Theory of Automata

Closure properties and Pumping Lemma for Regular Languages

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CLOSURE PROPERTIES OF REGULAR LANGUAGES

Closure properties for Regular Languages (RL)

This is different from Kleene closure

- ***Closure property:***
  - If a set of regular languages are combined using an operator, then the resulting language is also regular
- Regular languages are ***closed*** under:
  - Union, intersection, complement, difference
  - Reversal
  - Kleene closure
  - Concatenation
  - Homomorphism
  - Inverse homomorphism

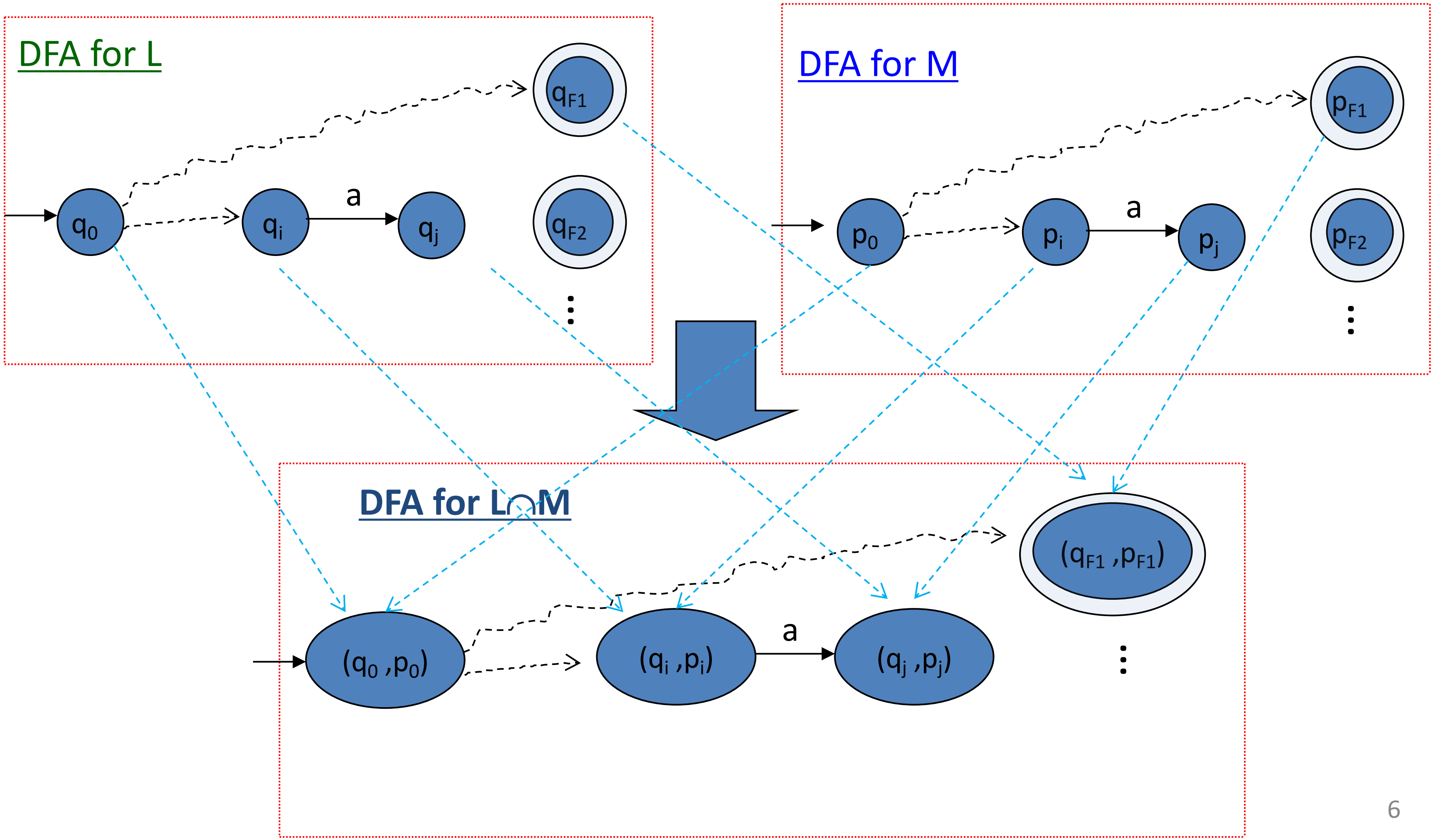
RLs are closed under intersection

- A quick, indirect way to prove:
  - By DeMorgan's law:
  - $L \cap M = \overline{(\overline{L} \cup \overline{M})}$
  - Since we know RLs are closed under union and complementation, they are also closed under intersection
