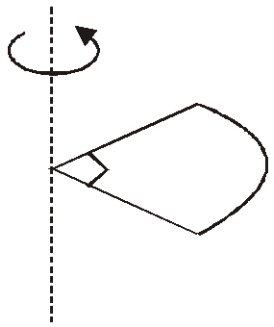
# PART-1 : PHYSICS

SECTION-I

1) One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original



disc. Its moment of inertia about the axis of rotation is

(A) $\frac{1}{2}MR^2$

(B) $\frac{1}{4}MR^2$

(C) $\frac{1}{8}MR^2$

(D) $\sqrt{2}MR^2$

2) From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:-

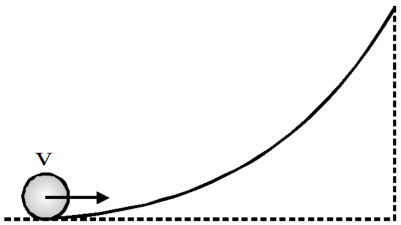
(A) $\frac{4MR^2}{9\sqrt{3}\pi}$

(B) $\frac{4MR^2}{3\sqrt{3}\pi}$

(C) $\frac{MR^2}{32\sqrt{2}\pi}$

(D) $\frac{MR^2}{16\sqrt{2}\pi}$

3) A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is :-




(A) ring

(B) solid sphere

(C) hollow sphere

(D) disc

4) A particle performing uniform circular motion has angular momentum L. If its angular velocity is doubled and its kinetic energy halved, then the new angular momentum is-

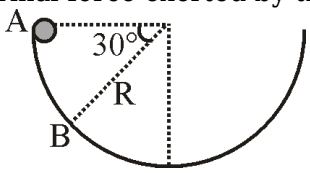
(A) $\frac{L}{4}$

(B) 2L

(C) 4L

(D) $\frac{L}{2}$

5) A small solid sphere *A* of mass *m* and radius *r* rolls without slipping inside a large fixed hemispherical bowl of radius *R* (*R* >> *r*) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the point *B* of the hemisphere.



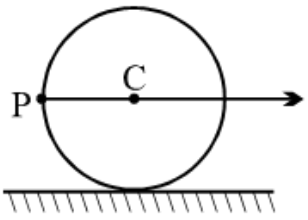

(A) $\frac{17}{14}mg$

(B) $\frac{17}{7}mg$

(C) $\frac{5}{7}mg$

(D) $\frac{7}{5}mg$

6) A disc of radius *R* is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point *P* is



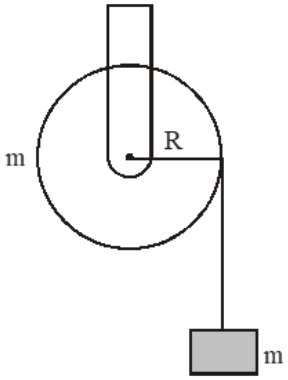

(A) zero

(B) 45°

(C) 135°

(D) $\tan^{-1}(1/2)$

7) A mass '*m*' is supported by a massless string wound around a uniform hollow cylinder of mass *m* and radius *R*. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



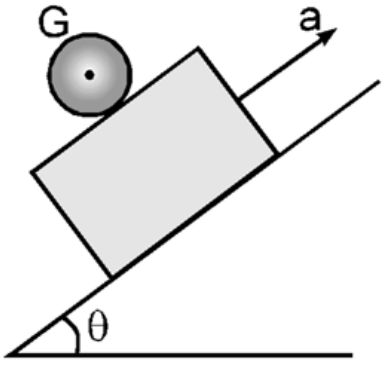

(A) $\frac{5g}{6}$

(B) g

(C) $\frac{2g}{3}$

(D) $\frac{g}{2}$

8) Determine the acceleration a of the supporting surface required to keep the centre G of the circular pipe in a fixed position during the motion. No slipping takes place between pipe and its support.



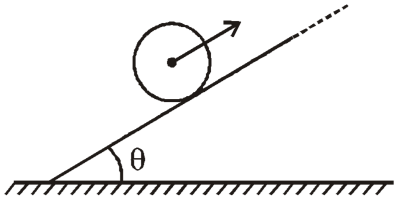

(A) g sin θ

(B) 2g sin θ

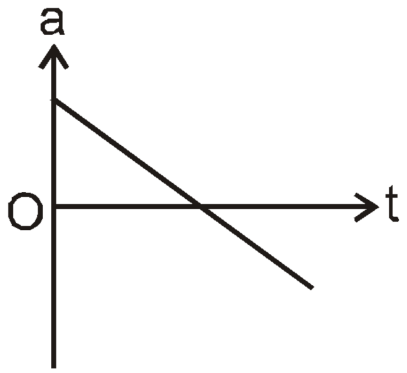
(C) g/2sin θ

(D) $\sqrt{2}$ g sin θ

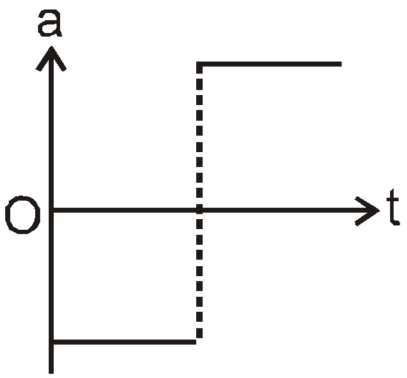
9) A uniform solid sphere rolls up (without slipping) the rough fixed inclined plane, and then back down. Which is the correct graph of acceleration 'a' of centre of mass of solid sphere as function of time t (for the duration sphere is on the incline) ? Assume that the sphere rolling up has a positive velocity.



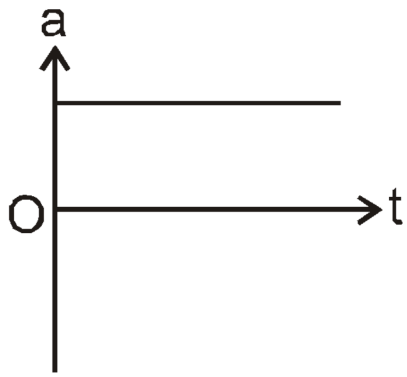
(A)



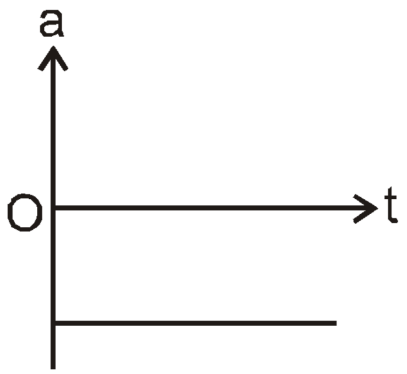

(B)



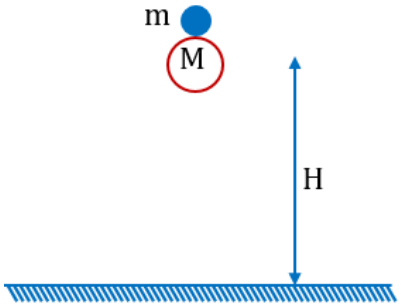

(C)




(D)




10) A small ball of mass $m$ is placed on a super ball of mass $M$ and the two balls are dropped from height $H$ ($H$ is very large compared to radius of balls). How high (in $m$) does small ball rise after collision? All collisions are head on elastic and ($m << M$). ($H = 1m$ and $g = 10\ m/s^2$)




(A) 3

(B) 9

(C) 11

(D) 15

11) A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :-

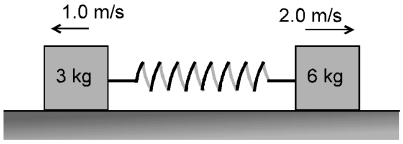
(A) $\frac{2v^2}{7g}$

(B) $\frac{v^2}{g}$

(C) $\frac{2v^2}{5g}$

(D) $\frac{v^2}{2g}$

12) Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. Find the maximum extension of the spring.



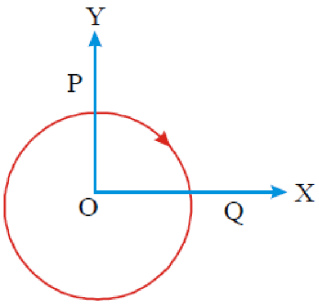

(A) 30 cm

(B) 10 cm

(C) $15\sqrt{2}$cm

(D) 15 cm

13) A particle moves in a circle of radius 4cm clockwise at constant speed 2 cm/sec. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along x and y-axes respectively (in cm/sec$^2$), the acceleration of the particle




at the instant halfway between P and Q is given by:

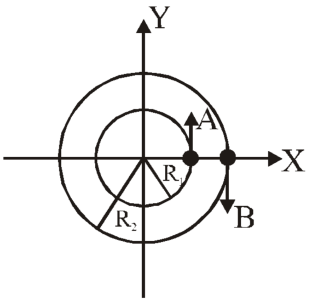
(A) $-4(\hat{x}+\hat{y})$

(B) $4(\hat{x}+\hat{y})$

(C) $-(\hat{x}+\hat{y})\frac{1}{\sqrt{2}}$

(D) $(\hat{x}-\hat{y})4$

14) Two particles A, B are moving on two concentric circles of radii $R_1$ and $R_2$ with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure :




The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by :

(A) $-\omega(R_1+R_2)\hat{i}$

(B) $\omega(R_1+R_2)\hat{i}$

(C) $\omega(R_1-R_2)\hat{i}$

(D) $\omega(R_2-R_1)\hat{i}$

15) A particle of mass m starts undergoing a circular motion of radius R. The speed of particle is increasing as V = Kt (K is a constant), (t is time in seconds). The magnitude of net acceleration at t = 1 second is

