Malik Zohaib Mustafa

Enrollment: 01-134192-030

Section: BSCS-4B

# Q#1

$$Ax^2 + By^2 = 1$$

$$\frac{d}{dx}\left[Ax^2 + By^2\right] = \frac{d}{dx} 1$$

$$2Ax + 2Byy' = 0$$

$$2(Ax + Byy') = 0$$

$$Ax + Byy' = 0 \quad \text{——} \quad (1)$$

Again Differentiating

$$\frac{d}{dx}\left[Ax + Byy'\right] = \frac{d}{dx}(0)$$

$$A + B\left[yy'' + y'y'\right] = 0$$

$$A + B\left[(y')^2 + yy''\right] = 0 \quad \text{——} \quad (2)$$

From eq (2) value of A

$$A = -B\left[(y')^2 + yy''\right]$$

$$-B\left[(y')^2 + yy''\right]x + Byy' = 0$$

$$\cancel{B}yy' = \cancel{B}\left[(y')^2 + yy''\right]x$$

$$yy' = (y')^2x + xyy''$$

$$xyy'' + x(y')^2 - yy' = 0$$

## Q# 3

$$y' = y - xy^3 e^{-2x}$$

$$y - y = -xe^{-2x} y^3$$

$$y' + (-1)y = (-xe^{-2x})y^3$$

$$\frac{dy}{dx} + (-1)y = (-xe^{-2x})y^3$$

Divide by $y^3$

$$\frac{1}{y^3}\frac{dy}{dx} + (-1)\frac{y}{y^3} = (-xe^{-2x})\frac{y^3}{y^3}$$

$$y^{-3}\frac{dy}{dx} + (-1)\frac{1}{y^{+2}} = -xe^{-2x}$$

$$y^{-3}\frac{dy}{dx} + (-1)y^{-2} = -xe^{-2x} \quad \text{— (1)}$$

Let $u = y^{-2}$ — (A)

$$\frac{du}{dx} = -2y^{-3}\frac{dy}{dx}$$

$$\frac{du}{dx} = \frac{-2}{y^3}\frac{dy}{dx}$$

$$\frac{-1}{2}\frac{du}{dx} = y^{-3}\frac{dy}{dx} \quad \text{— (B)}$$

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Put eq (A) and eq (B) in eq (1)

$$-\frac{1}{2}\frac{du}{dx}+(-1)u=-xe^{-2x}$$

$$-\frac{1}{2}\frac{du}{dx}+(-1)u=-xe^{-2x}$$

Multiply b/s by -2

$$\frac{du}{dx}+2u=2xe^{-2x}$$

Now it is a linear Equation

Compare with

$$\frac{dy}{dx}+P(x)y=Q(x)$$

$$P(x)=2$$

$$Q(x)=2xe^{-2x}$$

$$I.F=e^{\int P(x)dx}$$

$$I.F=e^{\int 2dx}=e^{2x}$$

$$u.I.F=\int I.F\,Q(x)\,dx+c$$

$$u.e^{2x}=\int e^{2x}.2xe^{-2x}dx+c$$

$$ue^{2x}=\int 2xe^{2x-2x}dx+c$$

$$ue^{2x}=\int 2xe^{0}dx+c$$

$$ue^{2x}=\int 2x\,dx+c$$

$$ue^{2x}=\frac{2x^2}{2}+C$$

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$$ue^{2x} = x^2 + C$$

As $u = y^{-2}$

$$y^{-2} e^{2x} = x^2 + C \Rightarrow \frac{1}{y^2} e^{2x} = x^2 + C$$

## Q# 4

$$(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy = 0$$

Let,

$$M = 2xy + y - \tan y \quad , \quad N = x^2 - x\tan^2 y + \sec^2 y$$

$$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y \quad , \quad \frac{\partial N}{\partial x} = 2x - \tan^2 y \neq 0$$

$$= 2x - \tan^2 y$$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So given diff eq is Exact

$$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = C$$

$$\int (2xy + y - \tan y)\,dx + \int \sec^2 y\,dy = C$$

$$\frac{2x^2 y}{2} + xy - x\tan y + \tan y = C$$

$$x^2 y + xy - x\tan y + \tan y = C$$

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Q#5

$$\frac{dP}{dt} = kP$$

$$\int \frac{1}{P}\,dP = \int k\,dt$$

$$\ln P = kt + c$$

$$e^{\ln P} = e^{kt} \cdot e^{c}$$

$$P = Ae^{kt} \quad \therefore \; e^{c} = A$$

Initially the population

$P = 5750000$

$A = 5000000$

$t = 10$ years

$$5750000 = 5000000\, e^{10k}$$

$$\frac{5750000}{5000000}$$

$$(1.15) = e^{10k}$$

Take ln on both sides

$$\ln(1.15) = \ln(e^{10k})$$

$$k = \frac{\ln(1.15)}{10}$$

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$\boxed{k = 0.0140}$

Population after 30 years, $t = 30$

$$P = 5000000\,e^{30(0.0140)}$$

$$\boxed{P = 7609807.78}$$

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Q# 2

$$2x + 6yy' = \left(\frac{x^2+3y^2}{y}\right)y'$$

$$2xy + 6y^2y' = (x^2+3y^2)y'$$

$$y'[6y^2 - x^2 - 3y^2] = -2xy$$

$$y'(3y^2 - x^2) = -2xy$$

$$y' = \frac{-2xy}{3y^2 - x^2}$$

$$\boxed{y' = \frac{2xy}{x^2 - 3y^2}}$$