- A more direct way would be construct a finite automaton for $L \cap M$

DFA construction for $L \cap M$

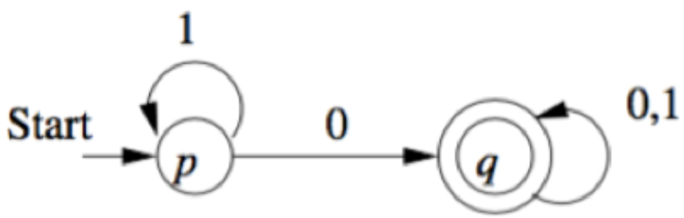
- A_L = DFA for L = $\{Q_L, \Sigma, q_L, F_L, \delta_L\}$
- A_M = DFA for M = $\{Q_M, \Sigma, q_M, F_M, \delta_M\}$
- Build $A_{L \cap M} = \{Q_L x\ Q_M, \Sigma, (q_L, q_M), F_L x\ F_M, \delta\}$ such that:
  - $\delta((p,q),a) = (\delta_L(p,a), \delta_M(q,a))$, where p in Q_L, and q in Q_M
- This construction ensures that a string w will be accepted if and only if w reaches an accepting state in <u>both</u> input DFAs.

DFA construction for L ∩ M

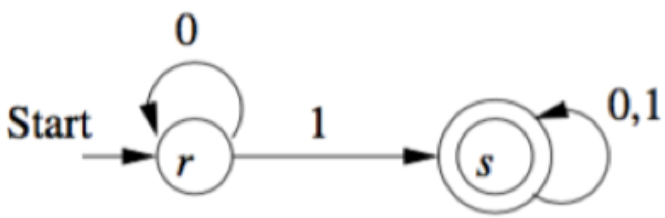


Example

- Design DFA for
  - The set of all the strings that contain at least one 1 and at least one 0.



(a)

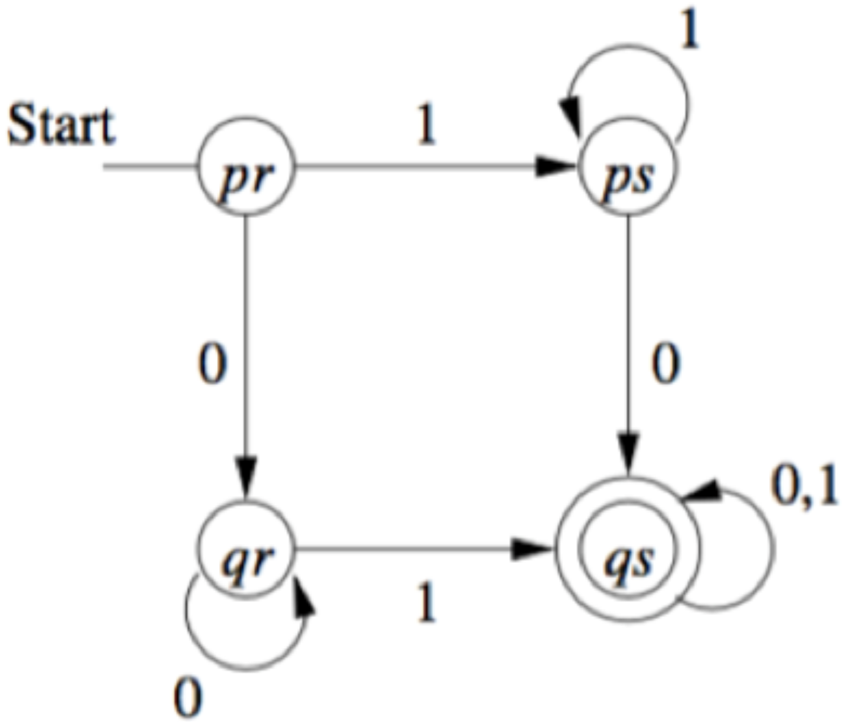


(b)

- Show L $\cap$ M using a DFA.

