


## PART-1 : PHYSICS

### SECTION-I

1)

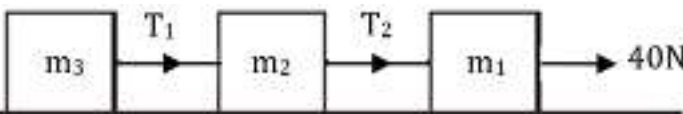
An elevator weighing 6000 kg is pulled upward by a cable with an acceleration of  $5 \text{ m/s}^2$ , then the tension in the cable is (Taking  $g$  to be  $10 \text{ m/s}^2$ )

- (A) 6000 N
- (B) 9000 N
- (C) 60000 N
- (D) 90000 N

2) What is the normal force applied on block of mass  $m$ . 

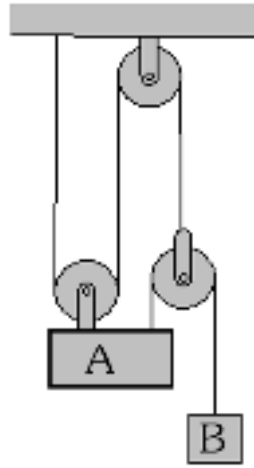
- (A)  $\frac{Fm}{M+m}$
- (B)  $\frac{FM}{M+m}$
- (C)  $\frac{F}{M+m}$
- (D)  $\frac{Fm}{M}$

3) Three blocks of masses  $m_1$ ,  $m_2$  and  $m_3$  are connected by massless strings as shown in the figure on a frictionless table. They are pulled with a force of 40 N. If  $m_1 = 10 \text{ kg}$ ,  $m_2 = 6 \text{ kg}$  and  $m_3 = 4 \text{ kg}$ ,

then tension  $T_2$  will be : 

- (A) 10 N
- (B) 20 N
- (C) 32 N
- (D) 40 N

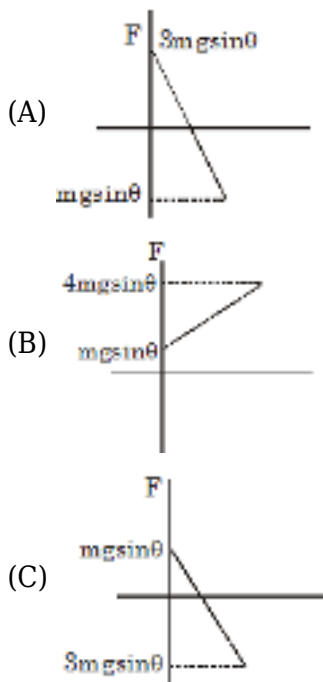
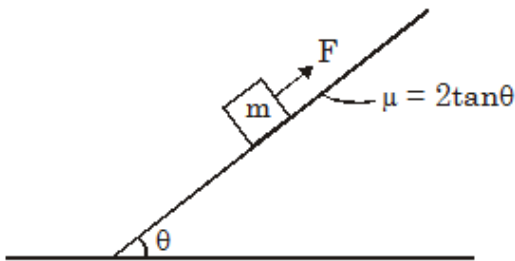
4) In arrangement shown the block A of mass 15 kg is supported in equilibrium by the block B. Mass

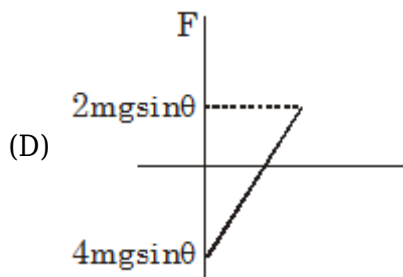


of the block B is closest to

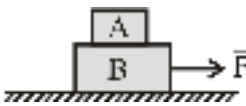
- (A) 2 kg
- (B) 3 kg
- (C) 4 kg
- (D) 5 kg

5) A block of mass  $m$  is placed on a rough inclined plane. A force  $F$  is applied on block, whose magnitude and direction can be changed. If force is up the plane take it positive, if down the plane take it negative. Which of the following variation of force ensure zero acceleration of block.



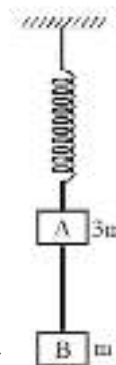


6) Block A of mass 4 kg and block B of mass 6 kg are resting on a horizontal surface as shown in the figure. There is no friction between the block B and the horizontal surface. The coefficient of friction between the blocks is 0.2. If the value of  $g = 10 \text{ ms}^{-2}$ , the maximum horizontal force  $F$  that can be

applied on block B without any relative motion between A and B is 

- (A) 20 N
- (B) 40 N
- (C) 60 N
- (D) 100 N

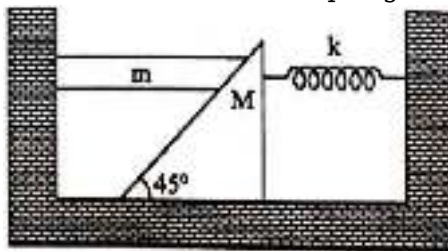
7) Two blocks A and B of masses  $3m$  and  $m$  respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The



magnitudes of acceleration of A and B immediately after the string is cut, are respectively

- (A)  $\frac{g}{3}, g$
- (B)  $g, g$
- (C)  $\frac{g}{3}, \frac{g}{3}$
- (D)  $g, \frac{g}{3}$

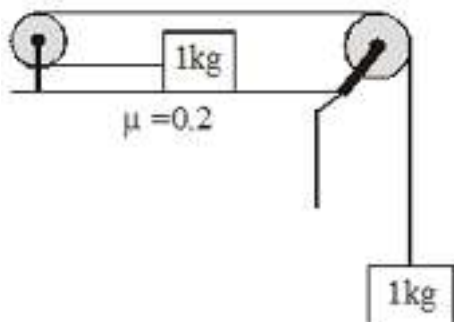
8) All surfaces shown in figure are smooth. System is released with the spring unstretched. In



equilibrium, compression in the spring will be :-

- (A)  $\frac{mg}{\sqrt{2}k}$   
 (B)  $\frac{2mg}{k}$   
 (C)  $\frac{(M+m)g}{\sqrt{2}k}$   
 (D)  $\frac{mg}{k}$

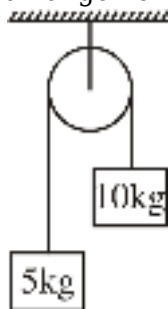
9) A block of mass 1kg is placed on a rough horizontal surface connected by a light string passing over two smooth pulleys as shown. Another block of 1kg is connected to the other end of the string. The acceleration of the block on table is (Coefficient of friction  $\mu = 0.2$ ) (Assume string and pulley



are ideal)

- (A) 0.8 g  
 (B) 0.4 g  
 (C) 0.5 g  
 (D) Zero

10) If this arrangement is set free at  $t = 0$  from rest. The work done by tension on 10 kg block in first



3 sec. is :-

- (A) + 500 J  
 (B) - 1100 J  
 (C) - 500 J  
 (D) - 1000 J

11) The over-bridge of a river is in the form of a circular arc of radius of curvature  $r$ . If  $m$  is the combined mass of the motorcycle and the rider crossing the bridge at a speed  $v$  at highest point. The normal on the bridge at the highest point will be :

- (A)  $\frac{mv^2}{r}$   
 (B)  $mg$

(C)  $\frac{mv^2}{r} - mg$

(D)  $mg - \frac{mv^2}{r}$

12) A hollow right circular cone is fixed with its axis vertical and vertex down. A particle is describing circular motion in contact with the smooth inside surface of the cone in a horizontal plane at a height  $h$  above the vertex. Its velocity is

(A)  $\sqrt{gh}$

(B)  $\sqrt{2gh}$

(C)  $\sqrt{\frac{gh}{2}}$

(D)  $\sqrt{\frac{2gh}{3}}$

13) A particle moves along a path. The angular acceleration of particle as a function of time is given by  $\alpha = 6t - 12$ . Initial angular velocity of particle is 9 rad/s then calculate angular displacement for first 4sec ?

(A) 8 rad

(B) 6 rad

(C) 4 rad

(D) 2 rad

14) A particle is moving in circle of radius  $R$  with constant angular acceleration  $\alpha$  at the time when centripetal acceleration is  $2\sqrt{2}$  times of tangential acceleration then net acceleration of particle is :-

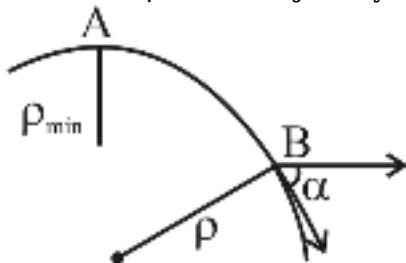
(A)  $3R\alpha$

(B)  $2R\alpha$

(C)  $R\alpha$

(D)  $4R\alpha$

15) If  $\rho_{\min}$  is radius of curvature of trajectory of a projectile at highest point A, then the radius of curvature  $\rho$  of the trajectory at B where tangent on it forms  $\alpha$  angle with horizontal is.