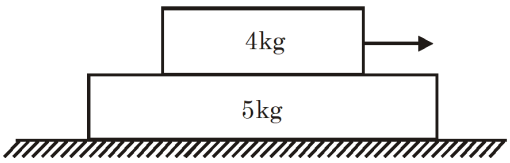
(A) K

(B) $\frac{K^2}{R}$

(C) $K\sqrt{1+\frac{K^2}{R^2}}$

(D) $2K\sqrt{1+\frac{K^2}{R^2}}$

16) A large slab of mass 5 kg lies on a smooth horizontal surface, with a block of mass 4 kg lying on the top of it. The coefficient of friction between the block and the slab is 0.25. If the block is pulled horizontally by a force of F = 6 N, then the work done by the force of friction on the slab, between the instants t = 2s to t = 3s is ($g = 10\ ms^{-2}$) (at t = 0 system is at rest)




(A) 2.4 J

(B) 5.55 J

(C) 4.44J

(D) 10 J

17) A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ where K is a positive constant, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0) and then parallel to the y-axis to the point (a, a). The total work done by the force $\vec{F}$ on the particle is :

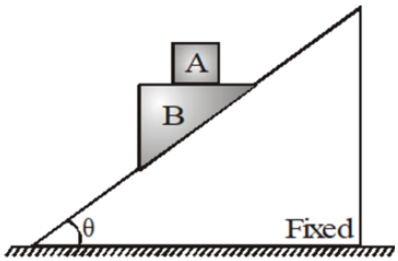
(A) $-2Ka^2$

(B) $2Ka^2$

(C) $-Ka^2$

(D) $Ka^2$

18) Block A of mass m is placed over a wedge B of same mass m. Assuming all surfaces to be smooth. The displacement of block A in 1 s if the system is released from rest is



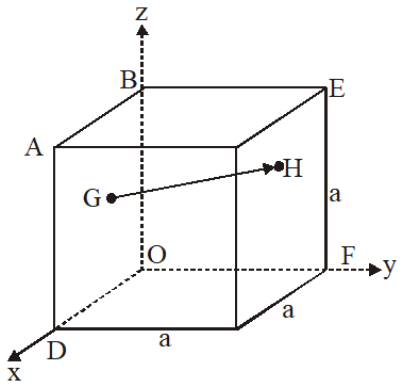

(A) $g\frac{(1+\sin^2\theta)}{(1-\sin^2\theta)}$

(B) $\frac{g\sin\theta}{2}$

(C) $g\frac{\cos^2\theta}{1+\sin^2\theta}$

(D) $g\frac{\sin^2\theta}{1+\sin^2\theta}$

19) In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to




the central point of the face BEFO will be:

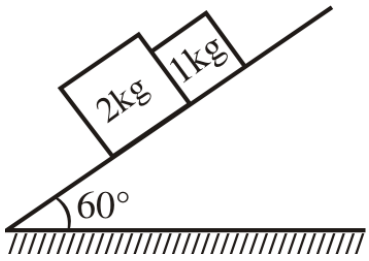
(A) $\frac{1}{2}a\left(\hat{i}-\hat{k}\right)$

(B) $\frac{1}{2}a\left(\hat{j}-\hat{i}\right)$

(C) $\frac{1}{2}a\left(\hat{k}-\hat{i}\right)$

(D) $\frac{1}{2}a\left(\hat{j}-\hat{k}\right)$

20) In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is $\mu_1$=0.5 and




$\mu_2$ = 0.4 respectively, then

(A) both block will move together.

(B) both block will move separately.

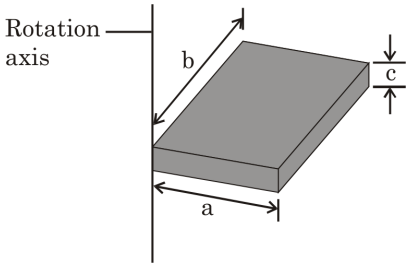
(C) there is a non zero contact force between two blocks.

(D) none of these

SECTION-II

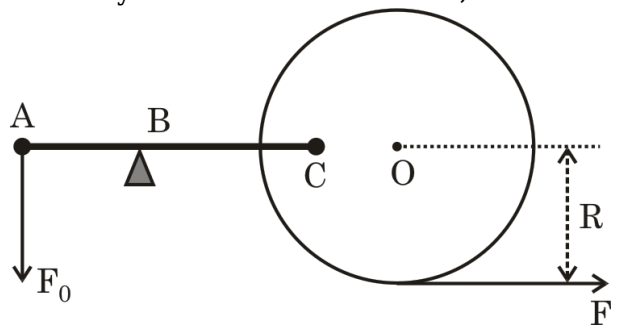
1) Find $\int_0^{\pi} \sin x\, dx$

2) The uniform solid block in figure has mass 3.0 kg and edge lengths a = 1m, b = 2m and c = 10cm. Calculate its moment of inertia about an axis through one corner and perpendicular to the large

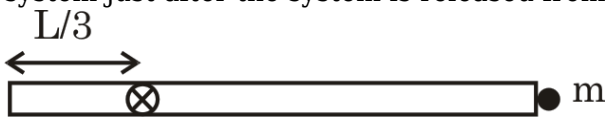



faces (in kg $m^2$).

3) A disc of radius 12 cm is free to rotate about point O, a rope is wound which is pulled with a force F = 10 N as shown in the figure. At point C, a light rod AC is attached to the disc, which rests on the fixed support at the point B. If force $F_0$ = 100k N is required to apply to the left end of the rod so that whole system remains in balance, find the value of k. Given BC = 30 cm, AB = 15 cm OC = 4 cm.

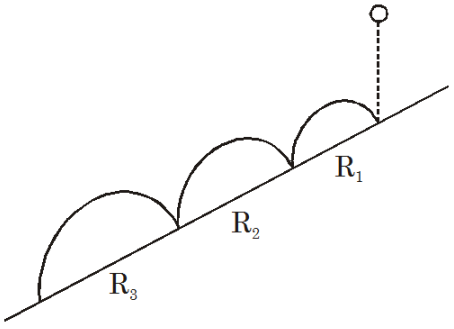



4) A particle of mass m (2kg) is attached to a rod of mass M(5kg) and length (3m) which is free to rotate in vertical plane about axis as shown in the figure. Find angular acceleration (in rad/$s^2$) of system just after the system is released from rest from the shown position.

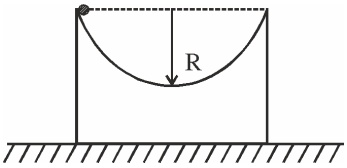



5) A particle of mass 1 kg is moving along the line
y = x + 2 (here x and y are in metres) with speed 2 m/s.
The magnitude of angular momentum of particle about origin is (in kg-$m^2$/s) (use $\sqrt{2} = 1.41$)

6) A ball is dropped on an inclined plane, its strikes the plane elastically. After each collision it's range is $R_1$, $R_2$ and $R_3$ respectively. If $R_2 = \frac{(R_1 + R_3)}{a}$, find a.

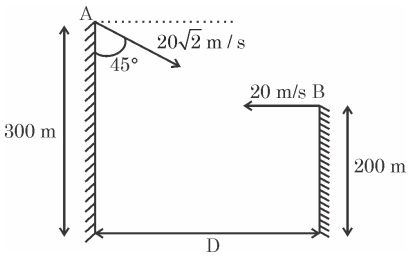



7) A particle of mass m Released from top of wedge. Mass of wedge is 4m. All the surface assumed to be frictionless and radius of circle R = 16 meter then find speed (in m/s) of particle when it reach at lowest point.




8) Two ball A and B are projected from top of towers located D distance apart as shown in figure. If

particles collide in mid air then value of D (in meter) is :




9) A boat moves relative to still water with a velocity which is n times greater than the river flow velocity. Boat moves at an angle 120° to the stream direction to minimize drifting. Find the value of n.

10) A passenger on a ship travelling due east with a speed of 18 km/hr observes that the smoke from the ship makes an angle of 30° with the ship's direction of motion. The wind is blowing from south to north. Assume that the smoke acquires a velocity (with respect to the earth) equal to the velocity of the wind, as soon as it leaves the ship. If the speed of the wind is $x\sqrt{3}$ km/hr. Find value of x

## PART-2 : CHEMISTRY

SECTION-I

1) A container when is empty weighs 50 gm. After certain liquid of density 25 gm/dm$^3$ is filled its mass becomes equal to 100 gm. The volume of the container will be :

(A) 0.25 dm$^3$

(B) 0.5 dm$^3$

(C) 1 dm$^3$

(D) 2 dm$^3$

2) Dalton's law cannot be applied for which gaseous mixture at normal temperatures:

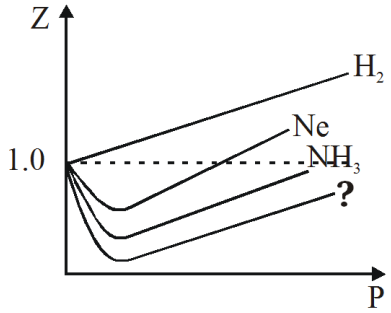
(A) $O_2$ and $N_2$

(B) $NH_3$ and HCl

(C) He and $N_2$

(D) $CO_2$ and $O_2$

3) Observe the following Z vs P graph.



The missing gas in the above graph can be :

(A) He

(B) Ar

(C) $C_5H_{12}$

(D) All are correct

4) Degree of Hydrolysis of $\frac{N}{100}$ solution of KCN is (Given Ka = $1.6 \times 10^{-9}$)

(A) $2.5 \times 10^{-3}$

(B) $2.5 \times 10^{-2}$

(C) $2.5 \times 10^{-4}$

(D) $2.5 \times 10^{-5}$

5) What volume of 0.2 M $NH_4Cl$ solution should be added to 100 ml of 0.1 M $NH_4OH$ solution to produce a buffer solution of pH = 8.7?
Given : $pK_b$ of $NH_4$ OH = 4.7 ; log 2 = 0.3

(A) 50 ml

(B) 100 ml

(C) 200 ml

(D) None of these

6) How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?