Solution

Solution

- 

(c)

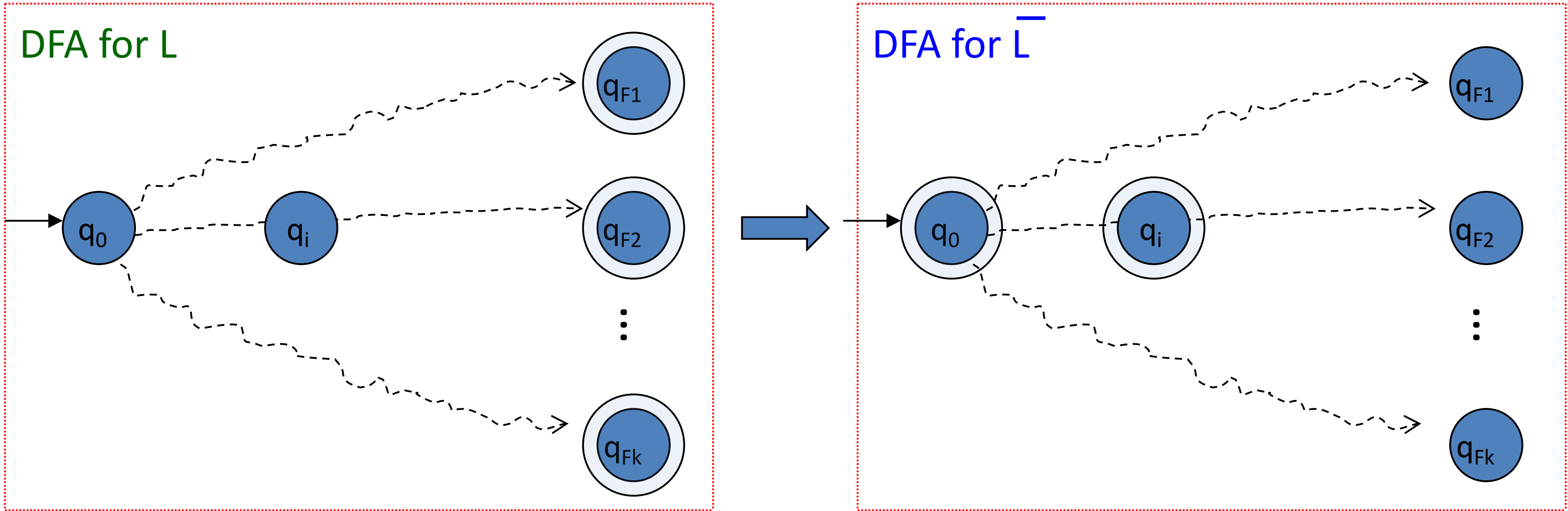
RLs are closed under union

- IF L and M are two RLs THEN:
  - they both have two corresponding regular expressions, R and S respectively
  - (L U M) can be represented using the regular expression R+S
  - Therefore, (L U M) is also regular

