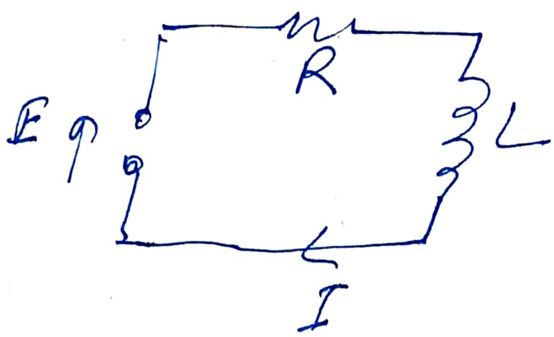
August 22, 2020 (2nd session)

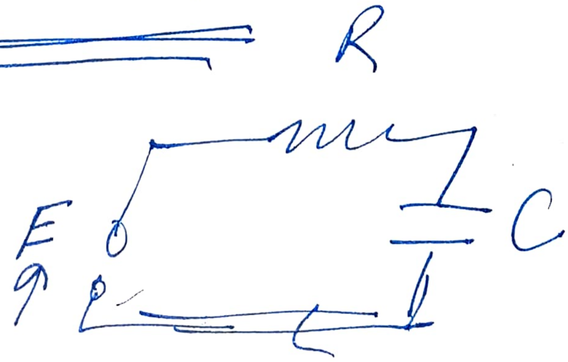
Application to the 1st order diff. equations:-

Electrical Circuits :- (R-L Circuit)



$$\frac{dI}{dt} + \frac{R}{L} I = \frac{E}{L}$$

(R-C circuit)



$$\frac{dq}{dt} + \frac{1}{RC} q = \frac{E}{R}$$

$$; \frac{dq}{dt} = I$$

Example :

$E = 5$ volts

$R = 50\,\Omega$

$L = 1$ henery

$$\frac{d}{dx}y + P(x)y = Q(x)$$

$$I\Big|_{t=0} = 0 \,, \quad I\Big|_{t_1=t} = ?$$

The fundamental equation of RL-circuit is:

$$\frac{d}{dt}I + \frac{R}{L}I = \frac{E}{L} \quad \text{——} (1)$$

$$\Rightarrow \quad \frac{d}{dt}I + \frac{50}{1}I = \frac{5}{1}$$

or

$$\frac{d}{dt}I + 50I = 5 \quad \text{——} (2)$$

This is a linear differential eq with $P = 50$ and $Q = 5$

$$I.F = e^{\int P(x)dx} = e^{\int 50 \cdot dx}$$

$$= e^{50t}.$$

The solution of the eq (2) is:

$$I\cdot\left(e^{50t}\right) = \int e^{50t} \cdot 5 \cdot dt + c$$

$$= \frac{1}{10}\int \underline{\underline{e^{50t} \cdot 50 \cdot dt}} + c$$

$$I\left(e^{50t}\right) = \frac{1}{10} e^{50t} + c$$

$$I = \frac{1}{10} + c e^{-50t} \longrightarrow (3)$$

at $t=0$, $I=0$

Eq (3) $\Rightarrow$ $0 = \frac{1}{10} + c \cdot e^{0}.$

$$\boxed{-\frac{1}{10} = c}$$

putting in Eq. 3

$$\Rightarrow \boxed{I\Big|_{t_1=t} = -\frac{1}{10}e^{-50t} + \frac{1}{10}}$$

Example 2

$E = 3.\sin 2t.$

$R = 10\ \Omega$

$L\ \cancel{t} = 0.5$ henery.

$I\Big|_{t=0} = 6$ amperes

$I\Big|_{t_1=t} = ?$

The governing eq. for RL-circuit

$$\frac{d}{dt}I + \frac{R}{L}I = \frac{E}{L}$$

$$\Rightarrow \frac{d}{dt}I + \frac{10}{0.5}I = \frac{3.\sin 2t}{0.5}$$

or $$\underbrace{\frac{d}{dt}I + 20}_{P}I = \underbrace{6.\sin 2t}_{Q} \quad \text{--- (1)}$$

$$\Rightarrow I.F = e^{\int 20.dt} = e^{20t}$$

The solution of eq. (1) is

$$I\left(e^{20t}\right) = \int e^{20t}.6.\sin 2t.dt + C$$

$$I\left(e^{20t}\right) = 6\underbrace{\int e^{20t}.\sin 2t.dt}_{P_1} + C \quad \text{--- (2)}$$

$$P_1 = \int \underbrace{e^{20t}}_{I}.\underbrace{\sin 2t}_{II}.dt$$

$$= e^{20t}.\frac{(-\cos 2t)}{2} - \int \left(\frac{-\cos 2t}{2}\right).e^{20t}.20.dt$$

$$P_1 = -\frac{1}{2} e^{20t} \cdot \cos 2t + 10 \int \underset{\mathrm{I}}{e^{20t}} \cdot \underset{\mathrm{II}}{\cos 2t} \cdot dt$$

$$= -\frac{1}{2} e^{20t} \cdot \cos 2t + 10 \left[e^{20t} \cdot \frac{\sin 2t}{2} - \int \frac{\sin 2t}{2} \cdot e^{20t} \cdot \cancel{20}\, dt \right]$$

$$= -\frac{1}{2} e^{20t} \cdot \cos 2t + 10 \left[\frac{1}{2} e^{20t} \cdot \sin 2t - 10 \underbrace{\int e^{20t} \cdot \sin 2t \cdot dt}_{P_1} \right]$$

or

$$P_1 = -\frac{1}{2} e^{20t} \cdot \cos 2t + 5 \cdot e^{20t} \cdot \sin 2t - 100 P_1$$

$$(101) P_1 = -\frac{1}{2} e^{20t} \cdot \cos 2t + 5 e^{20} \cdot \sin 2t$$

$$P_1 = -\frac{1}{202} e^{20t} \cdot \cos 2t + \frac{5}{101} e^{20t} \cdot \sin 2t$$

$$\boxed{P_1 = e^{20t} \left[-\frac{1}{202} \cos 2t + \frac{5}{101} \sin 2t \right]}$$

using value of P_1 in eq (2)

$$\Rightarrow I(e^{20t}) = 6\left[e^{20t}\left(-\frac{1}{202}\cos 2t + \frac{5}{101}\sin 2t\right)\right] + C$$

or

$$I(e^{20t}) = \left[-\frac{3}{101}\cos 2t + \frac{30}{101}\sin 2t\right]e^{20t} + C$$

$\div e^{20t}$

$$\Rightarrow \boxed{I = -\frac{3}{101}\cos 2t + \frac{30}{101}\sin 2t + Ce^{-20t}} \quad (2)$$

at $t = 0$, $I = 6$

Eq. (2)

$$\Rightarrow 6 = -\frac{3}{101}(1) + 0 + C\cdot(1)$$

$$C = 6 + \frac{3}{101} = \frac{606+3}{101} = \frac{609}{101}$$

$$(2) \Rightarrow \boxed{I = -\frac{3}{101}\cos 2t + \frac{30}{101}\sin 2t + \frac{609}{101}e^{-20t}}$$