CS/MATH 335: PROBABILITY, COMPUTING, AND GRAPH THEORY
HOMEWORK 1
DUE IN CLASS FRIDAY, SEPTEMBER 16, 2016

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Partner Problems

(1) See Matt's hand in for Partner Problems

Individual Problems

(5) Mitzenmacher 2.6

1.) Notice $\mathbf{E}[X|x_1 = 2] = 5.5, \mathbf{E}[X|x_1 = 4] = 7.5$, and $\mathbf{E}[X|x_1 = 6] = 9.5$. Since each of these are equally likely to occur if x_1 is even, our answer is $\mathbf{E}[X|x_1$ is even$] = \frac{5.5+7.5+9.5}{3} = 7.5$

2.) Notice the $P(x_1 = x_2) = \frac{1}{6}$. If $x_1 = x_2$, then the possible values for X are: $2, 4, 6, 8, 10, 12$. Hence, $E[X|x_1 = x_2] = \sum_{x=2}^{12} x(\frac{1}{6}) = 7$

3.) The possible ways for $X = 9$ are the combinations of $(3, 6), (4, 5), (5, 4), (6, 3)$, so $P(X = 9) = \frac{1}{9}$. So $E[x_1|X = 9] = \sum_{x=3}^{6} x(\frac{1}{9}) = 4.5$. This makes sense because x_1 can either be $3, 4, 5, 6$ and the average value of those 4 numbers is 4.5

4.) For this problem I wasn't completely sure how to go about solving it, so I just used reasoning to get an answer. First I noticed that the Probability of $X = k$ for k in the range $[2, 12] = 1.0$. This makes sense because the lowest values you can roll on a die is 1, so $1 + 1 = 2$ and the highest possible number is 6, so $6 + 6 = 12$, hence you're guaranteed to roll a number in the range of $[2, 12]$. Now looking at $x_1 - x_2$, if $x_1 = 1$, then the possible values of $x_1 - x_2$ are $0, -1, -2, -3, -4, -5$, if $x_1 = 2$ the possible values of $x_1 - x_2$ are $1, 0, -1, -2, -3, -4$. I noticed a pattern that as x_1 increases by 1, all possible values of X shift up one in the positive direction. This means that the $E[x_1 - x_2|X = k] = 0$.