Mathematics solutions

1 Elementary algebra

Problem 1.1.

$$\frac{x^{n+2}}{x^{n-2}} = x^{n+2-n-2} = x^4$$

Problem 1.2.

$$x^{-1} * 8 = 2$$
$$x \neq 0$$
$$\frac{8}{x} = 2$$
$$x = \frac{8}{2} = 4$$

Problem 1.3.

$$(a^b)^0 = (5^10)^0 = 5^0 = 1$$

Problem 1.4.

$$\frac{\sqrt{4x}}{\sqrt{x}} = 2\frac{\sqrt{x}}{\sqrt{x}} = 2$$

Problem 1.5.

$$x^2 + (x+1)^2 = (x+2)^2$$
$$x^2 + (x^2 + 2x + 1) = x^2 + 4x + 4$$
$$x^2 - 2x - 3 = 0$$
$$D = 4 + 4 * 3 = 16$$
$$x = \frac{2-4}{2} = -1$$
$$x = \frac{2+4}{2} = 3$$

Problem 1.6.

$$2^x > 1024$$
$$2^x > 2^10$$
$$x > 10$$

2 Functions of one variable

Problem 2.1 (Based on SYD 2.5.6).

$$y = kx + b$$

where x is the temperature in Celsius and y is temperature in Fahrenheit

$$32 = k * 0 + b$$

$$212 = k * 100 + b$$

$$b = 32$$

$$212 = 100k + 32$$

$$100k = 180$$

$$k = 1.8$$

$$x = y$$

$$x = 32 + 1.8x$$

$$-0.8x = 32$$

$$x = y = -40$$

Problem 2.2. $f(3) = 5x + 4 = 5 * 3 + 4 = 15 + 4 = 19$

Problem 2.3.

$$x^2 - 4x + 3 = 0$$

$$D = 4^2 - 4 * 3 = 16 - 12 = 4$$

$$x = \frac{4-2}{2} = 1$$

$$x = \frac{4+2}{2} = 3$$

Problem 2.4.

$$10 * (1 + 0.02)^9 0) = 60858$$

Problem 2.5.

$$e^{\ln 5} = 5$$

3 Calculus

Problem 3.1.

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{12 * \frac{1}{6}}{1 - \frac{1}{6}} = \frac{2}{\frac{5}{6}} = \frac{2 * 6}{5} = 2.4$$

Problem 3.2.

$$\lim_{x \to 1} \frac{6^{1-x}}{x} = \lim_{x \to 1} \frac{6^{1-x} * 6^x}{x * 6^x} = \lim_{x \to 1} \frac{6}{1 * 6} = 1$$

Problem 3.3. $f(x) = x^5 - 8$

$$f'(x) = 5x^4$$

$$f'(3) = 5 * 3^4 = 405$$

Problem 3.4.

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\frac{x^3+2x-1}{x-2}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\frac{(3x^3+2x)(x-2)-(x^3+2x-1)}{(x-2)^2}=\frac{3x^3-6x^2+2x-4-x^3-2x+1}{(x-2)^2}=\frac{2x^3-6x^2-3}{(x-2)^2}$$

Problem 3.5.

$$\frac{\mathrm{d}}{\mathrm{d}\,x}4x^4+4x^2=16x^3+8x$$

$$\frac{\mathrm{d}^2}{\mathrm{d}\,x^2}4x^4+4x^2=\frac{\mathrm{d}}{\mathrm{d}\,x}16x^3+8x=48x^2+8$$

Problem 3.6.

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\frac{\ln x}{e^x}=\frac{\mathrm{d}}{\mathrm{d}\,x}\frac{\frac{e^x}{x}-\frac{e^x}{\ln x}}{e^{2x}}=\frac{\frac{e^x}{x}-\frac{e^x}{\ln x}}{e^{2x}}=\frac{1}{xe^x}-\frac{1}{x\ln x}$$

Problem 3.7.

$$f(x)=3x^2-5x+2$$

$$f'(x)=6x-5=$$

$$6x=5$$

$$x=\frac{5}{6}$$

x	$<\frac{5}{6}$	$\frac{5}{6}$	$>\frac{5}{6}$
$f'x$	-	0	+
Slope	decreasing	min	increasing
$f''x$	+	+	+
Convexity	convex	convex	convex

Problem 3.8. $f(x,y)=x^2+y^3$

$$f(2,3)=2^2+3^3=4+9=13$$

Problem 3.9. $f(x,y)=\ln(x-y)$

$$x-y>0$$

$$x>y$$

Problem 3.10.

$$\frac{\partial}{\partial\,x}x^5+xy^3=5x^4+y^3$$

Problem 3.11.

$$f(x,y)=x^2y^2+10$$

$$\frac{\partial}{\partial\,x}x^2y^2+10=2xy^2$$

$$\frac{\partial}{\partial\,y}x^2y^2+10=2yx^2$$

$$2xy^2=2yx^2=0$$

$$y=x=0$$

As the function is convex, the stationary point (0,0) is a local minima.

Problem 3.12. $\max x^2y^2$ s.t. $x + y = 10$

$$\frac{\partial}{\partial x}x^2y^2 = 2xy^2$$

$$\frac{\partial}{\partial y}x^2y^2 = 2yx^2$$

$$2xy^2 = 2yx^2$$

$$x = y$$

$$2x = 10$$

$$x = y = 5$$

4 Linear algebra

Problem 4.1.

$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

Problem 4.2. Take the following matrices:

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

Problem 4.3.

$$\begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix}^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

Problem 4.4.

$$\begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix}$$

$D = 1 * 8 - 2 * 9 = 8 - 18 = -10$

5 Probability theory

Problem 5.1.

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

Problem 5.2+5.3.

	Uses Drugs (D)	Doesn't Use Drugs (D')	Total
Positive Test Results (T)	P(T$\cap D$)	P(T$\cap D'$)	P(T)
Negative Test Results (T')	P(T'$\cap D$)	P(T'$\cap D'$)	P(T')
Total	0.01	0.99	

$$P(T) = P(T \cap D) + P(T \cap D')$$

$$P(T|D) = 0.99$$

$$P(T'|D') = 0.995$$

$$P(T \cap D) = P(D)P(T|D) = 0.01 * 0.99 = 0.0099$$

$$P(T' \cap D') = P(D')P(T'|D') = 0.99 * 0.995 = 0.98505$$

$$P(D') = P(T' \cap D') + P(T \cap D') = 1 - 0.01 = 0.99$$

$$P(T \cap D') = P(D') - P(T' \cap D') = 0.99 - 0.98505 = 0.00495$$

$$P(T) = P(T \cap D) + P(T \cap D') = 0.0099 + 0.00495 = 0.01485$$

	Uses Drugs (D)	Doesn't Use Drugs (D')	Total
Positive Test Results (T)	0.0099	0.00495	0.01485
Negative Test Results (T')	0.0001	0.98505	0.98515
Total	0.01	0.99	1

$$P(D|T) = \frac{P(D \cap T}{P(T)} = \frac{0.0099}{0.1485} = 0.0667$$