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Q1

$$(D^2 - 2D + 3)y = \cos x + x^2$$

In operator form

$$(D^2 - 2D + 3)y = 0$$

Auxilary Eq

$$D^2 - 2D + 3 = 0$$

$$a = 1, \quad b = -2, \quad c = 3$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{2 \pm \sqrt{4 - 12}}{2} \Rightarrow \frac{2 \pm \sqrt{-8}}{2}$$

$$\frac{2 \pm 2\sqrt{2}i}{2}$$

$$D = 1, \pm \sqrt{2}i$$

$$y_c = e^x (c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x)$$

For y_p

$$y_p = \frac{1}{D^2 - 2D + 3} x^2 + \cos x$$

$$y_p = \frac{1}{(D+2)^2 - 2(D+2) + 3} x^2 + \cos x$$

$$y_p = \frac{1}{D^2-2D+3}\cos x + \frac{1}{D^2-2D+3}x^2$$

$$= \frac{1}{D^2-2D+3}\cos x + \frac{1}{D^2-2D+3}x^2$$

$$= \frac{1}{-1-2D+3}\cos x + \left[\frac{1}{3\left[\frac{D^2-2D}{3}+1\right]}\cdot x^2\right]$$

$$= \frac{1}{2-2D}\cdot\cos x + \left[\frac{1}{3}\left[1+\frac{D^2-2D}{3}\right]^{-1}x^2\right]$$

$$= \frac{2+2D}{4-4D^2}\cos x + \left[\frac{1}{3}\left[1-\frac{D^2-2D}{3}+\left(\frac{D^2-2D}{3}\right)^2\right]x^2\right]$$

$$= \frac{2+2D}{8}\cos x + \left[\frac{1}{3}\left[1-\frac{D^2-2D}{3}+\left[\frac{D^4+4D^2-4D^3}{4}\right]\right]x^2\right]$$

$$= \frac{\cos x}{4} + \left(\frac{-\sin x}{4}\right) + \left[\frac{1}{3}\left[x^2-\left(\frac{2-4x}{3}\right)+\left(\frac{0+8-0}{4}\right)\right]\right.$$

$$= \frac{\cos x}{4} - \frac{\sin x}{4} + \left[\frac{1}{3}\left[x^2-\left(\frac{2-4x}{3}\right)+\frac{8}{4}\right]\right.$$

$$= \frac{\cos x}{4} - \frac{\sin x}{4} + \left[\frac{1}{3}\left[\frac{9x^2-6+12x+8}{9}\right)\right]$$

$$= \frac{\cos x}{4} - \frac{\sin x}{4} = \frac{9x^2+12x+2}{27}$$

$$y = y_c + y_p$$

$$y = e^{2x}\left[\cos\sqrt{2}x + \sin\sqrt{2}x\right] + \left[\frac{\cos x}{4} - \frac{\sin x}{1} + \frac{9x^2 + 12x - 2}{27}\right]$$

Q 2

$$\frac{1}{D^3+D^2+D+1}(\sin 2x)$$

$$y_p = \frac{1}{D^3+D^2+D+1}(\sin 2x)$$

$$= \frac{1}{D^2.D+D^2+D+1}(\sin 2x) = \frac{\sin 2x}{-4D-4+D+1}$$

$$= \frac{\sin 2x}{-3D-3} = -\left[\frac{\sin 2x}{3D+3}\right]$$

$$= -\left[\frac{3D-3\ (\sin 2x)}{9D^2-9}\right]$$

$$= -\left[\frac{(3D-3)(\sin 2x)}{-36-9}\right]$$

$$= +\left[\frac{3D-3}{45}(\sin 2x)\right] = \frac{3D.\sin 2x}{45} - \frac{3}{45}\sin 2x$$

$$= \frac{1}{3}.\frac{2\cos 2x}{45_{15}} - \frac{3}{45}\sin 2x$$

$$y_p \Rightarrow \frac{2\cos 2x}{15} - \frac{\sin 2x}{15}$$