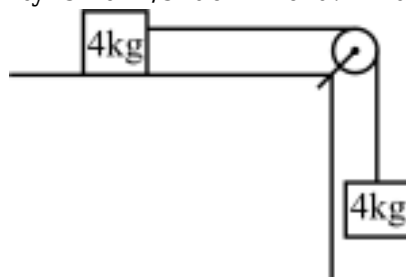
(A)  $\rho = \frac{\rho_{\min}}{\cos^3 \alpha}$

(B)  $\frac{\rho_{\min}}{\cos^2 \alpha}$

(C)  $\frac{\rho_{\min}}{\cos \alpha}$

(D) None of these

16) In the given arrangement, there is no friction anywhere. Pulley and string are ideal. Acceleration due to gravity is  $10 \text{ m/s}^2$  downward. Find out total work done by gravity in 2 sec, if



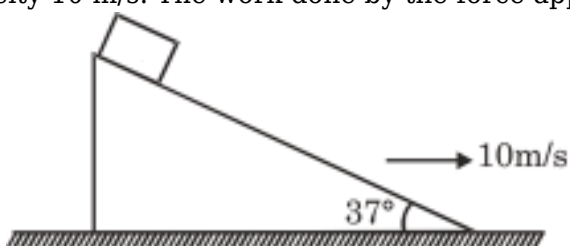
system starts from rest.

- (A) 100 J
- (B) 400 J
- (C) 200 J
- (D) 800 J

17) The work done by a force  $F = kx^2$  acting on a particle at an angle  $60^\circ$  with x-axis to displace it from  $x = 2\text{m}$  to  $x = 3\text{m}$  is :

- (A)  $\frac{5}{6}k$
- (B)  $\frac{5}{2}k$
- (C)  $\frac{19}{2}k$
- (D)  $\frac{19}{6}k$

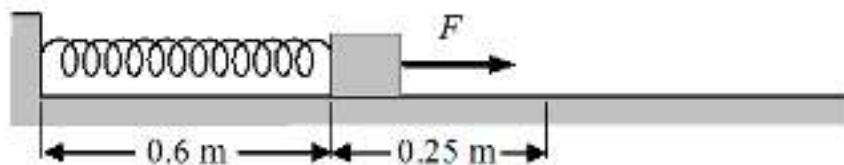
18) A block of mass 10 kg is released from the top of smooth wedge (whose inclination at  $\theta = 37^\circ$ ) which is moving with a constant velocity 10 m/s. The work done by the force applied by wedge on



the block in 2 sec will be (in joules) :-

- (A) 1600
- (B) 960
- (C) Zero
- (D) 720

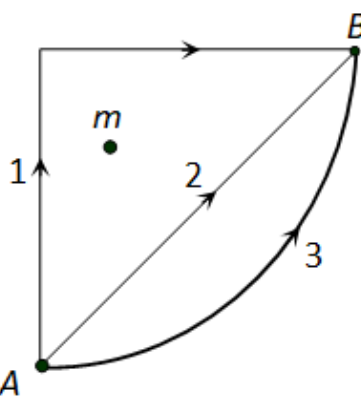
19) A 0.5 kg block is affixed to one end of a spring of relaxed length 0.6 m and force constant 40 N/m. The other end of the spring is affixed to the wall. The block rests at distance 0.6m away from the wall on frictionless floor. A constant force  $F = 20 \text{ N}$  is applied on the block. This force is removed when block covers a distance of 0.25 m further away from the wall. In subsequent motion how close,



will the block get to the wall ?

- (A) 10 cm
- (B) 15 cm
- (C) 20 cm
- (D) More information is required to decide.

20) If  $w_1$ ,  $w_2$  and  $w_3$  represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass  $m$ , find the correct

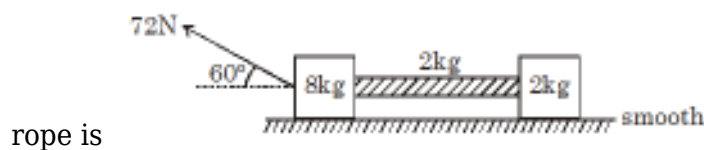


relation between  $w_1$ ,  $w_2$  and  $w_3$

- (A)  $W_1 > W_2 > W_3$
- (B)  $W_1 = W_2 = W_3$
- (C)  $W_1 < W_2 < W_3$
- (D)  $W_2 > W_1 > W_3$

## SECTION-II

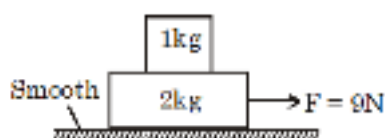
1) In the figure shown if mass of the rope is 2 kg, then tension (in Newton) at the midpoint of the



rope is

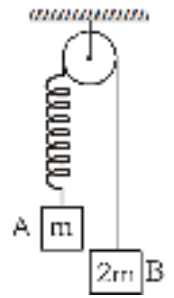
2)

In the figure, there is no relative motion between the two blocks. Force of friction acting on 1 kg block (in N) is :-



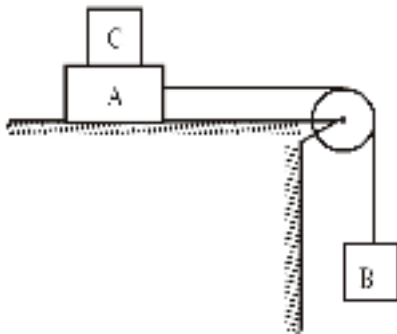
3) In the figure a block 'A' of mass 'm' is attached at one end of a light spring and the other end of

the spring is connected to another block 'B' of mass  $2m$  through a light string. 'A' is held and B has obtained equilibrium. Now A is released. The acceleration of A just after that instant is 'a'. The same



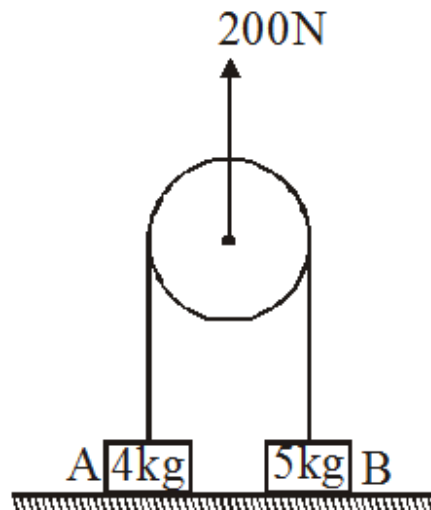
thing is repeated for 'B'. In that case the acceleration of 'B' is 'b', then find value of  $a/b$ .

4) Two masses A and B of 16 kg and 4 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown in figure. The coefficient of friction of A with the table is 0.2. Find the minimum mass (in kg) of C that may be placed on A to prevent it from



moving.

5) Two blocks of masses 4 kg and 5 kg are at rest on the ground. The masses are connected by a massless string passing over a smooth and light pulley. A 200 N force is applied on the pulley. Find



the mod. of difference in accelerations of blocks in  $\text{m/s}^2$ .

6) An object of mass 4 kg falls from rest through a vertical distance of 10 m and reaches ground with a speed of 14 m/s. How much work is done against air friction in joules ? [  $g = 10 \text{ m/s}^2$  ]

7) A block of mass 2.0 kg is given an initial speed along the floor towards a spring as shown. The coefficient of kinetic friction between the floor and the block is 0.4 and force constant of the spring is  $5.6 \times 10^3 \text{ N/m}$ . The block compresses the spring by 10 cm before it stops for a moment. What is





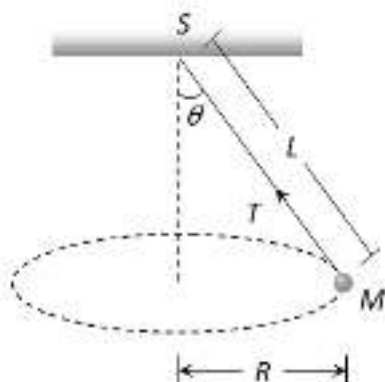
the initial speed (cm/s) of the block?

8) An object of mass 2kg is released at origin of x-y plane, where its potential energy changes as  $U = (2x - 4y)$  joules. Find speed of object after 3 sec of release. (use  $\sqrt{5} = 2.24$ )

9) A rod is rotating with an angular velocity of  $3\pi^3$  rad/s. The angle rotated by the rod from  $t = 0$  to the time when it has an angular velocity of  $24\pi$  rad/s is given by  $\frac{\alpha}{2}$  radian. Fill  $\frac{\alpha}{2}$  in OMR sheet :-

10)

A string of length  $L$  is fixed at one end and carries a mass  $M$  at the other end. The string makes  $2/\pi$  revolutions per second around the vertical axis through the fixed end as shown in the figure, then tension in the string is  $xML$ , then value of  $x$  is



## PART-2 : CHEMISTRY

### SECTION-I

1) Dalton's law cannot be applied for which gaseous mixture at normal temperatures :

- (A)  $O_2$  and  $N_2$
- (B)  $NH_3$  and  $HCl$
- (C)  $He$  and  $N_2$
- (D)  $CO_2$  and  $O_2$

2) A closed vessel contains helium and ozone at a pressure of  $P$  atm. The ratio of He and oxygen atoms is 1 : 1. If helium is removed from the vessel, the pressure of the system will reduce to (Assume constant temperature)

- (A)  $0.5 P$  atm
- (B)  $0.75 P$  atm

- (C) 0.25 P atm  
(D) 0.33 P atm

3) What is the average kinetic energy per gram  $\left( \text{in } \frac{\text{cal}}{\text{gm}} \right)$  of sample of  $\text{CH}_4$  at  $47^\circ\text{C}$ .

