August 22, 2020 (1st Session)

Applications to 1st order D. Eqs.: (1)

If P denotes the population of a country at any time t - Then

$$\frac{d}{dt}P \propto P$$

$$\Rightarrow \quad \frac{d}{dt}P = KP \quad ; \; K > 0.$$

Example:-

If N denotes the number of bacteria in the culture. Then

$$\frac{d}{dt}N \propto N$$

$$\Rightarrow \quad \frac{dN}{dt} = KN$$

$$\Rightarrow \quad \int \frac{dN}{N} = K\int dt$$

$$\Rightarrow \quad \ln N = Kt + C$$

$$\Rightarrow \quad N = Ae^{Kt} \quad \text{--- (i)}$$

at $t=1$, $N=1000$.

Eq. ① $\Rightarrow$

$\cancel{\frac{dN}{dt}} \cancel{= kt}$

$N = Ae^{kt}$

$\Rightarrow$ $1000 = Ae^{k(1)}$ —— ②

at $t=4$, $N=3000$.

Eq. ① $\Rightarrow$ $3000 = Ae^{4k}$ —— ③

② $\div$ ③

$\Rightarrow$ $\frac{1000}{3000} = \frac{\cancel{A}e^{k}}{\cancel{A}e^{4k}}$

$\frac{1}{3} = 0.333 = e^{-3k}$

$\Rightarrow$ $\frac{\ln(0.333)}{-3} = k$ $\quad \because \ln e = 1$

$\boxed{k = 0.366}$

Eq ② $\Rightarrow$ $1000 = Ae^{(0.366)\cdot t}$ —— ④

(3)

$$N = \frac{1000}{e^{0.366}} = 693.5 \simeq \underline{694}$$

a) using above in Eq. (1)

$$\Rightarrow \quad N = 694 \cdot e^{(0.366)t} \quad \text{———} \quad (5)$$

b) at $t = 0$

$$\text{Eq. (5)} \Rightarrow \quad N = 694 \cdot e^{(0.366)0}$$

$$= 694(1) = \underline{\underline{694}}$$

Example :- let P be number of people living in the country at any time t. Then according to given

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \quad P = Ae^{kt} \quad \text{———} \quad (1)$$

let the people initially living in the country be P_0.

at $t=0$, $P=P_0$

$$\Rightarrow \quad P_0 = Ae^{0} = A$$

$$(1) \Rightarrow \quad P = P_0 e^{Kt} \quad \text{——} \quad (2)$$

at $t=2$, $P = \underline{2P_0}$

$$\text{Eq}(2) \Rightarrow \quad 2\cancel{P_0} = \cancel{P_0}\cdot e^{K(2)}$$

$$\Rightarrow \quad K = \frac{\ln(2)}{2} = \underline{0.34657}$$

$$(2) \Rightarrow \quad P = P_0 \cdot e^{(0.34657)t} \quad \text{——} \quad (3)$$

at $t=3$, $P = 20{,}000.$

$$\text{Eq}(3) \Rightarrow \quad \underline{20{,}000 = P_0 \cdot e^{(0.34657)(3)}}$$

$$P_0 = \frac{(20{,}000)}{e^{\underline{(0.34657)(3)}}} =$$

$$P_0 = \underline{7062}$$

Example :- let the money deposited in the bank initially be P.

$$\Rightarrow \quad \frac{dP}{dt} \propto P$$

$$\Rightarrow \quad P = Ae^{kt} \quad \text{——— (1)} \quad ; k > 0$$

@ 5%

$$(1) \Rightarrow P = Ae^{0.05t} \quad \text{——— (2)}$$

at $t = 0$, $P = 20,000$.

$$(2) \Rightarrow \quad 20,000 = Ae^{0} = A$$

$$(2) \Rightarrow \quad P = 20,000 \cdot e^{(0.05)t} \quad \text{——— (3)}$$

(a) at $t = 3$

$$(3) \Rightarrow \quad P = 20,000 \cdot e^{(0.05)(3)}$$

$$\boxed{P = \$\, 23,236.68}$$

(b) at $P = 40{,}000$

$(3) \Rightarrow 40{,}000 = 20{,}000 \cdot e^{(0.05)t}$

$2 = e^{(0.05)t}$

$\Rightarrow t = \dfrac{\ln(2)}{0.05} = \underline{\underline{13.86 \text{ yrs}}}$

Example :-

let N denote the mass of the material at any time t.

$\Rightarrow \dfrac{d}{dt}N \propto N$

$\Rightarrow N = c\,e^{kt}$ ——— (1)

at $t = 0$, $N = 50$

$(1) \Rightarrow 50 = c\,e^{0} = c$

$(1) \Rightarrow N = 50 \cdot e^{kt}$ ——— (2)

10% of 50 = 5

remaining = 50 − 5 = 45

at $t = 2$, $x = 45$

(2) $\Rightarrow$ $45 = 50 \cdot e^{K(2)}$

$$K = \frac{\ln(45/50)}{2} = -0.053$$

(a) So (2) $\Rightarrow$ $x = 50 \cdot e^{(-0.053)t}$ —— (3)

(b) at $t = 4$

(3) $\Rightarrow$ $x = 50 \cdot e^{(-0.053)(4)}$

$$\boxed{x = 40.5 \text{ mg}}$$

(c) at $x = 25$

(3) $\Rightarrow$ $25 = 50 \cdot e^{(-0.053)t}$

$$t = \frac{\ln(\frac{1}{2})}{-0.053} = \boxed{13 \text{ hrs}}$$