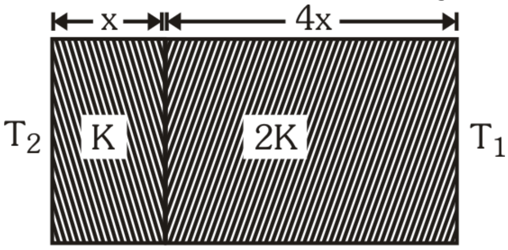
PART-A-PHYSICS

SECTION-I(i)

1) The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively are T_2 and $T_1(T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, with f equals to:



(A) 1
(B) 1/2
(C) 2/3
(D) 1/3

2) A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 3R made of material of thermal conductivity K_2 . The ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity is

(A) $K_1 + K_2$
(B) $\frac{K_1K_2}{K_1+K_2}$
(C) $\frac{K_1+3K_2}{4}$
(D) $\frac{K_1+8K_2}{9}$

3) Two rods of the same length, have radii in the ratio 3:4. Their densities are respectively 8000 and 9000 kg/m^3. Their specific heats are in the ratio of 2:3. When the same amount of heat is supplied to both, the changes in their lengths are in the ratio. (If their linear coefficients are in the ratio 5 : 6)

(A) 1 : 1
(B) 5 : 2
(C) 5 : 12
(D) 12 : 5

4) The root mean square velocity of all molecules in a mixture containing CO_2 and N_2 in the mole

ratio 1 : 7 is found to be x times the root mean square velocity of CO_2 molecules at the same temperature. The value of x is :

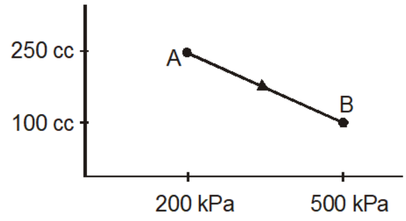
(A) $\sqrt{\frac{4}{5}}$

(B) $\sqrt{\frac{3}{2}}$

(C) $\sqrt{\frac{2}{3}}$

(D) None

5) A gas is taken along the path AB as shown in figure. If 70 cal of heat is extracted from the gas in the process, calculate the change in the internal energy of the system.



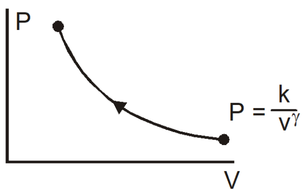
(A) –196 J

(B) –221 J

(C) –241 J

(D) –250 J

6) The molar heat capacity for the process shown in figure is



(A) C = Cp

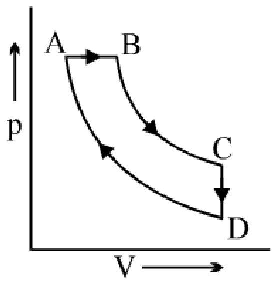
(B) C = Cv

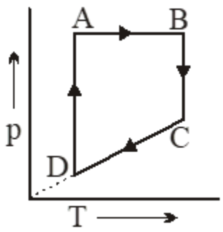
(C) C > Cv

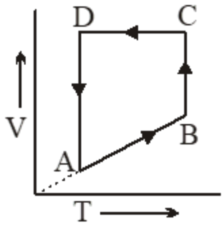
(D) C = 0

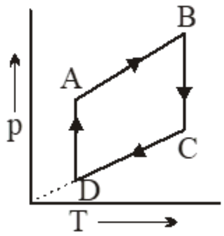
SECTION-I(ii)

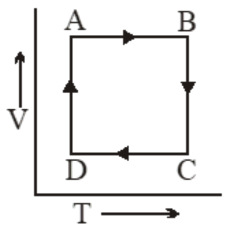
1) A cyclic process ABCD is shown in the p-V diagram. Which of the following curves represents the same process if BC and DA are isothermal processes?



(A) 

(B) 

(C) 

(D) 

2) Two gases have the same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally

(A) The final temperature is greater for the isothermal process

(B) The final pressure is greater for the isothermal process

(C) The work done by the gas is greater for the isothermal process

(D) All the above options are incorrect

3) Two moles of a monatomic ideal gas undergoes a thermodynamic process $\frac{V^3}{T^2}$ = constant, if the temperature is raised by 300K then

(A) work done by the gas is 400 R

(B) change in internal energy is 900 R

(C) molar heat capacity of the gas for the process is $\frac{13}{6}R$

(D) molar heat capacity of the gas for the process is $\frac{3}{2}R$

4) Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B, corresponding to the maximum spectral radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

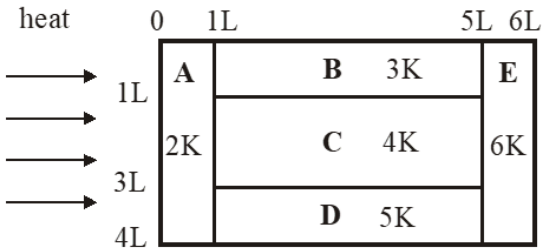
(A) the temperature of B is 1934 K.

(B) λ_B =1.5 μm.

(C) the temperature of B is 11604 K.

(D) the temperature of B is 2901 K.

5) A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(A) heat flow through A and E slabs are same

(B) heat flow through slab E is maximum among all.

(C) temperature difference across slab E is smallest

(D) heat flow through C = heat flow through B + Heat flow through D

6) A composite rod consists of a steel rod of length 25 cm and area 2A and a copper rod of length 50 cm and area A. The composite rod is subjected to an axial load F. If the Young's modulus of steel and copper are in the ratio 2 : 1.

(A) the extension produced in copper rod will be more .

(B) the extension in copper and steel parts will be in the ratio 2 : 1.

(C) the stress applied to the copper rod will be more.

(D) no extension will be produced in the steel rod.