- (A) 960  
(B) 60  
(C) 249.42  
(D) 2.463

4) A flask containing gas at  $107^\circ\text{C}$  and 722 mm of Hg is cooled to 100 K and 760 mm of Hg. If initial density of gas was  $1 \frac{\text{g}}{\text{ml}}$ , then find the final density  $\left( \text{in } \frac{\text{g}}{\text{cm}^3} \right)$

- (A) 4  
(B) 4000  
(C) 1.12  
(D) 1120

5) At what temperature  $U_{\text{mp}}$  of  $\text{SO}_2$  has the same value of  $U_{\text{rms}}$  of  $\text{O}_2$  at 300 K

- (A) 150 K  
(B) 600 K  
(C) 750 K  
(D) 900 K

6) A gas at a pressure of 5.0 atm is heated from  $0^\circ\text{C}$  to  $546^\circ\text{C}$  and simultaneously compressed to one-third of its original volume. Hence final pressure is

- (A) 10 atm  
(B) 30 atm  
(C) 45 atm  
(D) 5 atm

7)

Which of the following gas will have the highest value of translational kinetic energy per g at the same temp ?

- (A)  $\text{CH}_4$   
(B) He  
(C)  $\text{N}_2$   
(D) Same for all

8)

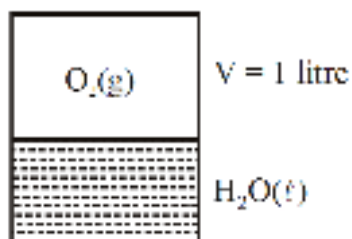
Initially the root mean square velocity (rms) of  $\text{N}_2$  molecules at certain temperature is  $u$ . If this temperature is doubled and all the nitrogen molecules dissociate into nitrogen atoms, then the new rms velocity will be :

- (A)  $u/2$
- (B)  $4u$
- (C)  $14u$
- (D)  $2u$

9) Assuming ideal gas behaviour, the ratio of density of ammonia to that of helium at same temperature & pressure is :

- (A) 1.64
- (B) 0.42
- (C) 4.25
- (D) 5.4

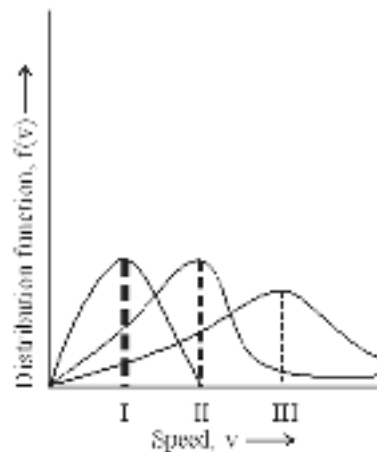
10)  $\text{O}_2$  gas is placed in a 4 litre container containing 3L of liquid water as shown & total pressure exerted by gases is 720 mm Hg. What will be the pressure of  $\text{O}_2(\text{g})$  if given container is attached to a empty container of 3 litre at same temperature. Given : [V.P. of  $\text{H}_2\text{O}$  at  $27^\circ\text{C}$  is = 20 mm of Hg].



- (A) 175 mm of Hg
- (B) 350 mm of Hg
- (C) 200 mm of Hg
- (D) 800 mm of Hg

11) A 4 : 1 molar mixture of He &  $\text{CH}_4$  kept in a vessel at 20 bar pressure. Due to a hole in the vessel, gas mixture leaks out. What is the composition of mixture effusing out initially -

- (A) 8 : 1
- (B) 4 : 1
- (C) 1 : 4
- (D) 4 : 3



12) Points I, II and III in the following plot respectively correspond to :  
( $V_{mp}$  ; most probable velocity)

- (A)  $V_{mp}$  of  $N_2$  (300K);  $V_{mp}$  of  $H_2$ (300K);  
 $V_{mp}$  of  $O_2$ (400K)
- (B)  $V_{mp}$  of  $H_2$  (300K);  $V_{mp}$  of  $N_2$ (300K);  
 $V_{mp}$  of  $O_2$ (400K)
- (C)  $V_{mp}$  of  $O_2$  (400K);  $V_{mp}$  of  $N_2$ (300K);  
 $V_{mp}$  of  $H_2$ (300K)
- (D)  $V_{mp}$  of  $N_2$  (300K);  $V_{mp}$  of  $O_2$ (400K);  
 $V_{mp}$  of  $H_2$ (300K)

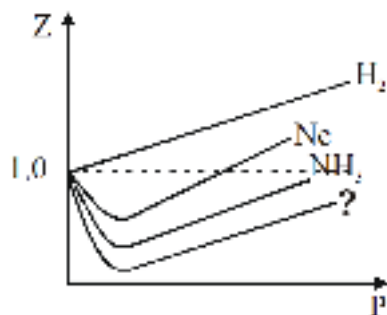
13) If the critical temperature of the gas is  $T_c$  and  $T_b$  is the Boyle's temperature, then which of the following is the correct relation between  $T_c$  and  $T_b$  ?

- (A)  $T_c = \frac{8}{27}T_b$
- (B)  $\frac{T_c}{27} = \frac{1}{8}T_b$
- (C)  $T_c = \frac{4}{27}T_b$
- (D)  $T_c = \frac{27}{4}T_b$

14) Which of the following is incorrect for real gas at high pressure and room temperature

- (A)  $Z > 1$
- (B)  $Z = 1 + \frac{a}{RTV_m}$
- (C)  $Z = 1 + \frac{Pb}{RT}$
- (D) Repulsive forces dominant

15) Observe the following Z vs P graph.



The missing gas in the above graph can be :

- (A) He
- (B) Ar
- (C)  $C_5H_{12}$
- (D) All are correct

16) When 1 mol gas is heated at constant volume, temp. is raised from 298 to 308 K. Heat supplied to the gas is 500 J. Then which statement is correct :-

- (A)  $q = w = 500J, \Delta U = 0$
- (B)  $q = \Delta U = 500J, w = 0$
- (C)  $q = w \neq 500 J, \Delta U = 0$
- (D)  $\Delta U = 0, q = w = - 500 J$

17) The work done by a system is 8 J when 40 J heat is supplied to it. The change in internal energy of the system during the process :-

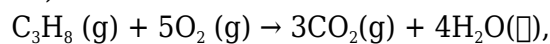
- (A) 32 J
- (B) 40 J
- (C) 48 J
- (D) -32 J

18)

According to first law of thermodynamics (where  $q$  = heat supplied to system &  $W \rightarrow$  work done by the system)

- (A)  $\Delta U = q - W$
- (B)  $\Delta U = q + W$
- (C)  $\Delta U = \Delta q + \Delta W$
- (D)  $\Delta U = \Delta q + W$

19) For the reaction



at constant temperature,  $\Delta H - \Delta U$  is

- (A)  $-RT$
- (B)  $+RT$

- (C)  $-3RT$   
(D)  $+3RT$

20) For isothermal expansion of an ideal gas, the correct combination of the thermodynamic parameters will be

- (A)  $\Delta U = 0$ ,  $Q = 0$ ,  $w \neq 0$  and  $\Delta H \neq 0$   
(B)  $\Delta U \neq 0$ ,  $Q \neq 0$ ,  $w \neq 0$  and  $\Delta H = 0$   
(C)  $\Delta U = 0$ ,  $Q \neq 0$ ,  $w = 0$  and  $\Delta H \neq 0$   
(D)  $\Delta U = 0$ ,  $Q \neq 0$ ,  $w \neq 0$  and  $\Delta H = 0$

## SECTION-II

1) If at 200 K & 500 atm density of  $\text{CH}_4$  is 0.246 gm/ml then its compressibility factor (Z) is  $2.0 \times 10^x$ , then x is :

2) A gas behaves ideally over a range of pressure at 270 K. Find the critical temperature (in K) of the gas

3) A 20 ml mixture of  $\text{C}_2\text{H}_4$  and  $\text{C}_2\text{H}_2$  undergoes sparking in gas eudiometer with just sufficient amount of  $\text{O}_2$  and shows contraction of 37.5 ml. Volume (in ml) of  $\text{C}_2\text{H}_2$  in the mixture is.

