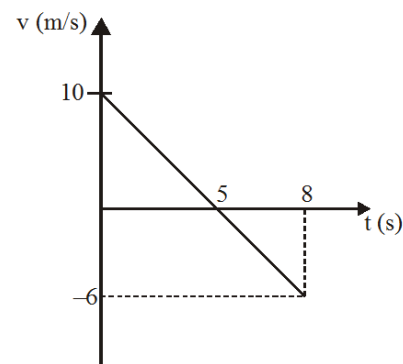


PART-A-PHYSICS

SECTION-I(A)



1) For the given v-t graph, the distance travelled in 8 seconds will be :-

- (A) 16 m
- (B) 25 m
- (C) 34 m
- (D) 50 m

2) If $y = \sin x + \cos x$, then $\frac{d^2y}{dx^2}$ equals :

- (A) $\sin x + \cos x$
- (B) $-\sin x + \cos x$
- (C) $-(\sin x + \cos x)$
- (D) None of these

3) A golf ball is released from rest from the top of a very tall building. Calculate the position in m of the ball after 2.00 s

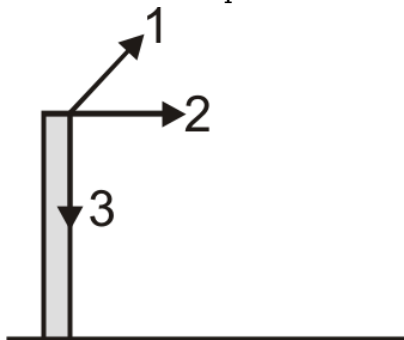
- (A) -32.1
- (B) -19.6
- (C) -22.2
- (D) -20.9

4) A balloon is moving vertically with a velocity of 4 ms^{-1} . When it is at a height h , a body is gently released from it. If it reaches the ground in 4 second, the height of the balloon, when the body is released, is : ($g = 9.8 \text{ m/s}^2$)

- (A) 62.4 m
- (B) 2.4 m
- (C) 78.4 m

(D) 82.2 m

5) Three bodies are projected from the top of a tower with same speeds but in the directions as shown. If their speeds on reaching the ground are v_1 , v_2 and v_3 respectively, then



(A) $v_1 > v_2 > v_3$

(B) $v_1 > v_2 < v_3$

(C) $v_1 = v_3 < v_2$

(D) $v_1 = v_2 = v_3$

6) Find the first derivative of given function with respect to x, $y = \ln x + e^{x^2} + \frac{1}{\sqrt{x}}$

(A) $\frac{1}{x} + e^{x^2} + \frac{1}{2\sqrt{x^3}}$

(B) $\frac{1}{x} + e^{x^2} \frac{-1}{2\sqrt{x^3}}$

(C) $\frac{1}{x} + 2xe^{x^2} + \frac{1}{2\sqrt{x^3}}$

(D) $\frac{1}{x} + 2xe^{x^2} - \frac{1}{2\sqrt{x^3}}$

SECTION-I(B)

1) A body moving with uniform acceleration in a straight line describes 25 m in the 5th second and 33 m in 7th second.

(A) Initial velocity is 6 m/s

(B) Initial velocity is 7 m/s

(C) Acceleration is 2 m/s^2

(D) Acceleration is 4 m/s^2

2) A person, standing on the roof of 40m high tower, throws a ball vertically upwards with speed 10 m/s. Two seconds later, he throws another ball again in vertical direction. Both the balls hit the ground simultaneously

(A) The first ball hits the ground after 4 seconds.

(B) The second ball was projected vertically downwards with speed 10 m/s.

- (C) The distance travelled by the first ball is 10 m greater than the distance travelled by the second ball.
- (D) Both balls hit the ground with same velocities.

3) For a particle moving rectilinearly

- (A) Area under acceleration-time graph equals to the velocity.
- (B) Area under acceleration time graph equals to the change in velocity.
- (C) Area under the velocity time graph equals to the displacement.
- (D) Area under speed time graph equals to the distance travelled.

