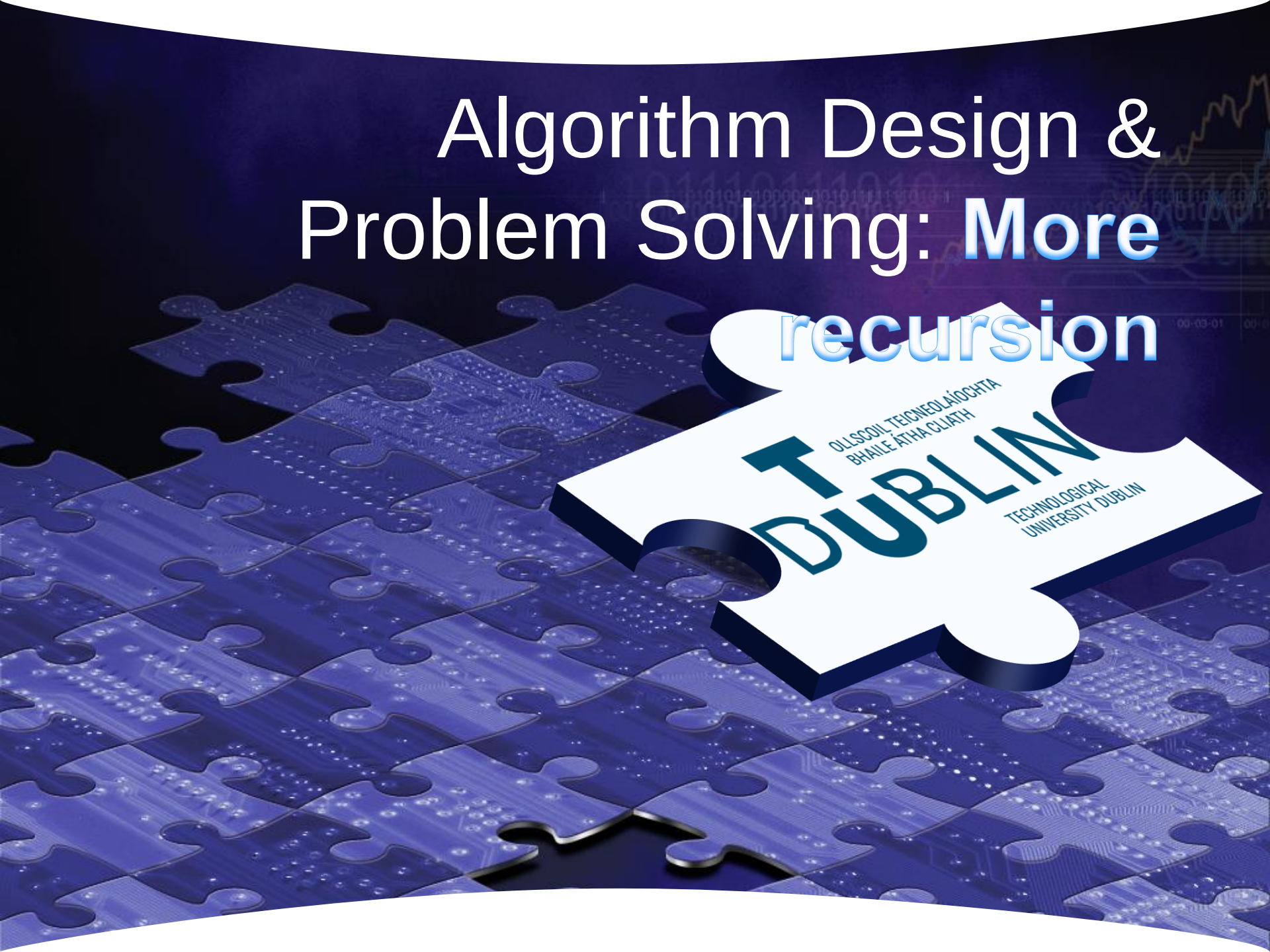
Algorithm Design & Problem Solving: More recursion



