LOGIC

* Truth table for conjunction – (and)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

When both individual propositions are true then the truth value of the conjunctions is true, otherwise false.
Note – In logic $\wedge$ & $\cap$, the intersection of sets

* Disjunction – (or)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note – The disjunction value will be true when either one or the other or both are true.
In logic $\vee$ & $\cup$, the union of sets, are equivalent.

* Implication – 'if ... then ...'.
p: It is raining
q: I carry an umbrella.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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* Equivalence. 'if and only if'.

p	q	$p \Leftrightarrow q$	
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Note : The truth value of equivalence is true only when all the proposition have same value

* Tautology. (Always true)

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

* Contradiction (Always false)

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

* Contingency

p	q	$\sim q$	$p \vee \sim p$	$(p \vee \sim q) \wedge p$
T	T	F	T	T

Q.)

p	q	$\sim p$	$p \rightarrow q$	$q \vee \sim p$	$(p \rightarrow q) \leftrightarrow (q \vee \sim p)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Q. $(p \wedge (\sim p \vee q)) \wedge \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$ (a)	$a \wedge \sim q$
T	T	F	F	T	T	F
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

F = Contradiction

Q.) $(p \wedge q) \vee (q \wedge \sim p)$

p	q	$p \wedge q$ (a)	$\sim p$	$q \wedge \sim p$ (b)	$a \vee b$
T	T	T	F	F	T
T	F	F	F	F	F
F	T	F	T	F	T
F	F	F	T	F	F

Contingency

Q.) $(p \vee \sim q) \wedge p$

p	q	$\sim q$	$(p \vee \sim q)$ (a)	$a \wedge p$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	T

Contingency

$$[(p \to q) \land (q \to r)] \to (p \to r)$$

p	q	r	ⓐ $p \to q$	ⓑ $q \to r$	ⓒ $p \to r$	ⓓ ⓐ $\land$ ⓑ	ⓓ $\to$ ⓒ
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Tautology

H.W. ① $[p \land (q \lor r)] \to [(p \land q) \lor (p \land r)]$

② $[p \lor (q \land r)] \to [(p \lor q) \land (p \lor r)]$

③ $[p \to (q \to r)] \to [(p \to q) \to r]$

Note:) $\to \; = \; \supset$

H.W. 1

Q.1)

p	q	r	i: $p \vee r$	$p \wedge i$	j: $p \wedge q$	k: $p \wedge r$
T	T	T	T	T	T	T
T	T	F	T	T	T	F
T	F	T	T	T	F	T
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	T	F	F	F
F	F	T	T	F	F	F
F	F	F	F	F	F	F

$j \vee k$	Final.
T	T
T	T
T	T
F	T
F	T
F	T
F	T
F	T

Tautology ✓

Q. 2)

p	q	r	i: $q \wedge r$	$p \vee i$	② $p \vee q$	③ $p \vee r$	② ∧ ③	Final
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

Tautology //

Q.3)

p	q	r	① $p \to r$	$p \to ①$	② $p \to q$	$② \to r$	Final
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Tautology.

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i) 3 Implications –
Converse $(p \to q)$ Converse $(q \to p)$

Eg. – If there is smoke then there is fire
$\underbrace{\text{there is smoke}}_{p}$ $\underbrace{\text{there is fire}}_{q}$

Converse – If there is fire then there is smoke.

ii) Inverse. $(p \to q)$ inverse $(\sim p \to \sim q)$

Eg. – If a metal is heated then it expands.
Inverse – If a metal is not heated then it does not expand

iii) Contrapositive
If both the premise and conclusion of an implication are first negated and then interchanged, we get CP.
$(p \to q)$ $(\sim p \to \sim q) \to (\sim q \to \sim p)$

CP – "If a metal does not expand it is not heated".

Laws of Logic –

1. Idempotent laws – $p \vee p \equiv p$ $\quad p \wedge p \equiv p$
2. De Morgans law – $\sim(p \wedge q) \equiv \sim p \vee \sim q$ $\quad \sim(p \vee q) \equiv \sim p \wedge \sim q$
3. Distributive – $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $\quad p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
4. Double negation – $\sim(\sim p) \equiv p$
5. Absorption – $p \vee (p \wedge q) \equiv p$ $\quad p \wedge (p \vee q) \equiv p$
6. Associative – $(p \vee q) \vee r \equiv p \vee (q \vee r)$ $\quad (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
7. Commutative – $p \vee q \equiv q \vee p$ $\quad p \wedge q \equiv q \wedge p$
8. Implication – $p \to q \equiv \sim p \vee q$ $\quad p \to q \equiv \sim q \to \sim p$
9. Biconditional – $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ $\quad p \leftrightarrow q \equiv \sim q \leftrightarrow \sim p$
10. Compliment – $p \vee \sim p \equiv T$ $\quad p \wedge \sim p \equiv F$
11. Identity – $p \wedge T \equiv p$ $\quad p \vee F \equiv p$
12. Dominaⁿ – $p \vee T \equiv T$ $\quad p \wedge F \equiv F$
13. Implicaⁿ – $p \to q \equiv \sim p \vee q$

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H. W. 2

(a) If It is cold, then he wears a hat
→ p: It is cold. q: He wears hat.
i) Converse - If he wears a hat, then it is cold
ii) Inverse - If It is not cold then he will not wear a hat
iii) Contrapositive - If he doesnt wear a hat then it ~~is~~ is not cold.