4) Two particles A & B projected along different directions from the same point P on the ground with the same velocity of 70 m/s in the same vertical plane . They hit the ground at the same point Q such that

PQ = 480 m. Then : [$g = 9.8 \text{ m/s}^2$] [$\sin 74^\circ = 0.96$] [$\sin 37^\circ = 0.6$]

- (A) Ratio of their times of flight is 4 : 5
- (B) Ratio of their maximum heights is 9 : 16
- (C) Ratio of their minimum speeds during flights is 4 : 3
- (D) The bisector of the angle between their directions of projection makes 45° with horizontal

5) Select correct statement(s)

(A) $\frac{d}{dx} (e^5) = 5e^4$

(B) If $y = \frac{1}{x^3}$, $\frac{dy}{dx}$ at $x = 2$ will be $\frac{3}{16}$

(C) $\int_{-\pi}^{\pi} \cos x dx = 0$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x dx = 0$

6) The position of the particle is given by $y = 2t^3 - 9t^2 + 12t + 4$ then :

- (A) Initial position of particle is on positive y-axis.
- (B) Initially particle is moving in positive y-direction.
- (C) Initially particle is moving in negative y-direction.
- (D) Acceleration of particle is constant.

SECTION-III

1) A man moves from A to B in a straight line and then moves from B to C where C is the mid point of the line. If the average velocity of the particle from A to C through B is v . Let the average speed of the particle for this motion is kv . The value of k .

2)

A stone is thrown horizontally from the top of a tower. At a time 0.5 second after the projection the magnitude of velocity of stone is found to be 1.5 times its initial speed. If initial velocity of the stone is given by $2\sqrt{X}$ (in m/s) then find the value of X. ($g = 10 \text{ m/s}^2$)

3)

If $y = \tan x$

Find the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

4) A ball is projected from ground such that maximum height achieved by ball is 20 m. Find the time of flight of the ball (in sec).

5) A ball is thrown from top edge of a vertical tower at $t = 0$. If the horizontal and vertical directions are taken x and y respectively and the origin is at the foot of the tower, the position of ball at any time t is given by $x = 2t \text{ m}$, $y = 80 - 5t^2 \text{ m}$. How far from foot of the tower on the horizontal ground does the ball strike if the height of the tower is 80 m

PART-B-CHEMISTRY

SECTION-I(A)

1) Two electromagnetic radiations have wave numbers in the ratio 2 : 3. Their energies per quanta will be in the ratio:

- (A) 3 : 2
- (B) 9 : 4
- (C) 4 : 9
- (D) 2 : 3

2) The wavelengths of the first Lyman lines of hydrogen, He^+ and Li^{2+} ions are λ_1 , λ_2 , λ_3 . The ratio of these wavelengths is

- (A) 1 : 4 : 9
- (B) 9 : 4 : 1
- (C) 36 : 9 : 4
- (D) 6 : 3 : 2

3) A metal surface having work function (ϕ) = 2.1 eV is irradiated with photon of energy = 4.00 eV. Find the maximum kinetic energy of photoelectron emitted?

- (A) 2.1 eV
- (B) 4.0 eV

(C) 1.9 eV

(D) 1.0 eV

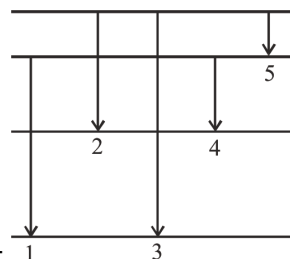
4) In order to increase the kinetic energy of ejected photoelectrons, there should be an increase in

(A) intensity of radiation

(B) wavelength of radiation

(C) frequency of radiation

(D) both wavelength and intensity of radiation



5) The correct order of energy of following spectral lines is -

(A) $1 > 3 > 2 > 5 > 4$

(B) $3 > 1 > 2 > 4 > 5$

(C) $3 > 2 > 1 > 4 > 5$

(D) $5 > 4 > 3 > 2 > 1$

6) Which of the following set of quantum numbers belong to highest energy ?

