We have thus far been concerned with solid bodies which maintain a definite shape even under an applied force, particularly when the force is small. In the liquid state, the application of even a small force results in a change in shape. If placed in a container, a liquid will move downward and flatten out at the bottom of the container in response to the downward pull of gravity, i.e. it will take on the shape of the container. Gravitational pull is not significant enough to make a gas settle at the bottom of a container; it will, instead, expand to fill the container.

Liquids and gases, which alter their shapes in response to an applied force, or to match the shape of the container, are called ***fluids***. Familiar examples of fluids are water (a liquid) and air (a gas). The study of fluid mechanics is usually classified into ***hydrostatics*** (fluids at rest) and ***hydrodynamics*** (fluids in motion).

10.1 Density and Specific Gravity

Density is the mass of a unit volume of a substance. If an object of mass m occupies volume V, the density (ρ) is

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{V} \qquad (10.1)$$

with units of kg/m^3. The density of water is 1,000 kg/m^3. Several substances, mostly solids, have higher densities than water, while a few solids, several liquids and all gases have lower densities.

The ***specific gravity*** (s.g.) or ***relative density*** of a substance is defined as the ratio of the density of that substance (ρ) to the density of water (ρ_w), i.e.

$$s.g = \frac{\rho}{\rho_w} \qquad (10.2)$$

from which it is clear that the specific gravity of water is 1.0. It should be noted that specific gravity, being a ratio of densities, is dimensionless.

10.2 Pressure

Consider a container which is filled to height h with a liquid of density ρ (Fig. 10.1). In response to gravity, the liquid presses down on the base of the container. This downward push is distributed evenly over the entire base. The pressure (P) on a surface of area A is defined as the normal force (F) on the surface divided by the area (A) of the surface, i.e.

$$P = \frac{F}{A} \qquad (10.3)$$

The pressure on the base of the container is thus the downward (vertical) force per unit area of the base, and it can easily be shown that this is given by

$$P = \rho g h \qquad (10.4)$$

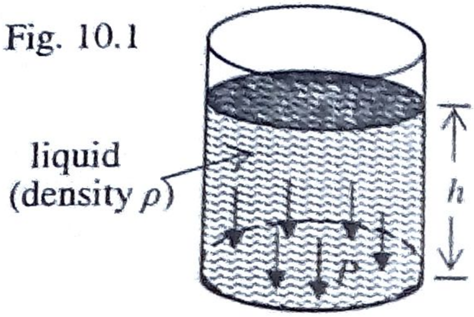
The unit of pressure is N/m^2, or the Pascal (Pa), with 1 N/m^2 = 1 Pa. It is clear from eq. (10.4) that pressure depends only o[n] the density and the height of the liqui[d] and not on the shape of the container o[r] the total volume of liquid.

The earth is surrounded by an air be[lt] which is generally referred to as t[he] ***atmosphere***. The earth's surface is th[us] situated beneath an "ocean" of air whi[ch] like all fluids, must be exerting so[me] pressure on it. The weight of [the] atmosphere supported by a unit area of [the] earth's surface, or the ***atmosphe*** *pressure* (P_a), is approximately 1.013×10^5 N/m^2. This is referred to as 1 *atmosphere* and it is equivalent to the pressure which will be exerted at the bottom of a mercury column of height 760 *mm* or a water column which is 10.3 *m* high.

The pressure given by eq. (10.4) is sometimes referred to as the gauge pressure, since it does not include the pressure exerted by the atmosphere when the surface of the liquid is exposed. To find the absolute pressure, the atmospheric pressure is added to the value determined from eq. (10.4). Thus

$$P(\text{absolute}) = P_a + \rho g h \qquad (10.5)$$

Fig. 10.1

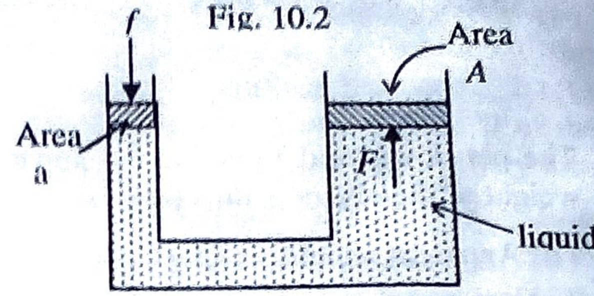


If a liquid is confined in a container and an additional pressure is exerted on the surface of the liquid, e.g. with the aid of a piston, the additional pressure is ***transmitted*** to every point in the liquid. In other words, ***the pressure exerted on a confined liquid is transmitted unchanged to every portion of the liquid.*** This is known as ***Pascal's principle***. This principle is utilized in the operation of a ***hydraulic press***, which consists essentially of a liquid-bearing container with two necks fitted with movable pistons of cross-sectional areas a and A (where $A > a$, see Fig. 10.2). Upon applying a force f on the small piston, pressure f/a is generated which is transmitted to the large piston. The latter thus feels an upward force (F) which is equal to *pressure* $\times$ *area*, or $(f/a)A$, i.e.

$$F = \frac{fA}{a} \qquad (10.6)$$

The hydraulic press is therefore a force multiplier, with a mechanical advantage of A/a.