4) 4.0g of argon gas has pressure, P and temperature, TK in a vessel. On keeping the vessel at  $50^\circ\text{C}$  higher temperature, 0.8g of argon gas escape out maintaining the pressure P. The original temperature was " $A$ "  $\times 10^2$  K. The value of A is

5) Root mean square speed of an unknown gas at  $727^\circ\text{C}$  is  $10^5$  cm/second. Calculate molar mass of unknown gas (in gram/mole) [Take  $R = \frac{25}{3}$  J/mole-K].

6) How many times is the rms speed of molecules in 8 gm  $\text{O}_2$  gas at 1200 K and 10 bar, relative to r.m.s speed of molecules in 16 gm  $\text{O}_2$  gas at 300K and 20 bar ?

7) 127 mL of a certain gas diffuses in the same time as 100 mL of chlorine under the same conditions. Calculate the molecular weight of the gas. (as nearest integer)

8) How many of the following physical properties are extensive :

- (i) Free energy    (ii) vapour pressure  
(iii) mole        (iv) Kinetic energy  
(v) Entropy       (vi) Internal energy  
(vii) Enthalpy    (viii) specific heat capacity  
(ix) Coefficient of viscosity  
(x) Total heat capacity

9) Change in internal energy for a cyclic process is

10) A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 at a constant temperature of 37.0 °C. As it does so, it absorbs 208 J of heat. The values of q for the process will be:

(R = 8.314 J/mol K) ( $\ln 7.5 = 2.01$ )

## PART-3 : MATHEMATICS

### SECTION-I

1) If roots of the equation  $ax^2 + bx + c = 0$

(where  $a \neq 0$ ) are  $\alpha, \beta$ . Then

$\frac{1}{(\alpha^2 + \beta^2)} \left[ \frac{1}{(a\alpha + b)^2} + \frac{1}{(a\beta + b)^2} \right]$  is equal to

(A)  $a^2$

(B)  $c^2$

(C)  $\frac{1}{c^2}$

(D)  $\frac{b^2 - 4ac}{2a^2}$

2) If  $3\alpha^2 - 6\alpha + 5 = 0$  &  $3\beta^2 - 6\beta + 5 = 0$ ,

$\alpha \neq \beta$ , then value of  $\frac{\alpha^2 + \beta^2}{\alpha^3 + \beta^3}$  is

(A)  $-\frac{1}{3}$

(B)  $\frac{2}{3}$

(C)  $\frac{3}{5}$

(D)  $-\frac{1}{5}$

3) If  $a, b, c \in \mathbb{Q}$ , then the roots of the equation

$$(a + b - c)x^2 + (b + c - a)x - 2b = 0$$

(where  $a + b \neq c$ ) are

(A) rational

(B) irrational

(C) imaginary

(D) real and lie on opposite side of unity

4) If equation  $2x^2 + 5x + 1 = 0$  and  $x^2 + bx + c = 0$ ,  
 $b, c \in \mathbb{Q}$  have a common root then value of  $b + c$  is

- (A) 5
- (B) 2
- (C) -4
- (D) 3

5) If  $\alpha, \beta, \gamma$  are roots of equation  $x^3 + 3x^2 + x - 1 = 0$   
then value of  $(3 + \alpha)(3 + \beta)(3 + \gamma)$  is

- (A) -4
- (B) -58
- (C) 58
- (D) 4

6) Maximum value of  $\frac{x^2 + 4x + 7}{x^2 + 4x + 5}$  is

- (A) 3
- (B) 4
- (C)  $\frac{17}{6}$
- (D)  $\frac{17}{4}$

7) The value of 'a' for which one root of the  
equation  $x^2 - (a + 1)x + (a^2 + a - 8) = 0$   
exceeds 2 and the other is less than 2 is given by

- (A)  $3 < a < 10$
- (B)  $-2 < a < 3$
- (C)  $a \geq 10$
- (D)  $a \leq -2$

8) An equation  $(2x^2 + 1 - 3x)(2x^2 + 1 + 5x) - 9x^2 = 0$  has

- (A) two real and two imaginary roots
- (B) all imaginary roots
- (C) four real roots
- (D) all roots are equal

9) If  $(a^3 - 3a - 2)x^2 + (a^2 - a - 2)x = 2a^4 - 10a + k$ ,  
is true for all real values of  $x$  then the value of  $k$  are

- (A) 6
- (B) -12



(C) - 6

(D) 12

10) Let  $p$  and  $q$  be real numbers such that  $p \neq 0$ ,  $p^3 \neq q$  and  $p^3 \neq -q$ . If  $\alpha$  and  $\beta$  are nonzero complex numbers satisfying  $\alpha + \beta = -p$  and  $\alpha^3 + \beta^3 = q$ , then a quadratic equation having  $\frac{\alpha}{\beta}$  and  $\frac{\beta}{\alpha}$  as its roots is

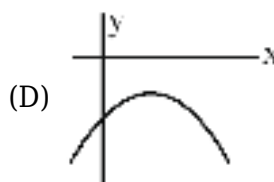
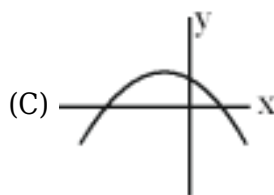
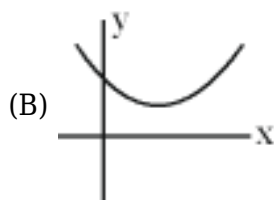
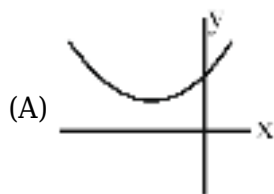
(A)  $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B)  $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C)  $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D)  $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

11) Let  $y = \left(\alpha + \frac{1}{\alpha}\right)x^2 + x + 5$ , where  $\alpha > 0$   
then which of the following can be graph of given expression



12) The equation  $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$  has

(A) No solution

(B) One solution

(C) Two solution

(D) More than two solutions

13)

Number of solutions of

$$\cos^4 2x + 2\sin^2 2x = 17(\cos x + \sin x)^8 \text{ is,}$$
$$0 < x < 2\pi$$

- (A) 2
- (B) 4
- (C) 6
- (D) 3

14) If  $\left(\cos^2 x + \frac{1}{\cos^2 x}\right)(1 + \tan^2 2y)(3 + \sin 3z) = 4$ , then

- (A) y can be a multiple of  $\pi/2$
- (B) x cannot be an even multiple of  $\pi$
- (C) z can be a multiple of  $\pi$
- (D) None of these

15) If  $5\cos\theta$ , 2 and  $-\cos 3\theta$  are in A.P,

then number of such value(s) of  $\theta$  in  $\left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]$ , is

- (A) 0
- (B) 2
- (C) 4
- (D) 6

16) If  $\sec x + \cos x = 2$ , then value of  $(\sec x)^6 + (\cos x)^6$ , is

- (A) 0
- (B) 1
- (C) 2
- (D) 8

17) Number of solution of the equation

$$(\sin x + \cos x)^{1+\cos 8x} = 2 \text{ in } \left(0, \frac{\pi}{2}\right) \text{ is}$$

- (A) 0
- (B) 1
- (C) 2
- (D) 4

18) Number of solutions of the equation

$$\sin x + \cos x = -\frac{3}{2} \text{ in } [0, 4\pi] \text{ is}$$

- (A) 1
- (B) 2
- (C) 3
- (D) 0

19) If  $\theta \in [0, 2\pi]$  then number of ordered pairs  $(x, \theta)$  which satisfies  $x^4 - 2x^2 \cos \theta + 1 = 0$  is/are

- (A) 1
- (B) 2
- (C) 3
- (D) 4

20) The number of integral value of  $k$  for which the equation  $7 \cos x + 5 \sin x = 2k + 1$  has a solution is

- (A) 4
- (B) 8
- (C) 10
- (D) 12

## SECTION-II

1) Let  $\alpha$  and  $\beta$  be the roots of  $x^2 - 5x - 3 = 0$  with  $\alpha > \beta$ .