RLs are closed under complementation

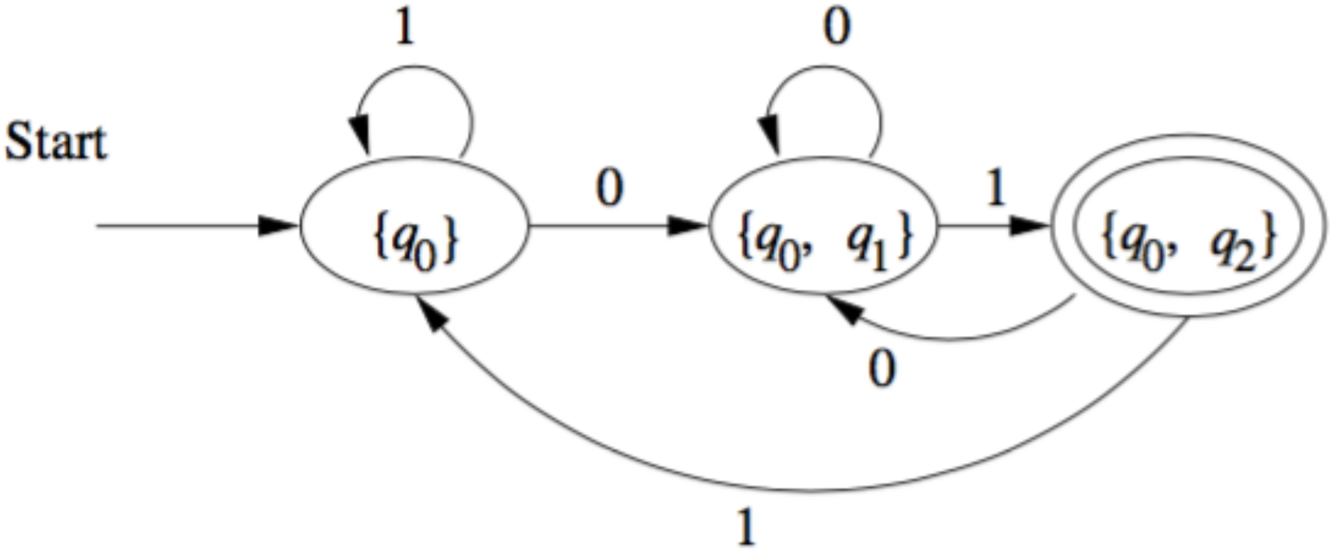
- If L is an RL over Σ, then $\overline{L}=\Sigma^*-L$
- To show $\overline{L}$ is also regular, make the following construction

Convert every final state into non-final, and every non-final state into a final state



Assumes q0 is a non-final state. If not, do the opposite.

Example



Example

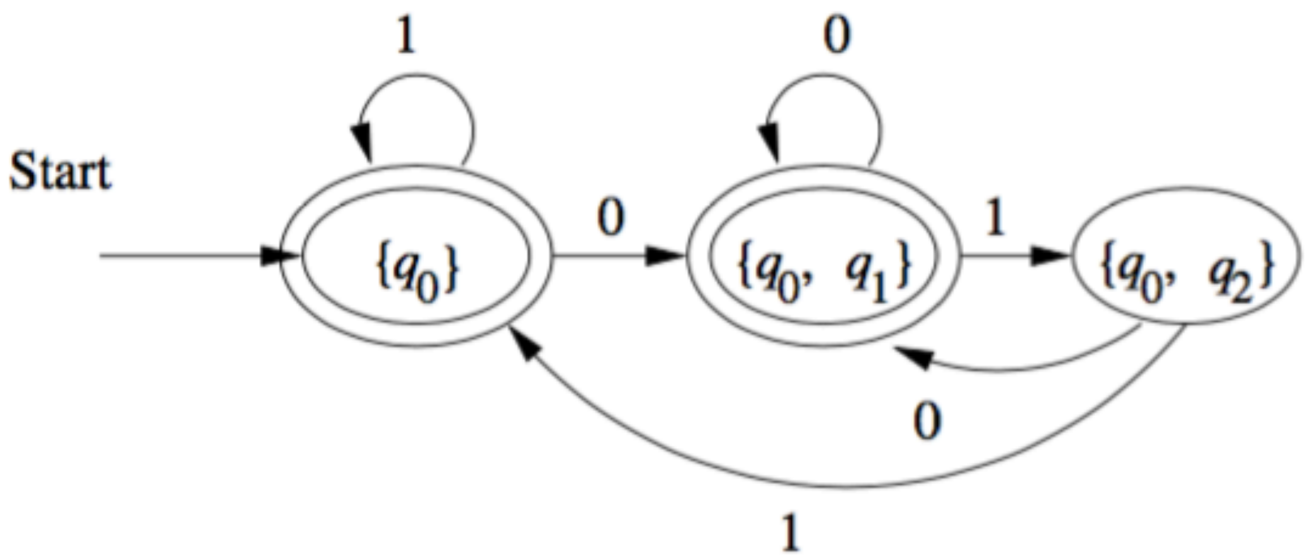


Figure 4.2: DFA accepting the complement of the language $(\mathbf{0}+\mathbf{1})^*\mathbf{01}$