(A) 0.1 L

(B) 0.9 L

(C) 10.0 L

(D) 9.0 L

7)

For the reaction
$CuSO_4.5H_2O_{(s)} \rightleftharpoons CuSO_4.3H_2O_{(s)} + 2H_2O_{(g)}$.
Which one is correct representation :–

(A) $K_p = (P_{(H_2O)})^2$

(B) $K_c = [H_2O]^2$

(C) $K_p = K_c(RT)^2$

(D) All

8)

For the following three reactions, 1, 2 and 3 equilibrium, constants are given :

(1) $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$ ; $K_1$
(2) $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g)$ ; $K_2$
(3) $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g)$ ; $K_3$
Which of the following relations is correct ?

(A) $K_1\sqrt{K_2} = K_3$
(B) $K_2K_3 = K_1$
(C) $K_3 = K_1K_2$
(D) $K_3 = K_2^3K_1^2$

9) In the reaction $PCl_5 \rightleftharpoons PCl_3 + Cl_2$ the amount of each $PCl_5$, $PCl_3$ and $Cl_2$ is 2 mole at equilibrium and total pressure is 3 atmosphere. The value of $K_p$ will be :-

(A) 1.0 atm.
(B) 3.0 atm.
(C) 2.9 atm.
(D) 6.0 atm.

10) Which of the following is not a redox reaction ?

(A) $BaO_2 + H_2SO_4 \rightarrow BaSO_4 + H_2O_2$
(B) $2BaO + O_2 \rightarrow 2BaO_2$
(C) $2KClO_3 \rightarrow 2KCl + 3O_2$
(D) $SO_2 + 2H_2S \rightarrow 2H_2O + 3S$

11) The volume of 0.1N dibasic acid sufficient to neutralize 1 g of a base that furnishes 0.04 mole of $OH^-$ in aqueous solution is :

(A) 400 mL
(B) 200 mL
(C) 600 mL
(D) 800 mL

12) An example of a disproportionation reaction is :

(A) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$
(B) $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$
(C) $2CuBr \rightarrow CuBr_2 + Cu$
(D) $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$

13) 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is :-
(molar mass of calcium bicarbonate is 162 g $mol^{-1}$ and magnesium bicarbonate is 146 $gmol^{-1}$)

(A) 1,000 ppm
(B) 10,000 ppm

(C) 100 ppm

(D) 5,000 ppm

14) Which of the following equation represents a reaction that provides the heat of formation of ethane ($CH_3CH_3$)?

(A) 2 C(s) + 6 H(g) → $CH_3CH_3$(g)

(B) 2 C(s) + 3 $H_2$(g) → $CH_3CH_3$(g)

(C) $CH_2$ = $CH_2$(g) + 2 $H_2$(g) → $CH_3CH_3$(g)

(D) CH≡CH(g) + 2 $H_2O$(g) → $CH_3CH_3$(g) + $O_2$(g)

15) If 100 mole of $H_2O_2$ decomposes at 1 bar and 300 K, the work done (kJ) by of $O_2$(g) as it expands against 1 bar pressure is:
$2H_2O_2(l) \rightleftharpoons H_2O(l) + O_2(g)$
(R = 8.3 $JK^{-1}$ $mol^{-1}$)

(A) 124.50

(B) 249.00

(C) 498.00

(D) 62.65

16) A green bulb and a red bulb are emitting the radiations with equal power. The correct relation between numbers of photons emitted by the bulbs per second is

(A) $n_g = n_r$

(B) $n_g < n_r$

(C) $n_g > n_r$

(D) unpredictable

17) Which of the following contain largest number of carbon atoms ?

(A) 15 gm ethane, $C_2H_6$

(B) 40.2 gm sodium oxalate, $Na_2C_2O_4$

(C) 72 gm glucose, $C_6H_{12}O_6$

(D) 35 gm pentene, $C_5H_{10}$

18) Among the following, the molecule with highest dipole moment is :

(A) $CH_3Cl$

(B) $CH_2Cl_2$

(C) $CHCl_3$

(D) $CCl_4$

19) An ionic bond between A and B is most likely to be formed when :

(A) the ionization energy of A is high and the electron gain enthalpy of B is low

(B) the ionization energy of A is low and the electron gain enthalpy of B is high

(C) the ionization energy of A and the electron gain enthalpy of B both are high

(D) the ionization energy of A and the electron gain enthalpy of B both are low

20) The structure of $XeF_2$ involves hybridization of the type :

(A) $sp^3$

(B) $sp^2$

(C) $sp^3d$

(D) $sp^3d^2$

SECTION-II

1) In a rigid closed vessel, the pressure exerted by gas is 57 cm Hg at 40°C. If it is heated to 80°C, the final pressure of gas (in cm Hg) becomes.

2) What will be the resultant pH when 200 ml of an aqueous solution of HCl (pH = 2.0) is mixed with 300 ml of an aqueous solution of NaOH (pH = 12.0) ?

3) $3A(g) \rightleftharpoons 2B(g)$ $K_p = 2\ bar^{-1}$
At what total pressure , the A is 75% dissociated

4) For the reaction 3 A (g) + B (g) $\rightleftharpoons$ 2 C (g) at a given temperature, $K_C$ = 9.0. What must be the volume (in litre) of the flask, if a mixture of 2.0 mol each of A, B and C exist in equilibrium?

5) An aq. solution of 0.5M $KMnO_4$ is divided into two parts. One part of it requires 125 ml of 1.5M aq. solution of oxalate ions in acidic medium, while another part requires 270 ml of 0.5M aq. solution of iodide ions in neutral medium which are converted into $I_2$ only. Calculate total volume (mL) of the initial $KMnO_4$ solution.

6) 2.0 moles of an ideal diatomic gas $\left(C_{v,m} = \frac{5}{2}R\right)$ undergoes adiabatic reversible compression from 20L to 4L, at an initial temperature of 27ºC. The change in entropy of gas, $\Delta S$ is: (in cal/mole)

7) A gas behaves ideally over a range of pressure at 270 K. Find the critial temperature of the gas (in K).

8) How many s-electrons are present in $Ba^{2\oplus}$?

9) Find total no. of elements, which has more $IE_1$ than Nitrogen (N) atom.
B, C, O, F, Ne, P, Si

10) Number of carbon atom present linearly with sp hybridisation in $C_2(CN)_2$:-

## PART-3 : MATHEMATICS

SECTION-I

1) The medians of a triangle meet at (0,–3) and its two vertices are at (–1,4) and (5,2). Then the third vertex is at

(A) (4,15)
(B) (–4,–15)
(C) (–4,15)
(D) (4,–15)

2) Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is

(A) $\left(\frac{8}{5}, \frac{29}{5}\right)$
(B) $\left(\frac{29}{5}, \frac{11}{5}\right)$
(C) $\left(\frac{11}{5}, \frac{28}{5}\right)$
(D) $\left(\frac{29}{5}, \frac{8}{5}\right)$

3) The pair of lines represented by
$3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ are perpendicular to each other for

(A) two values of a
(B) ∀ a
(C) for one value of a
(D) for no values of a

4) If the equation $ax^2 - 6xy + y^2 + 2gx + 2fy + c = 0$ represents a pair of lines whose slopes are m and $m^2$, then sum of all possible values of a is

(A) 17
(B) –19
(C) 19
(D) –17

5) The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point

(A) (–5, 2)

(B) (2, –5)

(C) (5, –2)

(D) (–2, 5)

6) $y - 1 = m_1(x - 3)$ and $y - 3 = m_2(x - 1)$ are two family of straight lines, at right angled to each other. The locus of their point of intersection is

(A) $x^2 + y^2 - 2x - 6y + 10 = 0$

(B) $x^2 + y^2 - 4x - 4y + 6 = 0$

(C) $x^2 + y^2 - 2x - 6y + 6 = 0$

(D) $x^2 + y^2 - 4x - 4y - 6 = 0$

7) The smallest distance between the circle
$(x - 5)^2 + (y + 3)^2 = 1$ and the line $5x + 12y - 4 = 0$, is

(A) 1/13

(B) 2/13

(C) 3/15

(D) 4/15

8) The equation of the image of the circle
$x^2 + y^2 + 16x - 24y + 183 = 0$
by the line mirror $4x + 7y + 13 = 0$ is

(A) $x^2 + y^2 + 32x - 4y + 235 = 0$

(B) $x^2 + y^2 + 32x + 4y - 235 = 0$

(C) $x^2 + y^2 + 32x - 4y - 235 = 0$

(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

9) Combined equation to the pair of tangents drawn from the origin to the circle $x^2 + y^2 + 4x + 6y + 9 = 0$ is

(A) $3(x^2 + y^2) = (x + 2y)^2$

(B) $2(x^2 + y^2) = (3x + y)^2$

(C) $9(x^2 + y^2) = (2x + 3y)^2$

(D) $x^2 + y^2 = (2x + 3y)^2$

10) If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is

(A) $\sqrt{57}$

(B) 4

(C) $2\sqrt{5}$

(D) 5

11) The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point

(A) (–4, 6)

(B) (6, –2)

(C) (–6, 4)

(D) (4, –2)

12) Locus of the image of the point (2, 3) in the line $(2x-3y+4)+k(x-2y+3)=0$, $k \in R$, is a

(A) circle of radius $\sqrt{2}$

(B) circle of radius $\sqrt{3}$

(C) straight line parallel to x-axis

(D) straight line parallel to y--axis

13) If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ (where $\alpha \neq \beta$) then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is

(A) 19/3

(B) 25/3

(C) –19/3

(D) none of these

14) If the roots of equation $(4p - p^2 - 5)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unity then the number of integral values of p is

(A) 4

(B) 2

(C) 3

(D) 1

15)

Let a, b, c are roots of equation $x^3 + 8x + 1 = 0$,
then the value of

$$\frac{bc}{(8b+1)(8c+1)} + \frac{ac}{(8a+1)(8c+1)} + \frac{ab}{(8a+1)(8b+1)}$$

is equal to

(A) 0

(B) –8

(C) –16

(D) 16

16) In triangle ABC, If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ then angle C is equal to [Note: All symbols used have usual meaning in triangle ABC.]