If  $a_n = \alpha^n - \beta^n$  for  $n \in \mathbb{N}$ , then the value of  $\frac{a_{10} - 3a_8}{4a_9}$  is

2) If the number of positive integral solutions

of  $|x^2 - 3x| + |6 - 2x| = |x^2 - 5x + 6|$  is  $k$  then the value of  $\frac{k}{4}$  is

3) Let  $a, b, c \in \mathbb{R}$  and  $\alpha, \beta$  are the real roots of the equation  $ax^2 + bx + c = 0$  (where  $a \neq 0$ ) and if  $a + b + c < 0$ ,  $a - b + c < 0$ ,  $c > 0$  then  $|[\alpha] + [\beta]|$  is equal to :  
(Where  $[.]$  denotes the greatest integer function)

4) Number of integral values of  $\lambda$  for which  $x^2 - 2\lambda x < 41 - 6\lambda \forall x \in (1, 6]$ , is

5) In  $\Delta PQR$ ,  $\angle R = \frac{\pi}{2}$ . If  $\tan\left(\frac{P}{2}\right)$  and  $\tan\left(\frac{Q}{2}\right)$  are roots of the quadratic equation  $ax^2 + bx + c = 0$  if  $a, b, c \in \{1, 2, 3, 4\}$  then

total number of ordered pairs of (a, b) are

6)

Number of solutions of the equation,  
 $|\sin x| = \cos x$ , in  $x \in [0, 6\pi]$  is

7) If the number of distinct solutions of equation

$\frac{5}{4}$

$$\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

in the interval  $[0, 2\pi]$  is  $40k$  then the value of  $k$  is

8) Number of solution(s) of the equation

$$\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = 0$$

in the interval  $\left(0, \frac{\pi}{4}\right)$  is \_\_\_\_\_

9) If  $0 \leq x < 2\pi$ , then the number of real values of  $x$ ,  
which satisfy the equation

$$\cos x + \cos 2x + \cos 3x + \cos 4x = 0, \text{ is } \underline{\hspace{2cm}}$$

10) Number of roots of the equation

$$\cos^7 x + \sin^4 x = 1 \text{ in the interval } [0, 2\pi] \text{ is}$$

## ANSWER KEYS

---

### PART-1 : PHYSICS

#### SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	B	A	A	A	D	B	D	D	A	C	A	A	B	D	B	A	B

#### SECTION-II

Q.	21	22	23	24	25	26	27	28	29	30
A.	9.00	3.00	2.00	4.00	5.00	8.00	600.00	6.72	6.00	16.00

---

### PART-2 : CHEMISTRY

#### SECTION-I

Q.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A.	B	C	B	A	D	C	B	D	C	A	A	D	A	B	C	B	A	A	C	D

#### SECTION-II

Q.	51	52	53	54	55	56	57	58	59	60
A.	0.00	80.00	5.00	2.00	25.00	2.00	44.00	7.00	0.00	208.00

---

### PART-3 : MATHEMATICS

#### SECTION-I

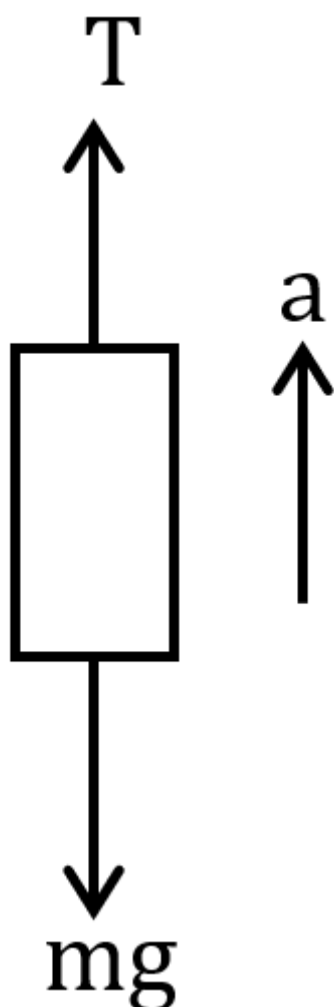
Q.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
A.	C	A	A	D	D	A	B	C	B	B	A	A	B	A	D	C	B	D	D	B

#### SECTION-II

Q.	81	82	83	84	85	86	87	88	89	90
A.	1.25	0.25	1.00	11.00	6.00	6.00	0.20	6.00	7.00	4.00

## SOLUTIONS

### PART-1 : PHYSICS



1)

$$m = 6000 \text{ kg}$$

$$a = 5 \text{ m/s}^2$$

equation of motion of elevator

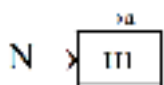
$$ma = T - mg$$

$$T = m(a + g)$$

$$T = (6000)(5 + 10)$$

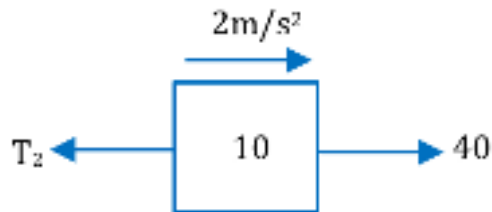
$$T = 90000 \text{ N}$$

$$2) a = \frac{F}{M + m}$$



$$N = ma$$

$$= \frac{Fm}{M + m} \quad \text{Ans (A)}$$



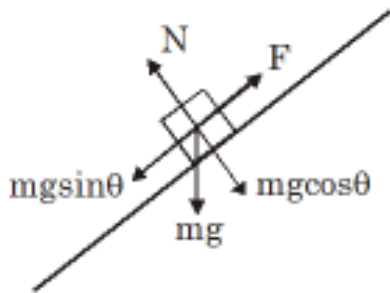
3)

$$a = 2 \text{ m/s}^2$$

$$40 - T_2 = 10 \times 2$$

$$T_2 = 20 \text{ N}$$

4) if A is in equilibrium B is also in equilibrium



5)

maximum value of friction

$$f_{\max} = \mu mg \cos \theta = 2 \tan \theta mg \cos \theta$$

$$= 2 mg \sin \theta$$

Hence maximum value of F up the incline, force for zero acceleration

$$F = mg \sin \theta + f_{\max}$$

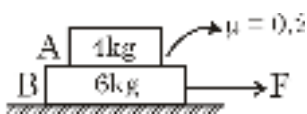
$$F = 3 mg \sin \theta$$

Down the incline force, for zero acceleration

$$F = f_{\max} - mg \sin \theta$$

$$= mg \sin \theta$$

6)



$$\text{max friction on A} = \mu m_A g$$

$$= (0.2) (4) (10) = 8 \text{ N}$$

$$a = \text{Common acceleration} = \frac{F}{10}$$

In B's frame :

$$f_{\text{pseudo}} = m_A a$$

□ For no slipping  $\Rightarrow$  pseudo force = max friction

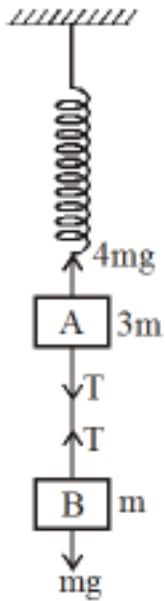
$$4(a) = 8 = \frac{4.F}{10}$$

$$\boxed{F_{\max} = 20 \text{ N}} \quad (\text{A) Ans.}$$

7)

initially tension in the spring =  $4mg$

after cutting the string block B  
will do freefall motion.

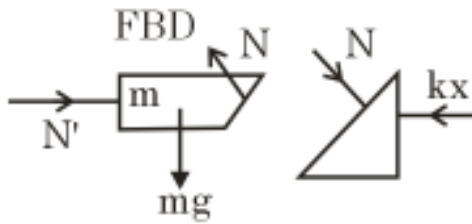
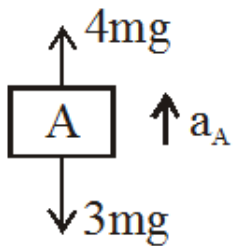


So  $a_B = g$

$$4mg - 3mg = m_A \cdot a_A$$

$$a_A = \frac{mg}{3m}$$

$$a_A = \frac{g}{3}$$



8)

Balancing  $mg = \frac{N}{\sqrt{2}} = kx \quad \& \quad x = \frac{mg}{k}$

9)  $f_{s_{\max}} = \mu mg = 2N$

for lower block  $= mg - T = ma$

for upper block  $= T - \mu mg = ma$

$= mg - \mu mg = 2ma$

$a = 0.4 g$

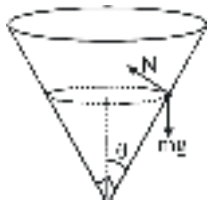
11)  $mg - N = \frac{mv^2}{r}$



12)

Ans. (1)

$$N \sin \theta = mg$$



$$N \cos \theta = \frac{mv^2}{h \tan \theta}$$

$$\therefore v = \sqrt{gh}$$

$$13) \alpha = \frac{d\omega}{dt} = 6t - 12$$

$$\int_9^\omega d\omega = \int_0^t (6t - 12) dt$$

$$\omega = 3t^2 - 12t + 9$$

$$\int_0^4 (3t^2 - 12t + 9) dt$$

$$d\theta = 0$$

$$\text{Angular disp.} = \Delta\theta = [t^3 - 6t^2 + 9t]_0^4$$

$$= 64 - 96 + 36 = 4 \text{ rad.}$$

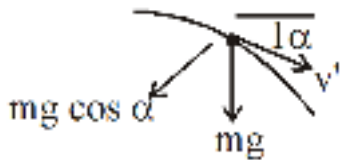
14) tangential acceleration =  $R\alpha$

centripetal acceleration =  $2\sqrt{2}R\alpha$

$$a_{\text{net}} = \sqrt{(R\alpha)^2 + (2\sqrt{2}R\alpha)^2} = 3R\alpha$$

15) **Case - I** → At highest pt.

$$mg = \frac{m.(v \cos \theta)^2}{\rho_{\min}} \Rightarrow \rho_{\min} = \frac{v^2 \cos^2 \theta}{g}$$