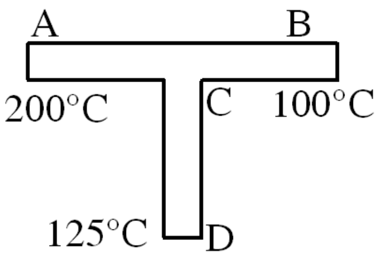
SECTION-III

1) Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is :-

2) A 200 gm mass of metal of specific heat 0.30 cal/(gm °C) is at 90°C. A 500 gm calorimeter of a specific heat of 0.10 cal/(gm °C) is at 20° C. The calorimeter is filled with 100 gm of water initially at 20°C. Now the metal piece is poured into the calorimeter. When the combination of metal, calorimeter and water reach equilibrium, the final temperature becomes 5x °C. Find numerical value of x.

3) A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure, The end A, B and D are maintained at 200°C, 100°C and 125°C respectively. The heat

current in CD is P watt. The value of P is ....



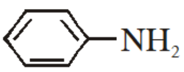
4) A monoatomic gas performs a work of $\frac{Q}{4}$ where Q is the heat supplied to it. The molar heat capacity of the gas will be ________R during this transformation. Where R is the gas constant.

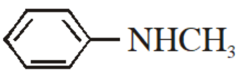
5) Estimate the change in the density of water in ocean at a depth of 400 m below the surface. The density of water at the surface = 1030 kg/m^3 and the bulk modulus of water = 2×10^9 N/m^2.

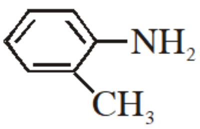
PART-B-CHEMISTRY

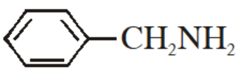
SECTION-I(i)

1) Which of the following is the strongest base -

(A) 

(B) 

(C) 

(D) 

2) Among the following acids which has the lowest pk$_a$ value-

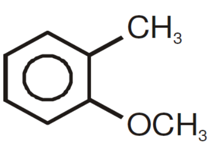
(A) CH_3CH_2COOH

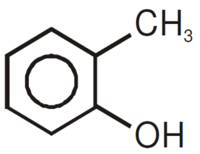
(B) $(CH_3)_2CHCOOH$

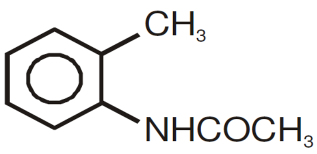
(C) HCOOH

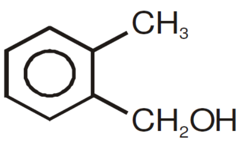
(D) CH_3COOH

3) Which one of the following benzene ring has maximum electron density?

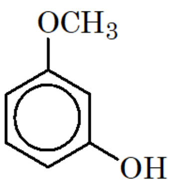
(A) 

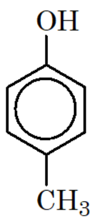
(B) 

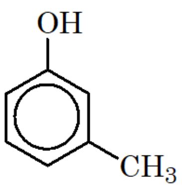
(C) 

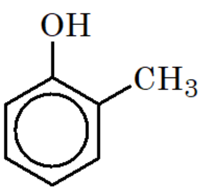
(D) 

4) Most acidic compound among these ?

(A) 

(B) 

(C) 

(D) 

5) Which of the following is not o, p-directing group

(A) $-NH_2$
(B) –OH
(C) –X(halogens)
(D) –CHO

6) Benzene reacts with acetyl chloride in presence of anhydrous aluminium chloride to form:

(A) Acetophenone
(B) Phenyl acetate
(C) Chlorobenzene
(D) Benzoic acid

SECTION-I(ii)

1) Benzene reacts with n-propyl chloride in the presence of anhydrous $AlCl_3$ to give predominantly :

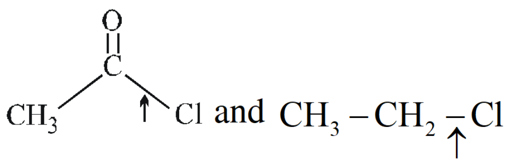
(A) n-Propylbenzene

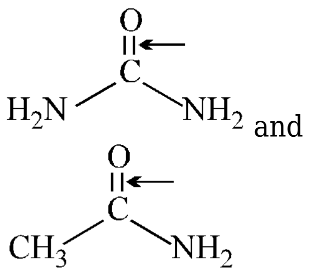
(B) Isopropylbenzene

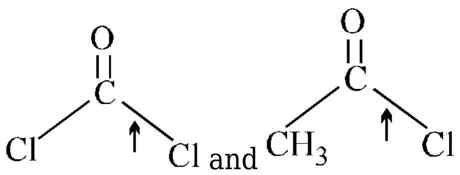
(C) 3-Propyl-1-chlorobenzene

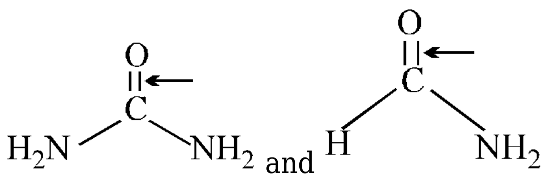
(D) Cumene

2) In which of the following pairs, indicated bond in II[nd] is of greater strength than in I[st]:

(A) 

(B) 

(C) 

(D) 

3) Select the correct statement.

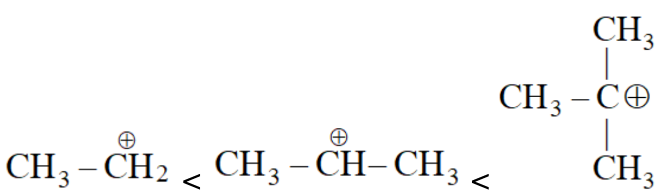
(A) Delocalisation of σ-electron is hyperconjugation.

