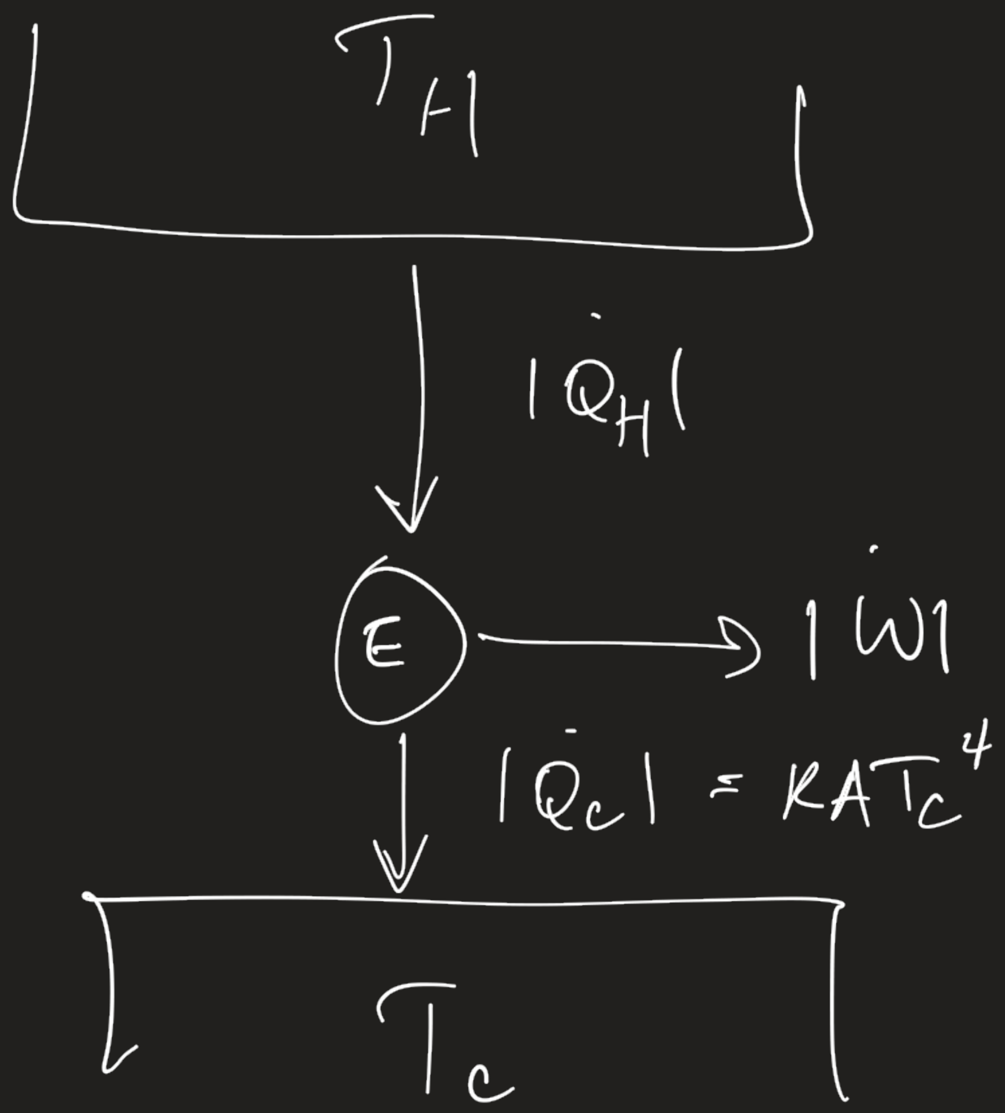
5.15



$$|\dot{W}| = \text{const.} = |\dot{Q}_H| - |\dot{Q}_c|$$

$$\frac{|\dot{Q}_H|}{|\dot{Q}_c|} = \frac{T_H}{T_c} = \frac{|\dot{W}| + KAT_c^4}{KAT_c^4}$$

