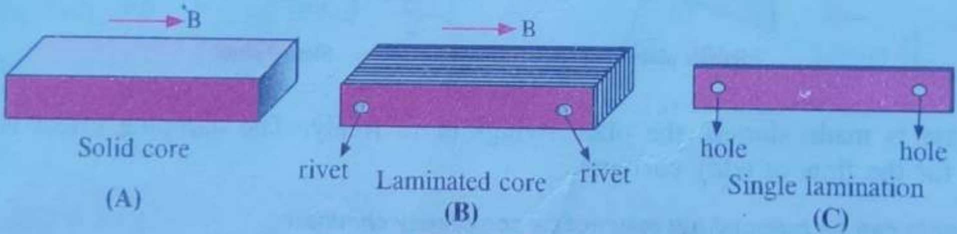


Figure 19

To minimise losses due to eddy currents, the solid metallic core (Figure 19A) is replaced with a large number of thin sheets (Figure 19B and 19C). These sheets are electrically insulated and are called ***laminations***. These laminations are tightly rivetted to form a core called ***laminated core***. These sheets are arranged parallel to the magnetic flux. The insulation breaks the paths of the eddy currents and keeps the eddy currents restricted to the individual sheets. As a result of this, eddy current produced in one sheet is not added in the current produced in the other sheet. In other words, paths of the eddy currents in the core as a whole are broken so the eddy currents are reduced to a large extent.

Hence, the loss ($H = I^2Rt$) of electrical energy due to eddy current is minimised.

SELF INDUCTION AND SELF INDUCTANCE

Self induction is the property of a coil by virtue of which it opposes the growth or decay of the current flowing through it.

Consider a coil (Figure 20) connected to a battery through a key (K).

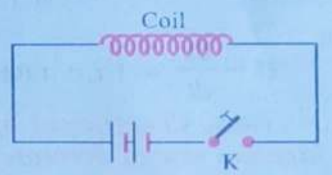


Figure 20

When the key (K) is pressed, current in the coil starts increasing. Due to increasing current in the coil, the magnetic field and hence flux linked around the coil also increases. As a result of this, induced e.m.f. is set up in the coil. According to Lenz' law, the direction of induced e.m.f. is such that it opposes the growth of current in the coil. This delays the current to acquire the maximum value in the coil.

When the key (K) is released, the current in the coil starts decreasing. So the magnetic flux linked with the coil decreases. As a result of this change in magnetic flux, ***induced e.m.f.*** *is set up in the coil itself*. According to Lenz' law, the direction of induced e.m.f. is such that it opposes the decay of current in the coil. This delays the current to acquire minimum or zero value in the coil.

This property of the coil which opposes the growth or decay of the current is called **Self Induction.**

Self Induction is also known as Inertia of electricity **as it opposes the growth or decay of the current in the circuit.**

Coefficient of Self Induction or Self Inductance

Let current I flows through a coil at any instant, then the magnetic flux (ϕ) linked with the coil is found to be proportional to the strength of the current (I)

i.e. $\phi \propto I$ or $\phi = LI$ ...(8)

where L is the constant of proportionality and is known as ***Co-efficient of self induction*** or simply ***self inductance***.

If $I = 1$, then from eqn. (8),

$$L = \phi \quad \ldots(9)$$

Thus, ***Co-efficient of Self Induction of a coil i.e., self inductance is defined as the magnetic flux linked with the coil when unit current flows through it.***

Also, according to *Faraday's law of electromagnetic induction*, induced e.m.f. in the coil is given by

$$\varepsilon = -\frac{d\phi}{dt}.$$

Using eqn. (8), we get

$$\varepsilon = -\frac{d}{dt}(\mathrm{LI})$$

or $$\varepsilon = -\mathrm{L}\,\frac{d\mathrm{I}}{dt} \qquad \ldots(10)$$

If $-\frac{d\mathrm{I}}{dt} = 1$ *i.e.* rate of decrease of current is unity, then from eqn. (10), we get, $\mathrm{L} = \varepsilon$

Thus, ***Co–efficient of self induction of a coil is defined as the induced e.m.f. produced in the coil through which the rate of decrease of current is unity.***

Units of Self Inductance

SI Unit of self inductance is ***henry*** **(H).**

Since $\mathrm{L} = \dfrac{\varepsilon}{\dfrac{d\mathrm{I}}{dt}}$ $\quad\therefore\quad$ $1 \text{ henry (H)} = \dfrac{1 \text{ volt}}{1 \text{ ampere second}^{-1}} = 1\mathrm{V\,A^{-1}s}$

Also $\mathrm{L} = \dfrac{\phi}{\mathrm{I}}$ $\quad\therefore\quad$ $1 \text{ henry (H)} = \dfrac{1 \text{ weber}}{1 \text{ ampere}} = 1\mathrm{Wb\,A^{-1}}$

i.e. $1\mathrm{H} = 1\,\mathrm{VA^{-1}s} = 1\,\mathrm{Wb\,A^{-1}}$

Smaller units of Self inductance are : $1\mathrm{mH} = 10^{-3}$ H and $1\,\mu\mathrm{H} = 10^{-6}$ H.