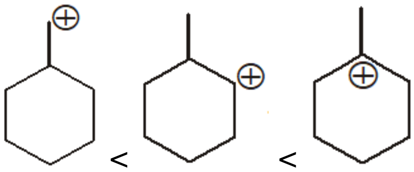
(B) Delocalisation of π-electron is resonance.

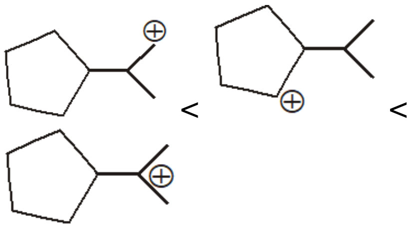
(C) Permanent partial displacement of σ-electron is inductive effect.

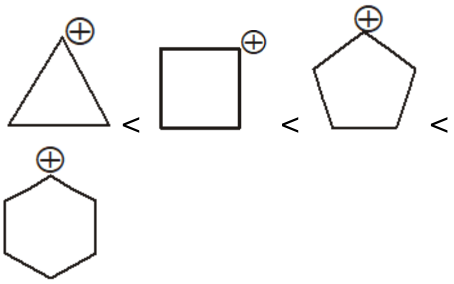
(D) Cyclopropyl methyl cation shows σ-resonance.

4) Which of the following represent correct stability order :

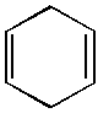
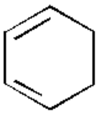
(A) 

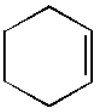
(B) 

(C) 

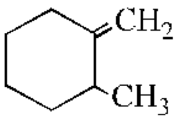
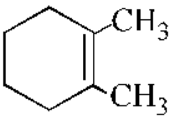
(D) 

5) Select the pair in which I[st] is having higher heat of hydrogenation than II[nd] :

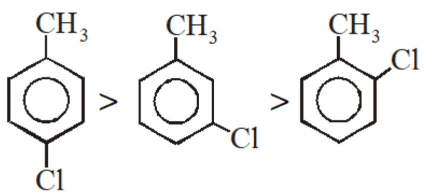
(A)  and 

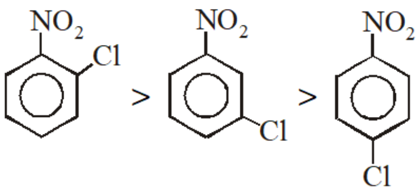
(B)  and 

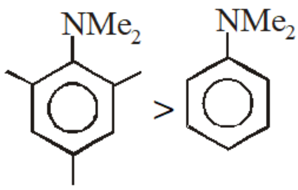
(C) CH_3-CH = CH-CH_3 and CH_3-CH_2-CH = CH_2

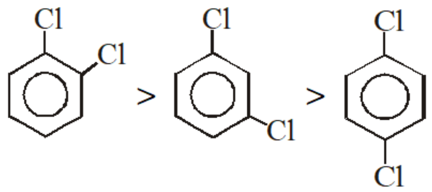
(D)  and 

6) Which of the following represent correct decreasing order of dipole moment :

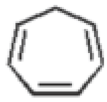
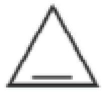
(A) 

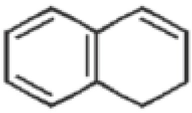
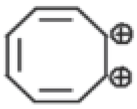
(B) 

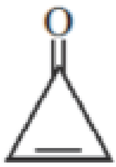
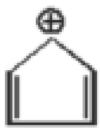
(C) 

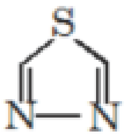
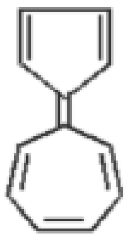
(D) 

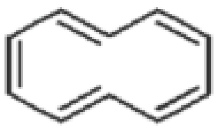
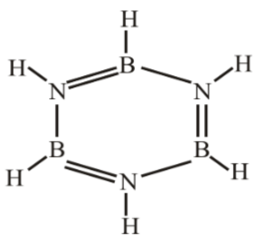
SECTION-III

1) How many of following compound are aromatic in nature (1)  (2) 

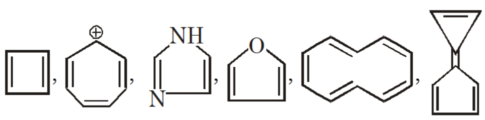
(3)  (4) 

(5)  (6) 

(7)  (8) 

(9)  (10) 

2) Find number of compound which are aromatic in nature :

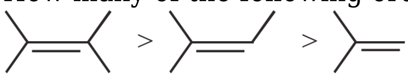


3) How many of the following are more acidic than CH_3-CH_2-OH

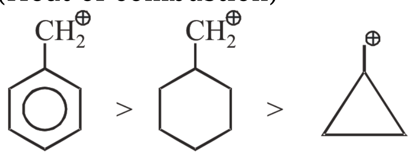
(a) CF_3-CH_2-OH (b) CH_3-CF_2-OH (c) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-OH$ (d) $CH_3-\underset{\underset{\displaystyle CH_3}{|}}{CH}-OH$

(e) $CH_3-\overset{\overset{\displaystyle CH_3}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-OH$ (f) O_2N-CH_2-OH (g) $CCl_3-\overset{\overset{\displaystyle O}{||}}{C}-OH$ (h) $Et-\overset{\overset{\displaystyle Et}{|}}{\underset{\underset{\displaystyle Et}{|}}{C}}-OH$

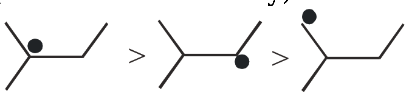
4) How many of the following orders are correctly matched?