**Case - II** At given pt.

Equating velocity in x = direction  $\Rightarrow v \cos \theta = v' \cos \alpha$

$$\Rightarrow v' = \frac{v \cos \theta}{\cos \alpha}$$

$$\Rightarrow mg \cos \alpha = \frac{m.(v')^2}{\rho}$$

$$\Rightarrow \rho = \frac{v^2 \cos^2 \theta}{g \cos^3 \alpha} \Rightarrow \rho = \frac{\rho_{\min}}{\cos^3 \alpha}$$

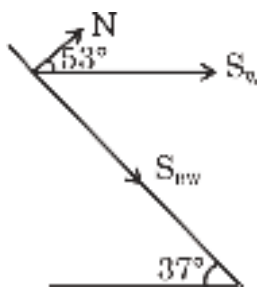
$$16) a = \frac{40}{8} = 5 \text{ m/s}^2$$

$$s = \frac{1}{2}at^2 = 10\text{m}$$

$$\square W_g = 40 \times 10 = 400 \text{ J}$$

$$\begin{aligned} 17) W &= \int \vec{F} \cdot d\vec{r} = \int_2^3 F \cdot dx \cdot \cos 60^\circ \\ &= \frac{1}{2}k \int_2^3 x^2 \cdot dx \\ &= \frac{1}{6}k(x)_2^3 = \frac{1}{6}(3^3 - 2^3) = \frac{19}{6}k \end{aligned}$$

18)



Displacement of the block in horizontal direction equals to the wedge displacement and also the displacement of block along inclined with respect to wedge

$$\vec{s}_B = \vec{s}_{BW} + \vec{s}_W$$

$$\square W_{\text{normal}} = NS_{W \cos 53^\circ}$$

$$= 10 \times 10 \times \frac{4}{5} \times 10 \times 2 \times = 960 \text{ J}$$

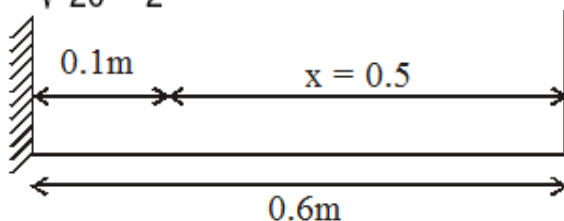
19) Use work energy theorem.

$$W_F + W_{SP} = \Delta KE$$

$$20 \times 0.25 + \frac{1}{2} K (0^2 - x^2) = 0 - 0$$

$$5 = \frac{1}{2} \times 40x^2$$

$$x = \sqrt{\frac{5}{20}} = \frac{1}{2} = 0.5$$



$$\text{Distance from wall} = 0.6 - 0.5 = 0.1\text{m}$$

20) Gravitational force is a conservative force and work done against it is a point function *i.e.* does not depend on the path

21)

$$a = \frac{36}{12} = 3 \frac{\text{m}}{\text{s}^2}$$

$$T = ma = 3 \times 3 = 9 \text{ N}$$

22) As no relative motion between blocks is the acceleration of both block will be same

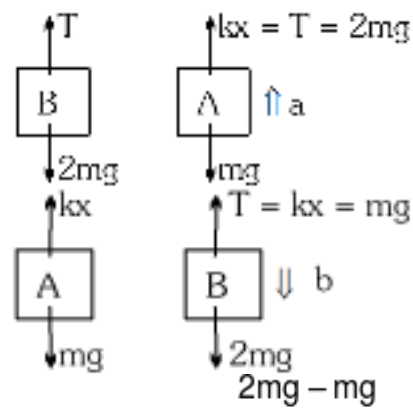
$$a = \frac{9}{3} = 3 \text{ m/s}^2$$

□ Force of friction between two blocks  
 $= 1 \times 3 = 3 \text{ N}$

23)

In 1<sup>st</sup> case

In 2<sup>nd</sup> Case



$$T = 2mg \quad \& \quad a = \frac{2mg - mg}{m} \Rightarrow a = g$$

$$kx = mg \quad \& \quad b = \frac{2mg - mg}{2m} \Rightarrow b = g/2 \downarrow$$

$$\square \frac{a}{b} = 2$$

24)

To prevent sliding  $m_B g \leq m (m_A + m_C) g$

$$\Rightarrow (m_C)_{\min} = \frac{m_B}{\mu} - m_A = \frac{4}{0.2} - 16 = 20 - 16 = 4 \text{ kg}$$

$$25) a_A = \frac{100-40}{4} = 15 \text{ m/s}^2$$

$$a_B = \frac{100-50}{5} = 10 \text{ m/s}^2$$

$$\Rightarrow a_A - a_B = 15 - 10 = 5 \text{ m/s}^2$$

26)

From work energy theorem  $W = \Delta KE$

$$W_{\text{gravity}} + W_{\text{air friction}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\Rightarrow W_{\text{air friction}} = \frac{1}{2}(4)(14)^2 - (4)(10)(10) = -8J$$

$$\Rightarrow \text{Work done against air friction} = 8J$$

27)

Use work energy theorem,

$$W_{\text{spring}} + W_{\text{friction}} = \Delta KE$$

$$-\frac{1}{2}kx^2 - \mu mg(\ell + x) = 0 - \frac{1}{2}mv_0^2$$

$$v_0^2 = \frac{kx^2}{m} + 2\mu g(\ell + x)$$

$$v_0^2 = \frac{5600}{2} \times \frac{1}{100} + 2(0.4)(10)(1)$$

$$v_0^2 = 28 + 8 = 36$$

$$v_0 = 6 \text{ m/s} = 600 \text{ cm/s}$$

$$28) U = 2x - 4y$$

$$F_x = -\frac{\partial U}{\partial x} = -2, F_y = -\frac{\partial U}{\partial y} = 4$$

$$\vec{F} = -2\hat{i} + 4\hat{j}$$

$$\vec{a} = -\hat{i} + 2\hat{j}$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{u} = 0 + (-\hat{i} + 2\hat{j}) \times 3$$

$$\vec{v} = -3\hat{i} + 6\hat{j}$$

$$|u| = \sqrt{9+36} = 3\sqrt{5} = 3 \times 2.24$$

$$= 6.72$$

29)

$$\omega = 3\pi t^3 = 24\pi$$

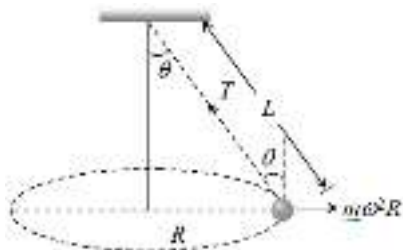
$$t = (8)^{\frac{1}{3}} \Rightarrow 2 \text{ sec}$$

$$\int_0^\theta d\theta = \int_0^2 \omega dt \Rightarrow \left[ \frac{3\pi t^4}{4} \right]_0^2$$

$$\theta = \frac{3\pi \times 16}{4} \Rightarrow 12\pi \text{ rad.}$$

$$30) T \sin \theta = M\omega^2 R \quad \dots (i)$$

$$T \sin \theta = M \omega^2 L \sin \theta \quad \dots (i)$$



From (i) and (ii)

$$\begin{aligned} T &= M \omega^2 L \\ &= M 4 \pi^2 n^2 L \\ &= M 4 \pi^2 \left( \frac{2}{\pi} \right)^2 L \\ &= 16 ML \end{aligned}$$

## PART-2 : CHEMISTRY

31)

On reacting mixture ( $\text{NH}_3 + \text{HCl}$ )

Dalton's law cannot be applied.

32)

Let N be the number of atoms of mole He and oxygen

Mole of He = N ; mole of  $\text{O}_3$  = N/3

Total number of moles =  $N + \frac{N}{3} = \frac{4N}{3}$  ; Total pressure = P

Hence,  $P_{\text{O}_3} = X_{\text{O}_3} P = \frac{N/3}{4N/3} P = 0.25 P$

33)

Ans.(2)

AV K.E. per gm =  $\left( \frac{3}{2} \times 2 \times 320 \right) \times \frac{1}{20} = 60 \text{ cal/gm}$

34)

Ans. (1)

$$d = \frac{PM}{RT}$$

35)

Ans.(4)

$$\frac{2 \times R \times T}{64} = \sqrt{\frac{3 \times R \times 300}{32}}$$

$$T = 900\text{K}$$

36)

$$5 \times v = n \times R \times (273) \dots \dots \dots (1)$$

$$P_f \times \frac{V}{3} = n \times R \times (273 + 546) \dots (2)$$

$$P_f = 45$$

37) Translational K.E. =  $\frac{3}{2} KT$  for 1 molecule  
for 1 mole molecule meaning NA number of molecules

$$\text{Translation KE per mol} = \frac{3}{2} RT$$

$$\text{Per gram translational KE} = \frac{\frac{3}{2} RT}{\text{weight}} = \frac{\frac{3}{2} RT}{(1) \times M}$$

M = molar mass of gas

So smallest molar mass has largest translational.

\* Since He has smallest molecular weight it will have most translational KE per gram.

