X-Rays and Atomic Spectra

Adam Fattal-Dakmak
L1 Discovery Labs, Lab Group A, Monday
Submitted: February 17, 2021, Date of Experiment: January 2021

When electrons change energy levels, a photon must be emitted or absorbed. For Molybdenum in the L shell, the values of the wavelengths are $\lambda_{K\alpha 1}$ = 0.632 $\mathring{A}$ and $\lambda_{K\alpha 2}$ = 0.709 $\mathring{A}$. These values are within the uncertainty of the results found in this experiment. This experiment also finds Planck's constant to be within 4 % of the value established in the literature.

I. INTRODUCTION

X-Rays are a form of electromagnetic radiation which have wavelengths ranging from 10^{-8}m to 10^{-12}m [1]. There are several applications to X-ray-related methods in fields ranging from material engineering to cancer treatment. A particularly notable use of X-Ray radiation is in spectronomy. One can utilize the apparatus shown on **Fig. 1** to use find the atomic spectra of a specific element (e.g. Molybdenum)

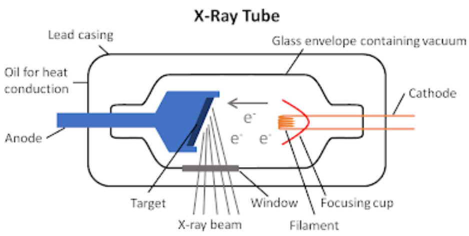

Fig. 1: Schematic of Experiment Apparatus [2]

The experiment works in the following way: a cathode (i.e a wire filament) is heated up until it reaches the temperature necessary to eject electrons, which then move towards the anode due to the potential difference. A target (e.g an Mo plate) will be placed in the anode so that the high energy electrons collide with the target, producing a wide range of X-Ray radiation which are targeted towards a crystal that diffracts them according to Bragg's Law (1)

$$n\lambda = 2d\sin(\beta) \tag{1}$$

where n is the order of diffraction ($n = 1$ for the sake of this report), λ is the wavelength of the X-Ray, d gap distance in the crystal lattice structure and β is the angle formed with the GM tube .

Another notable relation which will prove to be important in this report is the Duane–Hunt (DH) law [3], which states that the minimum wavelength of X-Ray radiation that can be emitted by the electron-target collision is inversely proportional to U, the potential difference. The following is the DH relationship:

$$\lambda_{min} = \frac{hc}{eU} \tag{2}$$

where e is the charge of the electron, U is the potential difference and h is Planck's Quantization Constant.

Rearranging *Eq. 2*, one gets that Planck's constant could be determined as following:

$$h = \frac{\lambda_{min} e U}{c} \tag{3}$$

II. METHODS

The experiment setup used is very similar to the one displayed in **Fig. 1**. The X-Ray machine used is a *Leybold 554-800 X-Ray Apparatus*, which takes in the following parameters: U (potential difference), I, (current), Δt (the time step), $\Delta\beta$ (the angle step) and β_{limits} (i.e minimum and maximum angle). For the first experiment, only two parameters were varied, the U and I. The other four parameters were set to be equal to $\Delta t = 4s$, $\Delta\beta = 0.1°$, $\beta_{min} = 2.5°$ and $\beta_{max} = 12.5°$. The target of the high energy electrons was a Molybdenum ($^{98}_{42}$Mo) tube. An NaCl crystal was then mounted where the X-rays are targeted.

At this point, the setup was complete and the experiment was ready to be performed. For the first part of the experiment, which is related to plotting the counts vs angle data and examining the peak count rates, four datasets were obtained. The first one where $U = 35kV$ and $I = 1.0mA$, the second where $U = 35kV$ and $I = 0.6mA$, the third where $U = 25kV$ and $I = 1.0mA$ and the fourth where $U = 25kV$ and $I = 0.6mA$ (in all cases, the default parameters mentioned above were used). Note that the data obtained was the counts that the Geiger-Muller Counter measured as a function of its angle (which was varying according to the default parameters)

The second part of the experiment, which is related to obtaining the data necessary to derive Planck's constant using a variation of (3), was performed with the same setup as the first part. The only difference is which parameters were held constant. In this case, I and $\Delta\beta$ were fixed at 1.0mA and 0.1 °, respectively. The datasets were obtained from the following setups ($U, \Delta t, \beta_{min}, \beta_{max}$):

$(22kV, 30s, 5.2°, 6.2°)$,$(26kV, 25s, 5.0°, 6.2°)$, $(30kV, 10s, 3.2°, 6.0°)$, and $(34kV, 10s, 2.5°, 6.0°)$

The data from this experiment was used to derive Planck's constant by using the values of e and c from [4].

