CS/MATH 335: PROBABILITY, COMPUTING, AND GRAPH THEORY
HOMEWORK 9
DUE IN CLASS THURSDAY, DECEMBER 1, 2016
ONE PROBLEM (YOUR CHOICE) DUE THURSDAY, NOVEMBER 17

Show all work to ensure full credit.

Partner Problems

Roughly in order of difficulty

(1) Mitzenmacher 6.16.

(2) Mitzenmacher 6.11.

(3) Section 6.3 analyzed the following algorithm for constructing an independent set: for every $v \in G$, delete v and all edges touching v with probability $1 - 1/d$. Let H be the set of vertices which survive this process.
   (a) Use the method of conditional expectations to turn this algorithm into a deterministic algorithm which always finds an independent set of size $n/2d$.
   (b) Let G be a 3-regular graph (i.e. all vertices have degree 3). Consider the randomized algorithm that deletes each vertex independently with probability 2/3 as above. For every edge that remains, delete one of its end-points randomly. Derive an upper bound on the probability that this algorithm finds an independent set smaller than $n(1 - \epsilon)/6$. Hint: Chernoff bounds.

(4) Mitzenmacher 6.15. Hint: this is the challenge problem. You can do each part independently of the others. So make sure to attempt all parts of this problem.

BONUS partner problem:

(5) Fix a constant $p \in (0, 1)$. Prove that almost no graph in $G(n, p)$ has a complete separating subgraph.

Individual Problems

(6) Mitzenmacher 6.8.

BONUS individual problem:

(7) Given $d \in \mathbb{N}$, is there a threshold function for the property of containing a d-dimensional cube? If so, what is the threshold function? If not, why not?