August 24, 2020 (1st session)

Case(3) :- $\frac{1}{f(D)} \cdot x^m$; m is a +ve integer.

Step 1 :- Take out the lowest degree term from f(D) to make the first term 1.

Step 2 :- The remaining factor will be of the form $[1 \pm \phi(D)]$

Step 3 : Take this factor in the numerator.

$$[1 \pm \phi(D)]^{-1}$$

Step 4 :- Use Binomial theorem,

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

The expansion is to be carried out upto the term D^m. As

$$D^{m+1}(x^m) = 0$$

Example :- Solve $y'' + y' - 6y = x$

For $y_c \Rightarrow D^2y + Dy - 6y = 0$

$$\Rightarrow (D^2 + D - 6)\, y = 0$$

The auxiliary equation is

$$D^2 + D - 6 = 0$$

$$(D+3)(D-2) = 0$$

$$\Rightarrow D = -3, +2$$

So,
$$\boxed{y_c = c_1 e^{-3x} + c_2 e^{2x}}$$

For $y_p \Rightarrow y_p = \frac{1}{D^2 + D - 6} \cdot x$

$$y_p = \frac{1}{-6\left[1 - \frac{D^2 + D}{6}\right]} \cdot x$$

$$= -\frac{1}{6}\left[1 - \frac{D^2 + D}{6}\right]^{-1} \cdot x$$

Formula: $\boxed{(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 \ldots}$

(2)

(3)

$$y_p = -\frac{1}{6}\left[1+\left(\frac{D^2+D}{6}\right)^1+\left(\frac{\cancel{D^2+D}}{\cancel{6}}\right)^2 \dots\right]x^1$$

$$= -\frac{1}{6}\left[1+\frac{D^2+D}{6}\right]x$$

$$= -\frac{1}{6}\left[x+\frac{1}{6}(D^2+D)x\right]$$

$$= -\frac{1}{6}\left[x+\frac{1}{6}(0+1)\right]$$

$$= -\frac{1}{6}\left[x+\frac{1}{6}\right]$$

$$\boxed{y_p = -\frac{1}{6}x-\frac{1}{36}}$$

$$\therefore \quad y = y_c + y_p$$

$$\Rightarrow \boxed{y = c_1e^{-3x}+c_2e^{2x}-\frac{1}{6}x-\frac{1}{36}}$$

Example :- Solve

$$(D^3-3D-2)y=x^2$$

For $y_c \Rightarrow (D^3-3D-2)y=0$

$A.E \Rightarrow \underline{\underline{D^3-3D-2=0}}$

$$\begin{array}{r|rrrr}
-1 & 1 & 0 & -3 & -2 \\
 & \downarrow & -1 & +1 & +2 \\
\hline
 & 1 & -1 & -2 & \underline{|0} \\
-1 & \downarrow & -1 & +2 & \\
\hline
 & 1 & -2 & \underline{|0} & \\
2 & \downarrow & 2 & & \\
\hline
 & 1 & \underline{|0} & &
\end{array}$$

$\Rightarrow D = \underline{\underline{-1,-1}}, \underline{\underline{+2}}$

$\therefore \Rightarrow \boxed{y_c = (c_1+c_2x)e^{-x}+c_3e^{2x}}$

For $y_p \Rightarrow$

$$y_p = \frac{1}{D^3-3D-2}\cdot x^2$$

⑤

$$y_p = \frac{1}{-2\left[1 - \frac{D^3 - 3D}{2}\right]} \cdot x^2$$

$$= -\frac{1}{2}\left[1 - \frac{D^3 - 3D}{2}\right]^{-1} \cdot x^2$$

$$= -\frac{1}{2}\left[1 + \left(\frac{D^3 - 3D}{2}\right)^1 + \left(\frac{D^3 - 3D}{2}\right)^2\right] \cdot x^2$$

$$= -\frac{1}{2}\left[x^2 + \frac{1}{2}(D^3 - 3D)x^2 + \frac{1}{4}(D^6 + 9D^2 - 6D^4)x^2\right]$$