III. RESULTS AND DISCUSSION

The Counts versus β datasets obtained are plotted in **Fig. 2**. There are 4 datasets plotted, each one with a different $I - U$ combination. One can notice that in all four cases, the two peaks are aligned at the same β values, which suggests that those angle values must be the angles that describes the transition energies of Molybdenum. Two dotted vertical lines are plotted to emphasize these points.

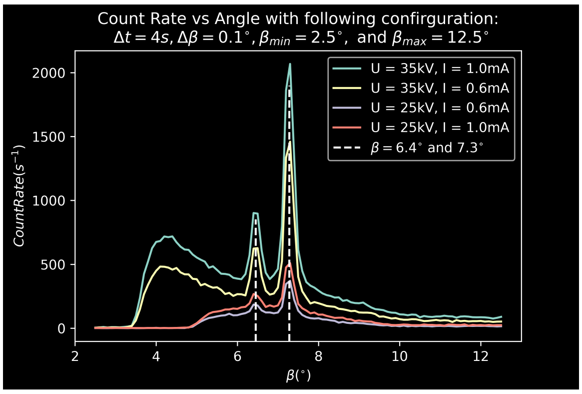

Fig. 2: Visualization of first experiment with peaks at β = 6.4 and 7.3

Looking at **Fig. 2**, one can notice that the peak energies occur when the angles are 6.4 ($\pm$ 0.3) $^\circ$ and 7.3 ($\pm$ 0.3) $^\circ$. Using Bragg's Law, one would find that this comes out to correspond to wavelengths of 6.28 ($\pm$ 1.3) $\times 10^{-11}$m and 7.26 ($\pm$ 1.5) $\times 10^{-11}$m, respectively. Comparing these values to the theoretical values of the wavelengths one would derive using the theoretical values of the energies of $K_{\alpha 1}$ and $K_{\alpha 2}$ (which are 0.632 $\mathring{A}$ and 0.709 $\mathring{A}$, according to $E = hc/\lambda$), one can notice that these values are within the standard error of the values obtained by the experiment. In other words, the energy at the two peaks would be the energy of transitions for Molybdenum in the L shell (6.4 $^\circ$ for $K_{\alpha 1}$ and 7.3 $^\circ$ for $K_{\alpha 2}$)

Fig. 3 represents the datasets obtained in the first experiment where $I = 1mA$. x and y error bars are added to visualize what the uncertainty is in both the wavelength and the number of counts

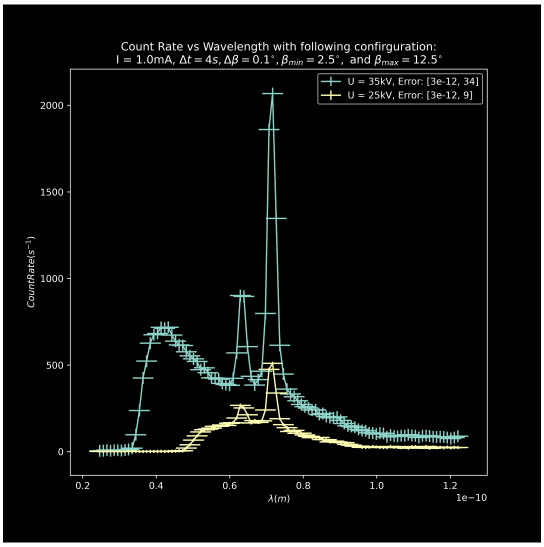

Fig. 4: Second Experiment Data. Subsets of this Data will be Used to Derive Planck's Constant

In this figure, wavelength is as the domain. One can find that the peaks are located in the wavelengths discussed earlier, and the error is around 3×10^{-12}m. The y-error bar is relatively small and require some effort to visualize.

The results of the next experiment, which performs the setup described in *Section 2* for four different $U, \Delta t, \beta_{min}$ and β_{max} values, are shown on **Fig. 4**. The purpose of this data is to use it to find Planck's constant, h using the BH relationship described in *Section 1*.

