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BS(CS) 4B

Assignment-03

QUESTION-01

$(D^2+3D+2)y = e^{2x}\sin x$

For $y_c \Rightarrow$

$(D^2+3D+2)y = 0$

$D^2+3D+2=0$

$D^2+D+2D+2=0$

$D(D+1)+2(D+1)=0$

$(D+2)(D+1)=0$

$D=1 \quad , \quad -2$

$y_c = c_1e^{-x}+c_2e^{-2x}$

For $y_p \Rightarrow$

$e^{2x}\dfrac{1}{D^2+3D+2}\sin x$

$= e^{2x}\cdot\dfrac{1}{(D+a)^2+3(D+a)+2}\sin x$

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$$= e^{2x} \cdot \frac{1}{D^2 + 2Da + a^2 + 3D + 3a + 2} \sin x$$

$$= e^{2x} \cdot \frac{1}{D^2 + 2D(2) + (2)^2 + 3D + 3(2) + 2} \sin x$$

$$= e^{2x} \frac{1}{D^2 + 4D + 4 + 3D + 6 + 2} \sin x$$

$$= e^{2x} \cdot \frac{1}{D^2 + 7D + 12} \sin x$$

$$= e^{2x} \cdot \frac{1}{-1^2 + 7D + 12} \sin x$$

$$= e^{2x} \cdot \frac{1}{7D + 11} \sin x$$

$$= e^{2x} \cdot \frac{7D - 11}{(7D + 11)(7D + 11)} \sin x$$

$$= e^{2x} \cdot \frac{7D - 11}{49D^2 - 121} \sin x$$

$$= e^{2x} \cdot \frac{7D - 11}{49[-(1)^2] - 121} \sin x$$

$$= e^{2x} \cdot \frac{7D - 11}{-170} \sin x$$

$$= \frac{e^{2x}}{-170} \left[7D \sin x - 11 \sin x \right]$$

$$= \frac{e^{2x}}{-170} \left[7 \cos x - 11 \sin x \right]$$

$$y = y_c + y_p$$

$$y = c_1 e^{-x} + c_2 e^{-2x} - \frac{e^{2x}}{170} \left[7 \cos x - 11 \sin x \right]$$

QUESTION-02

$(D^2-2D+1)y = x^2e^{3x}$

For $y_c \Rightarrow$

$$D^2-2D+1=0$$
$$D^2-D-D+1=0$$
$$D(D-1)-1(D-1)=0$$
$$(D-1)(D-1)=0$$
$$D=1\ ,\ 1$$
$$y_c = c_1e^x + c_2xe^x$$
$$y_c = e^x(c_1+c_2x)$$

For $y_p \Rightarrow$

$$e^{3x}\cdot\frac{1}{D^2-2D+1}x^2$$
$$= e^{3x}\cdot\frac{1}{D^2-2D+1}x^2$$
$$= e^{3x}\cdot\frac{1}{(D+3)^2-2(D+3)+1}x^2$$
$$= e^{3x}\cdot\frac{1}{D^2+6D+9-2D-6+1}x^2$$
$$= e^{3x}\cdot\frac{1}{D^2+4D+4}x^2$$
$$= e^{3x}\cdot\frac{1}{4\left[1+\frac{D^2+4D}{4}\right]}x^2$$
$$= e^{3x}\cdot\frac{1}{4}\left(1+\frac{D^2+4D}{4}\right)^{-1}x^2$$
$$= e^{3x}\cdot\frac{1}{4}\left[1-\frac{D^2+4D}{4}+\left(\frac{D^2+4D}{4}\right)^2\right]^{-1}x^2$$

Scanned with CamScanner

$$= \frac{e^{3x}}{4}\left[1 - \frac{D^2 + 4D}{4} + \frac{D^4 + 8D^3 + 16D^2}{16}\right] x^2$$

$$= \frac{e^{3x}}{4}\left[x^2 - \frac{D^2(x^2) + 4D(x^2)}{4} + \frac{D^4(x^2)}{16} + \frac{8D^3(x^2)}{16} + \frac{16D^2(x^2)}{16}\right]$$

$$= \frac{e^{3x}}{4}\left[x^2 - \frac{1}{2} - 2x + \frac{32}{16}\right]$$

$$= \frac{e^{3x}}{4}\left[x^2 - 2x + \frac{3}{2}\right]$$

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x)e^{x} + \frac{e^{3x}}{4}\left[x^2 - 2x + \frac{3}{2}\right]$$

