August 30, 2020 (1st session)

The method of Undetermined coefficients :-

Case (i) :- when RHS is a polynomial term.

Example :- Solve $y'' - y' - 2y = 4x^2$ ... (1)

For y_c

$$(D^2 - D - 2)y = 0$$

$\Rightarrow$ The auxiliary equation.

$\Rightarrow$

$$D^2 - D - 2 = 0$$

$$(D-2)(D+1) = 0$$

$$D = 2, -1$$

$\Rightarrow$

$$\boxed{y_c = c_1 e^{2x} + c_2 e^{-x}}$$

For y_p. $\quad y'' - y' - 2y = 4x^2$

Let $y_p = A_2 x^2 + A_1 x + A_0$ ——(2)

$\Rightarrow y_p' = 2A_2 x + A_1$

and $y_p'' = 2A_2$

Using above in Eq. (1)

$\Rightarrow (2A_2) - (2A_2 x + A_1) - 2(A_2 x^2 + A_1 x + A_0) = 4x^2$

or $2A_2 - 2A_2 x - A_1 - 2A_2 x^2 - 2A_1 x - 2A_0 = 4x^2$

or

$(-2A_2)x^2 + (-2A_2 - 2A_1)x + (2A_2 - A_1 - 2A_0) = 4x^2$

On comparing the coefficients of x^2, x, 1.

$\Rightarrow$

x^2: $\quad -2A_2 = 4 \quad \Rightarrow \boxed{A_2 = -2}$

x: $\quad -2A_2 - 2A_1 = 0 \quad \Rightarrow 4 - 2A_1 = 0 \Rightarrow \boxed{A_1 = 2}$

1: $\quad 2A_2 - A_1 - 2A_0 = 0$

$\Rightarrow 2(-2) - 2 - 2A_0 = 0$

$-4 - 2 - 2A_0 = 0$

$\boxed{A_0 = -3}$

Using values of A_2, A_1 and A_0 in Eq. (2)

$$\Rightarrow \boxed{y_p = -2x^2 + 2x - 3}$$

Thus, $y = y_c + y_p$

$$\Rightarrow \boxed{y = c_1 e^{2x} + c_2 e^{-x} - 2x^2 + 2x - 3}$$

Example: Solve $y'' + 3y' + 2y = 4x^2$ — (1)

For $y_c \Rightarrow$ $(D^2 + 3D + 2)y = 0$

The auxiliary equation is

$$D^2 + 3D + 2 = 0$$

$$(D+1)(D+2) = 0$$

$$D = -1, -2$$

$$\Rightarrow \boxed{y_c = c_1 e^{-x} + c_2 e^{-2x}}$$

For y_p.

Suppose $y_p = A_2x^2 + A_1x + A_0$ ——(2)

$\Rightarrow y_p' = 2A_2x + A_1 \quad y_p'' = 2A_2$

using above in Eq. (1)

$\Rightarrow (2A_2) + 3(2A_2x + A_1) + 2(A_2x^2 + A_1x + A_0) = 4x^2$

or $2A_2 + 6A_2x + 3A_1 + 2A_2x^2 + 2A_1x + 2A_0 = 4x^2$

or $(2A_2)x^2 + (6A_2 + 2A_1)x + (2A_2 + 3A_1 + 2A_0) = 4x^2$

On comparing

x^2: $2A_2 = 4 \Rightarrow \boxed{A_2 = 2}$ ✓

x: $6A_2 + 2A_1 = 0 \Rightarrow \boxed{A_1 = -6}$ ✓

1: $2A_2 + 3A_1 + 2A_0 = 0$

$2(2) + 3(-6) + 2A_0 = 0 \quad \boxed{A_0 = 7}$ ✓

using values of A_2, A_1 and A_0 in Eq. (2)

$\Rightarrow \boxed{y_p = 2x^2 - 6x + 7}$

$\therefore y = y_c + y_p$

$\Rightarrow \boxed{y = C_1e^x + C_2e^{-2x} + 2x^2 - 6x + 7}$