(i) 

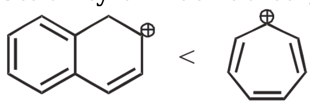
(Heat of combustion)

(ii) 

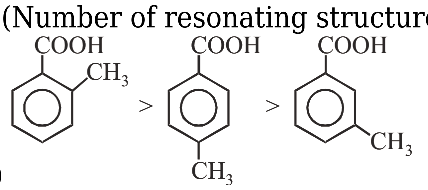
(Carbocation stability)

(iii) 

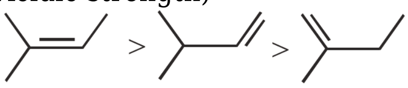
(Stability of free radical)

(iv) 

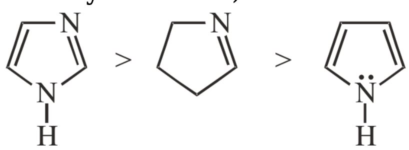
(Number of resonating structure involving monocation)

(v) 

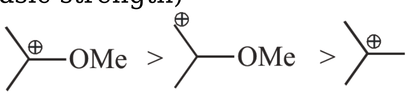
(Acidic strength)

(vi) 

(Stability of alkene)

(vii) 

(Basic strength)

(viii) 

(Stability of carbocation)

5)

How many of the following are –I effect showing groups :

(a) -Cl (b) $-CH_3$ (c) $-O^-$ (d) $-COO^-$

(e) $-CMe_3$ (f) –F (g) $-NH_2$ (h) –OH

(i) $-NO_2$ (j) -N = O (k) $-\overset{\oplus}{N}H_3$

PART-C-MATHEMATICS

SECTION-I(i)

1) Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by

7. Then $(\alpha^2 + \beta^2)$ is equal to

(A) 10
(B) 5
(C) 20
(D) 13

2) The coefficient of x^{101} in the expression $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ...x^{500}$, x > 0, is

(A) ${}^{501}C_{101}(5)^{399}$
(B) ${}^{501}C_{101}(5)^{400}$
(C) ${}^{501}C_{100}(5)^{400}$
(D) ${}^{500}C_{101}(5)^{399}$

3) In the expansion of $(x^3 + 2x^2 + x + 4)^{15}$, the coefficient of x^2 is not divisible by

(A) 8
(B) 25
(C) 27
(D) 64

4) If $2p^2 - 3q^2 + 4pq - p = 0$ and a variable line $px + qy = 1$ always touches a parabola whose axis is parallel to x-axis. Then the equation of the parabola is

(A) $(y-4)^2 = 24(x-2)$
(B) $(y-3)^2 = 12(x-1)$
(C) $(y-4)^2 = 12(x-2)$
(D) $(y-2)^2 = 24(x-4)^2$

5) The number of points with integral co-ordinates (2a, a–1) that fall in the interior of the larger segment of the circle $x^2 + y^2 = 25$ cut off by the parabola $x^2 + 4y = 0$ is

(A) One
(B) Two
(C) Three
(D) None of these

6) The algebraic sum of ordinates of points of contact of the tangents drawn from the point (–2, 3) to the parabola $y^2 = 8x$ is

(A) 6
(B) 8
(C) 10
(D) 8.5

SECTION-I(ii)

1) The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at T and N respectively the locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$

(B) directrix is $x = 0$

(C) latus rectum is $\frac{2a}{3}$

(D) focus is $(a, 0)$

2) Line $x + y = 7$ touches a parabola at P(3, 4). If the focus of the parabola is (1, 1), then

(A) equation of its directrix is $3x + 2y = 30$

(B) length of its latus rectum is $\frac{50}{\sqrt{13}}$

(C) its vertex is (6, 6)

(D) equation of its axis is $2x - 3y + 4 = 0$

3) If the normals to curve $y = x^2$ at the points P, Q and R pass through the point (0, 3/2) then

(A) radius of circum circle of ΔPQR is 1

(B) circumcenter of ΔPQR is (0, 1)

(C) centroid of ΔPQR is $\left(\frac{-1}{3}, \frac{1}{3}\right)$

(D) orthocenter of ΔPQR is (0, 0)

4) Let X = $\left({}^9C_1\right)^2 + 2\left({}^9C_2\right)^2 + 3\left({}^9C_3\right)^2 + \ldots + 9\left({}^9C_9\right)^2$, where ${}^9C_r$, $r \in \{1,2,\ldots,9\}$ denote binomial coefficients. Then $\frac{1}{1430}X$ is divisible by

(A) 9

(B) 3

(C) 17

(D) 19

5) If ${}^{100}C_6 + 4.{}^{100}C_7 + 6.{}^{100}C_8 + 4.{}^{100}C_9 + {}^{100}C_{10}$ has the value equal to ${}^xC_y$; then the possible value(s) of $x + y$ can be:

(A) 112

(B) 114

(C) 196

(D) 198

6) Let $(2 + x)(1 + (1 + x)^2)(1 + (1 + x)^4)(1 + (1 + x)^8)(1 + (1 + x)^{16}) = a_0 + a_1x + ... + a_nx^n$, then

(A) $a_r = a_{n-r}$, for each r = 0, 1 ,2, ... n

(B) $a_{14} = a_{16}$

(C) $\sum_{r=0}^{n} a_r = 2^{32} - 1$

(D) $a_0 + a_n = 2$

SECTION-III

1) Let $a = (4^{1/401} - 1)$ and let $b_n = {}^nC_1 + {}^nC_2 . a + {}^nC_3 . a^2 + ... + {}^nC_n . a^{n-1}$, then the value of $\frac{(b_{2006} - b_{2005})}{512}$ is

2) Let $P = (2 + \sqrt{3})^5$ and f = P – [P], where [P] denotes the greatest integer function, then the value of $\left(\frac{f^2}{1-f}\right) - 720$ is

3) The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3)... (1 + x^{100})$ is

4) Let a variable point A be lying on the directrix of parabola $y^2 = 4ax$. Tangents AB & AC are drawn to the curve where B & C are points of contact of tangent. The locus of centroid of ΔABC is a conic whose length of latus rectum is λ, then $\frac{\lambda}{a}$ is equal to