(A) $n = 2 \quad \ell = 1 \quad m = 0 \quad s = +\frac{1}{2}$

(B) $n = 3 \quad \ell = 0 \quad m = 0 \quad s = +\frac{1}{2}$

(C) $n = 4 \quad \ell = 0 \quad m = 1 \quad s = +\frac{1}{2}$

(D) $n = 4 \quad \ell = 1 \quad m = 1 \quad s = +\frac{1}{2}$

SECTION-I(B)

1) Out of the following pairs of electrons, identify the pairs of electrons present in degenerate orbitals :

(A) (a) $n = 3, \ell = 2, m_\ell = -2, m_s = -\frac{1}{2}$
(b) $n = 3, \ell = 2, m_\ell = -1, m_s = -\frac{1}{2}$

(B) (a) $n = 3, \ell = 1, m_\ell = 1, m_s = +\frac{1}{2}$
(b) $n = 3, \ell = 2, m_\ell = 1, m_s = +\frac{1}{2}$

- (C) (a) $n = 4, \ell = 1, m_\ell = 1, m_s = +\frac{1}{2}$
 (b) $n = 3, \ell = 2, m_\ell = 1, m_s = +\frac{1}{2}$
- (D) (a) $n = 3, \ell = 2, m_\ell = +2, m_s = -\frac{1}{2}$
 (b) $n = 3, \ell = 2, m_\ell = +2, m_s = +\frac{1}{2}$

2) For which of the following species, Bohr's theory is applicable ?

- (A) Li^+
 (B) H
 (C) Li^{+2}
 (D) He^+

3) As the orbit number increases, the K.E. and P.E. for an electron

- (A) K.E. increases.
 (B) K.E. decreases.
 (C) P.E. decreases.
 (D) P.E. increases

4) In a H-like sample, electrons make transition from 4th excited state upto 2nd state. Then :

- (A) 10 different spectral lines are observed
 (B) 6 different spectral lines are observed
 (C) Number of lines belonging to the Balmer series is 3
 (D) Number of lines belonging to Paschen series is 2.

5) Identify the pair(s) of elements in which first element having first ionization enthalpy greater than second element.

- (A) (N, O)
 (B) (Be, B)
 (C) (Ga, Al)
 (D) (Pb, Sn)

6) Select the **CORRECT** order of radii :-

- (A) $\text{Na}^+ > \text{Al}^{3+}$
 (B) $\text{N}^{3-} > \text{F}^-$
 (C) $\text{Y} > \text{La}$
 (D) $\text{Sc} > \text{Mn}$

SECTION-III

1) A transition for H-atom from second to first orbit has the same wavelength as from n^{th} orbit to second orbit for He^+ ion. The value of n is _____.

2) The maximum number of electrons present in 'P' (Phosphorus) having $(l + m = 0)$ is/are :

3) The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
$\phi(\text{eV})$	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

4) How many Balmer lines in the spectrum will be observed when electrons return from 7^{th} shell to 2^{nd} shell ?

5)

Find the number of INCORRECT orders :-

(i) I.E. of $\text{O}(\text{g}) < \text{I.E. of } \text{O}^-(\text{g})$	(ii) I.E. of $\text{Ne}(\text{g}) < \text{I.E. of } \text{Ne}^+(\text{g})$
(iii) I.E. of $\text{Na}^+(\text{g}) < \text{I.E. of } \text{Ne}(\text{g})$	(iv) Size of $\text{P}(\text{g}) > \text{size of } \text{N}(\text{g})$
(v) Size of $\text{Zn} > \text{size of } \text{Hg}$	(vi) $(n + \square)$ of $5p < (n + \square)$ of $6s$

PART-C-MATHEMATICS

SECTION-I(A)

1) If $\log_5 2, \log_5(2^x - 5), \log_5(2^x - 7/2)$ are in A.P., then x is equal to

- (A) 2
- (B) 3
- (C) 2 or 3
- (D) 1

2) The number of integral value of x satisfying the equation $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$