Inductor : An element of an electric circuit like a tightly wound coil of insulated wire which opposes the change in current flowing through it is called an **inductor**. The symbol of an inductor in an electric circuit is shown as follows :

NOTE

An ideal inductor has high value of self inductance and zero ohmic resistance.

NUMERICAL ILLUSTRATION. Calculate the magnitude of induced e.m.f. in a coil of inductance 1 mH in which current is changing from 0·1 A to 2·1 A in 0·01s.

Solution. Here $\mathrm{L} = 1\mathrm{mH} = 10^{-3}\,\mathrm{H},\ d\mathrm{I} = 2{\cdot}1 - 0{\cdot}1 = 2{\cdot}0\,\mathrm{A},$

$= 0{\cdot}01$ s

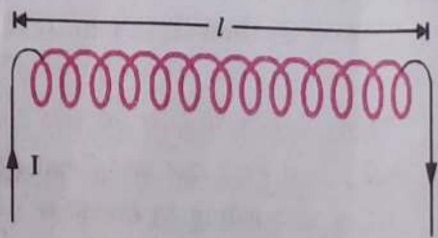
using , $|\varepsilon| = \mathrm{L}\dfrac{d\mathrm{I}}{dt}$, we get

$$|\varepsilon| = \frac{10^{-3}\times 2}{0{\cdot}01} = \mathbf{0{\cdot}2\ V.}$$

SELF INDUCTANCE OF A SOLENOID

Derive a relation for self inductance of a solenoid.

Consider a long solenoid of length l, area of cross section A and number of turns per unit length n. Let I be the current flowing through the solenoid.



The magnetic field inside this solenoid is uniform and given by

$$\mathrm{B} = \mu_0 n\mathrm{I}$$

Total number of turns in the solenoid $\mathrm{N} = nl$.

Now the magnetic flux linked with each turn of the solenoid = B × A = $\mu_0 n$I A

∴ Total magnetic flux linked with the whole solenoid,

$$\phi = \text{magnetic flux linked with each turn} \times \text{number of turns in the solenoid}$$

or $$\phi = \mu_0 n\mathrm{IA} \times nl = \mu_0 n^2\,\mathrm{IA}l \qquad \ldots(i)$$

Also, $$\phi = \mathrm{LI} \qquad \ldots(ii)$$

From (i) and (ii), we get

$$\mathrm{LI} = \mu_0 n^2 \mathrm{IA}l$$

∴ $$\mathrm{L} = \mu_0 n^2 \mathrm{A}l \qquad \ldots(11)$$

Since $n = \frac{\mathrm{N}}{l}$. Hence eqn. (11) becomes

$$\mathrm{L} = \mu_0 \frac{\mathrm{N}^2}{l} \mathrm{A} \qquad \ldots(12)$$

Thus, self inductance of an air cored solenoid (L) depends on *(i) the total number of turns (N) of the solenoid and (ii) the length (l) of the solenoid and (iii) the area of cross-section (A) of the solenoid.*

Solenoid Wound on Magnetic Material

When a solenoid is wound on a rod of magnetic material of permeability μ_r (Figure 24), the self inductance of the solenoid is given by

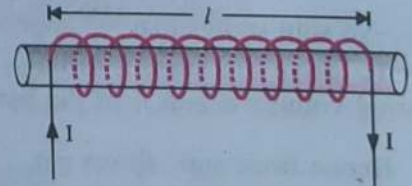


Figure 24

$$\mathrm{L}' = \frac{\mu_0 \mu_r \mathrm{N}^2 \mathrm{A}}{l} = \mu_r \mathrm{L} \qquad \ldots(13)$$

Thus, self inductance of a solenoid also depends upon the nature of the material of the core of the solenoid .

NOTE

(i) Self inductance increases with the increase in the number of turns of the coil and vice–versa.

(ii) Self inductance of a coil increases if air core of the coil is replaced by an iron core.

MUTUAL INDUCTION AND MUTUAL INDUCTANCE

Mutual induction is the phenomenon of inducing e.m.f. in a coil due to the change of current with time in a nearby coil.

Consider a coil connected to a battery through a key (K). This coil is called ***primary coil*** (P). Another coil placed near the primary coil and connected to the galvanometer is called ***secondary coil*** (S) (Figure 28). When key (K) is pressed, galvanometer shows a deflection.

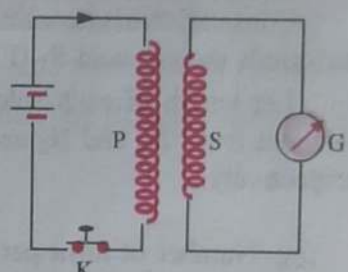


Figure 28

When key (K) is pressed, current through P begins to increase. As a result of this, magnetic field around P increases, so magnetic flux linked with secondary coil also changes. The induced *e.m.f.* is produced in the secondary coil due to the change in magnetic flux linked with the coil. Hence the current flows through the secondary coil which is indicated by the deflection in the galvanometer. This phenomenon of inducing *e.m.f.* is called *mutual induction*.