5) A tangent to the parabola $y^2 = 8x$ makes an angle of 45° with the straight line y = 3x + 5. If point of contact is integral point then its abscissa is

ANSWER KEYS

PART-A-PHYSICS

SECTION-I(i)

Q.	1	2	3	4	5	6
A.	D	D	B	B	C	D

SECTION-I(ii)

Q.	7	8	9	10	11	12
A.	A,B	A,B,C	A,B,C	A,B	A,C,D	A,C

SECTION-III

Q.	13	14	15	16	17
A.	2	8	2	2	2

PART-B-CHEMISTRY

SECTION-I(i)

Q.	18	19	20	21	22	23
A.	D	C	B	A	D	A

SECTION-I(ii)

Q.	24	25	26	27	28	29
A.	B,D	B,C,D	A,B,C,D	A,B,C,D	A,B,D	A,B,D

SECTION-III

Q.	30	31	32	33	34
A.	6	4	5	3	7

PART-C-MATHEMATICS

SECTION-I(i)

Q.	35	36	37	38	39	40
A.	B	A	C	C	B	A

SECTION-I(ii)

Q.	41	42	43	44	45	46
A.	A,D	A,B,C	A,B,D	A,B,C	B,D	B,C

SECTION-III

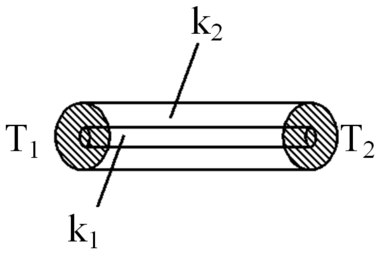
Q.	47	48	49	50	51
A.	**2**	**2**	**8**	**3**	**8**

SOLUTIONS

PART-A-PHYSICS

1) $R_{H_2} = R_1 + R_2 = \frac{x}{kA} + \frac{4x}{2kA} = \frac{3x}{kA}$

$\frac{dQ}{dt} = \frac{\Delta T}{\frac{3x}{kA}} = \frac{(T_2 - T_1)\,kA}{3x} = \frac{1}{3}\left[\frac{A\,(T_2 - T_1)\,k}{x}\right]$

2) 

⇒ Both are in parallel connection as ΔT across k_1 and k_2 are same

⇒ In parallel

$k_{eq} A = k_1 A_1 + k_2 A_2$

$k_{eq}\, \pi\, (3R)^2 = k_1 \pi R_2 + k_2 \pi((3R)^2 - R^2)$

$k_{eq} \times 9 = k_1 + 8k_2$

$k_{eq} = \frac{k_1 + 8k_2}{9}$

3) $\Delta Q = mS\Delta T = \rho A\ell s\Delta T$

$\Delta\ell = \ell\alpha\Delta T = (\ell\alpha)\left(\frac{\Delta Q}{\rho A\ell s}\right)$

$\Delta\ell = \frac{(\Delta Q)\,\alpha}{\rho AS}$

$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{\alpha_1}{\alpha_2} \times \frac{\rho_2 A_2 S_2}{\rho_1 A_1 S_1}$

$= \frac{5}{6} \times \frac{9000}{8000} \times \left(\frac{4}{3}\right)^2 \left(\frac{3}{2}\right) = \frac{5}{2}$

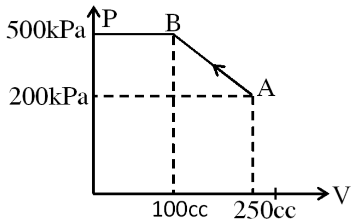
4) $V_{rms} = \sqrt{\frac{3RT}{M}}$ $\quad M_{CO_2} = 44g$

$M_{N_2} = 28g$

$M_{av} = \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2} = \frac{44 + 7 \times 28}{8} = 30g$

$XV_{rms}(CO_2) = V_{rms}(mix)$

$X = \frac{V_{rms}(mix)}{V_{rms}(CO_2)} = \sqrt{\frac{44}{30}} = \sqrt{\frac{22}{15}}$

5) 

Work done in AB process

$$W_{AB} = -\left(\frac{1}{2} \times 150 \times 10^{-6} \times 700 \times 10^{3}\right)$$

$$= -\frac{105}{2} = -52.5J$$

Q = -70 × 4.2 = – 294 J

ΔU = Q – W = -294 + 52.5 = -241.5 ~ -241 J

6) ΔQ = nCΔT

ΔQ = 0 for adiabatic process

⇒ C = 0

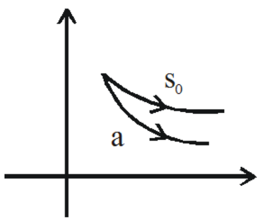
7)

$A \rightarrow B$ isobaric

$C \rightarrow D$ isochoric

in PT graph CD will be the line passing from origin

in VT graph AB will be the line passing from origin

8) 

9) $\frac{V^3}{T^2} =$ constant, $dV = \frac{2}{3}\frac{V}{T}dT$, $P = \frac{2RT}{V}$, $W = \int PdV$ ⇒ 400 R

$\Delta U = nC_V\Delta T = 900R$, $\Delta Q = nC\Delta T$ and $\Delta Q = \Delta U + W = 1300R$

$$C = \frac{\Delta Q}{n\Delta T} = \frac{13}{6}R$$

10)