Option B is correct.

$$38) u = \sqrt{\frac{3RT}{28}}$$

$$u' = \sqrt{\frac{3R \times 2T \times 2}{14 \times 2}}$$

$$= 2 \times \sqrt{\frac{3RT}{28}} = 2u$$

39)

$$PM = dRT$$

40)

$$P_T = P_{O_2} + P_{H_2O}$$

$$720 = P_{O_2} + 20 \Rightarrow P_{O_2} = 700$$

$$V \rightarrow 4V$$

$$P \rightarrow \frac{1}{4} = \frac{700}{4} = \frac{350}{2} = 175$$

$$\Rightarrow 175 \text{ mm of Hg}$$

$$41) \frac{r_{He}}{r_{CH_4}} = \frac{\frac{(\text{moles effused})_{He}}{t}}{\frac{(\text{moles effused})_{CH_4}}{t}} = \left( \frac{n_{He}}{n_{CH_4}} \right)_{\text{initial}} \sqrt{\frac{M_{CH_4}}{M_{He}}}$$

$$\frac{(\text{moles effused})_{He}}{(\text{moles effused})_{CH_4}} = \frac{4}{1} \sqrt{\frac{16}{4}} = \frac{8}{1}$$

42)

43)

$$T_C = \frac{8a}{27Rb}; T_B = \frac{a}{Rb}$$

$$T_C = \frac{8T_B}{27}$$

44) At high pressure

$$Z > 1$$

$$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

At higher pressure  $\frac{a}{V_m^2}$  can be neglected

$$P(V_m - b) = RT$$

$$PV_m - Pb = RT$$

$$\frac{PV_m}{RT} - \frac{Pb}{RT} = 1$$

$$Z = 1 + \frac{Pb}{RT}$$

45)

More is the 'a' value, more compressible is the gas. Intermolecular force when molecular weight .  
So,  $C_5H_{12}$  is most probable.

47)

$$\Delta E = q + w$$

$$= 40 - 8 = 32 \text{ J}$$

51)

$$\rho = \frac{PM}{ZRT}$$

$$Z = \frac{PM}{\rho RT} = \frac{500 \times 16 / 0.246 \times 10^3}{0.0821 \times 200}$$

$$\simeq 2$$

$$= 2 \times 10^0$$

$$x = 0$$

52)

$$T_B = 270 \text{ K} = \frac{a}{Rb}$$

$$T_C = \frac{8a}{27Rb} = \frac{8}{27} \times 270 = 80 \text{ K}$$

53)

$$54) P \times V = \frac{4}{M} \times R \times T \rightarrow (1)$$

**For More Material Join: @JEEAdvanced\_2026**

$$P \times V = \frac{3.2}{M} \times R \times (T + 50) \rightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow 1 = \frac{4T}{3.2(T + 50)}$$

$$T = 200 \text{ K} = 2.00 \times 10^2 \text{ K}$$

$$55) 10^5 \text{ cm/sec} = 10^3 \text{ m/sec}$$

$$10^3 = \sqrt{\frac{3 \times \frac{25}{3} \times 1000}{\frac{M}{1000}}}$$

$$56) u_{\text{rms}} = \sqrt{\frac{3RT}{m}} \propto \sqrt{T}$$

$$\frac{r_{\text{rms},1200}}{r_{\text{rms},300}} = \sqrt{\frac{1200}{300}} = 2$$

57)

$$\frac{V_1}{V_2} = \sqrt{\frac{d_2}{d_1}} = \sqrt{\frac{Mw_2}{Mw_1}}$$

$$\frac{127}{100} = \sqrt{\frac{71}{Mw_1}}$$

$$\therefore Mw_1 = 43.58$$

PART-3 : MATHEMATICS

61)

$$ax^2 + bx + c = 0$$

$$x(ax + b) = -c$$

$$ax + b = -\frac{c}{x}$$

$$\frac{1}{(\alpha^2 + \beta^2)} \left[ \frac{\alpha^2 + \beta^2}{c^2} \right]$$

62)

$$\alpha, \beta \text{ are roots of } 3x^2 - 6x + 5 = 0$$

$$\alpha + \beta = 2$$

$$\alpha\beta = \frac{5}{3}$$

63)

$$\text{Roots are } 1 \text{ and } \left( -\frac{2b}{a+b-c} \right)$$

64)

**For More Material Join: @JEEAdvanced\_2026**



For a common root  $\frac{2}{1} = \frac{5}{b} = \frac{1}{c}$   
 $\Rightarrow b = \frac{5}{2}, c = \frac{1}{2}$

65)

$x^3 + 3x^2 + x - 1 = (x - \alpha)(x - \beta)(x - \gamma)$   
 put  $x = -3$   
 $-27 + 27 - 3 - 1 = -(3 + \alpha)(3 + \beta)(3 + \gamma)$   
 $4 = (3 + \alpha)(3 + \beta)(3 + \gamma)$

66)  $\frac{x^2 + 4x + 7}{x^2 + 4x + 5} = \frac{x^2 + 4x + 5}{x^2 + 4x + 5} + \frac{2}{x^2 + 4x + 5}$   
 $= 1 + \frac{2}{x^2 + 4x + 5} \dots(1)$   
 $x^2 + 4x + 5 = (x + 2)^2 + 1$   
 $\Rightarrow x^2 + 4x + 5 \in [1, \infty)$   
 $\Rightarrow \frac{2}{x^2 + 4x + 5} \in (0, 2]$

$\square \frac{x^2 + 4x + 7}{x^2 + 4x + 5} \in (1, 3]$

67)

$f(2) < 0$   
 $(2)^2 - 2(a + 1) + a^2 + a - 8 < 0$   
 $a^2 - a - 6 < 0$   
 $(a - 3)(a + 2) < 0$   
 $-2 < a < 3$

68)

$\left(2x + \frac{1}{x} - 3\right) \left(2x + \frac{1}{x} + 5\right) - 9 = 0$

Put  $2x + \frac{1}{x} = P$

$(P - 3)(P + 5) - 9 = 0$

$P = -6$  or  $P = 4$

$2x + \frac{1}{x} = -6$  or  $2x + \frac{1}{x} = 4$

$x = \frac{-6 \pm \sqrt{28}}{4}$  or  $x = \frac{2 \pm \sqrt{2}}{2}$

69)

If equation true for all real values of  $x$  then

$a^3 - 3a - 2 = 0 \Rightarrow a = -1, 2, -1$   
 $a^2 - a - 2 = 0 \Rightarrow a = -1 \& 2$   
 $2a^4 - 10a + k = 0 \Rightarrow k = -12$

$$70) \alpha^3 + \beta^3 = q$$

$$\Rightarrow (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = q$$

$$\Rightarrow \alpha\beta = \frac{p^3 + q}{3p}$$

$$\text{sum of the roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{p^2 - 2\left(\frac{p^3 + q}{3p}\right)}{\frac{p^3 + q}{3p}} = \frac{p^3 - 2q}{p^3 + q}$$

Product of the roots = 1.

Required equation is

$$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$$

71)

$$\alpha + \frac{1}{\alpha} \geq 0$$

Parabola will be upward

$$D = 1 - 20\left(\alpha + \frac{1}{\alpha}\right)$$

of  $\alpha > 0$

$$\alpha + \frac{1}{\alpha} \geq 2$$

$$D < 0$$

$$= \frac{-1}{2\left(\alpha + \frac{1}{\alpha}\right)} < 0$$

Vertex

72)

$$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$$

squaring

$$2x - 2\sqrt{x^2 - 1} = 4x - 1$$

$$-2\sqrt{x^2 - 1} = 2x - 1 ; \text{ Again squaring}$$

$$4(x^2 - 1) = 4x^2 + 1 - 4x$$

$$x = \frac{5}{4} \Rightarrow \text{Rejected}$$

73) Let  $\sin 2x = y$

$$(1 - \sin^2 2x)^2 + 2\sin^2 2x = 17(1 + \sin 2x)^4$$

$$(1 - y^2)^2 + 2y^2 = 17(1 + y)^4$$

$$1 + y^4 = 17(1 + y)^4$$

$$16\left(y + \frac{1}{y}\right) + 68\left(y + \frac{1}{y}\right) + 70 = 0$$

$$\text{put } y + \frac{1}{y} = z$$

$$\text{we get } 8z^2 + 34z + 35 = 0$$

$$z = -\frac{7}{4}, -\frac{5}{2}$$

$$\Rightarrow y + \frac{1}{y} = -\frac{5}{2} \quad y = -\frac{1}{2}, -2$$

$$\sin 2x = -\frac{1}{2}$$

4 solution  $105^\circ, 165^\circ, 285^\circ, 345^\circ$

74)

$$\left( \cos^2 x + \frac{1}{\cos^2 x} \right) (1 + \tan^2 2y) (3 + \sin 3z) = 4$$