Fig. 10.2

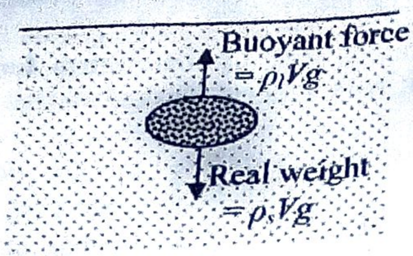


10.3 Buoyancy; Archimedes' Principle

An object immersed in a liquid experiences an upward force which is exerted by the liquid. This upward force is called ***buoyancy*** or ***upthrust***. The buoyant force is due to the difference in pressure between the upper and the lower faces of the object. (The pressure is greater at the lower face). ***Archimedes' principle*** states that ***a body immersed in a fluid experiences an upward force (buoyancy) which is equal to the weight of the fluid displaced.***

Any immersed body thus experiences two forces: its real weight acting vertically downwards and the buoyant force acting vertically upwards (Fig. 10.3). The net downward force is equal to the real weight minus the buoyant force, and this is referred to as the ***apparent weight*** of the body. For a solid object of density ρ_s and volume V which is fully immersed in a liquid of density ρ_l the real weight of the solid is $\rho_s Vg$ (downwards) and the buoyant force is $\rho_l Vg$ (upwards).

Fig. 10.3



The net downward force, or the apparent weight of the object is thus equal to

$$\text{Apparent weight} = \rho_s Vg - \rho_l Vg, \text{ or}$$
$$= (\rho_s - \rho_l)Vg.$$

If the solid is denser than the liquid (i.e. $\rho_s > \rho_l$), the solid experiences a net downward force (a positive apparent weight). If $\rho_s < \rho_l$ the solid feels a negative apparent weight, or a net upward force, and it therefore rises. The solid rises in the liquid until the weight of the liquid displaced becomes equal to the real weight of the solid (i.e. the solid floats, partly submerged).

10.4 Surface Tension and Capillarity

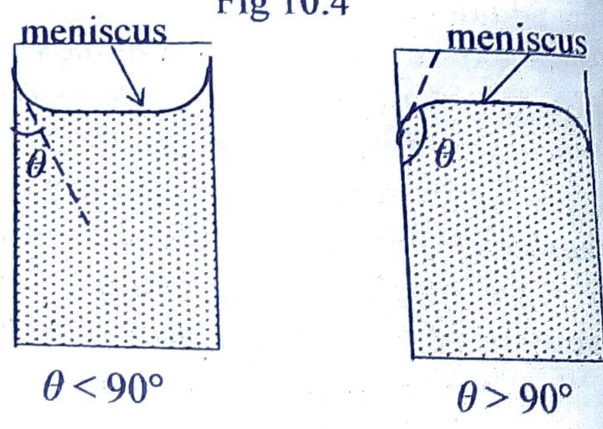
The molecules of a liquid are held together by intermolecular forces. Because of this, a liquid surface acts like a stretched elastic membrane, as if it is under tension. It is for this reason that a steel razor blade, for instance, can be floated on water surface, even though the density of steel is much higher than that of water. The surface forces which hold the molecules near the surface of a liquid together are described by the term ***surface tension.*** Specifically, the surface tension (γ) is defined as the surface force per unit length along a direction perpendicular to the force. Thus if a surface force (F) acts perpendicular to a line of length l, then

$$\gamma = \frac{F}{l} \qquad (10.7)$$

The magnitude of γ varies from one liquid to another, and for a given liquid it decreases with increasing temperature. The unit of γ is N/m.

The forces which the molecules of a liquid exert on each other are called ***cohesive forces.*** The molecules which are in contact with the wall of a container are also attracted to the wall. This force of attraction is termed ***adhesion.*** A liquid surface (which is also called a ***meniscus***) may curve upwards or downwards at the point of contact with the container wall, depending on which of the two forces is dominant. The angle (θ) between the container wall and the meniscus at the point of contact is the contact angle (Fig. 10.4). When adhesive forces outweigh cohesive forces, $\theta < 90°$ (meniscus curves upwards) and when cohesive forces are dominant, $\theta > 90°$ (meniscus curves downwards).

Fig 10.4



If a very thin tube is inserted into a liquid reservoir, there is a difference between the levels of the liquid inside and outside the tube. For $\theta < 90°$, the liquid rises higher inside the tube (e.g. water in a glass tube) while for $\theta > 90°$, the liquid level is depressed in the tube (as in mercury-in-glass tube).

Consider a liquid which rises to a height h inside a tube of radius r (Fig. 10.5). The cylindrical column of liquid experiences two counter-balancing forces: (1) the upward force F due to surface tension which acts around the circle of contact between the meniscus and the glass wall, and (2) the downward pull of gravity, or the weight (W) of the liquid column. The balancing of these forces at equilibrium yields

$$h = \frac{2\gamma \cos\theta}{\rho g r} \qquad (10.8)$$

for a liquid of surface tension γ and density ρ. From eq. (10.8), it is clear that h varies inversely as the radius r of the tube, i.e. a liquid will rise higher in a narrow tube than in a wide one. Tubes of very small cross-section, in which the rise of a liquid is quite noticeable, are referred to as ***capillary*** tubes, and the phenomenon is known as ***capillarity***.