$$= -\frac{1}{2}\left[x^2 + \frac{1}{2}(0 - 3(2x)) + \frac{1}{4}(0 + 9(2) - 0)\right]$$

$$= -\frac{1}{2}\left[x^2 - 3x + \frac{9}{2}\right]$$

$$y_p = -\frac{x^2}{2} + \frac{3}{2}x - \frac{9}{4}$$

$\therefore\ y = y_c + y_p =$ ______ + ______

Example :- Solve $y''' - 13y' + 12y = x$.

For y_c

$$(D^3 - 13D + 12)y = 0$$

The auxiliary eq.

$$D^3 - 13D + 12 = 0$$

$$\Rightarrow \cancel{D^3 + 12D - D + 12 = 0}$$

$$\cancel{D(D^2 + 12) - 1(D + 12) = 0}$$

$$\cancel{D(D}$$

$$\begin{array}{r|rrrr}
 & 1 & 0 & -13 & +12 \\
1 & \downarrow & 1 & 1 & -12 \\
\hline
3 & 1 & 1 & -12 & 0 \\
 & \downarrow & 3 & 12 & \\
\hline
-4 & 1 & 4 & 0 & \\
 & \downarrow & -4 & & \\
\hline
 & 1 & 0 & &
\end{array}$$

$$D = 1, 3, -4$$

$$\Rightarrow \boxed{y_c = c_1 e^{x} + c_2 e^{3x} + c_3 e^{-4x}}$$

For y_p

$$\Rightarrow y_p = \frac{1}{D^3-13D+12}\cdot x$$

$$y_p = \frac{1}{12\left[1+\frac{D^3-13D}{12}\right]}\cdot x$$

$$y_p = \frac{1}{12}\left[1+\frac{D^3-13D}{12}\right]^{-1}\cdot x$$

$$= \frac{1}{12}\left[1-\left(\frac{D^3-13D}{12}\right)\right]x$$

$$= \frac{1}{12}\left[1-\frac{1}{12}\left(D^3-13D\right)\right]x$$

$$= \frac{1}{12}\left[x-\frac{1}{12}\left(D^3-13D\right)x\right]$$

$$= \frac{1}{12}\left[x-\frac{1}{12}\left(0-13(1)\right)\right]$$

$$= \frac{1}{12}\left[x+\frac{13}{12}\right] = \frac{x}{12}+\frac{13}{144}$$

$$\therefore y = y_c + y_p = \_\_\_\_ + \_\_\_\_$$

Solve: $y''' - 3y'' - 6y' + 8y = x$

For y_c $(D^3 - 3D^2 - 6D + 8)y = 0$.

The auxiliary equation is

$$D^3 - 3D^2 - 6D + 8 = 0$$

1	1	−3	−6	8
	↓	1	−2	−8
	1	−2	−8	0
−2	↓	−2	+8	
	1	−4	0	
4	↓	4		
	1	0		

$$D = 1, -2, 4$$

$$\therefore \boxed{y_c = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{4x}}$$

For y_p

$$\Rightarrow y_p = \frac{1}{D^3-3D^2-6D+8}\cdot x$$

$$= \frac{1}{8\left[1+\frac{D^3-3D^2-6D}{8}\right]}\cdot x$$

$$= +\frac{1}{8}\left[1+\frac{D^3-3D^2-6D}{8}\right]^{-1}\cdot x$$

$$= \frac{1}{8}\left[1-\left(\frac{D^3-3D^2-6D}{8}\right)\right]x$$

$$= \frac{1}{8}\left[x-\frac{1}{\cancel{8}_{4}}\left(0-0-\cancel{6}^{3}\right)\right]$$

$$= \frac{x}{8}+\frac{3}{32}$$

$$\therefore\ y = y_c + y_p$$

$$\Rightarrow y = \_\_\_\_ + \_\_\_\_$$