

$$F = ma \quad (1)$$

$$a = \frac{dv}{dt} \quad (2)$$

$$F_{\text{gravity towards center of planet}} = -m \cdot g$$

$$F_{\text{resisting gravity}} = m \cdot g$$

$$F_{\text{resisting gravity}} = m \cdot g \propto v^2$$

$$F_{\text{resisting gravity}} = m \cdot g = k \cdot v^2$$

$$\sum Forces = F_{\text{resisting gravity}} + F_{\text{resisting gravity}}$$

$$\sum Forces = k \cdot v^2 - m \cdot g$$

$$m \cdot \frac{dv}{dt} = k \cdot v^2 - m \cdot g$$

$$\int \frac{m}{m \cdot g - k \cdot v^2} dv = \int dt$$

$$\frac{-\ln \left( \frac{\sqrt{k} \cdot v - \sqrt{m \cdot g}}{\sqrt{k} \cdot v + \sqrt{m \cdot g}} \right)}{2\sqrt{m \cdot g \cdot k}} = t + c$$

$$-\ln \frac{\sqrt{k} \cdot v - \sqrt{m \cdot g}}{\sqrt{k} \cdot v + \sqrt{m \cdot g}} = 2 \cdot t \sqrt{m \cdot g \cdot k} + c$$

$$e^{-\ln \frac{\sqrt{k} \cdot v - \sqrt{m \cdot g}}{\sqrt{k} \cdot v + \sqrt{m \cdot g}}} = e^{2 \cdot t \sqrt{m \cdot g \cdot k} + c}$$

$$\left( \frac{\sqrt{k} \cdot v - \sqrt{m \cdot g}}{\sqrt{k} \cdot v + \sqrt{m \cdot g}} \right)^{-1} = e^{2 \cdot t \sqrt{m \cdot g \cdot k}} e^c$$

$$\frac{\sqrt{k} \cdot v + \sqrt{m \cdot g}}{\sqrt{k} \cdot v - \sqrt{m \cdot g}} = c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}}$$

$$\begin{aligned}
\sqrt{k} \cdot v + \sqrt{m \cdot g} &= c e^{2 \cdot t \sqrt{m \cdot g \cdot k}} (\sqrt{k} \cdot v - \sqrt{m \cdot g}) \\
\sqrt{k} \cdot v + \sqrt{m \cdot g} &= \sqrt{k} \cdot v \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - \sqrt{m \cdot g} \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} \\
\sqrt{k} \cdot v - \sqrt{k} \cdot v \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} &= -\sqrt{m \cdot g} \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - \sqrt{m \cdot g} \\
v \cdot (\sqrt{k} - \sqrt{k} \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}}) &= -\sqrt{m \cdot g} \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - \sqrt{m \cdot g} \\
v &= \frac{-\sqrt{m \cdot g} \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - \sqrt{m \cdot g}}{\sqrt{k} - \sqrt{k} \cdot c \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}}}
\end{aligned}$$

Since the object was released from rest, the initial velocity at  $t = 0$  is  $v = 0$ . Therefore we can easily solve for  $c$ .

$$\begin{aligned}
&\text{At } t = 0 \text{ and } v = 0 \\
0 &= \frac{-c \cdot \sqrt{m \cdot g} \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - \sqrt{m \cdot g}}{\sqrt{k} - \sqrt{k} \cdot c} \\
&\rightarrow \frac{-\sqrt{m \cdot g} \cdot (c + 1)}{\sqrt{k} \cdot (1 - c)} \\
&\rightarrow c = -1
\end{aligned}$$

So

$$\begin{aligned}
v &= \frac{\sqrt{m \cdot g} \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - \sqrt{m \cdot g}}{\sqrt{k} + \sqrt{k} \cdot e^{2 \cdot t \sqrt{m \cdot g \cdot k}}} \\
v &= \frac{\sqrt{m \cdot g} \cdot (e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - 1)}{\sqrt{k} \cdot (1 + e^{2 \cdot t \sqrt{m \cdot g \cdot k}})} \\
\lim_{t \rightarrow \infty} \frac{\sqrt{m \cdot g} \cdot (e^{2 \cdot t \sqrt{m \cdot g \cdot k}} - 1)}{\sqrt{k} \cdot (1 + e^{2 \cdot t \sqrt{m \cdot g \cdot k}})} &= \frac{\sqrt{m \cdot g} \cdot \infty}{\sqrt{k} \cdot \infty} \\
\therefore \text{ As } t \rightarrow \infty \quad v &\rightarrow \frac{\sqrt{m \cdot g}}{\sqrt{k}}.
\end{aligned}$$