



### Subjective Part

## **Subject:** Discrete Structures

**Class: BSESS**

**Section(s): I-A**

Course Code: MA-216

Date : 10-01-26 (9-12)

**Time Allowed: 180 Minutes**

Max Marks: 100

**FM's Name: Sajid Muhammad Khan**

**FM's Signature:**

## INSTRUCTIONS

- Attempt responses on the answer book only.
  - Nothing is to be written on the question paper.
  - Rough work or writing on question paper will be considered as use of unfair means.
  - Calculators are allowed.
  - Solve all parts of a question together.

**Q1. (CLO-3, C3, GA 3)**

[10+10+5 = 25 marks]

- a. Suppose that  $a$  and  $b$  are integers,  $a \equiv 11 \pmod{19}$ , and  $b \equiv 3 \pmod{19}$ . Calculate integer  $c$  with  $0 \leq c \leq 18$  such that

$$C = \pm 1$$

$$a^3 + b^3 \mid (a+b)$$

87-93-19

- ii.  $(19^2 \bmod 41) \bmod 9$
  - iii.  $(99^2 \bmod 32)^3 \bmod 15$

- b. Construct list of ordered pairs in the relation  $R$  from  $A = \{0, 1, 2, 3, 4\}$  to  $B = \{0, 1, 2, 3\}$ , where  $(a, b) \in R$  if and only if

i. a | b

ii.  $\gcd(a,b)=1$

Q2. (CLO-2, C3, GA 2)

[10+10 = 20 marks]

- a. Make use of Mathematical induction, Prove that  $3 + 3 \cdot 5 + 3 \cdot 5^2 + \dots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$  whenever  $n$  is a non-negative integer.

$c_2$   $c$   
 $B - c$

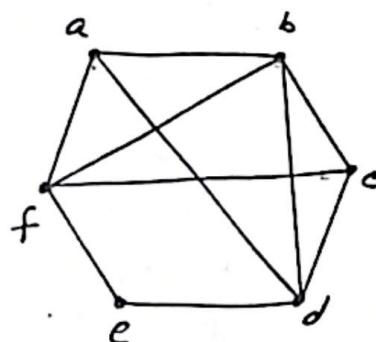
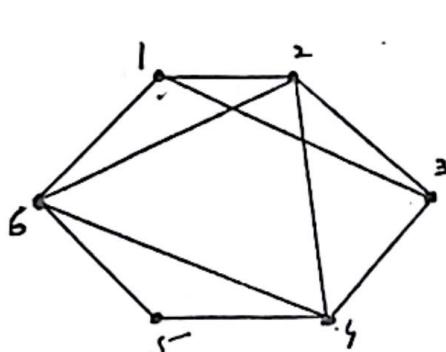
$\forall \in C \text{ such that}$

- b. Show that the premises "A student in this class has not read the book", and "Everyone in this class passed the final exam" imply the conclusion "Someone who passed the final exam has not read the book".

Q3. (CLO-3, C3, GA 3)

[10 marks]

For the given pair of graphs  $G_1$  and  $G_2$ , identify which of them are isomorphic. If the graphs are isomorphic, explicitly specify a vertex correspondence and write the corresponding adjacency matrices. If they are not isomorphic, provide a clear explanation justifying why no isomorphism exists.