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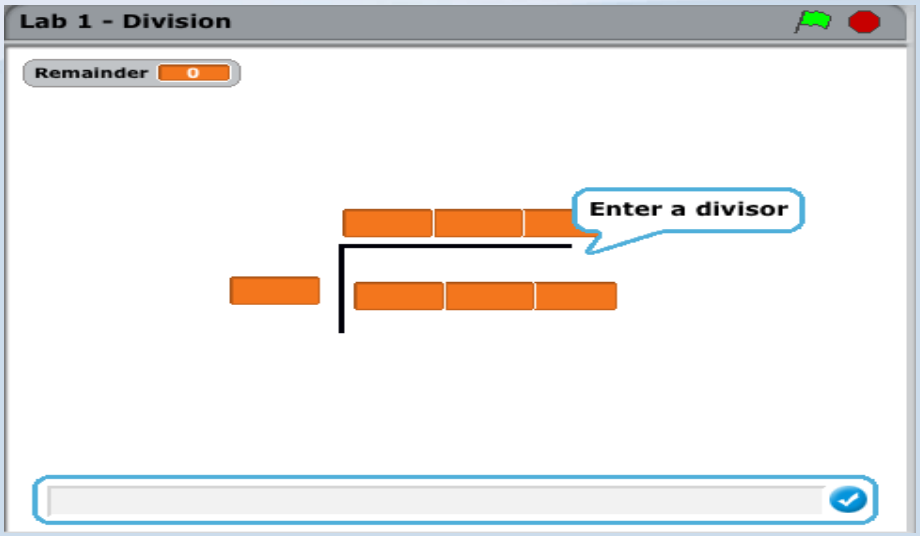
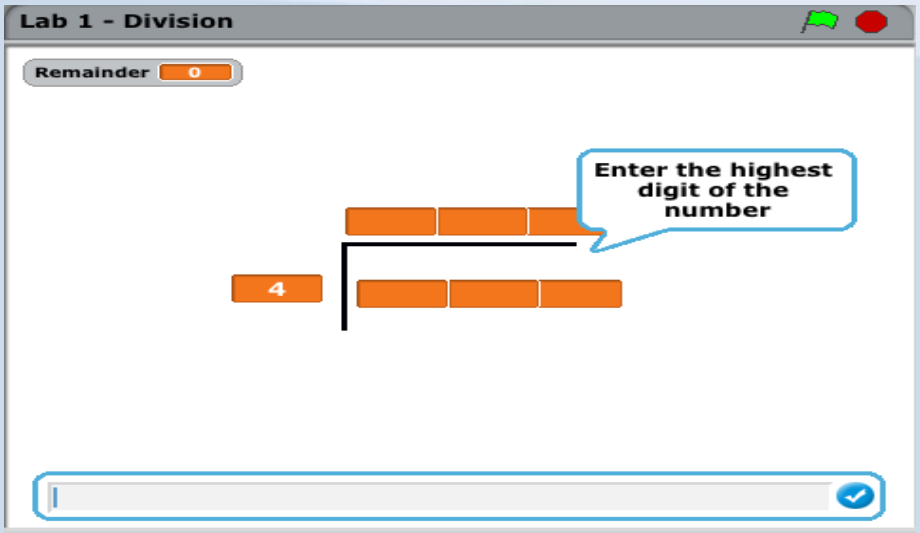
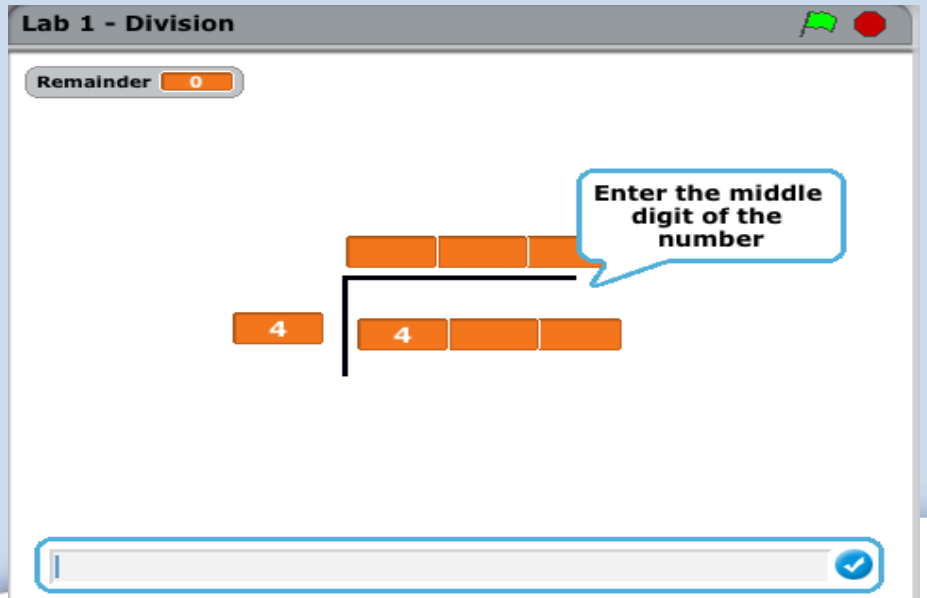
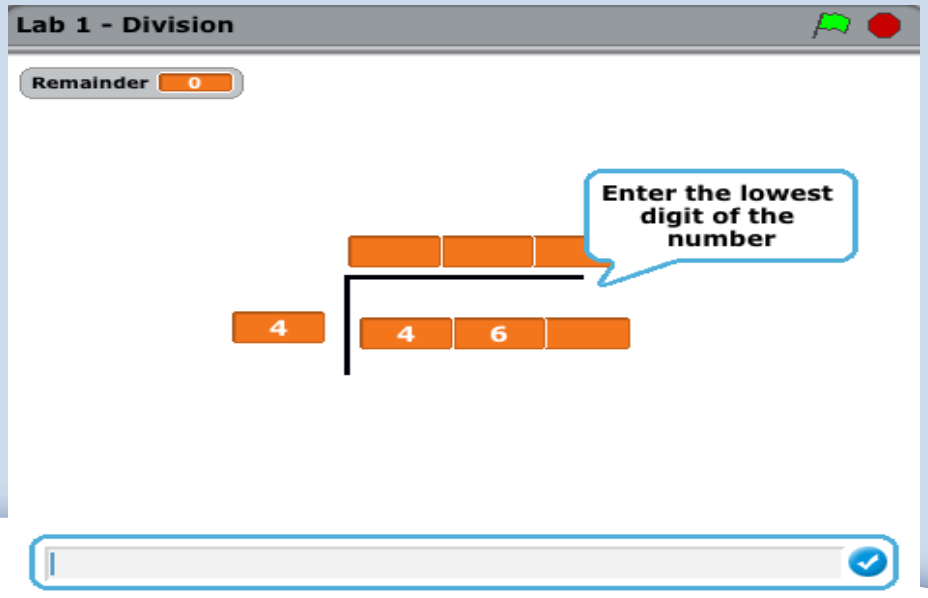
Last Monday's Lab