(A) 30°

(B) 45°

(C) 60°

(D) 90°

17) If sum of all the solutions of the equation

$8\cos x\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$

in $[0, \pi]$ is $k\pi$, then k is equal to

(A) $\frac{13}{9}$

(B) $\frac{8}{9}$

(C) $\frac{20}{9}$

(D) $\frac{2}{3}$

18) Let $a_1, a_2, a_3, \ldots$ be an A.P. such that

$\frac{a_1+a_2+\ldots+a_p}{a_1+a_2+a_3+\ldots+a_q}=\frac{p^2}{q^2}$ ; $p \neq q$.

Then $\frac{a_6}{a_{21}}$ is equal to

(A) $\frac{121}{1861}$

(B) $\frac{11}{41}$

(C) $\frac{121}{1681}$

(D) $\frac{41}{11}$

19) The sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots..$ upto 11-terms is

(A) $\frac{11}{4}$

(B) $\frac{60}{11}$

(C) $\frac{7}{2}$

(D) $\frac{11}{2}$

20) If $\log_2\left(x^2+1\right)+\log_{13}\left(x^2+1\right)=\log_2\left(x^2+1\right)\log_{13}\left(x^2+1\right)$,

($x \neq 0$), then $\log_7\left(x^2+24\right)$ is equal to

(A) 1
(B) 2
(C) 3
(D) 4

SECTION-II

1) If x, y, z ∈ R and $121x^2 + 4y^2 + 9z^2 - 22x + 4y + 6z + 3 = 0$ then value of $x^{-1} - y^{-1} - z^{-1}$ is equal to

2)

If $9^x + 6^x = 2.4^x$ then the value of $\frac{x^3+64}{5}$.

3) If n(A) = 10, n(B) = 15, n(A ∪ B) = 23 then n ( A ∩ B) is equals to

4)

The value of tan6°.tan42°.tan66°.tan78° is

5) Value of $\frac{n}{4}$ if n is the number of solution of cosx = |x| + 1

6) If the sum of n terms of an A.P. is $3n^2 + 5n$,
then which of its terms is 164

7) If α, β are roots of equation $6x^2 - 5x + 1 = 0$ then value of $\sum_{n=0}^{\infty}(\alpha^n+\beta^n)$ is

8) If two perpendicular tangents can be drawn from the origin to the circle $x^2 - 6x + y^2 - 2py + 17 = 0$ then the value of $p^2$ is

9) From the origin, chords are drawn to the circle
$(x-1)^2 + y^2 = 1$. The equation of the locus of the midpoints of these chords is $x^2 + y^2 - \lambda x = 0$,
then the value of λ is

10) The value of λ for which the equation
$2x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represents a pair of straight lines, must be

ANSWER KEYS

# PART-1 : PHYSICS

## SECTION-I

| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A. | A | A | D | A | A | B | D | A | D | B | C | A | C | D | C | B | C | D | B | B |

## SECTION-II

| Q. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| A. | 2.00 | 5.00 | 0.60 | 5.00 | 2.82 | 2.00 | 16.00 | 200.00 | 2.00 | 6.00 |

# PART-2 : CHEMISTRY

## SECTION-I

| Q. | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A. | D | B | C | B | C | D | D | C | A | A | A | C | B | B | A | B | D | A | B | C |

## SECTION-II

| Q. | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|
| A. | 64.28 | 11.3 | 6.00 | 6.00 | 240.00 | 0.00 | 80.00 | 10.00 | 2.00 | 4.00 |

# PART-3 : MATHEMATICS

## SECTION-I

| Q. | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A. | B | C | A | B | C | B | B | D | C | D | B | A | A | B | C | C | A | B | D | B |

## SECTION-II

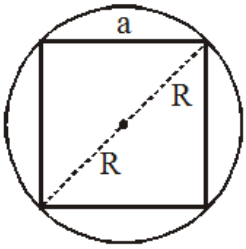
| Q. | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
|---|---|---|---|---|---|---|---|---|---|---|
| A. | 16.00 | 12.80 | 2.00 | 1.00 | 0.25 | 27.00 | 3.50 | 25.00 | 1.00 | 2.00 |

SOLUTIONS

PART-1 : PHYSICS

1)

$4I = \frac{4MR^2}{2}$ ; $I = \frac{MR^2}{2}$




2)

Let mass and side of cube be M' and a $\sqrt{3}$ a = 2R

M' = $\frac{M}{\frac{4}{3}2R^3}a^3$

Moment of Inertia of cube = $\frac{M'a^2}{6}$

$= \left(\frac{M}{\frac{4}{3}2R^3}a^3\right)\frac{a^2}{6} = \boxed{\frac{4MR^2}{4\sqrt{3}\pi}}$

3)

From the conservation of energy

loss in KE of body = Gain in potential energy

$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = mg\frac{3v^2}{4g}$

on solving

$I = \frac{mr^2}{2}$

 The body is a disc

4)

$L=I\omega$ ...(i)

$K=\frac{1}{2}I\omega^2$ ...(ii)

On replacing I from eq. (i) we get

$K = \frac{1}{2}\left(\frac{L}{\omega}\right)\omega^2 \Rightarrow L = \frac{2K}{\omega}$

$\frac{L_2}{L_1} = \left(\frac{K_2}{\omega_2}\right)\left(\frac{\omega_1}{K_1}\right) = \left(\frac{K_2}{K_1}\right)\left(\frac{\omega_1}{\omega_2}\right) = \frac{1}{4} \Rightarrow L_2 = \frac{L_1}{4}$

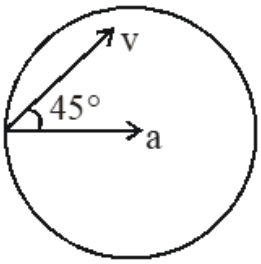
5)

$$N = mg + \frac{mv^2}{R}$$

and $\frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mv^2 = mgR$

$\Rightarrow N = mg + \frac{10}{7}mg = \frac{17}{7}mg$

6)



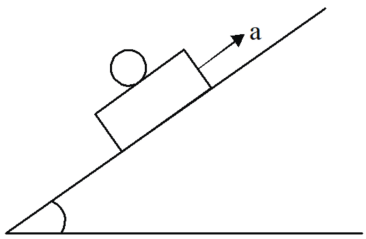

7)

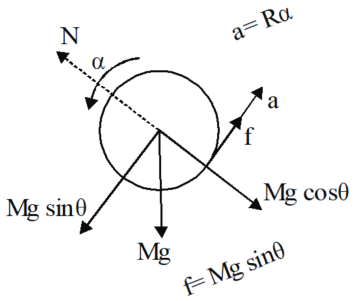
$mg - T = ma$

$T \times R = mR^2 \times \frac{a}{R}$

$\Rightarrow a = \frac{g}{2}$

8)



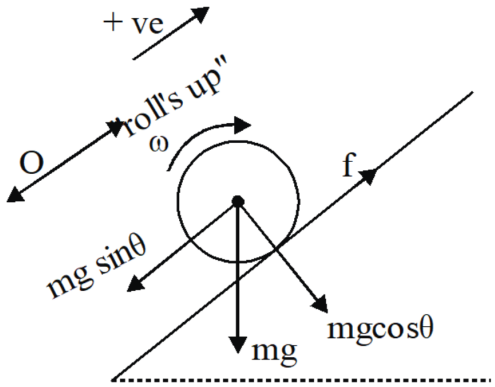

$fR = (MR^2)(\alpha)$

$f = M(R\alpha) = Ma$

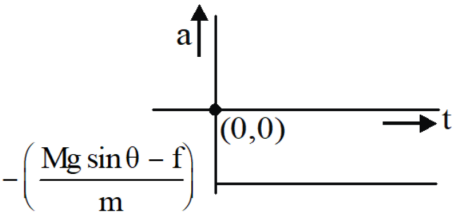
$Mg \sin\theta = Ma$

$a = g \sin\theta$

9)



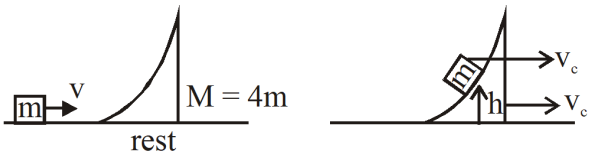

$$a = \frac{Mg\sin\theta - f}{m}$$




10)

After all collisions $v_m = 3\sqrt{2gH}$

11)




Applying Linear momentum conservation

$mv = (m + M)v_c$

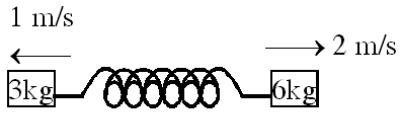
$v_c = \frac{v}{5}$

applying work energy theorem

$-mgh = \frac{1}{2}(m + M)v_c^2 - \frac{1}{2}mv^2$

solve, $h = \frac{2v^2}{5g}$

12)



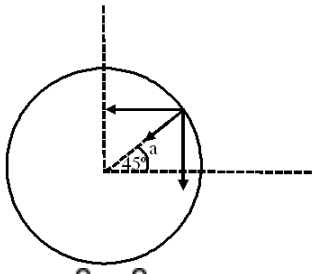

$$\frac{1}{2}kx_{max}^2 = \frac{1}{2}\mu v_{rel}^2$$

$$\frac{1}{2} \times 200 \times x_{max}^2 = \frac{1}{2} \times \left(\frac{3 \times 6}{3+6}\right)(3)^2$$

$$x_{max}^2 = \sqrt{\frac{2 \times 9}{200}} = \sqrt{\frac{9}{100}}$$

$$= \frac{3}{100}m = 30cm$$

13)