(b) If an integer is multiple of two then it is even.
→ p: An integer is multiple of 2 q: It is even.
i) Converse - If it is even then it is multiple of two.
ii) Inverse - If an integer is not a multiple of 2 then it is not even.
iii) Contrapositive - If the integer is not even then it is not a multiple of 2.

(c) If you are good in mathematics then you are good in sci.
→ p: You are good in maths, q: You are good in science.
i) Convers - If you are good in sci then you are good in maths.
ii) Inverse - If you are not good in math then you are not good in science
iii) Contra - If you are not good in science then you are not good in maths.

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Q) Prove
$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

$\sim(p \vee q) \vee r$ — (implication)
$(\sim p \vee r) \vee r$ — (DeMorgans)
$(\sim p \vee r) \wedge (\sim q \vee r)$ — (Distributive)
$(p \rightarrow r) \wedge (q \rightarrow r)$ — (Implication)

Q.) Prove
$(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$ is tautology.
$\sim q \wedge (\sim p \vee q) \rightarrow \sim p$ — (implication)
$\sim(\sim q \wedge (\sim p \vee q)) \vee \sim p$ — (implication)
$\sim\sim q \vee \sim(\sim p \vee q)) \vee \sim p$ — (DeMorgan)
$\sim\sim q \vee (\sim\sim p \wedge \sim q)) \vee \sim p$ — (DeMorgan)
$q \vee (p \wedge \sim q) \vee \sim p$ (Double Negaⁿ)
$(q \vee p) \wedge (q \vee \sim q) \vee \sim p$ (Distributive)
$(q \vee p) \wedge \text{True} \vee \sim p$ (Inverse) (compliment)
$p \vee \sim p$
T //

Q.) Prove $(p \wedge q) \rightarrow (p \vee q)$ is Tautology.
$\sim(p \wedge q) \vee (p \vee q)$ — (Implication)
$(\sim p \vee \sim q) \vee (p \vee q)$ — (DeMorgans)
$(\sim p \vee p) \vee (\sim q \vee q)$ — (Associative)
$T \vee T$
T //

q.) $(p \to q) \wedge (r \to q) \equiv (p \vee r) \to q$

LHS,

$(p \to q) \wedge (r \to q)$

$(\sim p \vee q) \wedge (\sim r \vee q)$ — implication

$(q \vee \sim p) \wedge (q \vee \sim r)$ — commutative

$q \vee (\sim p \wedge \sim r)$ — Distributive

$(\sim p \wedge \sim r) \vee q$ — commutative

$\sim (p \wedge r) \vee q$ — Demorgan.

$(p \vee r) \to q$ //

q.) Show that $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) = r$

$\to$ $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$

$(\sim p \wedge (\sim q \wedge r)) \vee (r \wedge q) \vee (p \wedge r)$ — commutative

$(\sim p \wedge \sim q) \wedge r) \vee (r \wedge q)(p \wedge r)$ — associative

$(r \wedge (\sim p \wedge \sim q)) \vee (r \wedge (q \vee p))$ — Distributive

$r \wedge ((\sim p \wedge \sim q)) \vee q \vee p)$ — Distributive

$r \wedge [(\sim p \wedge \sim q \vee q) \vee p]$ — associative

$r \wedge [\sim p \wedge T] \vee p$ — inverse

$r \wedge [\sim p \vee p]$ — complimentary

$r \wedge T$ identity

r //

Mathematical Induction

Step 1 : Verification (Basis of Induction)
- First verify $p(n)$ is true for n equal $n=1$ (i.e. first step of ladder).

Step 2 : Induction step (Inductive Property)
- Assuming $p(n)$ is true for some value $n=k$. also for $n=k+1$.

Step 3 : Conclusion.
- The result is true for any value $n \in N$.

Q.) Prove by induction -
$1+2+3+\cdots+n(n+1)$ for all natural no.
2

→ Let $p(n)$ be the statement

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

Step 1 Verification = for $n=1$ $p(n) = n(n+1)/2$

for $p(1) = 1(1+1)/2 = 1$

$p(1)=1$ $\therefore$ $p(1)$ is true.

Step 2 Induction step = Assume $p(k)$ is true.

$p(k) = 1+2+3+\cdots+k = k(k+1)/2$ — (1)

Prove $P(k+1)$ is also true.

$p(k+1) = 1+2+3+\cdots+k$

$= (k+1)((k+1)+1)/2$

$= (k+1)(k+2)/2$ — (2)

$1+2+3+\cdots+k+(k+1) = \frac{k(k+1)}{2}+k+1$ — (3)

Subs. (3) in (2)

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$$

$\therefore$ $P(n)$ is true for all natural no's.