$$\text{Since, } \cos^2 x + \frac{1}{\cos^2 x} \geq 2,$$

$$1 + \tan^2 2y \geq 1, \quad 2 \leq 3 + \sin 3z \leq 4$$

So, the only possibility is

$$\cos^2 x + \frac{1}{\cos^2 x} = 2, \quad 1 + \tan^2 2y = 1,$$

$$3 + \sin 3z = 2$$

$$\Rightarrow \cos x = \pm 1 \Rightarrow x = n\pi$$

$$\tan 2y = 0 \Rightarrow y = \frac{m\pi}{2}$$

$$\sin 3z = -1 \Rightarrow z = (4k-1)\frac{\pi}{6}; m, n, k \in \mathbb{I}.$$

75)

$$5\cos\theta - \cos 3\theta = 4$$

$$\Rightarrow \cos^3\theta - 2\cos\theta + 1 = 0$$

$$\Rightarrow (\cos\theta - 1)(\cos^2\theta + \cos\theta - 1) = 0$$

$$\text{valid values of } \cos\theta = 1, \frac{\sqrt{5}-1}{2}$$

$$\square \text{ 6 solution in } \left[ -\frac{\pi}{2}, \frac{5\pi}{2} \right]$$

76)

$$\sec x + \cos x = 2$$

$$\square \cos x = 1$$

$$\square (\sec x)^6 + (\cos x)^6 = 2.$$

77)

$$\text{Maximum value at } x = \frac{\pi}{4} \text{ only}$$

$$(\sqrt{2})^2 = 2$$

$$78) \sin x + \cos x \in [-\sqrt{2}, \sqrt{2}]$$

$$\text{and } \frac{-3}{2} \notin [-\sqrt{2}, \sqrt{2}]$$

$\Rightarrow$  No solution

79)

$$x^2 + \frac{1}{x^2} = 2 \cos \theta$$

$$\cos \theta = 1 \quad x^2 = 1$$

$$\theta = 0, 2\pi \quad x = \pm 1$$

80)

$$-\sqrt{74} \leq 7 \cos x + 5 \sin x \leq \sqrt{74}$$

$$-\sqrt{74} \leq 2k + 1 \leq \sqrt{74}$$

$$\square k = -4, \pm 3, \pm 2, \pm 1, 0$$

81)

$\because \alpha, \beta$  are roots of  $x^2 - 5x - 3 = 0$  then

$$\alpha^2 - 5\alpha - 3 = 0$$

$$\Rightarrow \alpha^2 - 3 = 5\alpha$$

$$\text{Similarly } \beta^2 - 3 = 5\beta$$

$$\square a_n = \alpha^n - \beta^n$$

$$\frac{a_{10} - 3a_8}{4a_9} = \frac{(\alpha^{10} - \beta^{10}) - 3(\alpha^8 - \beta^8)}{4(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 3) - \beta^8(\beta^2 - 3)}{4(\alpha^9 - \beta^9)} = \frac{\alpha^8(5\alpha) - \beta^8(5\beta)}{4(\alpha^9 - \beta^9)} = \frac{5}{4} = 1.25$$

82)

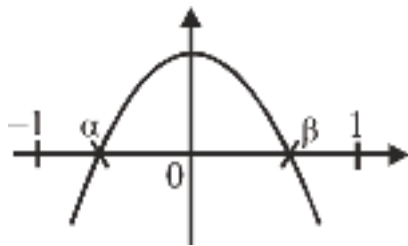
If  $|a| + |b| = |a + b|$  then  $ab \geq 0$

$$\Rightarrow (x^2 - 3x)(6 - 2x) \geq 0 \Rightarrow x(x - 3)^2 \leq 0$$

$$\Rightarrow x \in (-\infty, 0] \cup \{3\}$$

$\square$  only one positive integer solution  $x = 3$

83)



$$f(1) = a + b + c < 0$$

$$f(-1) = a - b + c < 0$$

$$f(0) = c > 0$$

$$\text{so } -1 < \alpha < 0, 0 < \beta < 1$$

$$\therefore |[\alpha] + [\beta]| = |-1 + 0| = 1$$

84)

$$\text{Let } Q(x) = x^2 - 2\lambda x + 6\lambda - 41$$

As  $Q(x) < 0 \forall x \in (1, 6]$ , so

$$Q(1) \leq 0 \quad \dots(1)$$

$$\& \quad Q(6) < 0 \quad \dots(2)$$

must be satisfied simultaneously.

$$\text{So, } (1) \cap (2) \rightarrow \lambda \in \left[ \frac{5}{6}, 10 \right]$$

85)

$$\angle R = \frac{\pi}{2} \Rightarrow \angle P + \angle Q = \frac{\pi}{2}$$

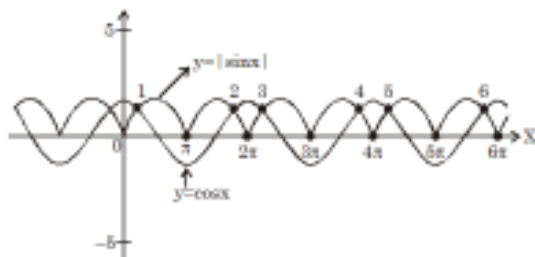
$$\frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$$

$$1 = \tan \frac{\pi}{4} = \frac{\tan \frac{P}{2} + \tan \frac{Q}{2}}{1 - \tan \frac{P}{2} \cdot \tan \frac{Q}{2}}$$

$$1 = \frac{-b/a}{1 - c/a} \Rightarrow a - c = -b$$

$$\Rightarrow a + b = c$$

$$\Rightarrow (a, b) \equiv \{(1, 1), (2, 2), (1, 2), (1, 3), (2, 1), (3, 1)\}$$



86)

Number of point of intersection = 6

for  $x \in [0, 6\pi]$

Number of solutions = 6

87)

$$\frac{5}{4} \cos^2 2x + 1 - 2 \sin^2 x \cos^2 x + 1 - 3 \sin^2 x \cos^2 x = 2$$

$$\frac{5}{4} \cos^2 2x + 2 - \frac{5}{4} \sin^2 2x = 2$$

$$\frac{5}{4} \cos 4x = 0$$

$$\cos 4x = 0$$

$$4x = (2n - 1) \frac{\pi}{2}$$

$$x = (2n - 1) \frac{\pi}{8}$$

8 solutions in  $[0, 2\pi]$

$$40k = 8$$

$$k = \frac{1}{5} = 0.20$$

88)

$$\begin{aligned}
& \frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = 0 \\
& \Rightarrow \frac{2 \sin x \cos x}{2 \sin x \cos x} + \frac{2 \sin 3x \cos 3x}{2 \sin 3x \cos 3x} + \frac{2 \sin 9x \cos 9x}{2 \sin 9x \cos 9x} = 0 \\
& \Rightarrow \frac{2 \cos 3x \cos x}{\sin(3x-x)} + \frac{2 \cos 9x \cos 3x}{\sin(9x-3x)} + \frac{2 \cos 27x \cos 9x}{\sin(27x-9x)} = 0 \\
& \Rightarrow \frac{2 \cos 3x \cos x}{\sin(3x-x)} + \frac{2 \cos 9x \cos 3x}{\sin(9x-3x)} + \frac{2 \cos 27x \cos 9x}{\sin(27x-9x)} = 0 \\
& \Rightarrow (\tan 3x - \tan x) + (\tan 9x - \tan 3x) + (\tan 27x - \tan 9x) = 0 \\
& \Rightarrow \tan 27x - \tan x = 0 \\
& \Rightarrow \tan x = \tan 27x \\
& \Rightarrow 27x = n\pi + x, n \in \mathbb{I} \\
& \Rightarrow x = \frac{n\pi}{26}, n \in \mathbb{I} \\
& \Rightarrow x = \frac{\pi}{26}, \frac{2\pi}{26}, \frac{3\pi}{26}, \frac{4\pi}{26}, \frac{5\pi}{26}, \frac{6\pi}{26} \\
& \text{Hence there are six solution}
\end{aligned}$$

89)

$$\begin{aligned}
& \cos x + \cos 2x + \cos 3x + \cos 4x = 0 \\
& 2 \cos \frac{5x}{2} \cdot \cos \frac{3x}{2} + 2 \cos \frac{5x}{2} \cdot \cos \frac{x}{2} = 0 \\
& 2 \cos \frac{5x}{2} \times 2 \cos \frac{x}{2} = 0 \\
& x = (2n+1) \frac{\pi}{5}, x = (2k+1) \frac{\pi}{2}, x = (2r+1)\pi \\
& \text{Hence } x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{2\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}
\end{aligned}$$

90)

$$\begin{aligned}
& \cos^7 x \leq \cos^2 x \text{ and } \sin^4 x \leq \sin^2 x \\
& \Rightarrow \cos^7 x + \sin^4 x \leq 1 \\
& \text{So, the given equation satisfies if and only if } \cos^7 x = 0 \text{ and } \sin^4 x = 1 \text{ or } \cos^7 x = 1 \text{ and } \sin^4 x = 0 \\
& \Leftrightarrow x = (2n+1) \frac{\pi}{2} \text{ or } x = 2m\pi \\
& \because 0 \leq x \leq 2\pi, \text{ so } x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi
\end{aligned}$$