Chapter 1

Names as Corecursive Trajectories

Slogan

A *name* is a trajectory-in-motion whose meaning is *witnessed* at every beat. Each inference step is a tick of logical time; coherence is the film that keeps running.

1.1 Motivation: the projection room

Imagine the projection booth of a cinema. The reel is not a single frame but a becoming—a sequence of cuts, pans, rewinds, and splices. A name behaves likewise. It is not merely a token at one instant; it is a *kept motion*. If the scene jumps (a rupture), the audience does not lose the character; the film gives us bridges so the identity travels across the cut. That bridge is the witness of coherence we aim to formalize.

Scope of this chapter. We work entirely inside DHoTT at a fixed world of contexts. We *do not* introduce agency, world-change, or Grothendieck indexing here. Those arrive later (Chapter 10). Our task is modest: define and analyze *names* as corecursive witnesses of meaning over time.

1.2 A light DHoTT recap (just what we need)

Let $\mathbb{T}$ be the temporal index (discrete or site-like) and let an *evolving type* be a presheaf $\mathsf{A}\colon \mathbb{T}^{op} \to \mathcal{U}$. A *trajectory* for A from time τ is a section

$$\alpha \ : \ \prod_{t\geq\tau} \mathsf{A}(t), \qquad \alpha(t) \in \mathsf{A}(t).$$

We assume a small generating basis of time steps $\mathcal{E}$ whose composites span all intervals. *Coherence across a step* $(t \to t') \in \mathcal{E}$ is witnessed internally by a DHoTT term

$$\mathsf{Coh}(t \to t',\, \alpha(t),\, \alpha(t')),$$

whose exact shape depends on the drift/transport rules fixed in Chapter 6. Intuitively: the frame advances and the identity of the name remains intelligible.

Definition 1 (Robust coherence)**.** *For a trajectory α we define the* robust coherence predicate

$$\mathsf{R}^{\star}(\alpha) \ \ :\equiv \prod_{(t\to t')\in\mathcal{E},\, \tau\leq t\leq t'} \mathsf{Coh}(t \to t',\, \alpha(t),\, \alpha(t')).$$

Equivalently, in guarded form,

$$\mathsf{R}^{\star}(\alpha) \ \ \simeq \ \ \nu X.\ \mathsf{Coh}\big(t \to \mathsf{next}(t),\, \alpha(t),\, \alpha(\mathsf{next}(t))\big) \ \times \ \triangleright X.$$

1.3 Names as corecursive witnesses

Definition 2 (The type of names in a family)**.** *Fix* A *and a start time* τ*. A* name of A from τ *is a pair*

$$\mathsf{Name}(\mathsf{A}, \tau) \ :\equiv \sum_{\alpha : \prod_{t\geq\tau} \mathsf{A}(t)} \mathsf{R}^{\star}(\alpha).$$

We write $(\alpha,\, \rho) : \mathsf{Name}(\mathsf{A}, \tau)$ *with* $\rho : \mathsf{R}^{\star}(\alpha)$ *the corecursive witness.*

Remark (Meaning as kept motion). A single inhabitant $a \in \mathsf{A}(\tau)$ is only a *term-now*. A name is the *kept trajectory* (α, ρ): not where the token sits, but how it keeps going in a way we can witness at every cut.

1.3.1 Transport and equivalence invariance

Lemma 1 (Names respect equivalence of fibres)**.** *If* $\mathsf{A}(t) \simeq (t)$ *by an equivalence natural in* t *(transport in DHoTT), then* $\mathsf{Name}(\mathsf{A}, \tau) \simeq \mathsf{Name}(, \tau)$*.*

Idea. Transport carries α pointwise and sends Coh-witnesses along the same naturality, preserving the guarded product. Univalence turns these into identifications if desired. □

1.3.2 Names compose along time

Proposition 1 (Local-to-global via the basis)**.** *If* ρ *witnesses coherence on the generators* $\mathcal{E}$*, then (by closure under composition)* ρ *extends to all intervals* $[t, t']$ *with* $t \leq t'$*. Thus the basis choice fixes only the* editing tempo*, not the notion of name.*

1.4 Film-room intuition formalized

A cut $(t \to t')$ is a splice. A pan is an adiabatic drift where Coh is carried by transport. A jump cut is a rupture paired with an explicit *healing* witness Heal, which we model here simply as an admissible Coh term. In all cases, the audience keeps track of the character because the projectionist supplies ρ—the recursive reel of coherence proofs.

1.5 Elementary constructions on names

Restriction. Starting a little later forgets frames: there is a canonical map $\mathsf{Name}(\mathsf{A}, \tau) \to \mathsf{Name}(\mathsf{A}, \tau')$ for $\tau \leq \tau'$ given by tailing α and ρ.

Re-typing under soft change. If at some t we have an internal equivalence $\mathsf{A}(t) \simeq (t)$, we may *retime* the name without loss via the previous lemma; the film stock changes, the character persists.

Observation maps. Any fibrewise map $f : \mathsf{A} \to$ induces $f : \mathsf{Name}(\mathsf{A}, \tau) \to \mathsf{Name}(, \tau)$ by post-composition on trajectories and action on witnesses.

1.6 Examples

A lexical name in discourse

Let A record the sense of a word across turns of dialogue. A trajectory chooses one occurrence per turn; $\mathsf{R}^{\star}(\alpha)$ asserts that at every turn and step the use remains intelligible (allowing gentle drift). Then (α, ρ) is the *name* the conversation gives that word.

The projection-booth cut

Let A be the character state type across edits. In an adiabatic montage, ρ follows transports. At a hard cut, a repair is inserted (new establishing shot); the Coh-witness is a typed bridge from pre-cut to post-cut state. The name is precisely what the editor preserves.

1.7 What we defer (and why)

We have studiously avoided the global machinery: non-equivalence change of contexts, Grothendieck indexing, and the full definition of *agents*. Those belong to Chapter 10, where names become *carriers* of generativity and worlds themselves acquire topology. Here we needed only this: *a name is a robust, corecursively witnessed trajectory.*

1.8 Checklist of facts proved or required later

- **Equivalence invariance:** done above.
- **Basis independence:** sketched; full proof deferred to Chap. 10.
- **Closure under fibrewise maps:** immediate by functoriality.
- **Stability under guarded limits:** easy by coinduction (details deferred).
- **Interplay with rupture/heal rules:** admissible Coh-witnesses suffice here; full syntax/semantics in Chap. 6.

One-line moral

Name = trajectory + a reel of coherence proofs that never runs out.