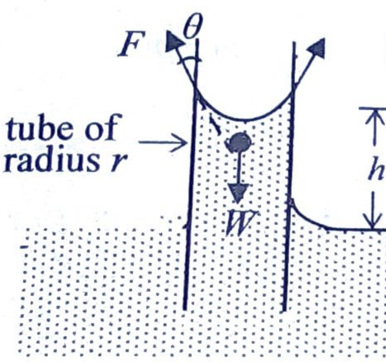


Fig. 10.5

Problem Solving Tips

1. The pressure at any point in a liquid depends only on the depth, not on the size or shape of the container.
2. Pressure is the same at all points at the same horizontal level within the same continuous liquid.
3. Absolute pressure = gauge pressure + atmospheric pressure.
4. Pascal's and Archimedes' principles should be remembered always.
5. For a body immersed in a liquid, apparent weight = true weight – buoyant force.
6. For problems involving surface tension and capillarity, identify the line of contact between the liquid and the containing vessel. Surface tension force = surface tension × total length of line of contact.
7. The equation for capillary rise (eq. 10.8) is derived by balancing the (upward) surface tension forces against the (downward) weight of the liquid column inside the capillary tube (see, for example, problem 10–20).
8. When $\theta > 90°$, h is negative (eq. 10.8), i.e. the liquid level is lower inside the capillary tube than in the liquid reservoir (see problem 10–23).

SOLVED PROBLEMS

10-1 *Show that the atmospheric pressure of 1.013 × 10^5 Pa will support (a) 760 mm of mercury and (b) 10.3 m of water (ρ = 13.6 × 10^3 kg/m^3 for mercury and ρ = 10^3 kg/m^3 for water).*

Solution

(a) Mercury: $P_a = \rho g h \Rightarrow h = P_a/\rho g$ or

$h = (1.013 \times 10^5)/(13.6 \times 10^3)(9.8)$
$= 0.76\ m$ (or 760 mm)

(b) Water: $h = (1.013 \times 10^5/(10^3)(9.8)$
$= 10.34\ m$

10-2 *Determine the gauge pressure and the absolute pressure inside a vessel if the height h of the mercury in an open-tube manometer attached to the vessel is 28.0 cm and the density of mercury* is 13.6 × 10^3 *kg/m^3*.

(OAU)

Solution

Gauge pressure, $P_g = \rho g h$
$= (13.6 \times 10^3)(9.8)(0.28)$
$= 3.73 \times 10^4\ Pa$

Absolute pressure $= P_g + P_a$
$= 3.73 \times 10^4 + 1.01 \times 10^5$
$= 1.38 \times 10^5\ Pa$

10-3 *A man's brain is approximately 0.33 m above his heart. If the density of human blood is $1.05 \times 10^3\ kg/m^3$, determine the pressure required to circulate blood between the heart and the brain.* (OAU)

Solution
Pressure required = difference in pressure between heart and brain
$\rho gh = (1.05 \times 10^3)(9.8)(0.33)$
$= 3.40 \times 10^3\ N/m^2$

10-4 *You have just purchased a chain claimed to be pure gold. The chain weighs 60 g and it displaces 4.0 cm^3 of water when fully immersed. Is it pure gold? (s.g. of gold = 19.3).*

Solution
Density of pure gold = (s.g. of gold)×(ρ of water)
$= 19.3\ (1) = 19.3\ g/cm^3$
Density of chain $= m/V = 60/4$
$= 15\ g/cm^3$.
Since the density of the chain is less than that of pure gold, the chain is not pure gold.

10-5 *The area of contact between the soles of the feet of a 70 kg man and the floor is 500 cm^2. (a) What pressure does he exert on the floor when standing? (b) If the man stands on stilts which make contact area of 2 cm^2 with the floor, what pressure does he exert? (Neglect the weight of the stilts).*

Solution

$$\text{Pressure } (P) = \frac{\text{force exerted}}{\text{contact area}} = \frac{F}{A}$$

F = weight of man = (70)(9.8)
$= 686\ N$

(a) $A = 500\ cm^2 = 5 \times 10^{-2}\ m^2$
$P = 686/(5 \times 10^{-2})$
$= 1.37 \times 10^4\ N/m^2$.

(b) $A = 2\ cm^2 = 2 \times 10^{-4}\ m^2$
$P = 686/(2 \times 10^{-4}) = 3.43 \times 10^6\ N/m^2$

10-6 *A swimmer whose body's surface area is approximately 1.6 m^2 lies at a depth of 3 m below the water surface. How much force is exerted on his body due to water pressure?*

Solution
Pressure at depth of 3 m =
$= (1000)(9.8)(3) = 2.94 \times 10^4\ N/m^2$
Force = pressure × area
$= (2.94 \times 10^4)(1.6)$
$= 4.7 \times 10^4\ N/m^2$

10-7 *An overhead water tank installed in a two-storey building is located at approximately 10 m above the water faucet in the ground floor and 5 m above the faucet in the first floor. Determine the water pressure at each faucet.*

Solution
Ground floor: $P = \rho gh = (1000)(9.8)(10)$
$= 9.8 \times 10^4\ N/m^2$
First floor: $P = (1000)(9.8)(5)$
$= 4.9 \times 10^4\ N/m^2$
(This explains why the water would issue out with greater force on the ground floor).

