Storing θ_{ijk} in arrays

- Assume that X_i takes values from 0 to $r_i - 1$.
- Although i, j, k are ordinals in the BN notation, here we consider that they range from 0 to the size of their domain -1.

counts

N_{ijk}

node X_i — parents of X_i — value of X_i

$\vec{pa}(X_i) = \{X_{i_1}, X_{i_2}, X_{i_3}\}$

$\longrightarrow$ Count in the data the number of times

(i_1, i_2, i_3, i) array of node indexes

takes the value

(j_1, j_2, j_3, k) array of values for that nodes

$$j \longleftarrow ? \longrightarrow (j_1, j_2, j_3)$$

<u>Example</u>:

X_0	X_1	X_2	X_3	X_4	X_5
1	0	0	1	0	0
0	0	1	0	1	0
0	0	0	0	0	0
1	0	1	1	1	1

X_i binary r.v.

$$\begin{cases} x_{50} = 0 \\ x_{51} = 1 \end{cases}$$

$$\begin{cases} w_{50} = 000 \\ w_{51} = 001 \\ w_{52} = 010 \\ \vdots \\ w_{57} = 111 \end{cases}$$

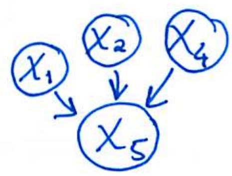
$w_{ij} = j_1 j_2 j_3$

$0 \leftrightarrow 000$

$1 \leftrightarrow 001$

$2 \leftrightarrow 010$

$\dots$



N_{500}

$j = 0 \longleftrightarrow (j_1, j_2, j_3) = (0, 0, 0)$

count in the data the number of times $(1, 2, 4, 5)$ takes the value $(0, 0, 0, 0)$

$\hookrightarrow$ <u>2</u>

→ Assume X_i has only one parent X_{i_1}

$$j \longleftrightarrow j_1$$

$$j_1 = 0, \dots, r_{i_1} - 1$$

→ Assume X_i has two parents X_{i_1} and X_{i_2}

$$j \overset{?}{\longleftrightarrow} (j_1, j_2)$$

$$j_1 = 0, \dots, r_{i_1} - 1$$

$$j_2 = 0, \dots, r_{i_2} - 1$$

$$
\begin{array}{cc|ccccc|}
 & & \multicolumn{5}{c}{\overbrace{\hspace{18em}}^{r_{i_2}}} \\
 & & 0 & 1 & 2 & \cdots & r_{i_2} - 1 \\
\hline
 & 0 & 0 & 1 & 2 & \cdots & r_{i_2} - 1 \\
 & 1 & r_{i_2} & r_{i_2}+1 & r_{i_2}+2 & \cdots & 2r_{i_2} - 1 \\
r_{i_1} & 2 & 2r_{i_2} & 2r_{i_2}+1 & 2r_{i_2}+2 & \cdots & 3r_{i_2} - 1 \\
 & \vdots & & & \cdots & & \\
 & r_{i_1}-1 & (r_{i_1}-1)r_{i_2} & (r_{i_1}-1)r_{i_2}+1 & & \cdots & r_{i_1} \times r_{i_2} - 1 \\
\hline
\end{array}
$$

$$
\underbrace{\boxed{0}\boxed{1}\boxed{2}\boxed{\cdots}\boxed{r_{i_2}-1}}_{r_{i_2}}\underbrace{\boxed{r_{i_2}}\boxed{r_{i_2}+1}\boxed{r_{i_2}+2}\boxed{\cdots}\boxed{2r_{i_2}-1}}_{r_{i_2}}\boxed{\cdots}\boxed{r_{i_1} \times r_{i_2}-1}
$$

Given $(j_1, j_2) \implies j = j_1 \times r_{i_2} + j_2$

Given $j \implies j_2 = j \,\%\, r_{i_2}$

$$j_1 = (j - j_2) / r_{i_2}$$

→ In general, when X_i has K parents $X_{i_1}, X_{i_2}, \dots, X_{i_K}$

$$j \xleftrightarrow{?} (j_1, j_2, \dots, j_K)$$

$$j_1 = 0, \dots, r_{i_1} - 1$$
$$j_2 = 0, \dots, r_{i_2} - 1$$
$$\dots$$
$$j_K = 0, \dots, r_{i_K} - 1$$

Given $(j_1, j_2, \dots, j_K) \implies ((j_1 \times r_{i_2} + j_2), j_3, \dots j_K)$

$$\downarrow$$

$$(((j_1 \times r_{i_2} + j_2) \times r_{i_3} + j_3), j_4, \dots, j_K)$$

$$\vdots$$
$$\downarrow$$

$$j = (((j_1 \times r_{i_2} + j_2) \times r_{i_3} + j_3) \times r_{i_4} + j_4) \times \dots + j_K$$

In java

we have $(j_1, j_2, \dots, j_K)$ and we want to find j

```
j = j1
for (l=2, l ≤ K, l++) {
    j = j × r_il
    j = j + j_l
}
```

Given $j \implies (j_1, j_2, j_3)$?

$$j_3 = j \,\%\, r_{i_3}$$
$$j' = (j - j_3) / r_{i_3}$$
$$j_2 = j' \,\%\, r_{i_2}$$
$$j_1 = (j' - j_2) / r_{i_2}$$

In java

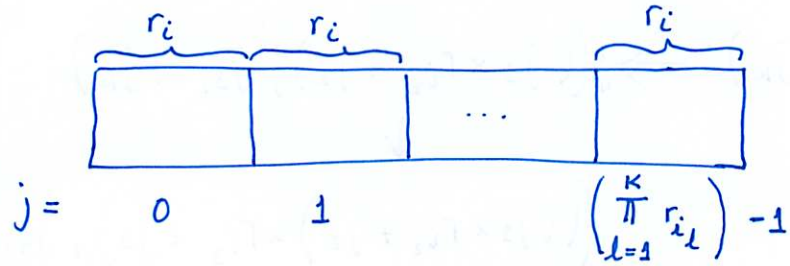
we have j and we want to find $(j_1, j_2, \dots, j_K)$

```
j' = j
for (l=K, l ≥ 2, l--) {
    j_l = j' % r_il
    j' = (j' - j_l) / r_il
}
j1 = (j' - j2) / r_i2
```

- N_{ijk} need not to be stored in memory!
- θ_{ijk} need to be stored in memory

$\downarrow$

for each X_i define an array of dimension $r_i \times \underbrace{\prod_{l=1}^{k} r_{i_l}}_{\text{parents of } X_i}$



$\rightarrow$ to find for θ_{ijk} in the array just retrieve the value at index $j \times r_i + k$