$e_A = 0.01$, $e_B = 0.81$

Area = same for both

$\frac{dQ}{dt} = e\sigma AT^4$ = same for both

$\sigma e_A A T_A^{\ 4} = \sigma e_B A T_B^{\ 4}$

$$\left(\frac{T_A}{T_B}\right)^4 = \frac{e_B}{e_A} = 81 = 3^4$$

$\Rightarrow \frac{T_A}{T_B} = 3 \Rightarrow T_B = \frac{T_A}{3} = 1934$

also

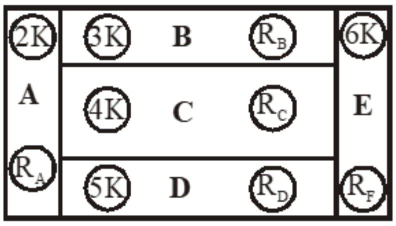
$\lambda_B T_B = \lambda_A T_A$

$\lambda_B \times 1934 = (\lambda_A - 1)5802$

$\lambda_B = 3\lambda_B - 3$

$2\lambda_B = 3$

$\Rightarrow \lambda_B = 1.5\ \mu m$

11) 

$Q_A = Q_B + Q_C + Q_D = Q_E$

$R = \frac{\ell}{kA}$

Let width of slabs be W

$R_A = \frac{L}{2k(4L)W} = \frac{1}{8kW}$

$R_B = \frac{4L}{3kLW} = \frac{4}{3kW}$

$R_C = \frac{4L}{4k(2LW)} = \frac{1}{2kW}$

$R_D = \frac{4L}{5k(LW)} = \frac{1}{5kW}$

$R_E = \frac{L}{6k(4LW)} = \frac{1}{24kW}$

as $Q = \frac{\Delta T}{R}$ and $\Delta T_B = \Delta T_C = \Delta T_D$

$Q_B : Q_C : Q_D = \frac{1}{R_B} : \frac{1}{R_C} : \frac{1}{R_D}$

12)

$Y_S : Y_{Cu} = 2 : 1$

i.e. $Y_{Cu} = Y$

$Y_S = 2Y$

$\frac{\Delta\ell_S}{\ell_S} = \frac{F}{2AY_S}$

$\Rightarrow \Delta\ell_S = \frac{F\ell}{2AY_S} = \frac{F\ell}{2A(2Y)}$

$\Delta\ell_{Cu} = \frac{F(2\ell)}{AY}$

& Stress in steel = $\frac{F}{2A}$

Stress in $Cu = \frac{F}{A}$

13)

$P = e\sigma AT^4$

λT = constant

$$\frac{P_A}{P_B} = \frac{e\sigma A_A T_A^4}{e\sigma A_B T_B^4} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{r_A^2}{r_B^2}\cdot\frac{\lambda_B^4}{\lambda_A^4}$$

$$10^4 = 400^2 \cdot \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

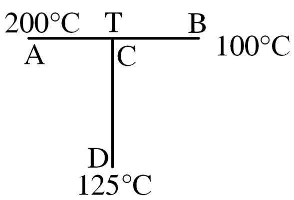
$$\frac{10^4}{16 \times 10^4} = \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

$$\frac{\lambda_A}{\lambda_B} = \left(\frac{16}{1}\right)^{\frac{1}{4}} = 2$$

14)

$$200 \times 0.30\,(T - 90) + 500 \times 0.10 \times (T - 20) + 100 \times 1 \times (T - 20) = 0$$

$T = 40°C \quad \Rightarrow \quad x = 8$



15) Rods are identical so

$R_{AB} = R_{CD} = 10\ Kw^{-1}$

C is mid-point of AB, so

$R_{AC} = R_{CB} = 5\ Kw^{-1}$

at point C

$$\frac{200 - T}{5} = \frac{T - 125}{10} + \frac{T - 100}{5}$$

$2(200 - T) = T - 125 + 2(T - 100)$

$400 - 2T = T - 125 + 2T - 200$

$$T = \frac{725}{5} = 145^oC$$

$$I_h = \frac{145 - 125}{10}w = \frac{20}{10}w$$

$$\boxed{I_h = 2w}$$

16)

$\Delta Q = \Delta E + W D \Rightarrow Q = \Delta E + \frac{Q}{4}$

$$\Rightarrow n\frac{3R}{2}\Delta T = \Delta E = \frac{3Q}{4}$$

$\Rightarrow n\Delta T = \frac{Q}{2R}$

 $C = 2R$

17) $\beta = \frac{dp}{-\frac{dv}{v}} = \frac{-vdp}{dv}$

We can also write it in terms of density as $\rho = \frac{m}{v}$

$\beta = \frac{-\rho dP}{d\rho}$

Change in pressure at a depth of 400 m = ρgh

= 1030 × 9.8 × 400

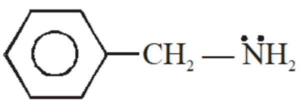
= 40.37×10^5 Pa

Or $d\rho = \frac{-\rho dP}{\beta}$

$d\rho = \frac{1030 \times (40.37 \times 10^5)}{2 \times 10^9}$

$d\rho = 2.07$ Kg/m$^3 \approx 2$ Kg/ m^3

PART-B-CHEMISTRY

18) 

 localised lone pair

19) Acidic strength ↑ pKa↓

$H-\underset{\underset{O}{\|}}{C}-OH$ most acidic due to no +I groups present.

20) Due to + M effect of – OH group and hyperconjugation of – CH_3 group

21) –OCH_3 shows –I effect.

22) $\left.\begin{array}{l}-NH_2\\-OH\\-X\end{array}\right\}$ o, p-directing group

–CHO → m-directing group

23)

Friedal crafts acylation of benzene yield acetophenone

33) (i), (iii) and (iv) are correct.