10-8 *If air were assumed to be uniformly distributed within the atmosphere, determine (a) the height of the atmosphere, and (b) the atmospheric pressure on the top of a mountain which is 1 km high. Take the atmospheric pressure at sea level as $1.01 \times 10^5\ N/m^2$ and density of air = 1.29 kg/m^3.*

Solution

(a) If h_a is the height of the atmosphere, the atmos. pressure at sea level = $\rho_a g h_a$, where ρ_a is the density of air. Hence

$1.01 \times 10^5 = (1.29)(9.8)(h_a)$,

$h_a = 7.99 \times 10^3\ m$ (or 7.99 km)

(b) Pressure at height 1 km

= Atmos. pressure at sea level − $\rho g h$

$= 1.01 \times 10^5 - (1.29)(9.8)(103)$

$= 8.84 \times 10^4\ N/m^2$

10-9 *A vacuum chamber has a door with surface dimensions 20 cm × 20 cm. How many 70 kg men, each exerting a pull equal to his own weight, will it take to pull the door open when the chamber is evacuated to a pressure of 0.1 atmosphere?*

Solution

Atmos. pressure = $1.013 \times 10^5\ N/m^2$

Pressure inside vacuum chamber

$= 0.1013 \times 10^5\ N/m^2$

Pressure difference

$= (1.013 - 0.1013) \times 10^5$

$= 9.117 \times 10^4\ N/m^2$.

Net force on door (which keeps door closed) = pressure difference × area of door

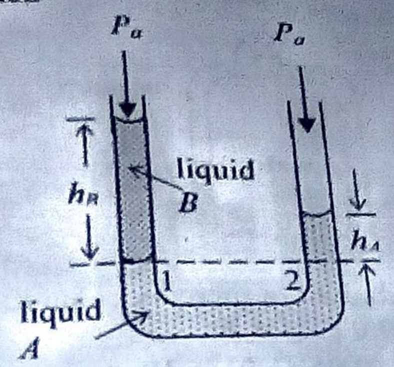
$= (9.117 \times 10^4)(4 \times 10^{-2})$

$= 3.65 \times 10^3\ N$

Each man can exert a pull of 70 × 9.8, or 686 N. Since $(3.65 \times 10^3)/686 = 5.3$, it will require six men to pull the door open.

10-10 *The right arm of a V-tube contains liquid A while the left arm is partly filled with liquid B. Calculate the ratio h_B/h_A of the liquid levels in the two arms when they are both exposed to the atmosphere (see Figure). The relative densities of A and B are 2.8 and 0.7 respectively.*

Solution



The pressure is the same at all points on the same horizontal plane within the same continuous liquid, i.e.

pressure at 1 = pressure at 2

$P_a + \rho_B g h_B = P_a + \rho_A g h_A$

where subscripts A, B refer to liquids A and B respectively.

Thus $h_B/h_A = \rho_A/\rho_B = 2.8/0.7 = 4.0$

10-11 *An inverted U-tube has one of its arms dipped into a pool of mercury and the other into water (see figure). Air is gradually pumped out of the tube through valve V. Calculate the heights to which the liquids will rise in the arms when the pressure drops to 95% of atmospheric. (Densities of water and mercury are $10^3\ kg/m^3$ and $1.36 \times 10^4\ kg/m^3$ respectively).*

Solution

Considering the left arm in the wate vessel:

Pressure at a = atmos. pressure = P_a

Pressure at b = pressure inside the tube $\rho_w g h_w$

$= P_i + \rho_w g h_w$

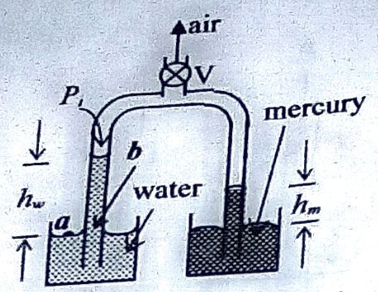
Pressure at a = pressure at b i.e.

$P_a = P_i + \rho_w g h_w \Rightarrow h_w = (P_a - P_i)/$

But $P_i = 0.95\ P_a \Rightarrow h_w = 0.05\ P_a/\ \rho_w g$

$h_w = (0.05)(1.013 \times 10^5)/(10^3)(9.8)$

$= 51.68\ cm$



Similarly, height of mercury column,

$$h_m = 0.05\, P_a / \rho_m g$$

$$= \frac{(0.05)(1.013\times105)}{(1.36\times10^4)(9.8)\ \text{m}}\ \text{m}$$

$$= 3.80\ cm.$$

10-12 *An airplane's altimeter which is designed to measure the pressure of the atmosphere reads 76 cm of mercury before the plane's take off and 40 cm when airborne. Determine (a) the altitude of the plane (b) the resultant force on a 25 × 25 cm window pane of the plane if the inside cabin is maintained at 76 cm of mercury. (Densities of mercury and air are 1.36 × 10⁴ kg/m³ and 1.29 kg/m³ respectively).*

Solution

(a) Assuming uniform distribution of air in the atmosphere,
Pressure at altitude = atmos. pressure $- \rho_a g h_a$. where subscript a stands for air.

