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Definition :-

When an equation involves one or more derivatives w.r.t a particular variable. Then that variable is called an independent variable. A variable is called dependent variable if a derivative of that variable occurs. The equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

has dependent variable V and two independent variables x and y.

Definition :- An equation involving one dependent variable and its derivative w.r.t one or more independent variables, is called a "Differential equation". e.g;

i) $\frac{d^2}{dx^2}y + 2\frac{d}{dx}y + y = 0$

ii) $\frac{\partial^2}{\partial x^2}u + \frac{\partial^2}{\partial y^2}u + \frac{\partial^2}{\partial z^2}u = 0$

There are two types of Differential equations:

i) Ordinary Differential equations

ii) Partial Differential equations.

1) Ordinary Differential equation

A differential equation which involves the derivatives of the dependent variable w.r.t a single independent variable is known as "Ordinary Differential equation". eg;

i) $\frac{d}{dx}y + 2y = \cos x$

ii) $\frac{d^3}{dx^3}y + xy \cdot \left(\frac{d}{dx}y\right)^2 = 0$

iii) $\left(\frac{d^2}{dx^2}y\right)^4 + x \cdot \frac{d}{dx}y - 4xy = 0$

are ordinary differential equations

ii) Partial Differential equation.

A differential equation which contains two or more independent variables and partial derivative w.r.t. them is called a Partial Differential equation". e.g;

i) $x \cdot \frac{\partial}{\partial x} z + y \cdot \frac{\partial}{\partial y} z = nz$

ii) $\frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V + \frac{\partial^2}{\partial z^2} V = 0.$

iii) $\frac{\partial^3}{\partial t^3} V = K \cdot \left(\frac{\partial^2}{\partial x^2} V\right)^2.$

are the partial differential equations.

Order of a Differential equation

The order of a differential equation is the order of the highest ordered derivative that occurs in the equation. e.g;

$$\frac{dy}{dx} + 2x = 0 \quad \text{has order 1.}$$

and

$$\frac{d^2}{dx^2}y + 2\left(\frac{d}{dx}y\right)^3 + y = 0$$

has order 2.

Degree of a differential equation

The degree of a differential equation is defined to be the exponent of the highest order derivative

e.g;

i) $\left(\frac{d}{dx}y\right)^1 = \cos x$

ii) $\left(\frac{d^2}{dx^2}y\right)^1 + 4y = 0$

iii) $\left(\frac{d^2}{dx^2}y\right)^1 + 7\left(\frac{d}{dx}y\right)^3 - 8y = 0.$

iv) $\left(\frac{d^3}{dx^3}y\right)^2 - 2\left(\frac{d}{dx}y\right)^4 + y = 0.$

has degrees 1, 1, 1, 2.

(7)

$$(y''')^3 + 2y' + 3y = 0 \quad \text{— (1)}$$

$$(y''')^2 + 2y'' + 4(y')^4 = 0 \quad \text{— (2)}$$

$$(y'')^2 + 2(y')^3 + 3y = 0 \quad \text{— (3)}$$

$$(y')^2 + 2y = 0 \quad \text{— (4)}$$

$$(y^{iv})^3 + 2y''' + 2x(y'')^4 2y' + 3y = 0 \quad \text{— (5)}$$

$$(y'')^3 + 2(y')^2 + 4y = \cos x \quad \text{— (6)}$$

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Solution of a differential equation:-

A solution of a differential equation is a relation between the dependent and independent variables not involving the derivatives such that this relation and the derivatives obtained from it satisfies the given differential equation. e.g;

$y = ce^{2x}$ is a solution of

$$y' - 2y = 0 = \frac{d}{dx}y - 2y \quad \text{---(1)}$$

Given is $y = ce^{2x}$

$$\Rightarrow \frac{d}{dx}y = 2ce^{2x}$$

putting the values of y and y' in equation (1)

$$\Rightarrow \frac{d}{dx}y - 2y = 0$$

$$\Rightarrow (2ce^{2x}) - 2(ce^{2x}) = 0$$

$$0 = 0$$

General Solution

A solution of a differential equation which contains the number of arbitrary constants equal to the order of the differential equation is called a "General Solution".

$$\frac{d^2}{dx^2}y + y = 0 \quad \text{---} (2)$$

the general solution of eq. (2) is

$$y = A\cos x + B\sin x$$

Particular Solution

A solution obtained from the general solution by giving particular values to the constants is called a "Particular Solution".

e.g; $\frac{d^2}{dx^2}y + y = 0$

$\rightarrow$ $y = 2\cos x + 3\sin x$.

Formation of a differential equation :-

Example :- Form the diff. eq. given is: $y = cx^2$ — (1)

$\Rightarrow y' = 2cx$ — (2)

From eq (1) $\Rightarrow c = \frac{y}{x^2}$

using in (2)

$\Rightarrow y' = 2\left(\frac{y}{x^2}\right)x$

$$\boxed{y' = \frac{2y}{x}}$$