- (A) 1
- (B) 2
- (C) 3
- (D) 4

3) The sum of series $1^2 + \frac{2^2}{3} + \frac{3^2}{3^2} + \frac{4^2}{3^3} + \frac{5^2}{3^4} + \dots$ upto infinite terms is

- (A) 9

- (B) 5
(C) $\frac{7}{2}$
(D) $\frac{9}{2}$

4) If $0 < x, y, a, b < 1$, then the sum of the infinite terms of the series $\sqrt{x}(\sqrt{a} + \sqrt{x}) + \sqrt{x}(\sqrt{ab} + \sqrt{xy}) + \sqrt{x}(b\sqrt{a} + y\sqrt{x}) + \dots$

- (A) $\frac{\sqrt{ax}}{1 + \sqrt{b}} + \frac{x}{1 + \sqrt{y}}$
(B) $\frac{\sqrt{x}}{1 + \sqrt{b}} + \frac{\sqrt{x}}{1 + \sqrt{y}}$
(C) $\frac{\sqrt{x}}{1 - \sqrt{b}} + \frac{\sqrt{x}}{1 - \sqrt{y}}$
(D) $\frac{\sqrt{ax}}{1 - \sqrt{b}} + \frac{x}{1 - \sqrt{y}}$

5)

The minimum value of the quantity $\frac{(a^2 + 3a + 1)(b^2 + 4b + 1)(c^2 + 5c + 1)}{abc}$, where a, b and c are positive integers, is

- (A) 125
(B) 210
(C) 60
(D) $\frac{11.13.15}{2^3}$

6) The sum of three numbers in G.P. is 21 and the sum of their squares is 189. If the sum to infinite terms for the given G.P. is finite, then (r = common ratio, a = first term)

- (A) $r = \frac{1}{2}$
(B) $r = 2$
(C) $a = 4$
(D) $a = 2$

SECTION-I(B)

1) Let M denotes the number of digits in 11^{50} and N denotes the number of cyphers (zeros) after decimal but before the first significant digit in $(0.11)^{50}$, then {consider $\log_{10} 11 = 1.041$ }

- (A) $M = 52$
(B) $M = 53$
(C) $N = 47$

(D) $N = 48$

2)

If $|x|^{x+1} = 10^{\log_2 |x|}$, then

(A) The number of real solutions is 3.

(B) Equation has one irrational and two rational solutions.

(C) Equation has one rational and two irrational solutions.

(D) The sum of the solutions is $\log_2 5$.

3) If $a_1, a_2, a_3, \dots, a_n$ are n arithmetic means inserted between 7 and 2015 whose sum is 56616, then

(A) $n = 56$

(B) $n = 28$

(C) $a_{19} = \frac{2029}{3}$

(D) $a_{19} = \frac{36543}{57}$

4) a_1, a_2, a_3, \dots are distinct terms of an A.P. We call (p, q, r) an increasing triad if a_p, a_q, a_r are in G.P. where $p, q, r \in \mathbb{N}$ such that $p < q < r$. If $(5, 9, 16)$ is an increasing triad, then which of the following option is/are correct

(A) if a_1 is a multiple of 4 then every term of the A.P. is an integer

(B) $(85, 149, 261)$ is an increasing triad

(C) If the common difference of the A.P. is $\frac{1}{4}$, then its first term is $\frac{1}{3}$

(D) ratio of the $(4k + 1)^{\text{th}}$ term and $4k^{\text{th}}$ term can be 4

5)

If $a_i > 0$ for all $i \in \mathbb{N}$, then

(A) $(a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right) \geq 9$

(B) $\left(\frac{a_1}{a_2} + \frac{a_3}{a_4} + \frac{a_5}{a_6} \right) \left(\frac{a_2}{a_1} + \frac{a_4}{a_3} + \frac{a_6}{a_5} \right) \geq 9$

(C) $(a_1 a_2 + a_3 a_4)(a_1 a_3 + a_2 a_4) \geq 4 a_1 a_2 a_3 a_4$

(D) $(a_1^2 + a_2^2 + a_3^2) \left(\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_1} \right) \geq 9$