Long Division Steps

- **Step 1:** Take the first digit of the dividend from the left. Check if this digit is greater than or equal to the divisor.
- **Step 2:** Then divide it by the divisor and write the answer on top as the quotient.
- **Step 3:** Subtract the result from the digit and write the difference below.
- **Step 4:** Bring down the next digit of the dividend (if present).
- **Step 5:** Repeat the same process.

Long Division Steps

- Dividing number=divisor
- 3 input components of dividend (number1,number2,number3)
- 3 output components of quotient – quotient1, quotient2, quotient3

- For the highest number (number 1)

```
If number1<divisor
   set quotient1 to 0
   set overflow to number1 * 10
Else
   set quotient1 to number1/divisor,
   set overflow to (number1 mod divisor) * 10
End if
number2=number2+overflow
```

Long Division Steps

- Dividing number=divisor
- 3 input components of dividend (number1,number2,number3)
- 3 output components of quotient – quotient1, quotient2, quotient3

- For the middle number (number 2)

```
If number2<divisor
   set quotient2 to 0
   set overflow to number2 * 10
Else
   set quotient2 to number2/divisor,
   set overflow to (number2 mod divisor) * 10
End if
number3=number3+overflow
```

Long Division Steps

- Dividing number=divisor
- 3 input components of dividend (number1,number2,number3)
- 3 output components of quotient – quotient1, quotient2, quotient3

- For the smallest number (number 3)

```
If number3<divisor
   set quotient3 to 0
   set overflow to number3
Else
   set quotient3 to number3/divisor,
   set overflow to (number3 mod divisor)
End if
Remainder=overflow
```

```
Enter divisor
Enter highest digit                    *of number to be divided*
Enter middle digit                     *of number to be divided*
Enter smallest digit                   *of number to be divided*
If highest digit < divisor
        highest quotient = 0           *the highest number of the answer*
        remainder = highest digit * 10
Else
        highest quotient = highest digit /divisor
        remainder = (highest digit mod divisor)*10
middle digit = (middle digit + remainder)
```

```
If middle digit < divisor
        middle quotient = 0            *the middle number of the answer*
        remainder = middle digit * 10
Else
        middle quotient = middle digit /divisor
        remainder = (middle digit mod divisor)*10
smallest digit = (smallest digit + remainder)
```

```
If smallest digit < divisor
        smallest quotient = 0          *the smallest number of the answer*
        remainder = smallest digit
Else
        smallest quotient = smallest digit /divisor
        remainder = smallest digit mod divisor
```

Long division test

- Let's test with the 466/4:
  - Divisor =
  - Highest digit =
  - Middle digit =
  - Lowest digit =
- What is:
  - Highest quotient =
  - Middle quotient =
  - Lowest quotient =

Last Monday's Lab

- Any problems with this algorithm?
- Can it be made more efficient?
- How is Scratch as a programming language?

Last Monday's Lab

- How did you get rid of the decimal problem?
- Or.. The value of having data types …

Remainder = number1 mod Divisor
Quot1 = (Number1-Remainder)/Divisor
Remainder = Remainder * 10

Reminder: What is Recursion?

When one function calls ITSELF directly or indirectly.

When should I use Recursion?