Pressure at altitude

$$= (1.36 \times 10^4)(9.8)(0.40)$$
$$= 5.33 \times 10^4\ N/m^2$$

Atmos. pressure $(1.36 \times 10^4)(9.8)(0.76)$

$$= 1.013 \times 10^5\ N/m^2$$

Thus

$$5.33\times10^4 = 1.013\times10^5 - (1.29)(9.8)\, h_a, \text{ or}$$
$$h_a = 3.8 \times 10^3\ m \text{ (or 3.8 km).}$$

(b) Outside pressure = $5.33 \times 10^4\ N/m^2$
Inside pressure = $1.013 \times 10^5\ N/m^2$

Pressure difference

$$= 1.013 \times 10^5 - 5.33 \times 10^4$$
$$= 4.8 \times 10^4\ N/m^2$$

Force on window pane

$$= \text{pressure difference} \times \text{area of pane}$$
$$= (4.8 \times 10^4)(25)(25)(10^{-4}) = 3000\ N$$

10-13 *The pistons of a hydraulic press are of diameters 5 cm and 25 cm. The smaller piston is subjected to a force of 200 N. Determine (a) the pressure transmitted by the fluid, and (b) the weight which can be lifted by the larger piston.*

Solution

Area of small piston,

$$a = \tfrac{1}{4}\pi(0.05)^2 = 1.96 \times 10^{-3}\ m^2$$

Area of large piston,

$$A = \tfrac{1}{4}\pi(0.25)^2 = 4.91 \times 10^{-2}\ m^2$$

(a) Pressure applied on fluid by small piston

$$= \frac{\text{force on piston}}{\text{area of piston}} = \frac{200}{1.96\times10^{-3}}$$

$$= 1.02\times10^5\ Nm^{-2}$$

= pressure transmitted by fluid to large piston.

(b) Force on large piston

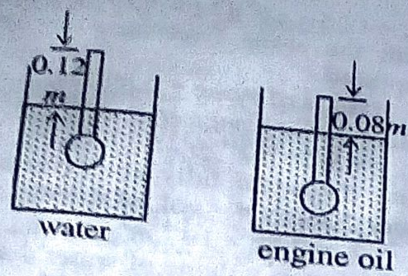
= pressure on piston × area

$= 1.02 \times 10^5 \times 4.91 \times 10^{-2} = 5{,}000\ N.$

10-14 *A hydrometer consists of a spherical bulb and a cylindrical stem of cross-section 0.4 cm². The total volume of bulb and stem is 13.2 cm³. When immersed in water the hydrometer floats with 0.12 m of stem above the water. In engine oil 0.08 m of the stem is above the surface. Find the density of the engine oil.*

(OAU)

Solution

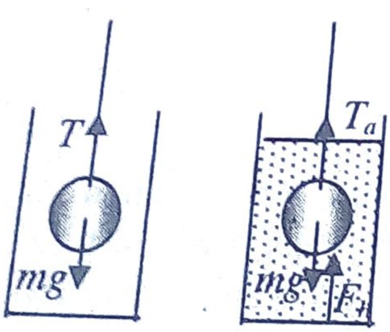


In water:
vol. of stem above water
$= (0.4 \times 10^{-4})(0.12) = 4.8 \times 10^{-6}\ m^3$
vol. of water displaced
$= (13.2 - 4.8)10^{-6} = 8.4 \times 10^{-6}\ m^3$
mass of water displaced
$= (10^3)(8.4 \times 10^{-6}) = 8.4 \times 10^{-3}\ kg$
= mass of hydrometer
In oil:
vol. of oil displaced
$= 13.2 \times 10^{-6} - (0.4 \times 10^{-4})(0.08)$
$= 10^{-5}\ m^3$
mass of oil displaced = mass of hydrometer

$\rho_{oil} \times (10^{-5}) = 8.4 \times 10^{-3}$, or
$\rho_{oil} = 0.84 \times 10^3\ kg/m^3$

10-15 *A boy tries to lift a 10 kg stone (of s.g. 4.0) out of a container by pulling on a rope attached to the stone. Determine the tension in the rope if the boy pulls vertically upwards (a) directly, and (b) after first filling the container with water. (Density of water = $10^3\ kg/m^3$).*

Solution



(a) (b)

(a) Tension in rope T = true weight of stone
$= (10)(9.8) = 98\ N$
(b) Let T_a = apparent weight and
F_b = buoyant force (see figure).
volume of stone $= m/\rho = 10/(4 \times 10^3)$
$= 2.5 \times 10^{-3}\ m^3$
= volume of water displaced
F_b = weight of water displaced
$= (2.5 \times 10^{-3})(10^3)(9.8) = 24.5\ N$
$T_a = mg - F_b = (10)(9.8) - 24.5 = 73.5\ N$

10-16 *The true weight of a piece of metal of s.g. 2.7 as measured with a spring balance is 4.9 N. Determine the reading on the spring balance when the metal is completely submerged in (a) water, (b) oil of s.g. 0.8, and (c) glycerine of s.g. 1.26.*