PART-C-MATHEMATICS

35)

$(22)^{2022} + (2022)^{22}$
divided by 3
$(21 + 1)^{2022} + (2022)^{22}$
$= 3k + 1$ $(\alpha = 1)$
Divided by 7
$(21 + 1)^{2022} + (2023 - 1)^{22}$
$7k + 1 + 1$ $(\beta = 2)$
$7k + 2$
So $\alpha^2 + \beta^2 \Rightarrow 5$

36) $(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + ... + x^{500}$
$= \dfrac{(5+x)^{501} - x^{501}}{(5+x) - x} = \dfrac{(5+x)^{501} - x^{501}}{5}$
$\Rightarrow$ coefficient x^{101} in given expression
$= \dfrac{{}^{501}C_{101}5^{400}}{5} = {}^{501}C_{101}5^{399}$

37) $(x^3 + 2x^2 + x + 4)^{15}$
$= a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{45}x^{45}$
Differentiate :
$15(x^3 + 2x^2 + x + 4)14(3x^2 + 4x + 1)$
$= a_1 + 2a_2x + 3a_3x^2 + ...$
Differentiate :
$15[(x^3 + 2x^2 + x + 4)^{13}(3x^2 + 4x + 1)^2$
$+ (x^3 + 2x^2 + x + 4)^{14}(6x + 4)]$
$= 2a_2 + 6a_3x + ...$
Put $x = 0$:
$15(14(4)^{13} + (4)_{14}(4)) = 2a_2$
$\Rightarrow$ $15(4)^{13}(14 + 16) = 2a_2$
$\Rightarrow$ $a_2 = 15^2 \times 2^{26}$
Not divisible by 3^3.

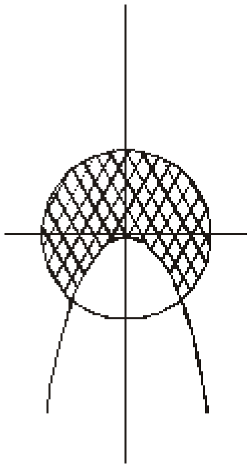
38)

The parabola be $(y - a)^2 = 4b(x - c)$
Equation of tangent is $y - a = -\dfrac{p}{q}(x - c) - \dfrac{bq}{p}$
Comparing it with $px + qy = 1$, we get $cp^2 - bq^2 + apq - p = 0$
$\therefore \dfrac{c}{2} = \dfrac{b}{3} = \dfrac{a}{4} = 1$

$\Rightarrow$ The equation is $(y-4)^2 = 12(x-2)$

39)

$x^2 + 4y = 0$
$(2a)^2 + 4(a - 1) > 0$
$4a^2 + 4a - 4 > 0$
$a^2 + a - 1 > 0$
$a = \dfrac{-1 \pm \sqrt{5}}{2}$



$a \in \left(-\infty, \dfrac{-1-\sqrt{5}}{2}\right) \cup \left(\dfrac{-1+\sqrt{5}}{2}, \infty\right)$ .....(1)
$4a^2 + a^2 + 1 - 2a - 25 < 0$
$5a^2 - 2a - 24 < 0$
$5a^2 - 12a + 10a - 24 < 0$
$(a + 2)(5a - 12) < 0$
$a \in \left(-2, \dfrac{12}{5}\right)$ .....(2)

By taking intersection of (1) & (2) $a \in \left(-2, \dfrac{-1-\sqrt{5}}{2}\right) \cup \left(\dfrac{-1+\sqrt{5}}{2}, 2.4\right)$
$a \in (-2, -1.62) \cup (0.62, 2.4)$
$a = 1, 2 \rightarrow$ Only two integers

40) Let $yt = x + 2t^2$ be the tangent passing through $(-2, 3)$
$\therefore 3t = -2 + 2t^2 \Rightarrow 2t^2 - 3t - 2 = 0$
$t_1 + t_2 = \dfrac{3}{2}$
$\therefore 4t_1 + 4t_2 = 6$

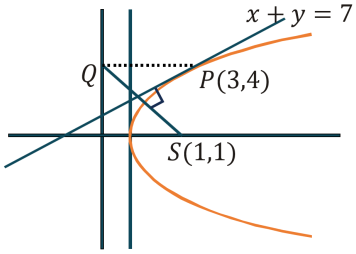
41) Equation of tangent : $ty = x + at^2$
$\Rightarrow T \equiv (-at^2, 0)$
equation of normal : $y = -tx + 2at + at^3$
$\Rightarrow N \equiv (2a + at^2, 0)$

If centroid of ΔPTN is (h,k) then

$3h = at^2 - at^2 + 2a + at^2$, $3k = 2at$

$t = \frac{3k}{2a}$

$3h = 2a + a\left(\frac{3k}{2a}\right)^2 \Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2a}{3}\right)$

42) 

Q lies on directrix, which is image of $(1, 1)$ with respect to line $x + y = 7 \Rightarrow Q \equiv (6, 6)$

$PQ ||$ axis

$m_{PQ} = \frac{2}{3} \Rightarrow$ axis $2x - 3y + 1 = 0$

Slope of the directrix is $-\frac{3}{2}$, which passes through $(6, 6)$

$\Rightarrow$ Directrix is $3x + 2y = 30$

$LR = 2.\frac{25}{\sqrt{13}}$

43) Normal is $x + yt = 2at + at^2$ cuts parabola at $(0,0)(-1,1)\ \&\ (1,1)$

44) $X = \sum_{r=0}^{n=9} r.\left({}^nC_r\right)^2;\ n = 9$

$\Rightarrow X = n.\sum_{r=0}^{n} {}^nC_r.{}^{n-1}C_{r-1}$

$\Rightarrow X = n.\sum_{r=1}^{n} {}^nC_{n-r}.{}^{n-1}C_{r-1}$

$\Rightarrow X = n.{}^{2n-1}C_{n-1};\ n = 9$

$\Rightarrow X = 9\ .\ {}^{17}C_8$

$\Rightarrow \frac{X}{1430} = 153$

45) $\left({}^{100}C_6 + {}^{100}C_7\right) + 3\left({}^{100}C_7 + {}^{100}C_8\right) + 3\left({}^{100}C_8 + {}^{100}C_9\right) + \left({}^{100}C_9 + {}^{100}C_{10}\right)$