Coefficient of Mutual Induction or Mutual Inductance

It is known that the magnetic flux linked with the secondary coil is directly proportional to the current flowing through the primary coil.

i.e. $$\phi_s \propto \mathrm{I}_p \quad \text{or} \quad \phi_s = \mathrm{MI}_p \qquad \ldots(20)$$

where M is a constant of proportionality called ***Co–efficient of Mutual induction*** or ***Mutual Inductance***.

If $$\mathrm{I}_p = 1, \text{ then } \mathrm{M} = \phi_s \qquad \ldots(21)$$

Thus, *magnetic inductance of two coils or circuits is defined as the magnetic flux linked with the secondary coil due to the flow of unit current in the primary coil*

According to Faraday's law of electromagnetic induction,

$$\varepsilon_s = -\frac{d\phi_s}{dt}$$

Using equation (21), we get $\varepsilon_s = -\frac{d\mathrm{MI}_p}{dt} = -\mathrm{M}\frac{d\mathrm{I}_p}{dt}$

i.e.,
$$\varepsilon_s = -M \frac{dI_p}{dt} \quad \text{...(22)}$$

and
$$M = -\varepsilon_s \Big/ \frac{dI_p}{dt}. \text{ If } -\frac{dI_p}{dt} = 1, \text{ then } M = \varepsilon_s$$

Thus, ***mutual inductance of two coils can be defined as the induced e.m.f. produced in the secondary coil due to unit rate of decrease of current in the primary coil.***

SI unit of mutual inductance is *henry* (H).

$$1H = 1Wb\ A^{-1} = 1VA^{-1}s$$

NUMERICAL ILLUSTRATION. Calculate the mutual inductance between two coils when a current of 2 A changes to 6 A in 2 s and induces an e.m.f. of 20 mV in the secondary coil.

Solution. E.m.f. induced is given by,

$$|\varepsilon| = M \frac{dI_p}{dt} \quad \text{or} \quad M = \frac{|\varepsilon|}{dI_p / dt}$$

Here,
$$|\varepsilon| = 20 \text{ mV and } \frac{dI_p}{dt} = \frac{(6-2)}{2} = 2 \text{ A s}^{-1}$$

$\therefore$
$$M = \frac{20 \times 10^{-3}}{2} = 10 \times 10^{-3} = 10 \text{ mH}$$

MUTUAL INDUCTANCE OF TWO LONG CO-AXIAL SOLENOIDS

Derive an expression for mutual inductance of two long co-axial solenoids.

Consider two solenoids S_1 and S_2 such that the solenoid S_2 completely surrounds the solenoid S_1 (Figure 29).

Let length of each solenoid be l and the area of cross-section of each solenoid is A. N_1 and N_2 are the total number of turns of solenoid S_1 and S_2 respectively.

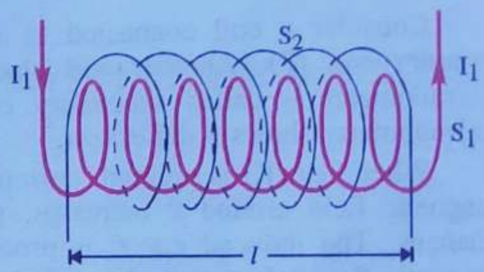


Figure 29

$\therefore$ Number of turns per unit length of solenoid S_1 is given by, $n_1 = \frac{N_1}{l}$

Number of turns per unit length of solenoid S_2 is given by, $n_2 = \frac{N_2}{l}$.

Let current I_1 flow through solenoid S_1. Then magnetic field inside the solenoid S_1 is given by, $B_1 = \mu_0 n_1 I_1 = \mu_0 \frac{N_1}{l} I_1$

$\therefore$ Magnetic flux linked with each turn of solenoid S_2 is given by, $B_1 A = \mu_0 \frac{N_1}{l} I_1 A$

Then total magnetic flux linked with N_2 turns of the solenoid S_2 is

$$\phi_2 = N_2(B_1 A) = \mu_0 \frac{N_1}{l} I_1 A \times N_2 = \frac{\mu_0 N_1 N_2 I_1 A}{l} \quad \text{...}(i)$$

But
$$\phi_2 = M_{12} I_1 \quad \text{...}(ii)$$

where, M_{12} is the mutual inductance of coil S_2 with respect to the coil S_1.

From *(i)* and *(ii)*, we get

$$M_{12} I_1 = \frac{\mu_0 N_1 N_2 I_1 A}{l} \quad \therefore \quad M_{12} = \frac{\mu_0 N_1 N_2 A}{l} \quad \text{...(23)}$$

Similarly, $M_{21} = \frac{\mu_0 N_1 N_2}{l} A$, where M_{21} is the mutual inductance of coil S_1 with respect to the coil S_2.

Clearly
$$M_{21} = M_{12} = M \quad \text{...(24)}$$

The above equation is treated as the general result.

If the two solenoids are wound on a magnetic substance of a relative permeability μ_r, then the mutual inductance is given by

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l} = \frac{\mu N_1 N_2 A}{l} \quad \text{...(25)}$$