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Q.) Show that n^3+2n is divisible by 3 for $n \geq 1$

→ Let $p(n)$ be the statement n^3+2n

Step 1 Basis of induction (Verification)

for $n=1$,

n^3+2n

$p(n) = n^3+2n$

$p(1) = 1^3+2(1)$

$= 1+2$

$= 3$ (Divisible by 3)

$p(2) = 2^3+2(2)$

$= 12$ (Divisible by 3)

Hence it is true for $n = 1, 2$.

Step 2 Assume that the result is true $n=k$

$p(k) =$ true

$= k^3+2k$ (is Divisible by 3)

$k^3+2k =$ multiple of 3 — (1)

$= 3m$

$p(k+1) = (k+1)^3 + 2(k+1)$

$= k^3+32k^2+3k+1+2k+3$

$= k^3+32k^2+5k+3$

$= k^3+32k^2+3k+2k+3$

$= (k^3+2k)+32k^2+3k+3$

$= 10$

$= 3m+3(k^2+k+1)$

$p(k+1)$ is divisible by 3.

Step 3 Conclusion — By Mathematical Induction the result is true for all n.

H. W. 3

Q) For $n \geq 1$, $1\cdot2 + 2\cdot3 + 3\cdot4 + \cdots n(n+1)$
$\dfrac{n(n+1)(n+2)}{3}$

step 1 Let $p(n) = 1\cdot2 + 2\cdot3 + 3\cdot4 + \cdots + n(n+1) = \dfrac{n(n+1)(n}{3}$

Basis of Induction

$p(1) = 1\cdot2 + 2\cdot3 + 3\cdot4 + n(n+1) = .$

LHS = $n(n+1)$

$= 1\ (1+1) = 2$

RHS $= \dfrac{1(1+1)(1+2)}{3} = \dfrac{6}{3} = 2$

$P(1)$ is true

step 2 $n(k) = 1\cdot2 + 2\cdot3 + 3\cdot4 + \cdots + k(k+1)$

$\dfrac{p(k+1)}{k^2} = 1$

$\dfrac{k(k+1)(k+2)}{3} + (k+1) + k + 2) = k+1)$

$\dfrac{k(k+1)(k+2)}{3}$

$\dfrac{k(k^2+2k+k+2)}{3}$

$\dfrac{k^3 + 2k^2 + k^2 + 2k}{3}$

$\dfrac{k^3 + 3k^2 + 2k}{3}$

$$\frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

$$\frac{(k+1)(k+2)(k+3)}{3} = \frac{(k+1)(k+2)(k+3)}{3}$$

Q. Prove $1^2 + 3^2 + 5^2 + \dots + (2n+(-1))^2$

$$= \frac{n(2n-1)(2n+1)}{3}$$

$$P(n) = 1^2 + 3^2 + 5^2 + (2n-1)^2$$

$$\frac{n(n-1)(2n+1)}{3}$$

$P(1) = 1$

LHS = RHS

$P(1)$ is true

$$P(k) = 1^2 + 3^2 + 5^2 + \dots + (2k-1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3}$$

$$P(k+1) = 1^2 + 3^2 + 3\,5^2 + \dots + (2(k-1)-1)^2$$

$$P(k+1) = \frac{(k+1)(2(k+1)-1)(2k+1)+1)}{3}$$

$$\frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$\frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$

$$\frac{(2k+1)\,[k(2k-1) + 3(2k+1)]}{3}$$

$$\frac{(2k+1)\,[2k^2 - k + 6k + 3]}{3}$$

$$\frac{(2k+1)\,[2k^2 + 5k + 3]}{3}$$

$$\frac{(2k+1)\,[2k(k+1)(3k+1)]}{3}$$

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$$\frac{(2k+1)(2k+3)(k-1)}{3}$$

$$\frac{(k+1)[(2(k+1)+1)][2(k+1)-1]}{3}$$

Conclusion — $P(k+1)$ is ~~pa~~ true for all n

q.) Prove that $2+5+8+\cdots+(3n-1)$
$= n\left(\frac{3n+1}{2}\right)$

→ $P(n) = 2+5+8 \cdots (3n-1)$
for $P(1)$
$3(n)-1) = n(3n+1/2)$
$2 = 2$
$LHS = RHS$
$P(1)$ is true.
$n = k$, for $p(k)$
$p(k) = 2+5+8+\cdots+3(k-1) = \frac{1}{2}k\left(\frac{3k+1}{2}\right)$

$2+5+8+\cdots+(3k-1)+(3k+2)$
$= \frac{1}{2}k(3(k+1)+1)$

$P_{k+1} = P(k) + T_{\dots}$
$\frac{1}{2}k(3k+1) + [3(k+1)-1]$
$\frac{1}{2}[3k^2+k+3k+3-1)$
$\frac{1}{2}[3k^2+k+6k+6-2]$
$\frac{1}{2}[3k^2+7k+4]$
$1/2\,(k+1)(3k+4)$
$1/2\,(k+1)[3(k)+1]$
& for $p(k+1) = \frac{1}{2}(k+1)[3(k+1)]$

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