6) Let a_1, a_2, \dots be in harmonic progression with $a_1 = \frac{1}{25}$ & $a_{10} = \frac{1}{7}$. The positive integer n for which $a_n > 0$ is

- (A) 12
- (B) 13
- (C) 14
- (D) 15

SECTION-III

1) Let $a, b, c \in \mathbb{R}^+$ and $(a + b + c)^3 \leq 27abc$ and $4a + 5b + 7c = 16$, then the value of $a^4 + b^5 + c^6$ is

2) Let a and b be positive integers, the value of xyz is 55 and $\frac{343}{55}$ when a, x, y, z, b are in A.P. and H.P. respectively, then value of $a+b$ is

3) The number of common terms of two A.P.'s 3, 7, 11,407 and 2, 9, 16,709 equals to $\frac{28}{a}$. Find a .

4)

If $a + b + c + d = 6$ & $\frac{1}{a+b+c} + \frac{1}{b+c+d} + \frac{1}{a+b+d} + \frac{1}{a+c+d} = \frac{5}{3}$, then value of $\frac{a}{b+c+d} + \frac{b}{c+d+a} + \frac{c}{d+a+b} + \frac{d}{a+b+c}$ is

5)

If $x > 0, y > 0$ such that $2^{\lfloor ny \rfloor} = y^{\lfloor nx \rfloor}$ and $3^{\lfloor nx \rfloor} = y^{\lfloor n9 \rfloor}$, then minimum value of $(x^2 + y^2)$ is

ANSWER KEYS

PART-A-PHYSICS

SECTION-I(A)

Q.	1	2	3	4	5	6
A.	C	C	B	A	D	D

SECTION-I(B)

Q.	7	8	9	10	11	12
A.	B,D	A,B,C,D	B,C,D	B,C,D	C,D	A,B

SECTION-III

Q.	13	14	15	16	17
A.	3	5	2	4	8

PART-B-CHEMISTRY

SECTION-I(A)

Q.	18	19	20	21	22	23
A.	D	C	C	C	B	D

SECTION-I(B)

Q.	24	25	26	27	28	29
A.	A,D	B,C,D	B,D	B,C,D	A,B,C,D	A,B,D

SECTION-III

Q.	30	31	32	33	34
A.	4	9	4	5	4

PART-C-MATHEMATICS

SECTION-I(A)

Q.	35	36	37	38	39	40
A.	B	A	D	D	B	A

SECTION-I(B)

Q.	41	42	43	44	45	46
A.	B,C	A,B,D	A,C	A,B,C	A,B,C,D	A,B

SECTION-III

Q.	47	48	49	50	51
A.	3	8	2	6	2

SOLUTIONS

PART-A-PHYSICS

$$1) \text{ Area of v-t curve without sign} = \frac{1}{2} \times 5 \times 10 + \frac{1}{2} \times (8-5) \times (6) = 34$$

$$3) s = \frac{1}{2}gt^2$$

4)

$$h = ut + \frac{1}{2}at^2$$

$$h = 4 \times 4 - \frac{1}{2} \times 9.8 \times 4^2$$

$$h = -62.4 \text{ m or } 62.4 \text{ m}$$

6)

$$\frac{dy}{dx} = \frac{1}{x} + (e^{x^2}) (2x) + \left(\frac{-1}{2}\right) x^{-3/2}$$

$$7) d_5 = 25 \text{ m}, d_7 = 33 \text{ m}$$

$$d_n = u + (2n-1) \frac{a}{2}$$

$$25 = u + \frac{9}{2}a$$

$$33 = u + \frac{13}{2}a$$

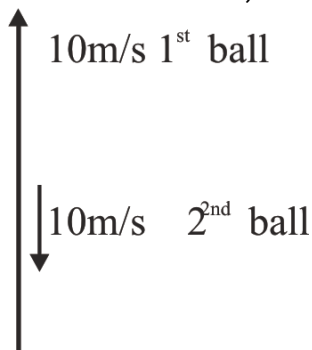
$$\Rightarrow u = 7 \text{ ms}^{-1}, a = 4 \text{ ms}^{-2}$$