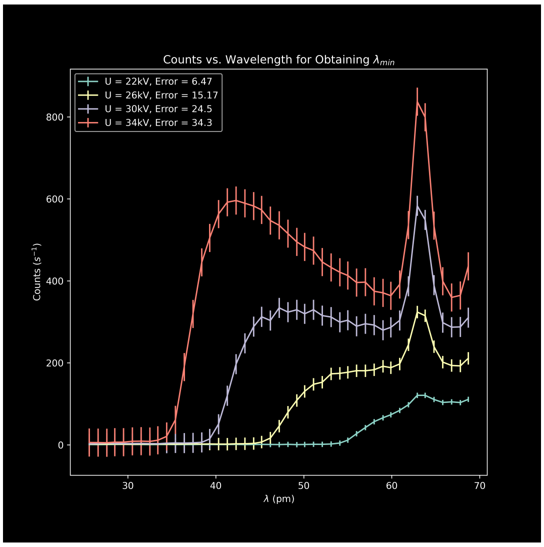

Fig. 3: Visualization of first experiment where $I = 1mA$ ($\alpha_\lambda = 3 \times 10^{-12}, \alpha_{counts_{25}} = 9, \alpha_{counts_{35}} = 34$)

If one's aim is to extract λ_{min} values from these datasets, one can get a good approximation by getting the x-intercept of the line of best fit for the subsets of these datasets with most linearity (e.g for the red plot, it's 34.4 to 40.3 pm). The following was given as a result of this method (34.4, 38.8, 43.0, 53.8) $\pm$ 4.1 pm. Using The DH relationship, one can realize that the slope of the line of best fit of λ_{min} vs $1/U$ would be a c/eth factor of h (where e and$c$ are given in [4]). Using that method, a value for Planck's constant was found to be 6.35 $(\pm 0.02) \times 10^{-34}$ J.s, as compared to the value established in the literature (6.62 $\times 10^{-34}$ J.s). This gives a percent error of 4%

IV. CONCLUSIONS

In conclusion, this experiment was based on the utilization of an X-Ray apparatus to obtain certain results related to Molybdenum. The key findings were the wavelengths of transitions of Mo in the L shell, which came out to be $\lambda_{K_{\alpha 1}} = 0.628(\pm 0.13)$ Å$and\lambda_{K_{\alpha 2}} = 0.726(\pm 0.15)\mathring{A}$ and the empirical finding of Planck's constant, which came out to be 6.35 $(\pm 0.02) \times 10^{-34}$ J.s. While all results are consistent with the literature, this is not to say that certain errors (such as the human error of approximating the linear subsections and the potential random error caused in the experiment) can't be reduced

V. DATASETS

All datasets, as well as data analysis methods can be found on https://github.com/theheavygluon/X-Ray-experiment

VI. REFERENCES

[1] Young, H. D., Freedman, R. A., amp; Ford, A. L. (2020). Chapter 33. In *Sears and Zemansky's university physics: With modern physics*. Harlow, United Kingdom: Pearson Education Limited.

[2] Roque, R., Luz, H. N., Carramate, L., Azevedo, C., Mir, J., amp; Amaro, F. (2017). Gain characteristics of a 100 m thick gem In Krypton-CO2 mixtures. *Journal of Instrumentation*, 12(12). doi:10.1088/1748-0221/12/12/c12061

[3] William Duane and Franklin L. Hunt (1915). "On X-Ray Wave-Lengths". *Physical Review.* 6 (2): doi:10.1103/PhysRev.6.166.

[4] Fundamental physical constants - frequently used constants. (2014). *CRC Handbook of Chemistry and Physics*, 2664-2664. doi:10.1201/b17118-27

Error Appendix

Given that all data was proccessed in python, it would be natural for one to use tools such as $numpy.polyfit(x, y, 1)$ for linear regression and $scipy.stats.sem(data)$ for calculating uncertainty. An important equation used to propagate the error of β to λ is the following:

$$\alpha_{\sin(\theta)} = \cos\theta\alpha_{\theta} \tag{4}$$

which can be found in Hughes and Hayes under *The Functional Method*

Note that for all error propagation, a numerical method was used by the python function $uncertainties.ufloat(data, error)$, which propagates the error with the operations that act on the data

Scientific Summary for a General Audience

Something that one might find notable about the history of science is that there are several world changing discoveries that could be highly attributed to coincidence and accident. A famous example of this phenomenon is the discovery of the X-Ray. In 1895, german physicist Wilhelm Roentgen had been experimenting with cathode rays (rays of high-speed electrons) and attempted to see if they could pass through glass. What he noticed, however, is that for unknown reasons, a florescent screen nearby was glowing. This was due to what he named X-Rays (now known to be light rays with wavelength of 0.01 to 10 nanometers). The experiment performed in this report utilizes X-Rays, which are caused by cathode rays targeting a Molybdenum plate, to target an NaCl crystal, which, due to its structure, diffuses the X-Ray. The data is obtained by a Geiger-Muller tube (a radiation detector, roughly speaking) that varies its angle to the crystal. The obtained can then be used to study the transition energy of the Molybdenum, as well as derive Planck's quantization constant using the Duane–Hunt law, a law that states that there is an inverse relationship between the minimum wavelength and the potential difference between the cathode and anode.