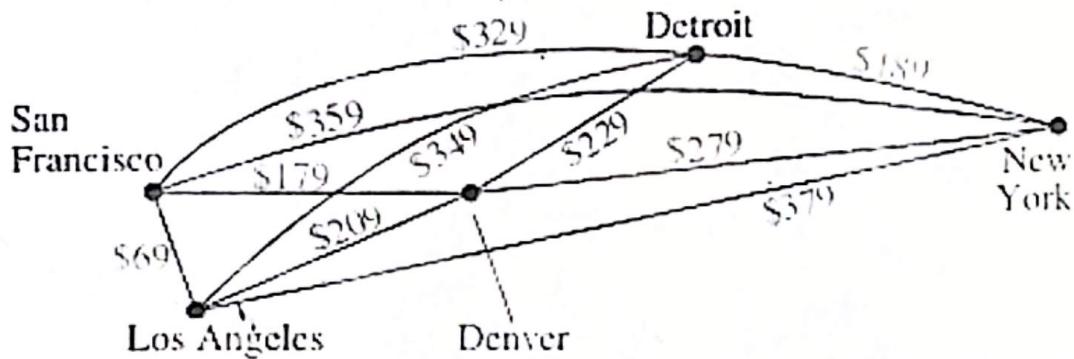
$G_1$

$G_2$

~~6~~  
[~~10 + 5 + 10 = 25 marks~~]

Q4. (CLO-1, C2, GA 2)

Consider the following graph:



i. Demonstrate a Hamiltonian circuit (Travelling Salesman Problem) using Repetitive nearest neighbor algorithm.

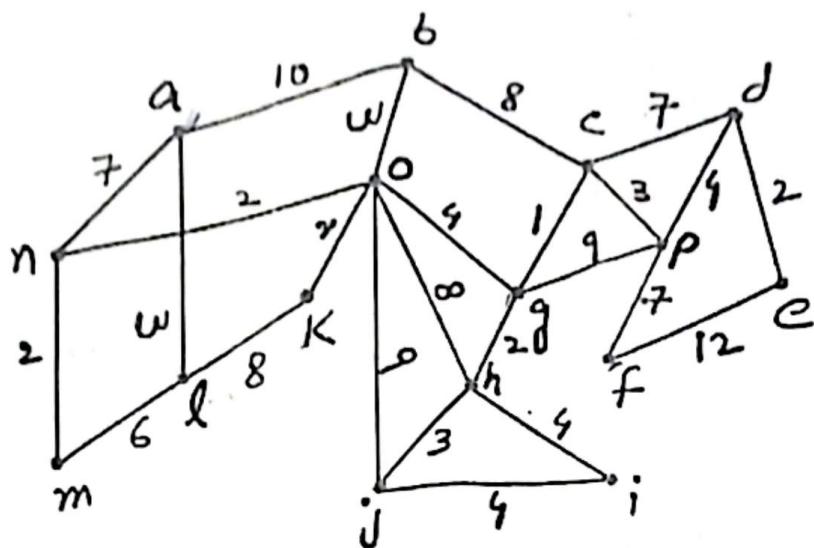
ii. Predict a minimum spanning tree of above graph using Prim's algorithm.

iii. Estimate a Prüfer sequence of the tree found in part ii.

Q5. (CLO-1, C2, GA 2)

[10 marks]

Apply Dijkstra's algorithm to the graph  $G$  below, predict the shortest path, Start = "a" and Ends = "f".



G

Q 6. (CLO-4, C4, GA 3)

[10 marks]

Seven committees must elect a chairperson to represent them at the end of year board meeting; however, some people serve on more than one committee and so cannot be elected chairperson for more than one committee. Based on the membership lists below, examine a system of distinct representatives for the board meeting.

Committee	Members			
Benefits	Agatha	Dinah	Evan	Vlad
Computing	Evan	Nancy	Leah	Omar
Purchasing	George	Vlad	Leah	
Recruitment	Dinah	Omar	Agatha	
Refreshments	Nancy	George		
Social Media	Evan	Leah	Vlad	Omar
Travel Expenses	Agatha	Vlad	George	

10.3

84  
1

10.6  
S PGP  
SD  
CFD

$Q_1 = 1$   
 $Q_2 = 2$   
 $Q_3 = 0$   
 $Q_4 = 5$   
 $Q_5 = 10$   
 $Q_6 = 1$

$Q_1 = 2$   
 $Q_2 = 1$   
 $Q_3 = 5$   
 $Q_4 = 2$   
 $Q_5 = 1$   
 $Q_6 = 2$