8)

$$\text{For 1}^{\text{st}} \text{ ball} \Rightarrow -40 = 10t - 10t^2$$

$$t = 4 \text{ s}$$

$$\text{For 2}^{\text{nd}} \text{ ball } t = 2 \text{ s}, S = 40 \text{ m}, g = 10$$



$$40 = u \times 2 + \frac{1}{2} \times 10 \times 2^2$$

$$10 \text{ m/s} = u$$

9) Area under acceleration time graph equals to the change in velocity.
 Area under the velocity time graph equals to the displacement.
 Area under speed time graph equals to the distance travelled.

10)

We are given $u = 70$, $R = 480$, $g = 9.8$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \frac{70^2 \sin 2\theta}{9.8} = 480$$

$$\Rightarrow \sin 2\theta = \frac{24}{25}$$

This implies, $\sin \theta = \frac{3}{5}$ or $\frac{4}{5}$

Now $\frac{T_1}{T_2} = \frac{3}{4}$

$$\frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2} = \frac{9}{16} \text{ [B is correct]}$$

$$\frac{v_{\min 1}}{v_{\min 2}} = \frac{\cos \theta_1}{\cos \theta_2} = \frac{4}{3} \text{ [C is correct]}$$

D is correct and θ_1 and θ_2 will be complimentary.

12)

At $t = 0$; $y = 4$

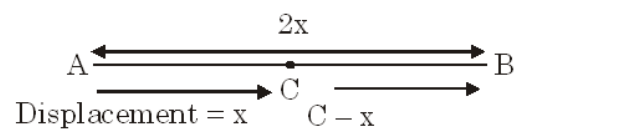
$$v = \frac{dy}{dt} = 6t^2 - 18t + 12$$

at $t = 0$; $v = 12$

$$a = \frac{dv}{dt} = 12t - 18$$

(acceleration is variable)

13)



(i) Average velocity = $\langle V \rangle = \frac{\text{displacement}}{\text{time}}$

$$V = \frac{x}{t}$$

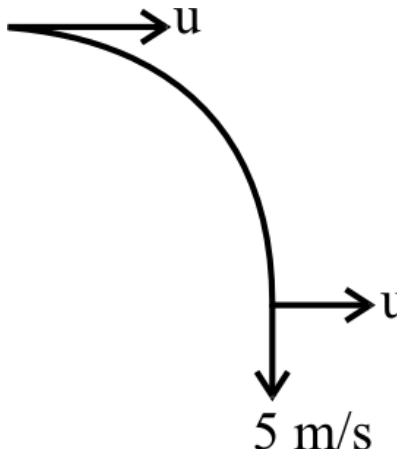
(ii) Average speed = $\frac{\text{distance}}{\text{time}}$

$$\Rightarrow kv = \frac{(2x + x)}{t}$$

$$kv = \frac{3x}{t}$$

$$kv = 3v$$

$$\square k = 3$$



14)

$$\sqrt{u^2 + 25} = 1.5u$$

$$u = \sqrt{20}$$

$$u = 2\sqrt{5}$$

15)

$$\frac{dy}{dx} = \sec^2 x = \sec^2 \frac{\pi}{4} = 2$$

16)

$$\frac{u_y^2}{20} = 20 \Rightarrow u_y = 20$$

$$\frac{2u_y}{g} = T$$

PART-B-CHEMISTRY

18)

$$\bar{\nu} = \frac{1}{\lambda} \propto E \Rightarrow \frac{E_1}{E_2} = \frac{\bar{\nu}_1}{\bar{\nu}_2} = \frac{2}{3}$$

24) All orbitals of np subshell are degenerate.
All orbitals of nd subshell are degenerate.

25)

Bohr theory is applicable for one electron system only.