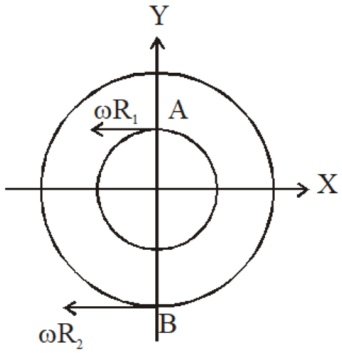



$$a_c = \frac{2 \times 2}{4} = 1 cm/s^2$$

$$\vec{a} = -a_c \cos 45 \hat{i} - a_c \sin 45 \hat{j}$$

$$= -\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

14)




$$\theta = \omega t = \omega \frac{\pi}{2\omega} = \frac{\pi}{2}$$

$$\vec{V}_A - \vec{V}_S = \omega R_1 \left(-\hat{i}\right) - \omega R_2 (-i)$$

15) $a_t = \frac{dv}{dt} = K$

$$a_c = \frac{v^2}{R} = \frac{K^2 t^2}{R} = \frac{K^2}{R}$$

$$a_{net} = \sqrt{a_t^2 + a_c^2}$$

16)

$$a = \frac{6}{9};\ f_{req} = \frac{2}{3} \times 4 = \frac{8}{3}$$

$f_L = 2.5 \times 4 = 10N$

No slipping

$W_{static\ friction} = Fs \times d$

$$d = \frac{1}{2}a(3^2 - 2^2) = \frac{1}{2}.\frac{2}{3}(5) = \frac{5}{3}$$

$$w = \frac{8}{3} \times \frac{5}{3} = 4.44J$$

17)

$$W = \int f_x\, dx + \int f_y\, dy$$

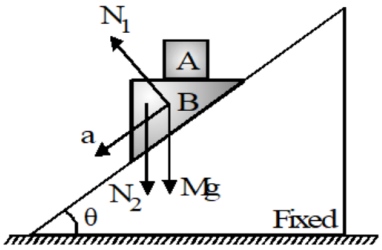
$f_x = -ky$

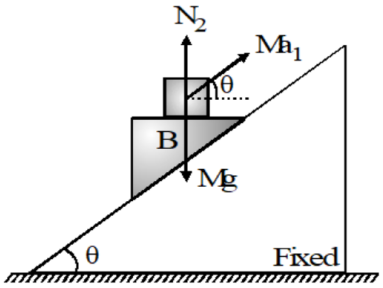
$f_y = -kx$

$$w = \int_0^a (-ky)dx + \int_a^{-a} (-ky)dx + \int_0^0 (-kx)dy + \int_0^a (-kx)dy$$

$W = 0 + 0 + 0 - ka^2 \Rightarrow -ka^2$

18)




$N_1 = (N_2 + Mg)\cos\theta$ ....(1)

$(N_2 + Mg)\sin\theta = Ma_1$ ....(2)

$N_2 + Ma_1 \sin\theta = Mg$ ....(3)

$Ma_1 \cos\theta = Ma_2$ ....(4)

$N_2 = Mg - Ma_1 \sin\theta$

$(2mg - ma_1 \sin\theta) \times \sin\theta = ma_1$

$(2g - a_1 \sin\theta)\sin\theta = a_1$

$2g \sin\theta = (1 + \sin^2\theta)\, a_1$

$$a_1 = \frac{2g\sin\theta}{1+\sin^2\theta}$$

$$a_1 \sin\theta = \frac{2g\sin^2\theta}{1+\sin^2\theta}$$

$$S = ut + \frac{1}{2}at^2$$

$$t = 1;\quad S = \frac{g\sin^2\theta}{1+\sin^2\theta}$$

19)

$$\vec{r}_g = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}$$

$$\vec{r}_H = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$$

$$\vec{r}_H - \vec{r}_g = \frac{a}{2}\left(\hat{j} - \hat{i}\right)$$

20) $\mu_1 = \frac{1}{2}$ $\mu_2 = \frac{4}{10}$

$f_2 = \mu mg\cos 60$ $f_1 = \frac{1}{2} \times 10 \times \frac{5}{2} = \frac{10}{4} = 2.5$

$f_2 = \frac{4}{10} \times 20 \times \frac{1}{2}$

$f_2 = 4$

21)

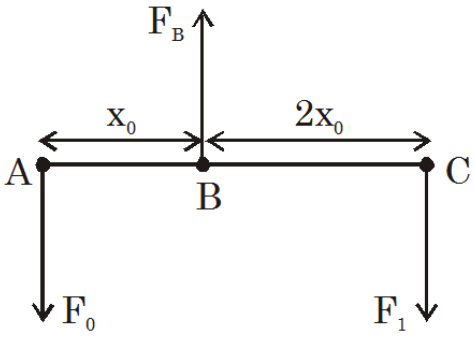
$$\int_0^{\pi} \sin dx = (-\cos x)_0^{\pi} = 2$$

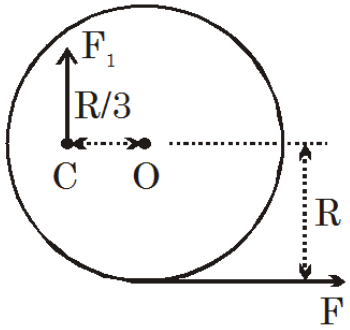
22)

$$I = \frac{m}{3}\left(a^2 + b^2\right)$$
$$= \frac{3}{3}\left(1^2 + 2^2\right)$$
$= 5 \text{ kg m}^2$

23)

$F_1 = 3F$ and $F_0 = 2F_1 = 6F = 60$ N



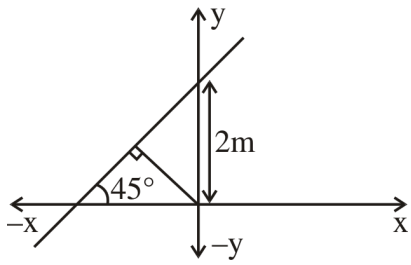




24)

$$Mg\left(\frac{L}{6}\right) + mg\left(\frac{2L}{3}\right) = \left(\frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 + m\left(\frac{2L}{3}\right)^2\right)\alpha$$

Put M = 5kg, m = 2kg, L = 3m

$\alpha = 5$

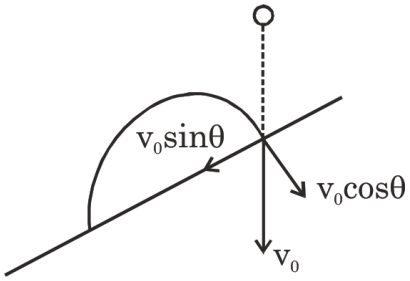



25)

$y = x + 2$

$L = 1 \times 2 \times \sqrt{2} = 2 \times 1.41 = 2.82 \text{ kg m}^2/\text{sec}$

26)




Since collision is elastic ball rebound with same speed, after collision motion is projectile on incline plane

Let t - time of flight

$$t = \frac{2u_y}{a_y} = \frac{2v_0\cos\theta}{g\cos\theta}$$

$$= \frac{2v_0}{g}$$

$$R = v_0\sin\theta t + \frac{1}{2}g\sin\theta t^2$$

As velocity along the plane is increasing linearly so $R_1$, $R_2$, $R_3$ are in A.M.

$$R_2 = \frac{R_1 + R_3}{2}$$

a = 2

27) When particle reach at bottom.

From momentum conservation :

$mv_1 = 4mv_2$

$(v_1 = 4v_2)$

From energy conservation :

$$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}4mv_2^2$$

$$gR = \frac{1}{2}v_1^2 + 2\left(\frac{v_1}{4}\right)^2$$

$$10 \times 16 = \frac{v_1^2}{2} + \frac{v_1^2}{8} = \frac{5v_1^2}{8}$$

$$v_1^2 = \frac{160}{5} \times 8 = 256$$

$v_1$ = 16 m/s.

28) Relative velocity $V_{A/B}$ must be along line of joining A and B

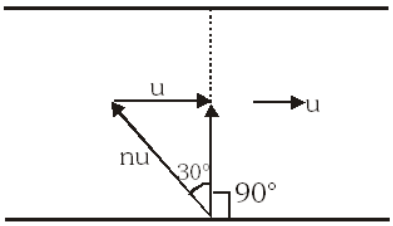
$$\tan\theta = \frac{100}{D} = \frac{20}{40}$$

$\Rightarrow$ D = 200 M

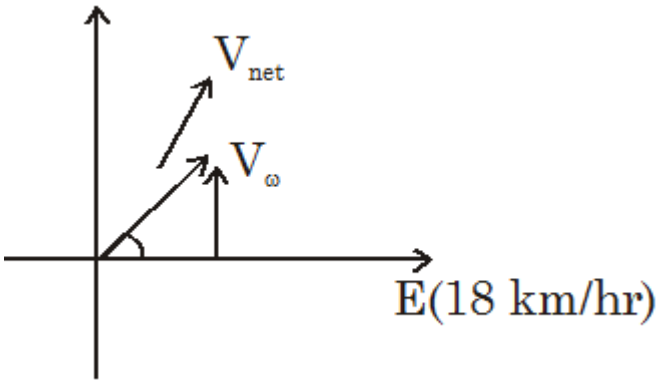
29)

In this case minimum drifting = 0

so $\sin 30° = \frac{u}{nu} \Rightarrow \frac{1}{2} = \frac{1}{n} \Rightarrow n = 2$




30)




$\tan 30^\circ = \frac{V_\omega}{18} = \frac{1}{\sqrt{3}}$

$V_\omega = \frac{18}{\sqrt{3}}$

$V_\omega$ = 10.39 km/hr

PART-2 : CHEMISTRY

31) As we can add 50 g of liquid

$v = \frac{w}{d} = \frac{50g}{25\,g/L} =$ 2L = 2 $dm^3$

32) Dalton's law of partial pressure is applicable on mixture of non reacting gases.
- $NH_3$ + HCl → NH4Cl
- $O_2$ & $N_2$ Non reating at normal temperature
- He & $N_2$ Non reacting at normal temperature

33)

More is the 'a' value, more compressible is the gas. Intermolecular force  when molecular weight  .
So, $C_5H_{12}$ is most probable.

