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Q1

$$\frac{dy}{dx} + Py = Q$$

where $P = \sec^2 x$, $Q = \tan x$

Integrating Factor

$$If = e^{\int P dx}$$
$$= e^{\int \sec^2 x \, dx}$$
$$= e^{\tan x}$$

$$y\,(If) = \int \tan x \sec^2 x \,(I.F)\, dx$$
$$y \cdot e^{\tan x} = \int \tan x \sec^2 x \cdot e^{\tan x} dx \rightarrow (1)$$

Now,

$$\int \tan x \sec^2 x \cdot e^{\tan x} dx$$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$$\int \tan x \sec^2 x \cdot e^{\tan x} dx = \int t e^t dt$$
$$t e^t - e^t$$
$$(\tan x - 1) e^{\tan x}$$

Putting in (1)

$$y e^{\tan x} = (\tan x - 1) e^{\tan x} + C$$

$+c$

Q2

Let $M = y + \frac{y^3}{3} + \frac{x^2}{2}$, $N = \frac{1}{4}(x + xy^2)$

$M_y = 1 + y^2$, $N_x = \frac{1}{4}(1 + y^2)$

$M_y \neq N_x$ (so it is Nonexact)

$M_y - N_x = (1+y^2) - \frac{1}{4}(1+y^2)$

$= 1+y^2 \left(\frac{3}{4}\right)$

$= \frac{3}{4}(1+y^2) = \frac{3+3y^2}{4}$

$\frac{M_y - N_x}{N} = \frac{3+3y^2/4}{x+xy^2/4}$

$= \frac{3+3y^2}{x+xy^2} = \frac{3\cancel{(1+y^2)}}{x\cancel{(1+y^2)}} = \frac{3}{x}$

$= 3x^{-1} = f(x)$

$IF = e^{\int f(x)\,dx} = e^{\int 3 \cdot \frac{1}{x}\,dx}$

$= e^{3\ln x} = e^{\ln x^3} = x^3$

×ing with (1)

$$\left(x^3y + \frac{x^3y^3}{3} + \frac{x^5}{2}\right)dx + \left(\frac{x^4}{4} + \frac{x^4y^2}{4}\right)dy = 0$$

$$M = x^3y + \frac{x^3y^3}{3} + \frac{x^5}{2}, \quad M_y = x^3 + x^3y^2$$

$$N = \frac{x^4}{4} + \frac{x^4y^2}{4}, \quad N_x = x^3 + x^3y^2$$

$$M_y = N_x \quad (\text{Exact})$$

$$\int\left(x^3y + \frac{x^3y^3}{3} + \frac{x^5}{2}\right)dx + 0 = C$$

$$\int x^3y\,dx + \int \frac{x^3y^3}{3}dx + \int \frac{x^5}{2}dx = C$$

$$\boxed{\frac{x^4y}{4} + \frac{x^4y^3}{12} + \frac{x^6}{12} = C}$$