26)

$$\text{K.E.} = \frac{1}{2} mv^2, \text{ P.E.} = -mv^2 \text{ and } V \propto \frac{1}{n}$$

Therefore, with increase in orbit number, K.E. decreased but P.E. increases.

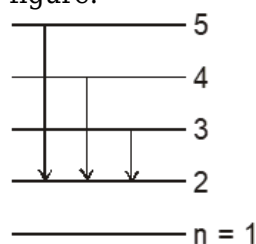
27)

Transition is taking place from $5 \rightarrow 2$

$$\Rightarrow \Delta n = 3$$

Hence maximum number of spectral line observed = $\frac{3(3+1)}{2} = 6$.

(C) number of lines belonging to the Balmer series = 3 ($5 \rightarrow 2, 4 \rightarrow 2, 3 \rightarrow 2$) as shown in figure.



Number of lines belonging to Paschen series = 2 ($5 \rightarrow 3, 4 \rightarrow 3$).

30)

$$R_H(1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = R_H(2)^2 \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

$$\Rightarrow \frac{1}{n^2} = \frac{1}{16} \Rightarrow n = 4$$

33)

$(7 \rightarrow 2), (6 \rightarrow 2), (5 \rightarrow 2)$

$(4 \rightarrow 2), (3 \rightarrow 2)$

Total = 5

PART-C-MATHEMATICS

35)

$$\log_5(2^x - 5) = \log_5 2 + \log_5 \left(2^x - \frac{7}{2} \right)$$

$$\Rightarrow (2^x - 5)^2 = 2 \cdot \left(2^x - \frac{7}{2} \right)$$

$$\Rightarrow (2^x)^2 - 10 \cdot (2^x) + 25 = 2 \cdot 2^x - 7$$

$$\Rightarrow (2^x)^2 - 12 \cdot (2^x) + 32 = 0$$

$$\Rightarrow 2^x = 4, 8$$

$$\Rightarrow x = 2, 3$$

but $x = 2$ is rejected.

37)

$$\text{Let } S = 1^2 + \frac{2^2}{3} + \frac{3^2}{3^2} + \frac{4^2}{3^3} + \frac{5^2}{3^4} + \frac{6^2}{3^5} + \dots \infty$$

$$S \left(\frac{1}{3} \right) = \frac{1^2}{3} + \frac{2^2}{3^2} + \frac{3^2}{3^3} + \frac{4^2}{3^4} + \frac{5^2}{3^5} + \dots$$

$$S\left(\frac{2}{3}\right) = 1^2 + \frac{3}{3} + \frac{5}{3^2} + \frac{7}{3^3} + \frac{9}{3^4} + \frac{11}{3^5} + \dots \infty$$

$$S\left(\frac{2}{9}\right) = \frac{1}{3} + \frac{3}{3^2} + \frac{5}{3^3} + \frac{7}{3^4} + \dots \infty$$

$$S\left(\frac{4}{9}\right) = 1 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \dots \infty$$

$$S\left(\frac{4}{9}\right) = 1 + \frac{2/3}{1 - 1/3}$$

$$\Rightarrow S = \frac{9}{2}$$

$$\frac{(a^2 + 3a + 1)(b^2 + 4b + 1)(c^2 + 5c + 1)}{abc}$$

$$= \left(a + \frac{1}{a} + 3\right) \left(b + \frac{1}{b} + 4\right) \left(c + \frac{1}{c} + 5\right)$$

39)

$$\therefore x + \frac{1}{x} \geq 2$$

So minimum value of expression

$$= (5)(6)(7) = 210$$

$$\log(11^{50}) = 50(1.041) = 52.05$$

$$\text{so } M = 53$$

41)

$$\text{and } \log(.11)^{50} = -47.930 = \overline{48.069}$$

$$\text{so } N = 47$$

$$|x|^{x+1} = 10^{\log_2 |x|}$$

$$\Rightarrow |x|^{x+1} = |x|^{\log_2 10}$$

$$42) \text{ (A) } x + 1 = \log_2 10 \Rightarrow x = \log_2 10 - 1$$

$$\text{(B) } |x| = 1 \Rightarrow x = \pm 1$$

$$\text{(C) } x \neq 0$$

$$\Rightarrow x = 1, -1, \log_2 10 - 1$$

43)