34)

For KCN, Normality = Molarity.

$K_h = \frac{10^{-14}}{1.6 \times 10^{-9}} = \frac{10^{-5}}{1.6} = \frac{10^{-4}}{16}$

h = $\sqrt{\frac{K_h}{C}} = \sqrt{\frac{10^{-4}}{16 \times \frac{1}{100}}} = \sqrt{\frac{10^{-2}}{16}} = \frac{0.1}{4} = 0.025$

$= 2.5 \times 10^{-2}$

35)

Let V ml of $NH_4Cl$ added into $NH_4OH$ solution

$[NH_4Cl]$ in resultant solution $= \frac{0.2 \times V}{100 + V}$

$[NH_4OH]$ in resultant solution $= \frac{100 \times 0.1}{100 + V}$

$pOH = \frac{[NH_4^{\oplus}]}{[NH_4OH]} + \log$

$5.3 = 4.7 + \log \left[\frac{\left(\frac{0.2 \times V}{100+V}\right)}{\left(\frac{100 \times 0.1}{100+V}\right)}\right]$

$\Rightarrow 4 = \frac{0.2 \times V}{10} \Rightarrow V = 200$ ml

36)

pH = 1, Volume = 1 Litre

$[H^{\oplus}] = 10^{-1}$ M moles = $10^{-1}$ mol

New volume = 1 + V

New $[H^{\oplus}] = 10^{-2}$ M

$\frac{10^{-1}}{1+V} = 10^{-2}$

$10 = 1 + V$

$V = 9L$

37) $K_p = (P_{H_2O})^2$ ; $K_c = [H_2O]^2$ ;

$K_p = K_c(RT)^2$ $\Delta s$ ($\Delta ng = 0$)

38) from Q

$K_1 = \frac{[CO_2][H_2]}{[CO][H_2O]}$

$K_2 = \frac{[CO][H_2]^3}{[CH_4][H_2O]}$

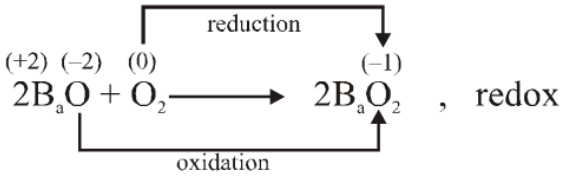
$K_3 = \frac{[CO_2][H_2]^4}{[CH_4][H_2O]^2}$

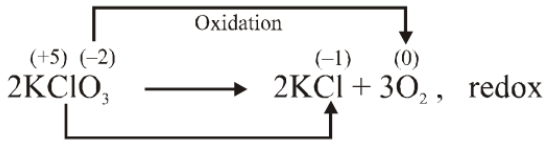
$K_1 \times K_2 = \frac{[CO_2][H_2]}{[CO][H_2O]} \times \frac{[CO][H_2]^3}{[CH_4][H_2O]} = K_3$

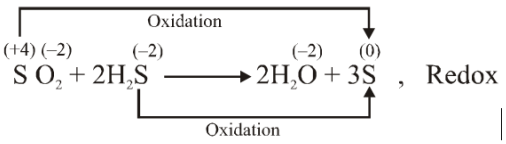
39) $K_p = \frac{2 \times 2}{2}\left(\frac{3}{6}\right)^1 = 1$

40) (A) $\overset{(+2)\ (-1)}{BaO_2} + \overset{(+1)\ (-6)\ (-2)}{H_2SO_4} \rightarrow \overset{(+2)\ (+6)\ (-2)}{BaSO_4} + \overset{(+1)\ (-1)}{H_2O_2}$

there is no change in oxidation number of any element, so this is not a redox reacton

(B) 


(C) 


(D) 


41) 1gm of base gives: 0.04 mole $OH^-$ ions.

No. of eq. of $OH^-$ = moles of $OH^-$ × n-factor

No. of eq. of $OH^-$ = 0.04 × 1 = 0.04

No. of eq. of $OH^-$ = No. of eq. of acid

0.04 = 0.1 × V(litre)

$$V = \frac{0.04\ell}{0.1} = 400m\ell$$

42) Disproportionate reaction is one in which 1 atom of a given compound oxidises & Reduces both in a redox reaction.

(1) $\overset{+7\ -2}{KMnO_4} \rightarrow K_2\overset{+6}{Mn}O_4 + \overset{+4}{Mn}O_4 + \overset{0}{O_2}$ only reduction of Mn & oxidation of oxygen.

(2) $2\overset{+7}{MnO_4^{\ominus}} + 10\overset{-1}{I^{\ominus}} + 16H^{\oplus} \rightarrow 2\overset{+2}{Mn^{2\oplus}} + 5\overset{o}{I_2} + 8H_2O$

Reduction of Mn & oxidation of Iodine

(3) $2\overset{+1}{Cu}Br \rightarrow \overset{+2}{Cu}Br_2 + \overset{o}{Cu}$

+1 +2

Both oxidation & reduction of copper hence disproportionate

(4) $2Na\overset{-1}{Br} + \overset{0}{Cl_2} \rightarrow 2NaCl^{1\ominus} + \overset{0}{Br_2}$

oxidation of Br & Reduction of chlorine

43) Hardness of $H_2O$ is expressed as equivalent amount of $CaCO_3$ as compared to actual hardening compound present in 106 gm of $H_2O$

So 1 mole $Ca(HCO_2) \equiv$ 1 mole $CaCO_3$

1 mole $Mg(HCO_3)_2 \equiv$ 1 mole $CaCO_3$

So $n_{Ca(HCO_3)_2} = \frac{0.81}{162} = 0.005$

$n_{Mg(HCO_3)_2} = \frac{0.73}{146} = 0.005$

So $n_{Total}$ of equivalent CaCO3

= 0.005 + 0.005 = 0.01

Hence wt of $CaCO_3$ = 0.01 × 100 = 1 gm in (100 ml ≡ 100 gm $H_2O$)

So in $10^6$ gm $H_2O$ wt $CaCO_3$ = $\frac{1 \times 10^6}{10^2}$ = 104 = 1000 ppm

44)

2 C(s) + 3 $H_2$(g) → $CH_3CH_3$(g)
1 mole of $C_2H_6$ is formed from elements in their stable standard state.

45)

$2H_2O_2(l) \rightleftharpoons 2H_2O(l) + O_2(g)$
$w = -P_{ext}(\Delta V) = -n_{O_2}RT$
 100 mole $H_2O_2$ on decomposition give 50 mole $O_2$.
 w = –(50) (8.3) (300) = –124500 J = –124.5 kJ.

46)

$E = nhn$

$\Rightarrow \frac{n_g h\nu_g}{t} = \frac{n_r h\nu_r}{t}$ $n_g$ = no. of photons (green)

$\Rightarrow n_g n_g = n_r n_r$ $n_r$ = no. of red photons

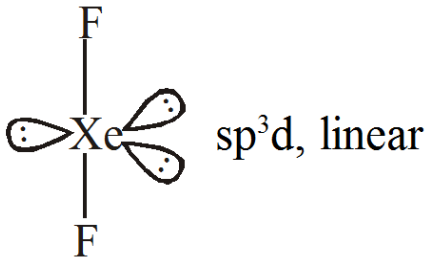
$\Rightarrow \frac{v_g}{v_r} = \frac{n_r}{n_g}$ v → frequency $v_g > v_r \Rightarrow n_r > n_g$

47) No. of C-atoms in 35 gm $C_5H_{10}$

$= \frac{35}{70} \times 5 \times 6.022 \times 10^{23}$

49) An ionic bond between A and B is most likely to be formed when:
(i) Ionization energy of A is low, because A+ will formed easily
(ii) Electron gain enthalpy of B is high, because $B^-$ will formed easily

50)



$sp^3d$, linear

51)

Rigid vessel so volume is constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$
$$\frac{57}{313} = \frac{P_2}{353}$$
$P_2 = 64.28$ cm Hg

52)

HCl (pH = 2)
$[H^{\oplus}] = 10^{-2}$ M
200 ml of HCl gives → $10^{-2} \times 200 \Rightarrow 2$ mmoles of $H^{\oplus}$
NaOH (pH = 12)
$[OH^{\ominus}] = 10^{-2}$ M
300 ml of NaOH gives → $10^{-2} \times 300 = 3$ mmoles

| HCl | + | NaOH | → | NaCl + $H_2O$ |
|---|---|---|---|---|
| 2 m mole | | 3 m mole | | 2 |
| × | | 1 m mole | | |

$$[OH^{\ominus}] = \frac{1 \times 10^{-3}}{0.5} \rightarrow 2 \times 10^{-3}$$
⇒ pH = 14 – 2.7 →11.3

53) $3A(g) \rightleftharpoons 2B(g)$

| At eq. (moles) | a(1–α) | $\frac{2a\alpha}{3}$ |
|---|---|---|

$$K_p = \frac{\left(\frac{2a\alpha}{3}\right)}{a^3(1-\alpha)^3} \times \left(\frac{P}{a(1-\alpha/3)}\right)^{-1}$$
$K_p = 2$ bar$^{-1}$
solving P = 6.00 bar

54)

$$KC = \frac{\left[\frac{C}{V}\right]^2}{\left[\frac{B}{V}\right]\left[\frac{A}{V}\right]^3}$$
$$\Rightarrow 9 = \frac{\left[\frac{2}{V}\right]^2}{\left[\frac{2}{V}\right]\left[\frac{2}{V}\right]^3}$$
⇒ V = 6 L

55)

Let $V_1$ & $V_2$ are volumes of $KMnO_4$ in 1st part & 2nd part
Reaction (1) $KMnO_4 + C_2O_4^{2-} \xrightarrow{\text{Acidic}} CO_2 + Mn^{2+}$
$\Rightarrow 0.5 \times V_1 \times 5 = 1.5 \times 125 \times 2$
$\Rightarrow V_1 = 150$ mL
Reaction (1) $KMnO_4 + I^- \xrightarrow{\text{Basic}} I_2 + MnO_2$

$\Rightarrow 0.5 \times V_2 \times 3 = 0.5 \times 270 \times 1$
$\Rightarrow V_2 = 90$ mL
So initial volume = 150 + 90 = 240 mL

56)

For reversible adiabatic process $\Delta S = 0$

57) $T_B = 270\ K = \frac{a}{Rb}$

$T_C = \frac{8a}{27Rb} = \frac{8}{27} \times 270 = 80K$

58)

10 s-electrons are present in $Ba^{2\oplus}$.

