CS/MATH 335: PROBABILITY, COMPUTING, AND GRAPH THEORY
HOMEWORK 1
DUE IN CLASS FRIDAY, SEPTEMBER 23, 2016

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Partner Problems

Roughly in order of difficulty

(1) $\boxed{\text{Mitzenmacher 3.8}}$

(2) $\boxed{\text{Mitzenmacher 3.23}}$

(3) $\boxed{\text{Mitzenmacher 3.12}}$

(4) $\boxed{\text{Mitzenmacher 3.11}}$

Individual Problems

(5) $\boxed{\text{Mitzenmacher 4.6}}$

a.)

We can look at this problem as a Poisson trial, with each ballot either going to person A(0) or person B(1). With the given information of $p = 0.02$ we can easily compute the expected number of misread ballots, which is $1,000,000 * .02 = 20,000$, so our mean $= 20,000$. Letting X be the number of misread ballots we want to find the $P(X \geq 40,000)$, since $.02(1,000,000) = 40,000$. Then we compute:

$\Rightarrow (1 + \delta)\mu = 40,000$

$\Rightarrow (1 + \delta)20,000 = 40,000$

$\Rightarrow 1 + \delta = 2$

$\Rightarrow \delta = 1$

Hence,

$\Rightarrow P(X \geq 40,000) < (\frac{e^1}{2^2})^{20,000}$

$\Rightarrow P(X \geq 40,000) < 4.89(10^{-3365})$

Which is essentially,

$\Rightarrow P(X \geq 40,000) = 0$

b.)

The votes for each candidate are $A = 510,000, B = 490,000$.

Now we'll let $x =$ the number of votes for A that are misread and $y =$ the number of votes for B that are misread.

So if $x - y > 10,000$ then candidate B should have won.

$\Rightarrow E[x - y] = E[x] - E[y]$

$\Rightarrow 10,200 - 9,800 = 400$

$\Rightarrow 10,000 = (1 + \delta)400$

Now we must find δ

$\Rightarrow 25 = 1 + \delta$

$\Rightarrow \delta = 24$

$\Rightarrow Pr(x - y > 10,000) < (\frac{e^{24}}{25^{25}})^{400}$

$\Rightarrow Pr(x - y > 10,000) < 10^{-9810.2}$

Which is essentially $Pr(x - y > 10,000) = 0$