RLs are closed under set difference

- We observe:
  - $L - M = L \cap \overline{M}$

Closed under intersection

Closed under complementation

- Therefore, L - M is also regular

RLs are closed under reversal

Reversal of a string w is denoted by w^R

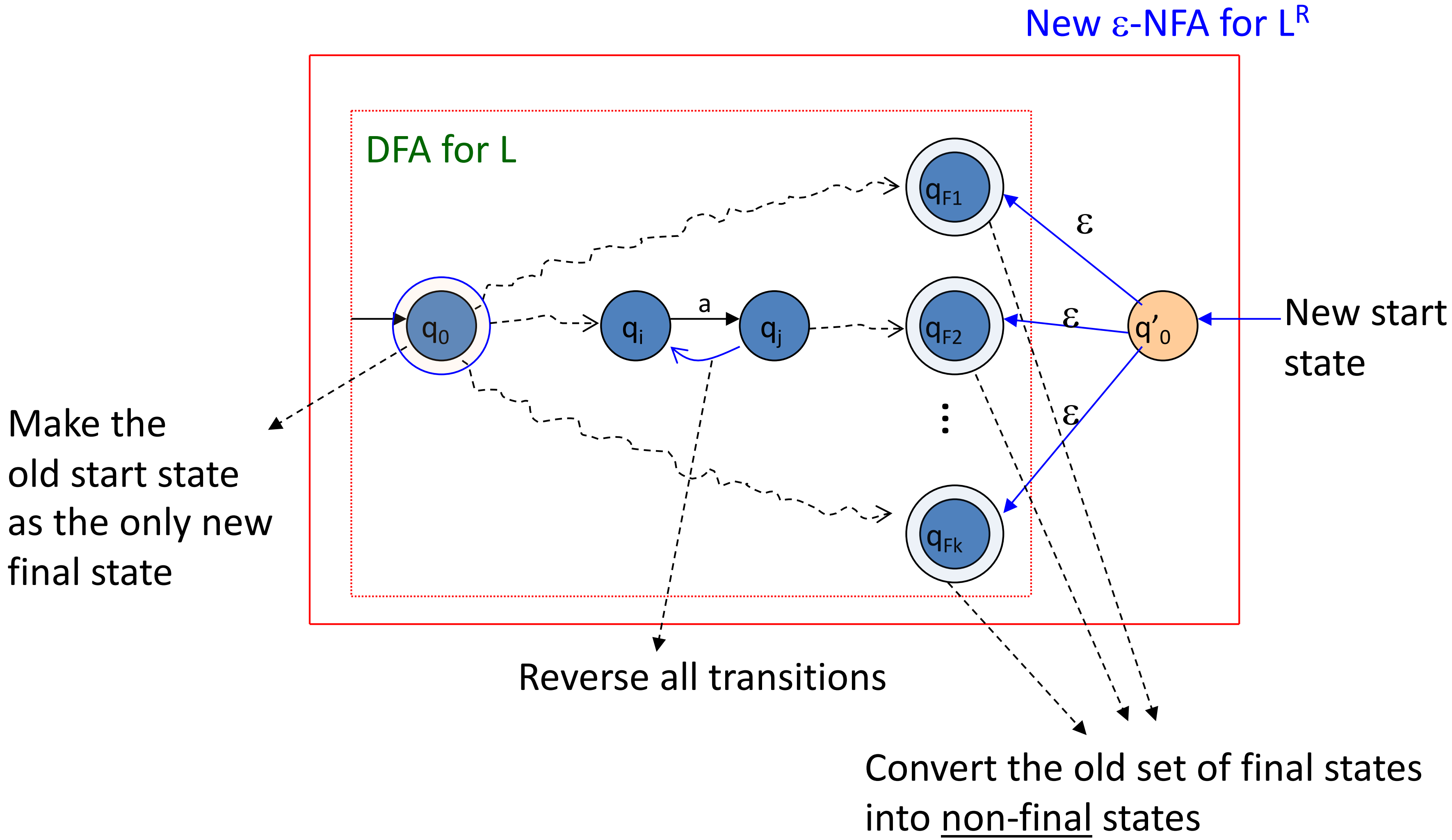
- E.g., $w=00111$, $w^R=11100$

Reversal of a language:

- L^R = The language generated by reversing all strings in L

Theorem: If L is regular then L^R is also regular

ε -NFA Construction for L^R



Class Activity

1.4 Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

a. $\{w|\ w$ has at least three a's and at least two b's$\}$

[A]**b.** $\{w|\ w$ has exactly two a's and at least two b's$\}$

c. $\{w|\ w$ has an even number of a's and one or two b's$\}$

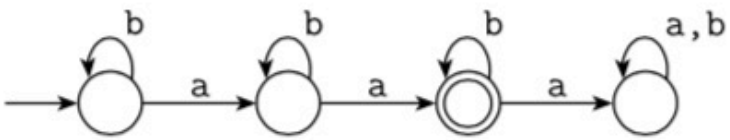
[A]**d.** $\{w|\ w$ has an even number of a's and each a is followed by at least one b$\}$

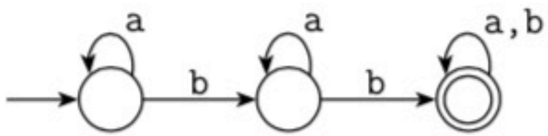
e. $\{w|\ w$ starts with an a and has at most one b$\}$

f. $\{w|\ w$ has an odd number of a's and ends with a b$\}$

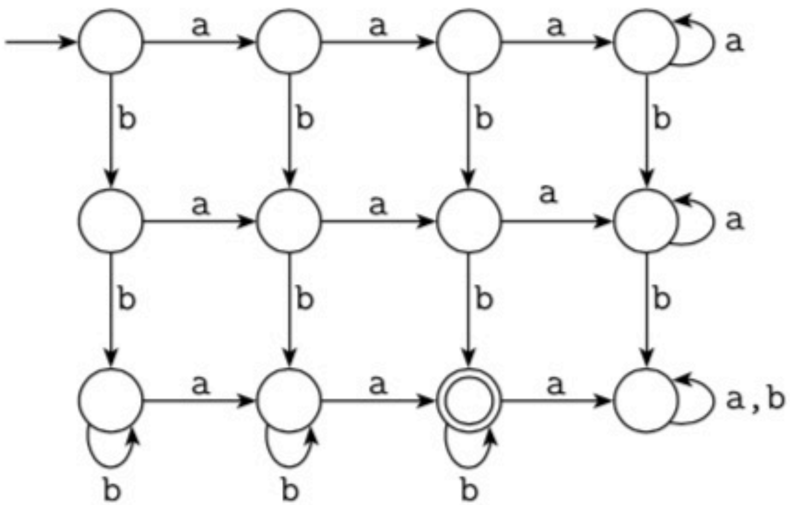
g. $\{w|\ w$ has even length and an odd number of a's$\}$

Solution part b





Combining them using the intersection construction gives the following DFA.



Class Activity

1.5 Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{\texttt{a}, \texttt{b}\}$.

[A]**a.** $\{w \mid w \text{ does not contain the substring } \texttt{ab}\}$

[A]**b.** $\{w \mid w \text{ does not contain the substring } \texttt{baba}\}$

c. $\{w \mid w \text{ contains neither the substrings } \texttt{ab} \text{ nor } \texttt{ba}\}$

d. $\{w \mid w \text{ is any string not in } \texttt{a}^*\texttt{b}^*\}$

References

- Book Chapter 4
- Lectures from Washington State University
  - http://www.eecs.wsu.edu/~ananth/CptS317/Lectures/
- Lectures from Stanford University
  - http://infolab.stanford.edu/~ullman/ialc/spr10/spr10.html#LECTURE%20NOTES