59)

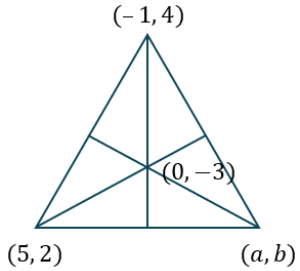
F, Ne have more $IE_1$ than nitrogen

60)

SP hybridised carbon → linear

PART-3 : MATHEMATICS

61)




$\frac{a+5-1}{3} = 0 \Rightarrow a = -4$

$\frac{b+4+2}{3} = -3 \Rightarrow b = -15$

$\therefore (\alpha, \beta) = (-4, -15)$.

62)

**Ans. (C)**

$L : \frac{x}{3} + \frac{y}{1} = 1$

$x + 3y - 3 = 0$

image of point (–1, –4) is :

$\frac{x+1}{1} = \frac{y+4}{3} = \frac{-2(-1-12-3)}{10}$

$$x+1=\frac{y+4}{3}=\frac{16}{5} \Rightarrow (x,y)=\left(\frac{11}{5},\frac{28}{5}\right)$$

63)

**Ans. (A)**

$3ax^2 + 5xy + a^2 - 2y^2 = 0$ represent pair of two perpendicular lines

$\Rightarrow (3a) + (a2 - 2) = 0$

$\Rightarrow a^2 + 3a - 2 = 0$

$$a=\frac{-3\pm\sqrt{17}}{2}$$

64)

$m + m^2 = 6, m^3 = a$

$\Rightarrow m^3 + m^6 + 3m^3 (m + m^2) = 216$

$\Rightarrow a^2 + a + 3a (6) = 216$

$\Rightarrow a^2 + 19a - 216 = 0$

sum = –19

65) Let equation of circle be $(x - 3)^2 + (y + r)^2 = r^2$

 it passes through (1, –2)

$\Rightarrow r = 2$

$\Rightarrow$ circle is $(x - 3)^2 + (y + 2)^2 = 4$

$\Rightarrow$ (5, –2)

Aliter :

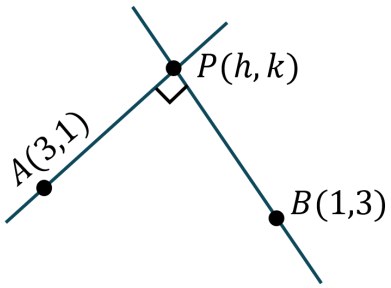
$(x - 3)^2 + y^2 + \lambda y = 0$ ....(1)

Putting (1, –2) in (1)

$\Rightarrow \lambda = 4$

Required circle is $x^2 + y^2 - 6x + 4y + 9 = 0$ point (5, –2) satisfies the equation the equation.

66)




First family passes through (3,1)

and second family passes through (1,3)

Let point of intersection is P(h,k)

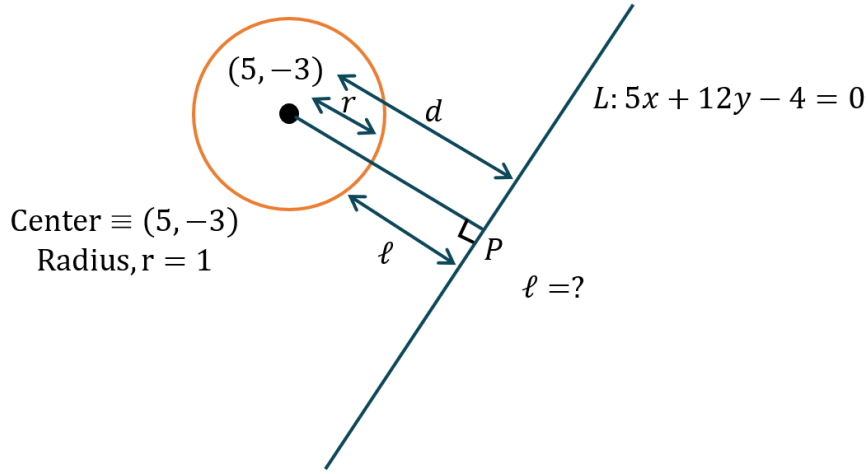
so, $m_{AP}.m_{BP} = -1$ (for locus)

$$\Rightarrow \left(\frac{k-1}{h-3}\right)\cdot\left(\frac{k-3}{h-1}\right)=-1$$

$\Rightarrow (k-1)(k-3) = -(h-3)(h-1)$
$\Rightarrow k^2 - 4k + 3 = -(h^2 - 4h + 3)$
$\Rightarrow h^2 + k^2 - 4h - 4k + 6 = 0$
$\Rightarrow x^2 + y^2 - 4x - 4y + 6 = 0$

67)

$$d = \left|\frac{5(5) + 12(-3) - 4}{\sqrt{5^2 + 12^2}}\right| = \left|\frac{25 - 36 - 4}{13}\right| = \frac{15}{13}$$




□ = |d - r| = minimum distance between line & circle

$$\Rightarrow \ell = \left|\frac{15}{13} - 1\right| = \frac{2}{13}$$

68)

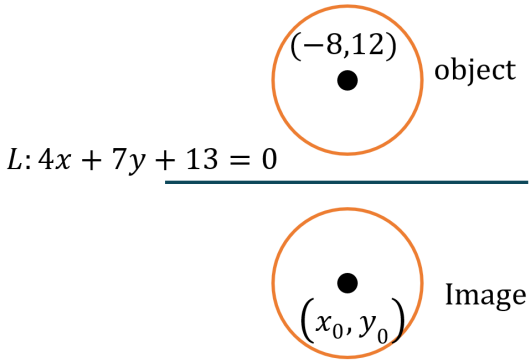



Image of a circle in a line is a circle with same radius and centre as the reflection of centre of the given circle.

Let image is $(x_0, y_0)$ of (-8,12) in line L : 4x + 7y + 13 = 0

so, $$\frac{x_0 - (-8)}{4} = \frac{y_0 - 12}{7} = (-2)\left(\frac{4 \times (-8) + 7 \times 12 + 13}{4^2 + 7^2}\right)$$

$$\Rightarrow \frac{x_0 + 8}{4} = \frac{y_0 - 12}{7} = -2$$

and radius, $r = \sqrt{(-8)^2 + (12)^2 - 183} = 5$

□ equation of the circle is

$(x + 16)^2 + (y + 2)^2 = 5^2$
$\Rightarrow x^2 + y^2 + 32x + 4y + 235 = 0$

69)

$SS_1 = T^2$ is formula for pair of tangents.
where $S = x^2 + y^2 + 4x + 6y + 9$
$S_1 = 9$
$T = 0.x + 0.y + 4\left(\frac{x+0}{2}\right) + 6\left(\frac{y+0}{2}\right) + 9$ $= 2x + 3y + 9$
so, $(x^2 + y^2 + 4x + 6y + 9)(9) = (2x + 3y + 9)^2$
$\Rightarrow 9(x^2 + y^2) + 18(2x + 3y) + 81 = (2x + 3y)^2 + 18(2x + 3y) + 81$
$\Rightarrow 9(x^2 + y^2) = (2x + 3y)^2$

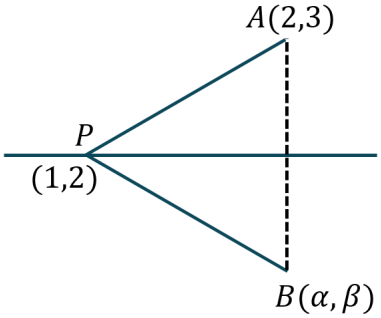
70)

$x^2 + y^2 + 4x - 6y - 12 = 0$
Equation of tangent at $(1, -1)$
$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$
$3x - 4y - 7 = 0$
 Equation of circle is
$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$
It passes through $(4, 0)$:
$(16 + 16 - 12) + \lambda(12 - 7) = 0$
$\Rightarrow 20 + \lambda(5) = 0$
$\Rightarrow \lambda = -4$
 $(x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$
or $x^2 + y^2 - 8x + 10y + 16 = 0$
Radius $= \sqrt{16 + 25 - 16} = 5$

71)

Circle touches internally
$C_1(0, 0)$; $r_1 = 2$
$C_2 : (-3, -4)$; $r_2 = 7$
$C_1C_2 = |r_1 - r_2|$
$S_1 - S_2 = 0 \Rightarrow$ eqn. of common tangent
$6x + 8y - 20 = 0$
$3x + 4y = 10$
$(6, -2)$ satisfy it

72)



P is the fixed point for given family of line

PB = PA ⇒ $(\alpha - 1)^2 + (\beta - 2)^2 = 1 + 1$

$$(x-1)^2 + (y-2)^2 = \left(\sqrt{2}\right)^2$$

∴ locus is a circle of radius $\sqrt{2}$.

