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1.

Let G be a connected graph

Consider a $u-v$ walk in G of length n

Then there exists $v \in V(G)$ and there exists $u \in V(G)$

Since G is connected there also exists a $u-v$ path of length m

Case 1: The $u-v$ walk and the $u-v$ path contain the same alternating sequence of vertices and edges as one another.

Since the $u-v$ walk and the $u-v$ path both share identical alternating sequences of vertices and edges

Then $m = n$

Case 2: The $u-v$ walk and the $u-v$ path contain different sequences of vertices as one another.

Then there exists a vertex $q \in$ the $u-v$ walk that is not in the $u-v$ path

Therefore the length of the $u-v$ walk, n, is made up of $d(u, q)$ and $d(q, v)$

By the definition of the triangle inequality length $m, d(u, v) \leq d(u, q) + d(q, v)$ for all u, v, q

Therefore, if a graph G has a $u-v$ walk of length n, then G has a $u-v$ path of length $m \leq n$

2.

Consider a graph G

Notice $rad(G)$ is the minimum eccentricity among all vertices in G and $diam(G)$ is the maximum eccentricity among all vertices in G

Hence $rad(G) \leq diam(G)$

Suppose the distance between two vertices $u, v = diam(G)$ and the eccentricity of vertex $t = rad(G)$

By definition of the triangle inequality the $diam(G) \leq d(u, t) + d(v, t)$ because $diam(G)$ is the length of the shortest path between u, v, any other path from $u-v$ must be as long or longer than $diam(g)$

$d(u,t) + d(v,t) \leq rad(G) + rad(G)$ because $rad(G)$ is the maximum length of a path from t to any other vertex $\in G$

Therefore $rad(G) \leq diam(G) \leq 2rad(G)$

3. Consider a tree T

Suppose a pair of vertices $u, v \in V(T)$ are joined by a diametral path

By definition of Theorem 4.2, a tree has wither 1 central vertex or 2 adjacent central vertices

Case 1: Graph T has 1 central vertex c

Suppose by contradiction that every diametral path in T does not include all central vertices.

Hence there exists a $u - v$ path that does not contain c

Since T is a tree we know the paths $c - u$ and $c - v$ exist

Therefore a $u - v$ path containing c exists, which forms a contradiction with our hypothesis and also forms a cycle which cannot occur in a tree.

Case 2: Graph T has 2 adjacent, central vertices $c1, c2$

Suppose by contradiction that every diametral path in T does not include all central vertices.

Hence there exists a $u - v$ path that does not contain $c1$ and $c2$

Since T is a tree we know the paths $c1 - u$ and $c2 - v$ exist, since $c1$ is adjacent to $c2$ the path $u - v$ containing $c1, c2$ exists

Therefore a $u - v$ path containing $c1, c2$ exists, which forms a contradiction with our hypothesis and also forms a cycle which cannot occur in a tree.

Therefore in any tree T, every diametral path includes all central vertices of T