Solution
ρ of metal $= 2.7 \times 10^3\ kg/m^3$
mass of metal, $m = 4.9/9.8 = 0.5\ kg$
volume of metal, $V = m/\rho = 0.5/(2.7 \times 10^3)$
$= 1.85 \times 10^{-4}\ m^3$
(a) In water:
mass of water displaced
$= (\rho \text{ of water})V = (10^3)(1.85 \times 10^{-4})$
$= 0.185\ kg$
wt. of water displaced
$= (0.185)(9.8) = 1.81\ N$
= buoyant force on metal (Archimedes)
Apparent weight of metal
= true weight – buoyant force
$= 4.9 - 1.81 = 3.09\ N$
= reading on balance
(b) In oil of s.g. 0.8
wt. of oil displaced
$= (0.8 \times 10^3)(1.85 \times 10^{-4})(9.8)$
$= 1.45\ N$
Apparent weight $= 4.9 - 1.45 = 3.45\ N$
In glycerine of s.g. 1.26
wt. of glycerine displaced
$= (1.26 \times 10^3)(1.85 \times 10^{-4})(9.8)$
$= 2.28\ N$
Apparent weight $= 4.9 - 2.28 = 2.62\ N$

10-17 *An iron ball (s.g. 7.2) and a plastic ball (s.g. 0.8) each of radius 5 cm are initially fully immersed in water. What is the acceleration on each ball upon release?*

Solution

Volume of balls $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.05)^3$

$= 5.24 \times 10^{-4}\ m^3$

= vol. of water displaced when completely immersed.

Weight of water displaced

$= (10^3)(5.24 \times 10^{-4})(9.8) = 5.14\ N$

= buoyant force on each ball.

(i) Iron ball:

mass of ball $= (7.2 \times 10^3)(5.24 \times 10^{-4})$

$= 3.77\ kg$

weight of ball

$= 3.77 \times 9.8 = 36.97\ N$

net force on ball

= weight – buoyant force

$= 36.97 - 5.14 = 31.83\ N$,

downward acceleration on ball

$=$ net force/mass $= 31.83/3.77$

$= 8.44\ m/s^2$, downwards

(i.e. it sinks).

(ii) Plastic ball

$m = (0.8 \times 10^3)(5.24 \times 10^{-4})$

$= 0.419\ kg$

$W = mg = 0.419 \times 9.8 = 4.11\ N$

$F = 4.11 - 5.14 = -1.03\ N$,

i.e. 1.03 N, upwards

$a = F/m = -1.03/0.419 = -2.46\ m/s^2$

(i.e. it rises).

10-18 *A spherical piece of stone of radius 4 cm and s.g. 2.6 is placed in a hollow pan which is floated in water inside a cylindrical container of radius 10 cm. Determine the change in the water level if the stone is lifted out of the pan and submerged in the water inside the big container.*

Solution

vol. of stone $= \frac{4}{3}\pi(0.04)^3 = 2.68 \times 10^{-4}\ m^3$

mass of stone $= \rho V$

$= (2.6 \times 10^3)(2.68 \times 10^{-4}) = 0.697\ kg$

With the stone floating, mass of water displaced

= mass of stone = 0.697 kg.

vol. of water displaced $= 0.697/10^3$

$= 6.97 \times 10^{-4}\ m^3$

With the stone fully submerged, vol. of water displaced = vol. of stone

$= 2.68 \times 10^{-4}\ m^3$

Reduction in volume of water displaced,

$\Delta V = (6.97 - 2.68) \times 10^{-4}$

$= 4.29 \times 10^{-4}\ m^3$

The water level inside the cylinder falls by $\Delta V/A$, where A is the cross-sectional area of cylinder

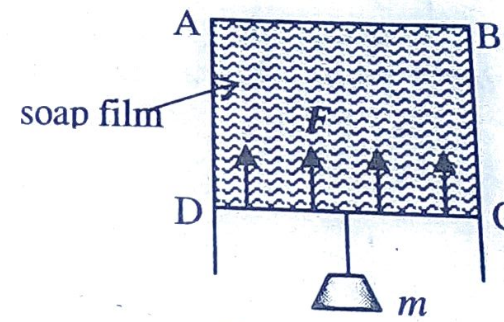
$= \pi(0.10)^2 = 3.14 \times 10^{-2}\ m^2$.

Reduction in water level

$= (4.29 \times 10^{-4})/(3.14 \times 10^{-2})$

$= 1.37 \times 10^{-2}\ m$ (or 1.37 cm).

10-19 *A film of soap solution is picked up by a rectangular wire frame ABCD with a movable side CD. The surface tension force (F) which tends to pull up the side CD is counterbalanced by hanging a mass (m) from it as shown. If the movable wire CD is 5 cm long and weighs 80 mg, while the mass m needed to maintain equilibrium is 170 mg, calculate the surface tension of the soap solution.*



Solution

The soap film (which has a finite thickness) has two surfaces, hence the total length of the surface along which the surface tension force F acts is $2l$, where l = length of CD.