$= \left({}^{100}C_7 + {}^{101}C_8\right) + 2\left({}^{101}C_8 + {}^{101}C_9\right) + \left({}^{101}C_9 + {}^{101}C_{10}\right)$

$= {}^{102}C_8 + 2.{}^{102}C_9 + {}^{102}C_{10}$

$= \left({}^{102}C_8 + {}^{102}C_9\right) + \left({}^{102}C_9 + {}^{102}C_{10}\right)$

$={}^{103}C_9+{}^{103}C_{10}={}^{104}C_{10}$

46) $\dfrac{1-(1+x)^{32}}{1-(1+x)}=\sum\limits_{r=0}^{n} a_r x^r$

$\Rightarrow \dfrac{(1+x)^{32}-1}{x}={}^{32}C_1+{}^{32}C_2x+\ldots..+{}^{32}C_{32}x^{31}=\sum\limits_{r=0}^{n=32} a_r x^{r-1}$

47) $a+1=4^{\frac{1}{401}}$

$b_n={}^nC_1+{}^nC_2a+{}^nC_3a^2+\ldots+{}^nC_na^{n-1}$

$\Rightarrow b_n\cdot a={}^nC_1a+{}^nC_2a^2+{}^nC_3a^3+\ldots+{}^nc_na^n$

$\Rightarrow b_n\cdot a+1=1+{}^nC_1a+{}^nC_2a^2+{}^nC_3a^3+\ldots+{}^nC_na^n$

$={}^nC_0+{}^nC_1a+{}^nC_2a^2+\ldots+{}^nC_na^n$

$\Rightarrow b_na+1=(1+a)^n\Rightarrow \boxed{b_n=\dfrac{(1+a)^n-1}{a}}$

Now, $b_{2006}-b_{2005}=\left[\dfrac{(1+a)^{2006}-1}{a}-\dfrac{(1+a)^{2005}-1}{a}\right]$

$=\dfrac{4^{\frac{2006}{401}}-4^{\frac{2005}{401}}}{a}$

$=\dfrac{4^{\frac{2005}{401}}\left[4^{\frac{1}{401}}-1\right]}{a}=4^5=2^{10}.$

48) $P=(2+\sqrt{3})^5=[P]+f$; $0<f<1$

observe, $0<2-\sqrt{3}<1$

So, let $(2-\sqrt{3})^5=f'$ where $0<f'<1$

Now, Add,

$$\begin{array}{l}(2+\sqrt{3})^5={}^5C_02^5+{}^5C_12^4(\sqrt{3})^1+{}^5C_22^3(\sqrt{3})^2+{}^5C_32^2(\sqrt{3})^3+{}^5C_42^1(\sqrt{3})^4+{}^5C_5(\sqrt{3})^5\\ (2+\sqrt{3})^5={}^5C_02^5-{}^5C_12^4(\sqrt{3})^1+{}^5C_22^3(\sqrt{3})^2-{}^5C_32^2(\sqrt{3})^3+{}^5C_42^1(\sqrt{3})^4-{}^5C_5(\sqrt{3})^5\\ \hline (2+\sqrt{3})^5+(2-\sqrt{3})^5=2({}^5C_02^5+{}^5C_22^3(\sqrt{3})^2+{}^5C_42^1(\sqrt{3})^4)\end{array}$$

$=2(32+10\times8\times3+5\times2\times9)$

$[P]+f+f'=2\times$ Integer $=2\times[32+240+90]=724$

Observe, $0<f+f'<2$

& $f+f'=2\times$ Integer $-[P]=$ integer

So, $f+f'=1\Rightarrow 1-f=f'$

□ Note: $f'\cdot P=(2-\sqrt{3})^5(2+\sqrt{3})^5=1$

$\Rightarrow(1-f)([P]+f)=1\Rightarrow(1-f)[P]+f-f^2=1$

$\Rightarrow(1-f)[P]=1-f+f_2\Rightarrow[P]=1+\dfrac{f^2}{1-f}$

2

□ $\overline{1-f}=[P]-1=(724-1)-1=722$

49) Coefficient of x^9 in

$(1+x)(1+x^2)$ -----$(1+x^{100})$

= Coefficient of x^9 in

$(1 + x)(1 + x^2)(1 + x^3)(1 + x^4)$----$(1 + x^9)$

$= 1 \cdot x^9 + x \cdot x^8 + x^2 \cdot x^7 + x^3 \cdot x^6 \cdot x^5 + x \times x^2 \times x^6 + x \times x^3 \times x^5 + x^2 x^3 x^4 = 8$

50) $A(-a, a(t_1+t_2))$, $B(at_1^2, 2at_1)$, $C(at_2^2, 2at_2)$

$h = \frac{a(a_1^2 + t_2^2) - 1}{3}$ & $k = a(t_1 + t_2)$

$\Rightarrow 3h = a((t_1 + t_2)^2 + 2) - a$

$\Rightarrow 3h = a\left(\frac{k^2}{a^2} + 1\right) \Rightarrow 3h = \frac{k^2}{a} + a$

$\Rightarrow k^2 = 3a(h - a) \Rightarrow \lambda = 3a$

51)

Let the slope of the tangent be m

 $\tan 45° = \left|\frac{3 - m}{1 + 3m}\right| \Rightarrow 1 + 3m = \pm(3 - m)$

 $m = -2$ or $\frac{1}{2}$

As we know that equation of tangent of slope m to the parabola $y^2 = 4ax$ is $y = mx\frac{a}{m}$ and point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

for $m = -2$, equation of tangent is $y = -2x - 1$ and point of contact is $\left(\frac{1}{2}, -2\right)$

for $m = \frac{1}{2}$, equation of tangent is $y = \frac{1}{2}x + 4$ and point of contact is (8, 8).