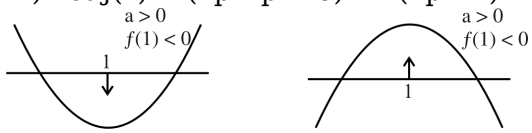
73)

Since, $\alpha^2 = 5\alpha - 3 \Rightarrow \alpha^2 - 5\alpha + 3 = 0$
and $\beta^2 = 5\beta - 3 \Rightarrow \beta^2 - 5\beta + 3 = 0$
These two equations shows that a and b are the roots of the equation
$x^2 - 5x + 3 = 0$
$\therefore \quad \alpha + \beta = 5$ and $\alpha\beta = 3$

Now $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$$= \frac{25-6}{3} = \frac{19}{3}$$

74) Let $f(x) = (4p - p^2 - 5)x^2 - (2p - 1)x + 3p \quad ...(1)$




∴ a.f(1) < 0
⇒ $(4p - p^2 - 5).[4p - p^2 - 5 - 2p + 1 + 3p] < 0$

$$\Rightarrow \underbrace{(p^2 - 4p + 5)}_{\substack{a>0 \\ D<0}} \underset{\text{positive}}{\therefore \text{ always}} (p^2 - 5p + 4) < 0$$

⇒ $(p - 4)(p - 1) < 0$
∴ $p \in (1, 4)$
∴ Number of integral value(s) of p is (2)

75) ∵ a, b, c are roots of $x^3 + 8x + 1 = 0$
∴ $a^3 + 8a + 1 = 0$
⇒ $8a + 1 = -a^3$
same as $8b + 1 = -b^3$
& $8c + 1 = -c^3$

find $\dfrac{bc}{(8b+1)(8c+1)} + \dfrac{ac}{(8a+1)(8c+1)} + \dfrac{ab}{(8a+1)(8b+1)}$

$$= \frac{bc}{b^3c^3} + \frac{ac}{a^3c^3} + \frac{ab}{a^3b^3}$$

$$= \frac{a^2 + b^2 + c^2}{b^3c^3} = \frac{(a+b+c)^2 - 2(ab+bc+ca)}{(abc)^2}$$

$$= \frac{(0)^2 - 2(8)}{(-1)^2} = -16$$

76)

$$\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$$

$$\frac{a+b+c}{a+c}+\frac{a+b+c}{b+c}=3$$

$$1+\frac{b}{a+c}+\frac{a}{b+c}+1=3$$

$$\frac{b}{a+c}+\frac{a}{b+c}=1$$

$b(b + c) + a(a + c) = (a + c)(b + c)$

$b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

$b^2 + a^2 = ab + c^2$

or $\frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}$ (By transforming)

$\cos C = \frac{1}{2}$ or $\angle C = 60°$

77)

$$8\cos x\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$$

$$\Rightarrow 8\cos x\left(\frac{1}{4}-\left(1-\cos^2 x\right)\right)=1$$

$$\Rightarrow 8\cos x\left(\cos^2 x-\frac{3}{4}\right)=1$$

$$\Rightarrow 2\cos 3x = 1 \Rightarrow \cos 3x = \frac{1}{2}$$

$\square\ 3x + 2n\pi \pm \frac{\pi}{3}$, $n \in I$

$$\Rightarrow x=\frac{2n\pi}{3}\pm\frac{\pi}{9}$$

In $x \in [0, \pi]$ : $x = \frac{\pi}{9}, \frac{2\pi}{3}+\frac{\pi}{9}, \frac{2\pi}{3}-\frac{\pi}{9}$ only

sum = $\frac{13\pi}{9}$

78)

$a_1, a_2, a_3$.... AP

$$\frac{a_1+a_2+\ldots+a_p}{a_1+a_2+a_3+\ldots+a_q}=\frac{p^2}{q^2}$$

$$\Rightarrow \frac{\frac{p}{2}(2a_1+(p-1)d)}{\frac{q}{2}(2a_1+(q-1)d)}=\frac{p^2}{q^2}$$

$$\Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d}=\frac{p}{q}$$

$$\Rightarrow \frac{a_1+\left(\frac{P-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d}=\frac{p}{q}$$ .....(i)

$$\frac{a_6}{a_{21}}=\frac{a_1+5d}{a_1+20d}$$ ($a_n = a_1 + (n - 1)d$)

From equation (i)

$\frac{p-1}{2} = 5$ and $\frac{q-1}{2} = 20$

p = 11 q = 41

put p = 11 and q = 41 in equation (i)

$\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41}$

$\frac{a_6}{a_{21}} = \frac{11}{41}$

79) $T_n = \frac{3 + (n-1)2}{\Sigma n^2} = \frac{6(2n+1)}{n(n+1)(2n+1)} = \frac{6}{n(n+1)}$

$S_n = 6\left[1 - \frac{1}{n+1}\right] \Rightarrow S_{11} = 6\left[1 - \frac{1}{12}\right] = 6\left(\frac{11}{12}\right) = \frac{11}{2}$

80)

Let $\log_2\left(x^2+1\right) = a$ & $\log_{13}\left(x^2+1\right) = b$

a + b = ab

$\frac{1}{a} + \frac{1}{b} = 1 \Rightarrow \frac{1}{\log_2(x^2+1)} + \frac{1}{\log_{13}(x^2+1)} = 1$

$\log_{(x^2+1)}2 + \log_{(x^2+1)}13 = 1$

$\log_{x^2+1}(26) = 1 \Rightarrow 26 = x^2 + 1$

$\Rightarrow x^2 = 25$

$\log_7\left(x^2+24\right) = \log_7 49 = \log_7 7^2 = 2$

81)

$(11x)^2 + (2y)^2 + (3z)^2 - 22x + 4y + 6z + 3 = 0$

$\Rightarrow (11x-1)^2 + (2y+1)^2 + (3z+1)^2 = 0$

$\Rightarrow 11x - 1 = 2y + 1 = 3z + 1 = 0$

$\Rightarrow x = \frac{1}{11}, y = \frac{-1}{2}, z = \frac{-1}{3}$

$\Rightarrow x^{-1} - y^{-1} - z^{-1} = 11 + 2 + 3 = 16.$

82)

$9^x + 6^x = 2 \times 4^x$

$\Rightarrow \left(\frac{9}{4}\right)^x + \left(\frac{6}{4}\right)^x = 2 \Rightarrow \left(\frac{3}{2}\right)^{2x} + \left(\frac{3}{2}\right)^x - 2 = 0$

Let $\left(\frac{3}{2}\right)^x = t\ (t > 0)$

so, $t^2 + t - 2 = 0$

$\Rightarrow t = 1$ or $t = -2$ (rejected)

$\Rightarrow \left(\frac{3}{2}\right)^x = 1 \Rightarrow x = 0$

$\frac{x^3 + 64}{5} = \frac{64}{5} = 12.8$

83)

use n (A ∪ B) = n(A) + n(B) – n(A ∩ B)

84)

Multiply and divide by tan 54º

$\Rightarrow \frac{(\tan 6^\circ . \tan 66^\circ . \tan 54^\circ)}{\tan 54^\circ} . \tan 42^\circ \tan 78^\circ$

w.r.t.

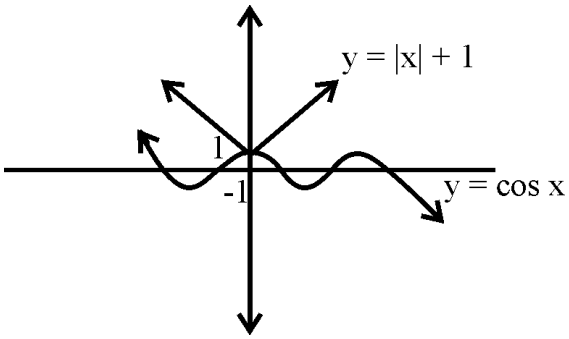
$\tan\theta.\tan(60 + \theta) \tan(60 - \theta) = \tan^3\theta$

so if θ = 6º, then

$\Rightarrow \frac{\tan 18^\circ . \tan 42^\circ . \tan 78^\circ}{\tan 54^\circ}$ (Now of θ = 18º)

$\Rightarrow \frac{\tan 54^\circ}{\tan 54^\circ} = 1$

85)




No. of solution = n =1

$\therefore \frac{n}{4} = \frac{1}{4} = 0.25$

86) $t_1 = 8$

$d = 6$

So, $8 + (n-1)6 = 164$

n = 27

87)

$1 + \alpha + \alpha^2 + \ldots \infty + 1 + \beta + \beta^2 + \ldots \infty$

$\frac{1}{1-\alpha} + \frac{1}{1-\beta}$

$= \frac{2-(\alpha+\beta)}{1-(\alpha+\beta)+\alpha\beta}$

$= \frac{2-\frac{5}{6}}{1-\frac{5}{6}+\frac{1}{6}}$

= 3.50

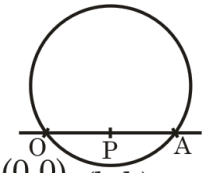
88)

origin lie on dir. circle

$(x - 3)^2 + (y - p)^2 = 2(p^2 - 8)$

$\Rightarrow 9 + p^2 = 2p^2 - 16$

89)




OP = PA

⇒ A(2h, 2k)

⇒ Point A lies on the circle,

Hence, $(2h - 1)^2 + (2k)^2 = 1$

$\Rightarrow x^2 + y^2 - x = 0$

$\therefore \lambda = 1$

90)

$$\Delta = 0 \Rightarrow \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

For represent a pair of straight line

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\Rightarrow 12 \times 2 \times \lambda + 2 \times \frac{-5}{2} \times \frac{11}{2} \times -5 - 12 \times \left(\frac{-5}{2}\right)^2$$

$$-2 \times \left(\frac{11}{2}\right)^2 - \lambda(-5)^2 = 0$$

On solving $\lambda = 2$