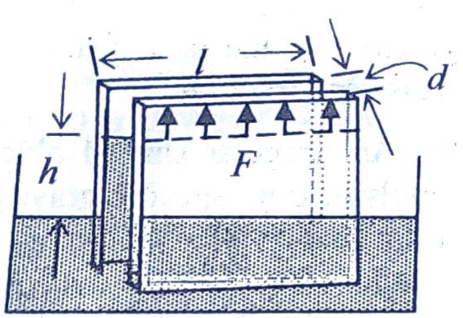
F = (mass of wire CD + mass m) g

$= (0.08 + 0.17)(10^{-3})(9.8)$
$= 2.45 \times 10^{-3}\ N$
$l \quad = 5\ cm = 0.05\ m$
Surface tension $\gamma = F/2l$
$= 2.45 \times 10^{-3}/(2)(0.05)$
$= 2.45 \times 10^{-2}\ N/m$

10-20 *Two parallel rectangular plates are partly immersed in a dish of water at 20°C. The plates, which lie in a vertical plane, are spaced 1 mm apart. Determine the height to which the water will rise in the space between the plates (above the water level in the dish) if the surface tension of water at 20°C is 7.3×10^{-2} N/m.*

Solution

Let the width of each plate = *l*.
From the figure, it can be seen that the weight of the water column between the plates is supported by surface tension forces *F* which act upwards around the line of contact (2*l*) between the water surface and the glass plates. This force acts vertically upwards since the contact angle between glass and water is 0°.



Thus

$F = 2l\gamma \qquad (1)$

This force is counterbalanced by the weight of the water column

$W = ldh\rho g \qquad (2)$

where ρ is the density of water. Since $F = W$,

$2l\gamma = ldh\rho g \Rightarrow h = 2\gamma/d\rho g$

$\gamma = 7.3 \times 10^{-2}\ N/m,\ d = 10^{-3}\ m,$
$\rho = 10^3\ kg/m^3,\ g = 9.8\ m/s^2$
$\Rightarrow h \quad = (2 \times 7.3 \times 10^{-2})/(10^{-3})(10^3)(9.8)$
$= 1.49 \times 10^{-2}\ m$ (or 1.49 cm)

10-21 *The xylem tubes which transport sap to the top of a tree can be considered as uniform cylinders. If the transport of sap is entirely due to capillarity, determine the diameter of the tubes which will move sap up a tree which is 25 m tall. Take the specific gravity and surface tension of sap as 1.0 and 5×10^{-2} N/m and the contact angle with the tubes as 45°.*

Solution

$\gamma \quad = 5 \times 10^{-2}\ N/m,\ \rho = 10^3\ kg/m^3,$
$h \quad = 25\ m,$
$\theta \quad = 45°$
$h \quad = 2\gamma \cos\theta/\rho g r$
$\Rightarrow r \quad = 2\gamma \cos\theta/(\rho g h)$ or

$$r = \frac{(2)(5 \times 10^{-2}) \cos 45°}{(10^3)(9.8)(25)}$$

$= 2.89 \times 10^{-7} m$
(or 2.89×10^{-4} mm).

10-22 *Two (glass) capillary tubes of diameters 0.05 mm and 2.00 mm are dipped in a pool of water. How high will the water rise in each of the tubes? (Contact angle between glass and water is 0°).*

Solution

(i) For $d = 0.05$ mm,
$r \quad = 0.025\ mm = 2.5 \times 10^{-5}\ m$
$h \quad = 2\gamma/(\rho g r)$
$= 2(7.3 \times 10^{-2})/(10^3)(9.8)(2.5 \times 10^{-5})$
$= 0.60\ m$ (or 60 cm)

(ii) For $d = 2.0$ mm, $r = 1\ mm = 10^{-3}\ m$
$h \quad = 2(7.3 \times 10^{-2})/(10^3)(9.8)(10^{-3})$
$= 1.5 \times 10^{-2}\ m$ (or 1.5 cm).

10-23 *A glass capillary tube of radius 1 mm is dipped into a container of* *mercury. Assuming a contact angle of 140° between mercury and glass, determine the level of the mercury column inside the tube. Take the specific gravity of mercury as 13.6 and its surface tension as 0.47 N/m.*

Solution

$h = 2\gamma \cos\theta/(\rho g r)$

$\gamma = 0.47\ N/m,\ \theta = 140°,$

$\rho = 13.6 \times 10^3\ kg/m^3,$

$g = 9.8\ m/s^2,\ r = 10^{-3}\ m$

$$h = \frac{2(0.47)\cos 140°}{(13.6 \times 10^3)(9.8)(10^{-3})}$$

$= -5.4 \times 10^{-3}\ m$ (or $-5.40\ mm$)

(The negative sign indicates that the mercury level inside the tube is depressed relative to the mercury level in the container).

