2023

COMPUTER SCIENCE

Paper : CSMC - 101

(Mathematics for Computing)

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** 1 & 2 and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) How do you distinguish between a *trail* and a *path* for a graph?

(b) State the four-color theorem for a graph.

(c) Find all solutions of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_1 = 0$, $a_2 = 6$.

(d) You toss a fair coin three times, given that a head is observed at least once, what is the probability that you observe at least two heads?

(e) Define Span and Spanning Set in context of Vector space V.

(f) What can you say about two non-zero vectors u and v that satisfy the equation $||u+v|| = ||u|| + ||v||$?

(g) Show that the standard unit vectors form a basis for $V = R^3$.

(h) Suppose the characteristic polynomial of a matrix A is found to be $P(\lambda) = (\lambda - 1)(\lambda - 3)^2(\lambda - 4)^3$. How many eigen values does A have? Is A invertible?

2. Answer ***any five*** questions : 4×5

(a) "$K_{3,3}$ is a planar graph."— Comment on the correctness of the statement and justify your opinion.

(b) "The degree of the unbounded region for a simple graph is at least 3."— Comment on the correctness of the statement and justify your opinion.

(c) "A 6-vertex graph cannot be self-complementary."— Comment on the correctness of the statement and justify your opinion.

(d) Consider the following function :

$$p_x(x) = \begin{cases} \frac{1}{10}x & \text{if } x \in \{1,2,3,4\} \\ 0 & \text{otherwise.} \end{cases}$$

Determine whether the above function is a valid probability mass function. Define and give an example of an Estimator.

(e) Show that the three vectors $u = \{0,3,1,-1\}$, $v = \{6,0,5,1\}$ and $w = \{4,-7,1,3\}$ form a linearly dependent set in R^4. Also, express each vector as a linear combination of other two.

(f) Show that the following matrix is not diagonizable.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 5 & 2 \end{bmatrix}$$

(g) Show that v is a subspace of the vector space of 2×2 real matrix with usual matrix addition and scalar multiplication.

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a + d = 0, a, b, c, d \in R \right\}.$$

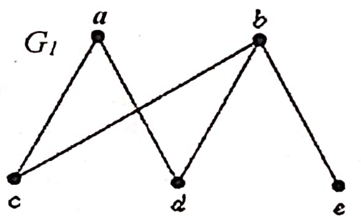
3. (a) State Euler's formula to find the number of regions for a planar graph.

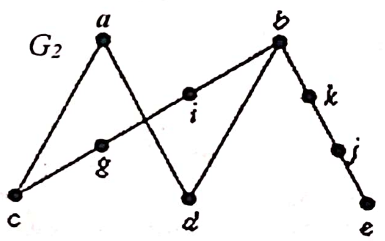
(b) Does Euler's formula stated above remain valid even if the graph is not planar? Justify your opinion.

(c) State the first theorem of Kuratowski.

(d) Determine the sequence of elementary subdivisions needed to obtain G_2 from G_1. Illustrate the intermediate states clearly.

1+(1+2)+2+4





4. (a) What would be the Chromatic number of a Cycle G with an odd number of nodes? Why?

(b) Distinguish between the maximal and maximum matching for a graph. Draw an example to indicate a maximum matching for a graph G having at least five edges.

(c) Schedule the final exams for Course-115, Course-116, Course-185, Course-195, Course-101, Course-102, Course-273 and Course-473, using the fewest number of different time slots, if no students are taking both Course-115 and Course-473, both Course-116 and Course-473, both Course-195 and Course-101, both Course-195 and Course-102, both Course-115 and Course-116, both Course-115 and course-185, and both Course-185 and Course-195, but there are students in every other pair of courses.

3+3+4

5. (a) What is a Particular Solution for a non-homogeneous recurrence relation?

(b) Prove that if $\{a_n^{(p)}\}$ is a particular solution of the non-homogeneous linear recurrence relation with constant coefficients

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k} + F(n),$$

then every solution is of the form $\{a_n^{(p)} + a_n^{(h)}\}$, where $\{a_n^{(h)}\}$ is a solution of the associated homogeneous recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \dots + c_ka_{n-k}$.

(c) What form does a particular solution of the linear non-homogeneous recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + F(n)$ have for $F(n) = n^2 2^n$ and for $F(n) = n3^n$? 1+4+(3+2)

6. (a) Determine the values of K so that the following system of equations has (i) no solution, (ii) more than one solution and (iii) a unique solution.

$$x + y - z = 1$$
$$2x + 3y + Kz = 3$$
$$x + Ky + 3z = 2.$$

(b) Find bases for the eigenspaces of

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

6+4

7. (a) Show that the set $\{(2, -3, 1), (4, 1, -5), (1, 1, 1)\}$ is a set of mutually orthogonal vectors in the inner product space R^3 with standard inner product. Is this set of vectors a basis of R^3? Justify.

(b) Prove that the set $S = \{(1,1,0), (1,0,1), (0,1,1)\}$ is a basis of vector space R^3. Show that the vector (1,0,1) of the set S may be replaced by (1,1,1) to form a new basis for R^3. However, the same is not true for vector (3,1,2). 5+5

8. (a) Let u and v are two vectors within an inner product space V such that $\langle \vec{u}, \vec{v} \rangle = \| \vec{u} \| \cdot \| \vec{v} \|$. Prove that u and v must be linearly independent.

(b) What is the maximum possible rank of an $m \times n$ matrix A that is not square?

(c) Find the rank and nullity of the matrix

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

4+1+5