- If the algorithm has a base case
- If a problem is iterative
- If the problem gets progressively smaller

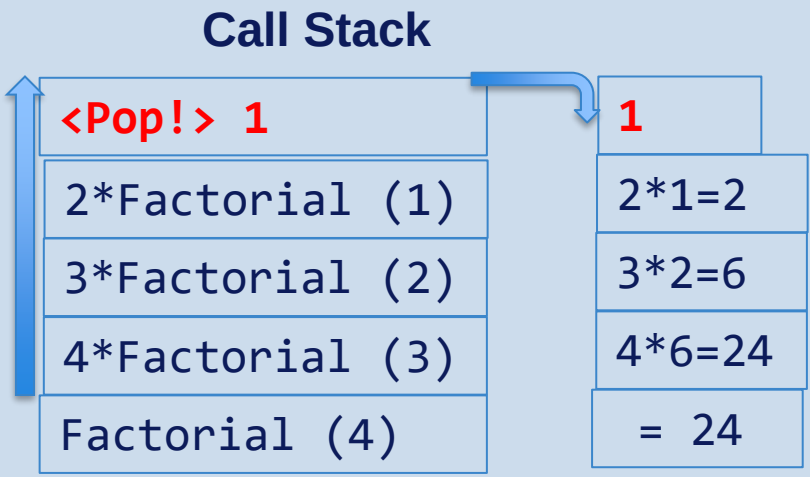
Reminder: Recursive Factorial

- What is the algorithm?

Reminder: Recursive Factorial

- Remember this …. 4!

```
Factorial (n)
    if n=1 or n=0
        return 1
    else
        return n*Factorial(n-1)
    end if
End
```



Recursive Power

- Write the recursive algorithm for Power.
- Illustrate the call stack for $\mathbf{2^5}$

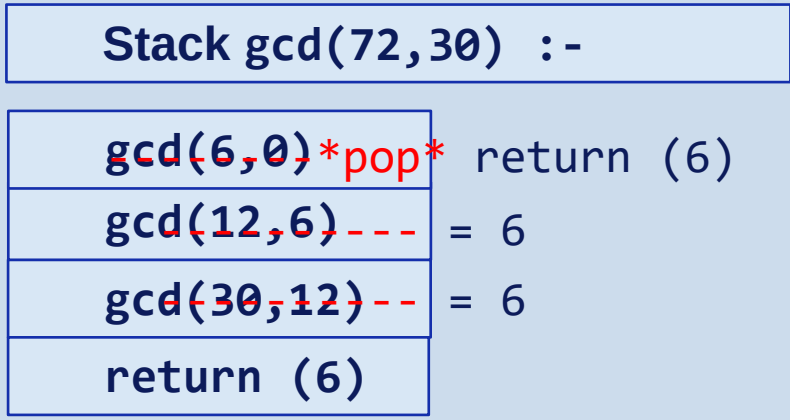
Recursive X ^ Y

```
Power(x,y)
   if (y=0) then
      return 1
   else
      return x*Power(x,y-1)
```

Stack power(2,5) :-	
~~power(2,0)~~	*pop* return 1
2*~~power(2,0)~~	=2*(1)=2
2*~~power(2,1)~~	=2*(2)=4
2*~~power(2,2)~~	=2*(4)=8
2*~~power(2,3)~~	=2*(8)=16
2*~~power(2,4)~~	=2*(16)=32
return (32)	

Recursive Euclid's Algorithm

```
gcd(a, b)
 if (b = 0) then
    return a
else
  return gcd(b,a mod b)
```



Illustrate this recursive algorithm using a call stack – **gcd (72, 30)**

Write an *iterative* GCD algorithm

An iterative solution ….

```
gcd(a, b)
if b=0 then
        return a
else
        while b!=0
                rem = a mod b
                If rem=0
                        return b
                else
                        a=b
                        b=rem
```

Does this work?
Test it with (72, 30)

Fibonacci

- The Fibonacci Sequence is the series of numbers:
  - 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...
- The next number is found by adding up the two numbers before it.

Fibonacci

n =	0	1	2	3	4	5	6
x_n =	0	1	1	2	3	5	8

Example: term 6 would be calculated like this:

$$x_6 = x_{6-1} + x_{6-2} = x_5 + x_4 = 5 + 3 = 8$$

- Can you calculate the following?
  - Term 7
  - Term 9

Fibonacci

- What is the base case?
- What is the recursive call?