SUPPLEMENTARY PROBLEMS

10-24 A 4500 *kg* accelerator magnet sits on a 35 *cm* × 40 *cm* base. Calculate the pressure on the floor due to the magnet. [Answer = $3.15\times10^5\ N/m^2$]

10-25 The density of a liquid used in a manometer is 780 kg/m^3. What height of the liquid in the manometer tube will correspond to 1 *atmosphere* ($1.013 \times 10^5\ Pa$)? [Answer = 13.25 *m*]

10-26 A plane flies at an altitude where the atmospheric pressure is 200 *mmHg*. If the pressure inside the cabin is maintained at 760 *mmHg*, calculate the net force on a window whose area is 800 cm^2. [Answer = $5.97 \times 10^3\ N$]

10-27 A 2.0 *kg solid* mass has an apparent weight 12.34 *N* when totally immersed in water. Calculate the apparent weight of the body when totally immersed in a liquid of relative density 0.78. [Answer = 13.94 *N*]

10-28 A 3.5 *kg* solid object has an apparent weight 9.60 *N* when wholly immersed in a liquid of relative density 0.90. Calculate the volume of liquid displaced when the object is placed in a liquid of relative density 1.50. [Answer = $2.33\times10^{-3}\ m^3$]

10-29 A canoe with a mass 350 *kg* is carrying 6 men, each having a mass 70 *kg*. Calculate the minimum volume of the canoe to keep them afloat in fresh water of specific gravity 1. [Answer = 0.77 m^3]

10-30 An alloy is made up of a material of specific gravity 7.87 and another material of specific gravity 4.50. the alloy of mass 750 g has an apparent weight of 6.6 *N* when totally immersed in a liquid of specific gravity 0.78. Calculate the ratio of the volumes of the constituents of the alloy. [Answer = 1/15]

10-31 A man escaping from a sinking ship climbs unto a piece of wood of length 4.5 *m* and thickness 3.5 *cm*. The man has a weight 637 *N*, and the wood has a mass 18 *kg*. Calculate the minimum width of the wood that will keep the man afloat. The specific gravity of the sea water is 1.03. [Answer = 5.12 *m*]

10-32 An irregular shaped object made purely of iron (specific gravity = 7.87) has a mass 1.8 *kg*. It has an apparent weight 14.89 *N* when wholly immersed in water. Calculate the volume of void (empty space containing air of specific gravity 1.29×10^{-3}) contained in the object. [Answer = $5.6\times10^{-5}\ m^3$]

10-33 Calculate the mass of lead of specific gravity 11.3 that must be attached to a slab of wood such that they both just float in water. The

10 $cm \times 3.5\ cm$ and its specific gravity is 0.096. [Answer = 0.521 *kg*]

10-34 A man attempts to suck juice from an open cup through a straw positioned vertically in the cup. The height of the straw above the surface of the liquid is 25 *cm* and the specific gravity of the liquid is 1.12. Calculate the pressure that the man has to maintain in his lungs to suck up the juice. Atmospheric pressure is $1.13 \times 10^5\ Pa$. [Answer = $1.10 \times 10^5\ Pa$]

10-35 A bottle is capped under vacuum. The pressure inside the bottle is $6.0 \times 10^4\ Pa$ and atmospheric pressure is $1.10 \times 10^5\ Pa$. Calculate the force that must be applied to open the cap which has a radius 2.5 *cm*. [Answer = 98 *N*]

10-36 A submarine is 50 *m* below the surface in the ocean. If the sailors must escape by forcefully opening a hatch of diameter 40 *cm*, how much force has to be applied? Atmospheric pressure is $1.13 \times 10^5\ Pa$ and the specific gravity of brine is 1.10. [Answer = $8.2 \times 10^4\ N$]

10-37 What minimum pressure must a pump maintain to pump water through a 2.54 *cm* diameter pipe into an open tank 15.0 *m* above the pump? [Answer = $2.6 \times 10^6\ Pa$]

10-38 A hydraulic lift has the a narrow cylinder of diameter 5 *cm* and wide cylinder of diameter 40 *cm*. Calculate the force that must be applied to the liquid in the small cylinder to lift a car 1950 *kg*. [Answer = 298.6 *N*]

10-39 A barometer is made up of a narrow-bore glass tube with inner diameter 3 *mm*. The height of the mercury column in the barometer is 750 *mm* when only vacuum exists above the column. Calculate the length of the mercury column when $1.44 \times 10^{-6}\ m^3$ of glycerin (specific gravity = 1.26) is introduced into the vacuum. Specific gravity of mercury is 13.6. [Answer = 731.5 *mm*]

10-40 A glass tube having an inner diameter 0.5 *mm* is dipped vertically into a liquid. If the surface tension between the tube and liquid is $7.5 \times 10^{-2}\ N/m$, calculate how high the liquid rises in the tube. [Answer = 6.12 *cm*]

10-41 A microscope slide of width 2.5 *cm* and thickness 2 *mm* is lowered into a liquid whose surface tension is $7.6 \times 10^{-2}\ N/m$. Calculate the force due to surface tension on the slide. [Answer = $4.1 \times 10^{-3}\ N$]

10-42 A capillary tube of 0.40 *mm* diameter is inserted vertically in a liquid of relative density 0.78 and surface tension $6.0 \times 10^{-2}\ N/m$. If the angle of contact is 20°, calculate how high in the stem the liquid rises [Answer = 7.38 *cm*]

10-43 When a capillary tube is inserte into water, the water rises in the tube height 7.20 *cm*. If the same tube dipped into mercury, specific grav 13.6 and angle of contact 140°, what depth is the mercury in capillary tube depressed below surface? Angle of contact in wate zero, surface tension of water is 7 $10^{-2}\ N/m$, surface tension of mer is 0.54 *N/m*. [Answer = 3.13 *cm*]