Sum of n inserted A.M. = $n(\text{single AM})$

$$\sum_{k=1}^n a_k = n.1011 = 56616$$

$$\Rightarrow k=1$$

$$n = 56 \quad (A)$$

$$a_{19} = 7 + 19d \Rightarrow 7 + 19 \cdot \frac{(2015-7)}{57}$$

$$\Rightarrow 7 + \frac{2008}{3} \Rightarrow \frac{2029}{3} \quad (B)$$

44)

Let R be the common ratio of the G.P. and D be the common difference of A.P.

$$a_5 = a_5, a_9 = Ra_5, a_{16} = a_5 R^2$$

$$a_9 - a_5 = 4D \Rightarrow (R-1)a_5 = 4D \dots\dots\dots(1)$$

$$a_{16} - a_9 = 7D \Rightarrow R(R-1)a_5 = 7D \dots\dots\dots(2)$$

$$\text{From equation (1)/(2), we get } \frac{1}{R} = \frac{4}{7} \Rightarrow R = \frac{7}{4}$$

$$\text{From equation (2) - (1), we get } (R-1)^2 a_5 = 3D \Rightarrow \frac{9a_5}{16} = 3D$$

$$\frac{3}{16}(a_1 + 4D) = D$$

$$\Rightarrow \frac{3}{16}a_1 = \left(1 - \frac{3}{4}\right)$$

$$D \Rightarrow a_1 = \frac{4D}{3}$$

(a) Using $AM \geq HM$

$$\frac{a_1 + a_2 + a_3}{3} \geq \frac{3}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)}$$

$$\Rightarrow (a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \geq 9$$

45)

(b) Similar to (a)

$$(c) \frac{a_1 a_2 + a_3 a_4}{2} \geq \sqrt{a_1 a_2 a_3 a_4}$$

$$\text{and } \frac{a_1 a_3 + a_2 a_4}{2} \geq \sqrt{a_1 a_2 a_3 a_4}$$

Multiplying, we get (3) is true.

47)

$$(abc)^{\frac{1}{3}} \geq \frac{a+b+c}{3}$$

But $GM \leq AM$

$$\Rightarrow AM = GM$$

$$\Rightarrow a = b = c$$

$$\Rightarrow 4a + 5a + 7a = 16$$

$$a = 1, b = 1, c = 1$$

48)

If $a, x, y, z, b \rightarrow AP$

$$x = \frac{3(a+b)}{4}, y = \frac{a+b}{2}, z = \frac{a+3b}{4}$$

if $a, x, y, z, b \rightarrow GP$

$$x = \frac{4ab}{3b+a}, y = \frac{2ab}{a+b}, z = \frac{4ab}{3a+b}$$

$$\text{if } \left(\frac{3a+b}{4}\right) \left(\frac{a+b}{2}\right) \left(\frac{a+3b}{4}\right) = 55$$

$$\text{and } \left(\frac{4ab}{3b+a}\right) \cdot \left(\frac{2ab}{a+b}\right) \cdot \left(\frac{4ab}{3a+b}\right) = \frac{343}{55}$$

$$\Rightarrow ab = 7$$

49)

No. of common terms = 14

Or L.C.M. (d_1, d_2) = L.C.M.(4, 7) = 28

$$51) \begin{cases} 2^{f_{ny}} = y^{f_{nx}} \\ 3^{f_{nx}} = y^{f_{ny}} \end{cases} = \begin{cases} 2^{f_{ny}} = x^{f_{ny}} \\ 3^{f_{nx}} = 3^{2f_{ny}} \end{cases}$$

$$\Rightarrow \begin{cases} x=2 \text{ or } y=1 \\ x=y^2 \end{cases} \Rightarrow \begin{cases} x^2=4, y^2=2 \\ x^2=1, y^2=1 \end{cases}$$