$$= \frac{|\dot{W}|}{KAT_c^4} + 1$$

$$\frac{T_H}{T_c} - 1 = \frac{|\dot{W}|}{kAT_c^4}$$

$$A = \frac{|\dot{W}|}{kT_c^4}\left(\frac{T_H}{T_c} - 1\right)^{-1} = f(T_c)$$

$$\frac{dA}{dT_c} = 0 \quad @ \text{ minimum}$$

$$= \frac{-4|\dot{W}|}{kT_c^5}\left(\frac{T_H}{T_c} - 1\right)^{-1} +$$

$$\frac{-|\dot{W}|}{kT_c^4}\left(\frac{T_H}{T_c} - 1\right)^{-2} \cdot \frac{-T_H}{T_c^2}$$

$$= \frac{|\dot{W}|T_H}{kT_c^6}\left(\frac{T_H}{T_c} - 1\right)^{-2} - \frac{4|\dot{W}|}{kT_c^5}\left(\frac{T_H}{T_c} - 1\right)^{-1}$$

$$= \frac{|\dot{W}|}{kT_c^5}\left[\frac{T_H}{T_c}\left(\frac{T_H}{T_c} - 1\right)^{-2} - 4\left(\frac{T_H}{T_c} - 1\right)^{-1}\right] = 0$$

$$\left(\frac{T_H}{T_C}-1\right)^{-1}\left[\frac{T_H}{T_C}\left(\frac{T_H}{T_C}-1\right)^{-1}-4\right]=0$$

$$\frac{T_H}{T_C}\left(\frac{T_H}{T_C}-1\right)^{-1}-4=0$$

$$\frac{T_H}{T_C}=4\left(\frac{T_H}{T_C}\right)-4$$

$$4=3\left(\frac{T_H}{T_C}\right)\Rightarrow\boxed{\frac{T_C}{T_H}=\frac{3}{4}}$$

6.3

$$U = H - PV$$

$$\left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P - \left(\frac{\partial}{\partial T}\right)_P (PV)$$

$$= C_P - P\left(\frac{\partial V}{\partial T}\right)_P \qquad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$= C_P - \beta P V$$

$$C_P = C_V + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial U}{\partial T}\right)_P = C_V + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial T}\right)_P$$

$$= C_V + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]\left(\frac{\partial V}{\partial T}\right)_P$$

$$\frac{dV}{V} = \beta\, dT - \kappa\, dP \Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{\beta}{\kappa}$$

$$\left(\frac{\partial U}{\partial T}\right)_P = C_V + \left[T\frac{\beta}{\kappa} - P\right]\beta V$$

$$= C_V + \frac{\beta}{\kappa}\left[\beta T - \kappa P\right]V$$

<u>incompressible</u>: $\beta \simeq 0$, $\kappa \simeq 0$

$$\frac{\beta}{\kappa}(\beta T - \kappa P) = \left(\frac{\beta^2}{\kappa}T - \beta P\right)$$

$$\lim_{\beta \to 0} \frac{\beta^2}{\kappa}T - \beta P = 0$$

$$\lim_{\kappa \to 0} 0 = 0 \Rightarrow \left(\frac{\partial U}{\partial T}\right)_P = C_V$$

Ideal gas: $\kappa = \frac{1}{P}$ $\beta = \frac{R}{VP}$

$$\left(\frac{\partial U}{\partial T}\right)_P = C_V + \frac{R/VP}{1/P}\left(\frac{RT}{PV} - \frac{P}{P}\right)V$$

$$= C_V + \frac{R}{V}\left(1 - 1\right)V$$

$$= C_V$$

$$C_p \stackrel{?}{=} C_v + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$$

$$dU = dQ + dW$$

$$C_v\, dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V \cancel{-P}\right] dV$$

$$= C_p\, dT + \cancel{-P\, dV}$$

$$C_v\, dT + T\left(\frac{\partial P}{\partial T}\right)_V dV = C_p\, dT$$

$$C_v + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P = C_p$$

$$\left(\frac{\partial U}{\partial T}\right)_V + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial T}\right)_P$$