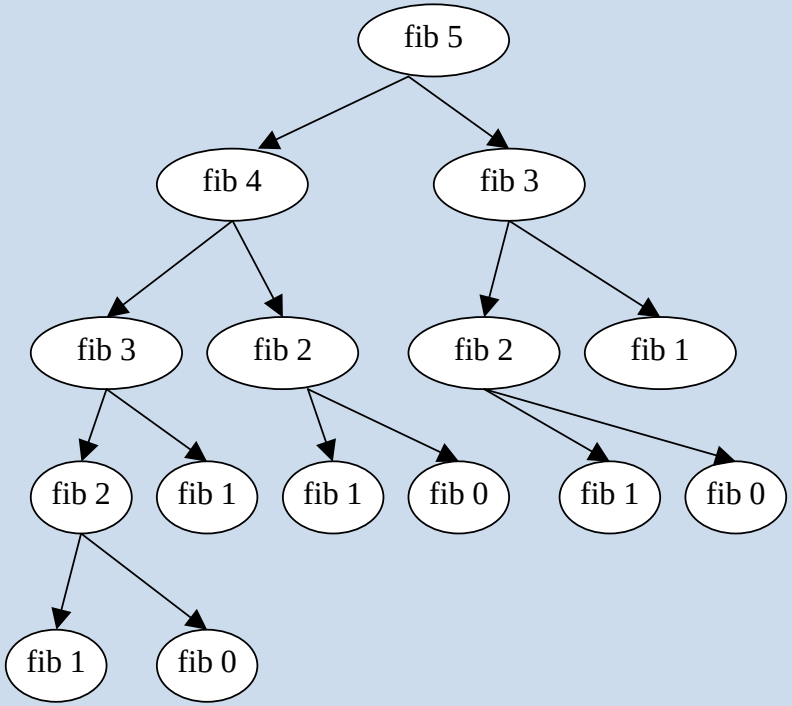
Recursive Fibonacci

- Let's try this with n=5

```
fibonacci(n)
if (n=0 or n=1)
    return n
else
    return fibonacci(n-1) + fibonacci(n-2)
```

