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Problem 8.

$\Leftarrow$ Let graph G have 1 vertex and 0 edges. By definition graph G is acyclic.

Notice that any connected graph has t vertices and $t-1$ edges, where $t \in \mathbb{Z}$. Accordingly the sum of all vertex degrees in G = 2(E(G)) = 2(V(G))-2. Therefore there exists a vertex with a degree ¡ 2. Similarly a vertex's degree can't = 0 or else the graph would be disconnected, so the degree = 1. Consider the graph $G - v$. $G - v$ has $V(G) - 1$ vertices and $E(G) - 1 = V(G) - 2$ edges. By the induction hypothesis it is acyclic. Adding v to $G - v$ can't create a cycle because the cycle would traverse v's edge twice. Therefore G is acyclic.

$\Rightarrow$ Let graph G have 1 vertex, this means there is at most 0 edges. By definition graph G is acyclic.

Consider an acyclic connected graph with more than 1 vertices. If we choose a random vertex v_0 and follow a path$(v_1, v_2..)$ by picking a vertex v_{i+1} that is adjacent to vertex v_i and is not already in the path. This path will end at some vertex v_k. If v_k is adjacent to some vertex v_t where $v_t \neq v_{k-1}$ then a cycle has been discovered. Also, if v_k has an adjacent vertex that isn't in the path, then the path wouldn't have ended at v_k it would have ended at v_{k+1} So v_k is adjacent to only v_{k-1} and therefore has a degree of 1. By removing v_k to form $G - v_k$ which is an acyclic graph with V(G)-1 vertices and E(G)-1 edges. By the induction hypothesis we have E(G)-1 = V(G)-2 which means the connected acyclic graph G has edges = V(G)-1

Problem 10.

$\Leftarrow$ Let T be a tree. By definition it is connected (any 2 vertices are joined by at least one path)

If any 2 vertices, $uandq$ of T are joined by 2 or more paths then a cycle is produced.

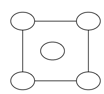
This creates a contradiction with the definition of a tree.

$\Rightarrow$ Let T be a graph where any 2 vertices are connected by a unique path.

Thus T is connected. However, for any 2 vertices,if a cycle contains $uandq$ then u, q are connected by more than 1 path. This produces a contradiction.

Problem 15.

If a graph G has n vertices and $n-1$ edges it does not have to be a tree. For a Graph to be a tree it must be connected and acyclic. If graph G has $n-1$ edges while also being connected, it is impossible for G to be cyclic. However, simply being acyclic does not make Graph G a tree. Graph G still must connected and since the question only defines graph G as having n vertices and $n-1$ edges we cannot make any assumptions of G being connected. The image below shows a graph that has 5 vertices and 4 edges. This graph is disconnected and cyclic therefore it is not a tree and proves that a graph having n vertices and $n-1$ edges does not make that graph a tree.



notTree.png

Problem 24.

Assume we have a nontrivial tree G By definition of a tree, every vertex,v in G has one unique path to every other vertex, q in G By deleting any edge in the path (v, q), vertex v and every vertex it still has a path to is no longer connected to vertex q and every vertex it still has a path to. Therefore there are now exactly 2 separate components after the deletion of any edge in graph G