

**CS/MATH 335: PROBABILITY, COMPUTING, AND GRAPH THEORY**  
**HOMEWORK 1**  
**DUE IN CLASS FRIDAY, SEPTEMBER 23, 2016**

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**Partner Problems**

Roughly in order of difficulty

- (1) Show that the components of a graph partition its vertex set.
- (2) Mitzenmacher 4.18
- (3) Mitzenmacher 4.23
- (4) Mitzenmacher 4.10

**Individual Problems**

- (5) □

	Edges	Max Deg	Min Degree	Av. Deg
$N_n$	0	0	0	0
$P_n(n > 2)$	$n-1$	2	1	$\frac{2(n-1)}{n}$
$C_n(n > 2)$	$n$	2	2	2
$K_n(n > 2)$	$\frac{n(n-1)}{2}$	$n-1$	$n-1$	$n-1$
$K_{n,n}(n > 1)$	$n^2$	$n$	$n$	$n$

In regards to  $P_n$ ,  $C_n$  and  $K_n$ , if  $n=2$  then there is only 1 edge and the max, min and average degree is 1. If  $n=1$  or 0, then there are no edges and the max, min and average degree is 0. For  $K_{n,n}$ , if  $n=1$  then there is only 1 edge and the max, min and average degree is 1. If  $n=0$  then there are 0 edges and the max, min and average degree is 0.

(Assuming all graphs have the same number of nodes) A subgraph is made by simply removing edges from the initial graph, thus it is clear to see that  $N_n$  is a subgraph of all the other graphs since removing all edges from any graph will get you  $N_n$

$P_n$  is a subgraph of  $C_n$ ,  $K_n$ ,  $K_{n,n}$ . Removing any edge from  $C_n$  gets you  $P_n$ . Since  $K_n$  has every edge possible we can obtain  $P_n$  by removing the edges

to create a path on  $n$  nodes. You can also obtain  $P_n$  from  $K_{n,n}$  by removing the proper edges.

$C_n$  is a subgraph of  $K_n$  and  $K_{n,n}$ . Since  $K_n$  has every edge possible we can obtain  $C_n$  by removing the edges to create a cycle on  $n$  nodes. We can also obtain  $C_n$  from  $K_{n,n}$  by removing the proper edges.

$K_{n,n}$  is a subgraph of  $K_n$ . Since  $K_n$  is maximally connected we can obtain  $K_{n,n}$  by removing the proper edges.

Lastly,  $K_n$  is only a subgraph of itself since it is maximally connected.

For any of the above graphs, excluding  $N_n$ , if  $n = 2$  then its compliment is  $N_n$ . This is because there will only be one edge connecting 2 nodes, so the compliment of that graph is  $N_2$ .

The compliment of  $N_n$  is  $K_n$  (and vice versa) because  $N_n$  has no edges while  $K_n$  is maximally connected.

$C_3$ 's compliment is  $N_3$