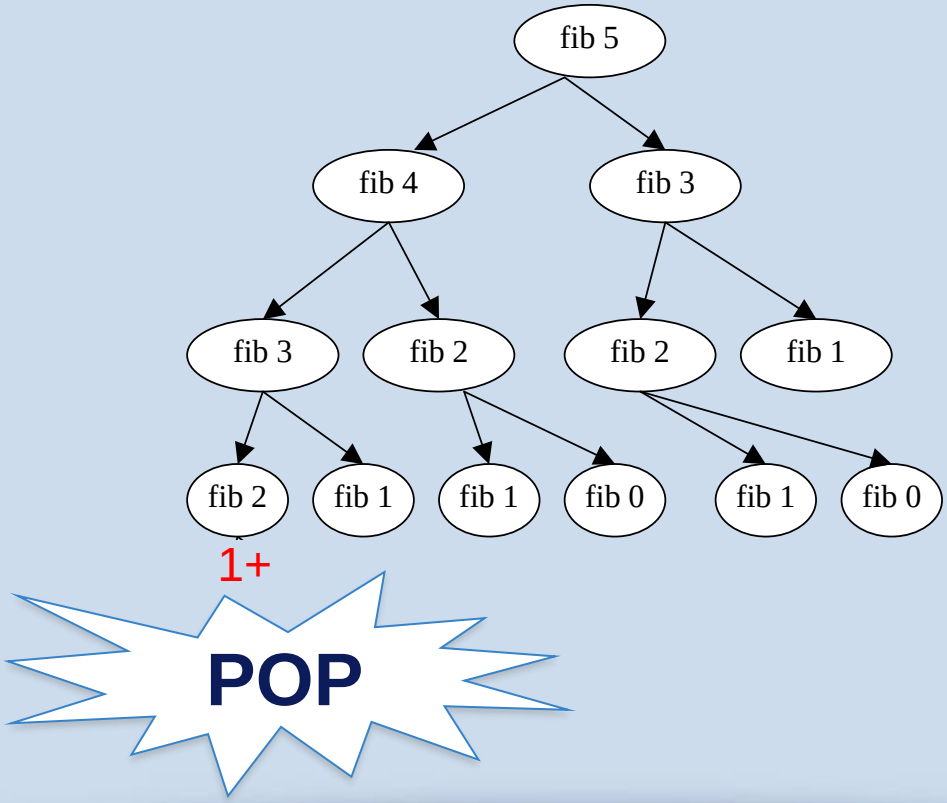
Fibonacci

- Let's try this with n=5



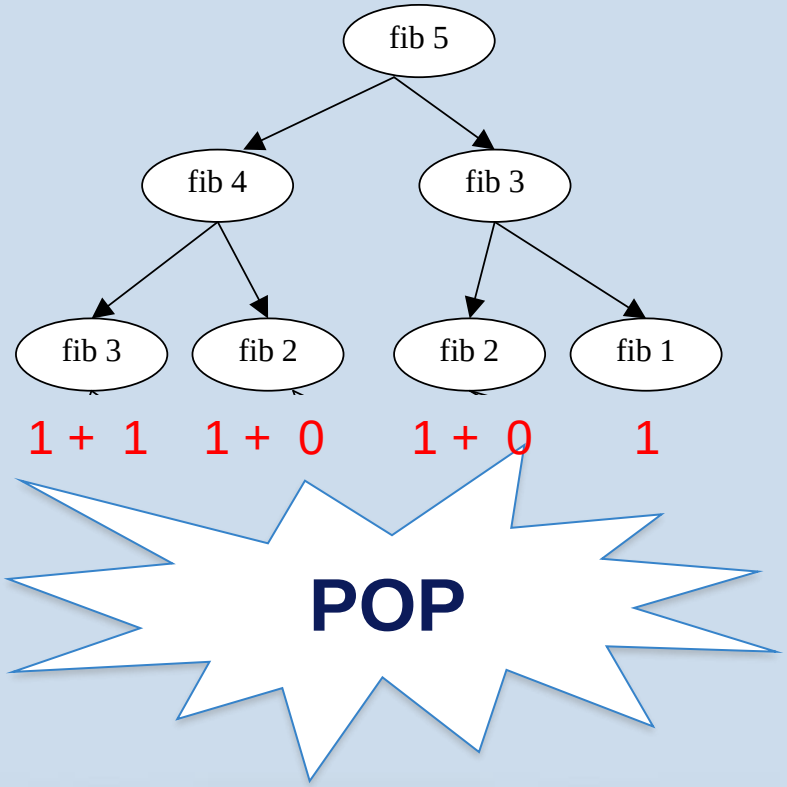
Fibonacci

- Let's try this with n=5



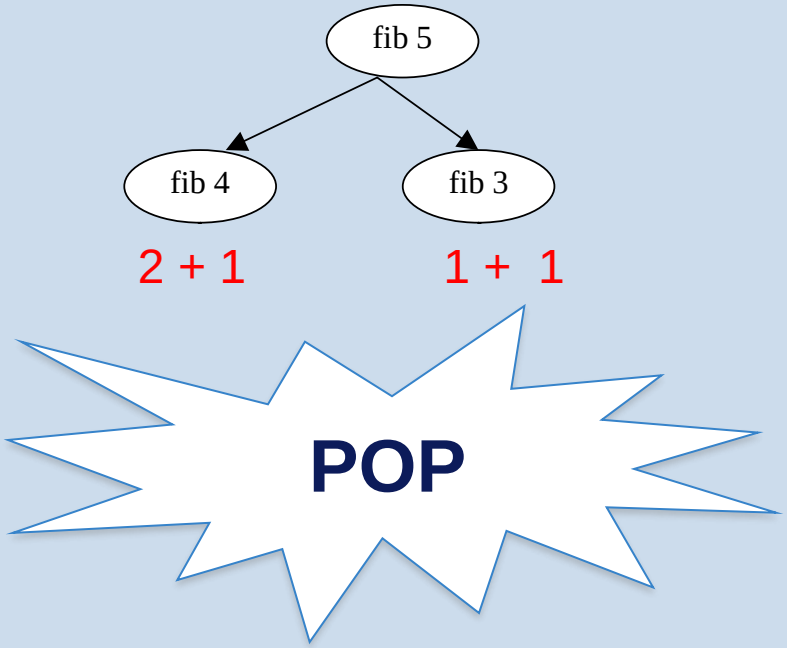
Fibonacci

- Let's try this with n=5



