Exercise to Solve

Bacteria grow in a nutrient solution at a rate proportional to the amount present. Initially, there are 250 ands of the bacteria in the solution which grows to 1 strands after 7 hours. Find

i) an expression for the approximate number of strands in the culture at any time t and

ii) the time needed for the bacteria to grow to 1600 strands.

Ans: i) $N = 250e^{0.166t}$ ii) 11.2 hrs.

A yeast grows at a rate proportional to its present size. If the original amount doubles in two hours, in how many hours will it tripple?

∴ 3.17 hrs.

The population of a certain state is known to grow at a rate proportional to the numbers of people presently living in the state. If after 10 yrs, the population has trebled and if after 20 yrs, the population is 50,000, find the number of people initially living in the state?

Ans: $N = 16,620e^{0.11t}$, $N_0 = 16,620$.

4) A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there are 100 milligrams of the material present and if after two years it is observed that 5% of the original mass has decayed. Find

i) an expression for the mass at any time t and

ii) the time necessary for 10% of the original mass to have decayed.

Ans: i) $N = 100e^{-0.026t}$, ii) 4.05 yr.

5) A depositor places $ 10,000 in a certificate of deposit which pays 6% interest per annum, compounded continuously. How much will be in the account at the end of seven years, assuming no additional deposits or withdrawls?

Ans: $ 15,219.62

6) Determine the interest rate required to double an investment in eight years under continuous compounding.

Ans: 8.67 %

) How long will it take a bank deposit to triple in value if interest is compounded continuously at a constt. rate of $5\frac{1}{4}$ % per annum?

Ans: 20.93 yr.

) A depositor currently has ~~$6000~~ $6000 and plans to invest it in an account that accrues (~~accrues~~) interest continuously. What interest rate must the bank pay if the depositor needs to have ~~$10,000 $13,506?~~ $ ~~10,000~~ $10,000 in 4 years?

Ans: 12.78 %

) An RC circuit has an emf of 5 volts, a resistance of 10 ohms, a capacitance of 10^{-2} farad, and initially a charge of 5 colombs on the capacitor. Find

i) the transient current and

ii) steady state current.

Ans: i) $-\frac{99}{2}e^{-10t}$ ii) 0 amp.

(10) A RC circuit has an emf of $10\sin t$ volts, a resistance of 100 ohms, a capacitance of 0.005 farad and no initial charge on the capacitor. Find i) the charge on the capacitor at any time t ii) the steady state current.

Ans: i) $q = \frac{1}{50}(2\sin t - \cos t + e^{-2t})$

ii) $I = \frac{1}{50}(2\cos t + \sin t)$.