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1. Combinatorial Argument Proof for Theorem 1.1 (The sum of the entries of row n of Pascal's triangle is 2^n)

   PF

   Notice $(a+b)^n = (a+b)(a+b).......(a+b)$, where $(a+b)$ is written out n times Example: $(a+b)^4 = (a+b)(a+b)(a+b)(a+b)$

   The terms in $(a+b)^n$ have the form of $a^{n-k}b^k$ where k is a number between 0 and n

   The coefficient of $a^{n-k}b^k$ for some k is the number of ways to choose k factors of b from n factors of $(a+b)$

   The factors of a come from the remaining $(n-k)$

   Example: $a^4 = 1(aaaa)$

   $a^3b^1 = 4(aaab)(aaba)(abaa)(baaa)$

   $a^2b^2 = 6(aabb)(abba)(bbaa)(baba)(baab)(abab)$

   This can be written as $\binom{n}{k}$

   Therefore $(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$

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2. Induction Proof

   PF

   Let $n = 1$

   $(a+b)^1 = \binom{1}{0}a^{1-0}b^0 + \binom{1}{1}a^{1-1}b^1 = \sum_{k=0}^{1}\binom{1}{k}a^{n-k}b^k$

   Assume $(a+b)^t = \sum_{k=0}^{t}\binom{t}{k}a^{t-k}b^k$

   We show that $(a+b)^{t+1} = \sum_{k=0}^{t+1}\binom{t+1}{k}a^{(t+1)-k}b^k$

   $(a+b)^{t+1} = (a+b)^t(a+b)$

   $= \sum_{k=0}^{t}\binom{t}{k}a^{t-k}b^k(a+b)$

$= \sum_{k=0}^{t} \binom{t}{k} a^{t-k} b^k (a+b)$

$= a \sum_{k=0}^{t} \binom{t}{k} a^{t-k} b^k + b \sum_{k=0}^{t} \binom{t}{k} a^{t-k} b^k$

$= \sum_{k=0}^{t} \binom{t}{k} a^{t-k+1} b^k + b \sum_{k=0}^{t} \binom{t}{k} a^{t-k} b^k + 1$

according to the summations, the coefficient of $a^{t+1-k} b^k$ is $\binom{t}{k} + \binom{t}{k-1}$

This can be rewritten as $\binom{t+1}{k}$ by Formula 1.6

By the Principle of Mathematical Induction, the Formula is proven.

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3. Induction Proof for Theorem 1.1

   PF

   Formula 1.6$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

   Let $n = 0$

   $\sum_{k=0}^{0} \binom{0}{k} = 2^0 = 1$

   Assume $n > 0$ and that the theorem is true for $n - 1$

   Then $\sum_{k \in \mathbb{Z}}^{n} \binom{n}{k} = \sum_{k \in \mathbb{Z}}^{n} [\binom{n-1}{k-1} + \binom{n-1}{k}$

   $=\sum_{k \in \mathbb{Z}}^{n} [\binom{n-1}{k-1} + \sum_{k \in \mathbb{Z}}^{n} \binom{n-1}{k}$

   $= 2^{n-1} + 2^{n-1}$

   $= 2^n$

   The Principal of Mathematical Induction the Theorem is proven

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4. Combinatorial Argument Proof for Theorem 1.1

   PF

   Notice that Pascal's Triangle has borders of all 1's

   This is true because the coefficients of the first and last elements in Pascal's Triangle can be written as

   $\binom{n}{0}$, where $n \in \mathbb{Z}$ for the first element.

The Last element can be written as $\binom{n}{n}$ where $n \in \mathbb{Z}$

$\binom{n}{0}$ and $\binom{n}{n}$ both $= 1$, hence every first and last element of Pascal's triangle is 1

Internally the numbers in Pascal's triangle are found by adding the two numbers above it

Because row two of Pascal's triangle is 1 1, the middle element of row three is 2

This can be written as $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Therefore to add an entire row of Pascal's triangle together we can use the formula $\sum_{k=0}^{n} \binom{n}{k}$

Notice $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

Therefore the sum of the entries of row n of Pascal's triangle is 2^n

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5. 1.1.29

   PF

   equation 1.14 $= (1+x)^n = \binom{n}{0} + \binom{n}{1}x^1 + \binom{n}{2}x^2 ..... \binom{n}{n}x^n$

   $[(1+x)^n]' = n(1+x)^{n-1}$

   $[\binom{n}{0} + \binom{n}{1}x^1 + \binom{n}{2}x^2 ..... \binom{n}{n}x^n]' = [\binom{n}{1} + \binom{n}{2}2x + \binom{n}{3}3x^2 ..... \binom{n}{n}nx^{n-1}$

   Which can be written as $n(1+x)^{n-1} = \sum_{k=1}^{n} k\binom{n}{k}x^{k-1}$

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