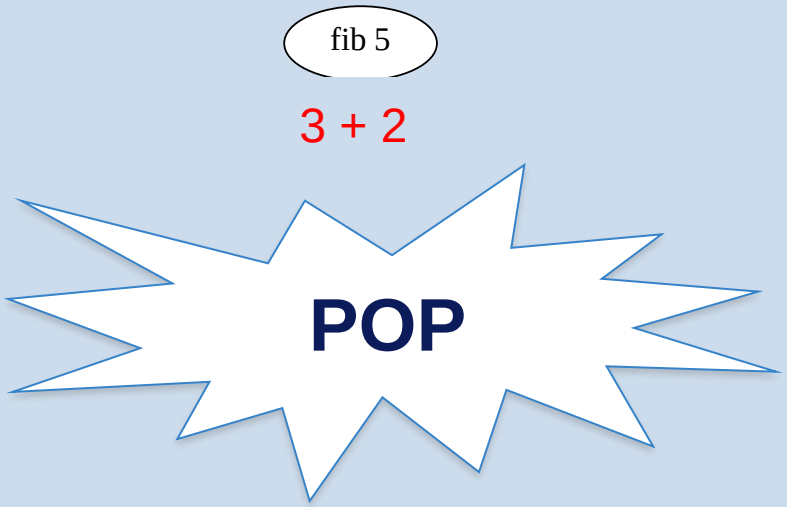
Fibonacci

- Let’s try this with n=5



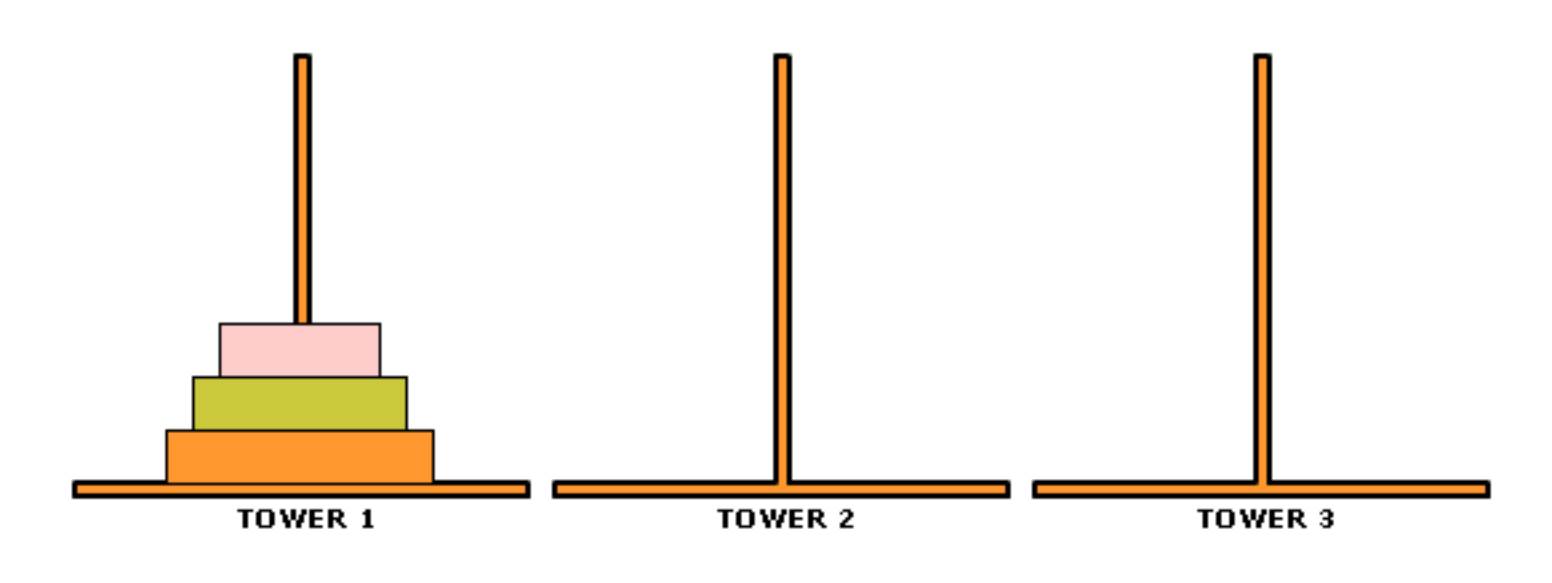
Fibonacci

- Let’s try this with n=5



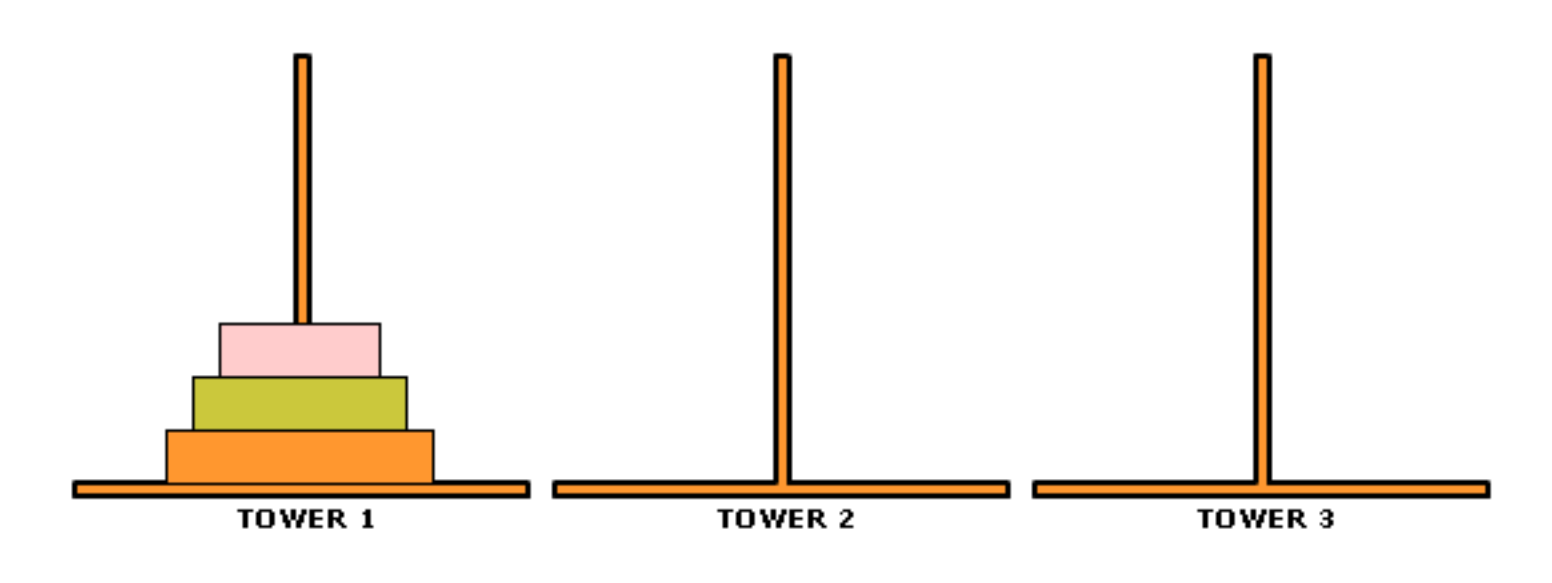
A more complicated use of recursion – *The Towers of Hanoi*