QUESTION-03

$$(D^3 + 3D^2 - 4)y = xe^{-2x}$$

For $y_c \Rightarrow$

$$D^3 + 3D^2 - 4 = 0$$

	1	3	0	-4
1	↓	1	4	4
	1	4	4	0
-2	↓	-2	-4	
	1	2	0	

$$D + 2 = 0$$

$$D = 2$$

$$D = 1, -2, -2$$

$$y_c = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$$

$$y_c = c_1 e^{x} + e^{-2x}(c_2 + c_3 x)$$

For $y_p \Rightarrow$

$$e^{-2x} \cdot \frac{1}{D^3 + 3D - 4} x$$

$$= e^{-2x} \cdot \frac{1}{(D-2)^3 + 3(D-2)^2 - 4} x$$

$$= e^{-2x} \cdot \frac{1}{D^3 - 8 - 6D^2 + 12D + 3(D^2 - 4D + 4) - 4} x$$

$$= e^{-2x} \cdot \frac{1}{D^3 - \cancel{8} - 6D^2 + \cancel{12D} + 3D^2 - \cancel{12D} + \cancel{12} - \cancel{4}} x$$

$$= e^{-2x} \cdot \frac{1}{D^3 - 3D^2} x$$

$$= e^{-2x} \cdot \frac{1}{-3D^2\left[1 - \frac{D^3}{3D^2}\right]} x$$

$$= e^{-2x} \cdot \frac{1}{-3D^2\left[1 - \frac{D}{3}\right]} x$$

$$= \frac{e^{-2x}}{-3D^2} \cdot \left(1 - \frac{D}{3}\right)^{-1} x$$

$$= \frac{e^{-2x}}{3D^2}\left[1 + \frac{D}{3}\right] x$$

$$= \frac{-e^{-2x}}{3D^2}\left[x + \frac{1}{3}\right]$$

$$= \frac{-e^{-2x}}{3}\left[\frac{1}{D^2}(x) + \frac{1}{D^2}\left(\frac{1}{3}\right)\right]$$

$$= \frac{-e^{-2x}}{3}\left[\frac{x^3}{6} + \frac{x^2}{6}\right]$$

$$= \frac{-e^{-2x}}{3}\left[\frac{x^3 + x^2}{6}\right]$$

$$= \frac{-e^{-2x}}{18}(x^3 + x^2)$$

$$= \frac{-xe^{-2x}}{18}(x+1)$$

$$y = y_c + y_p$$

$$y = c_1 e^x + e^{-2x}(c_2 + c_3 x) - \frac{1}{18} x^2 e^{-2x}(x+1)$$

QUESTION - 04

$$\frac{d^4 y}{dx} - y = e^x \cos x$$

For $y_c \Rightarrow$

$$D^4 y - y = 0$$

$$(D^4 - 1)y = 0$$

$$D^4 - 1 = 0$$

$$(D^2)^2 - (1)^2 = 0$$

$$(D^2 - 1)(D^2 + 1) = 0$$

$D^2 - 1 = 0$ $\qquad$ $D^2 + 1 = 0$

$D = \pm 1$ $\qquad$ $D = \pm 1$

$$y_c = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$$

For $y_p \Rightarrow$

$$e^x \cdot \frac{1}{D^4 - 1} \cos x$$

$$= e^x \cdot \frac{1}{(D+1)^4 - 1} \cos x$$

$$= e^x \cdot \frac{1}{[(D+1)^2]^2 - 1} \cos x$$

$$= e^x \cdot \frac{1}{(D^2 + 2D + 1)^2} \cos x$$

$$= e^x \cdot \frac{1}{[-(1)^2 + 2D + 1]^2} \cos x$$

$$= e^{x} \cdot \frac{1}{[-\cancel{1} + 2D + \cancel{1}]^2} \cos x$$

$$= e^{x} \cdot \frac{1}{4D^2 - 1} \cos x$$

$$= e^{x} \cdot \frac{1}{-4(1)^2 - 1} \cos x$$

$$= \frac{e^{x}}{-5} \cos x$$

$$y = y_c + y_p$$

$$y = c_1 e^{x} + c_2 e^{-x} + c_3 \cos x + c_4 \sin x - \frac{e^{x}}{5} \cos x$$