The Towers of Hanoi



- **Rules**
  - Move all disks to Tower 3
  - Only one disk can be moved at a time
  - A disk can never be put on a smaller disk

The Towers of Hanoi

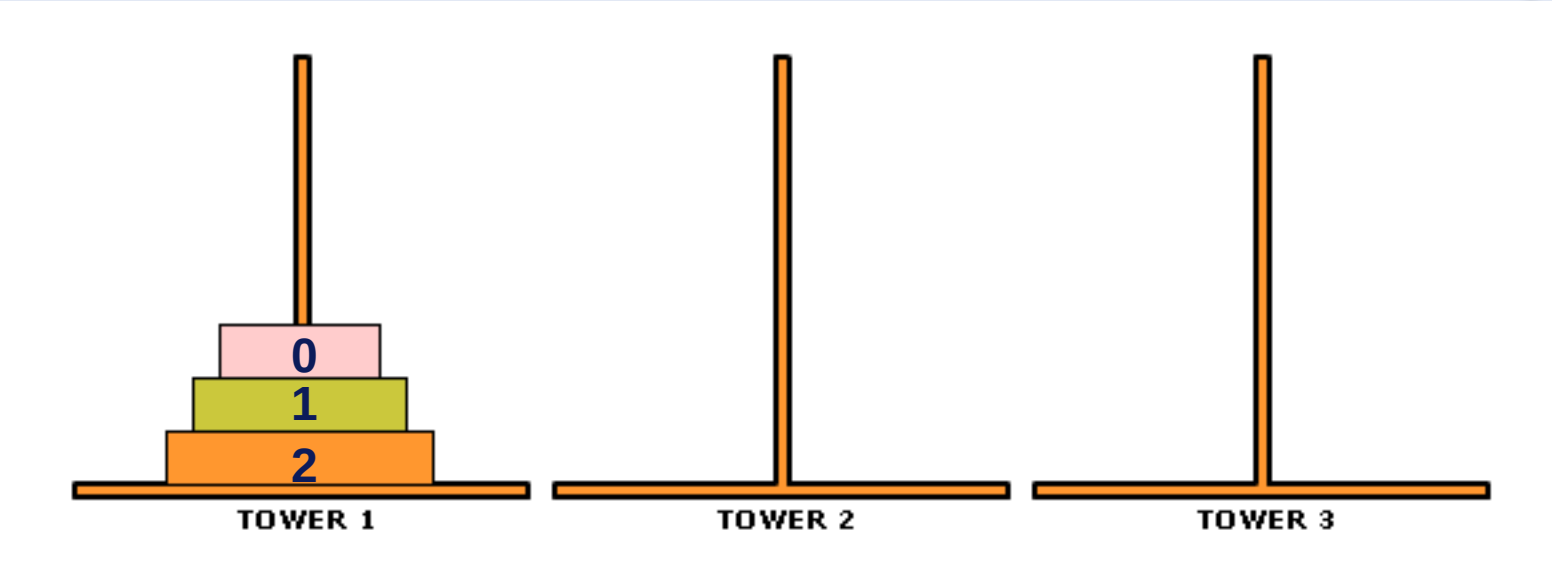


- What is the problem?
  - Move the largest disk, disk2, to Tower 3
  - Move the middle disk, disk1, to Tower 3
  - Move smallest disk, disk0, to Tower 3

The Towers of Hanoi

- Why is this suitable for **recursion**?
  - Because there is a base case
  - The problem is iteratively getting smaller

The Towers of Hanoi



Source

Spare

Dest

Disks = 2 (0,1,2)

The Towers of Hanoi: A recursive algorithm

moveTower (disks, source, dest, spare)

If disk = 0 ***if smallest disk*

Move disk from source to dest ***moves smallest disk*

else

moveTower (disk-1, source, spare, dest)

move disk from source to dest ***moves other 2 disks*

moveTower (disk-1, spare, dest, source)

Thank You !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