

Name: Taylor Heilman

Due:

Friday, Sept 18

Math 210: Homework 2 - Spring 2015

1. Problem 1.8: Let $A = \{n \in \mathbb{Z} \mid 2 \leq |n| < 4\}$, $B = \{x \in \mathbb{Q} \mid 2 < x \leq 4\}$,
 $C = \{x \in \mathbb{R} \mid x^2 - (2 + \sqrt{2})x + 2\sqrt{2} = 0\}$ and $D = \{x \in \mathbb{Q} \mid x^2 - (2 + 2\sqrt{2})x + 2\sqrt{2} = 0\}$

(a) Describe A by listing its elements.

$$A = \{-3, -2, 2, 3\}$$

(b) Give an example of three elements that belong to B but do not belong to A .

$$\{5/2, 7/3, 8/3\}$$

(c) Describe the set C by listing its elements.

$$C = \{\sqrt{2}, 2\}$$

(d) Describe the set D in another manner.

$$D = \{x \in \mathbb{Q} \mid -(\sqrt{2} - x)(x - 2) = 0\}$$

(e) Determine the cardinality of each of the sets A , B , C , and D .

$$|A| = 4, |B| = \infty, |C| = 2, |D| = 0$$

2. Problem 1.18: For $A = \{x \mid x = 0 \text{ or } x \in \mathcal{P}(\{0\})\}$, determine $\mathcal{P}(A)$.

$$\mathcal{P}(A) = \{\emptyset, 0, \{0\}, \{\{0\}\}\}$$

3. Problem 1.30: Let $A = \{x \in \mathbb{R} \mid |x - 1| \leq 2\}$, $B = \{x \in \mathbb{R} \mid |x| \geq 1\}$, and
 $C = \{x \in \mathbb{R} \mid |x + 2| \leq 3\}$.

(a) Express A , B , and C using interval notation.

$$A = [-1, 3] \quad B = (-\infty, -1) \cup (1, \infty) \quad C = [-5, 1]$$

(b) Determine each of the following sets using interval notation: $A \cup B$, $A \cap B$,

$$B \cap C, B - C.$$

$$A \cup B = (-\infty, \infty)$$

$$A \cap B = (1, 3]$$

$$B \cap C = [-5, -2]$$

$$B - C = (-\infty, -5) \cup (1, \infty)$$

4. For $i \in \mathbb{Z}$, let $A_i = \{i - 1, i + 1\}$. Determine the following:

$$(a) \bigcup_{i=1}^5 A_{2i}$$

$$A_2 \cup A_4 \cup A_6 \cup A_8 \cup A_{10}$$

$$A = \{1, 3, 5, 7, 9, 11\}$$

$$(b) \bigcup_{i=1}^5 (A_i \cap A_{i+1})$$

$$(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) \cup (A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6)$$

$$(A_i \cap A_{i+1}) = \{1, 2, 3, 4, 5, 6\}$$

$$(c) \bigcup_{i=1}^5 (A_{2i-1} \cap A_{2i+1})$$

$$(A_1 \cup A_3 \cup A_5 \cup A_7 \cup A_9) \cup (A_3 \cup A_5 \cup A_7 \cup A_9 \cup A_{11})$$

$$(A_{2i-1} \cap A_{2i+1}) = \{2, 4, 6, 8, 10\}$$

5. For $A = \{a \in \mathbb{R} \mid |a| \leq 1\}$ and $B = \{b \in \mathbb{R} \mid |b| = 1\}$, give a geometric description of the points in the xy -plane belonging to $(A \times B) \cup (B \times A)$.

The points in the set $(A \times B)$ will create two parallel, horizontal lines intersecting the Y-axis at 1 and -1. The points in the set $(B \times A)$ will create two parallel, vertical lines that intersect the X-axis at 1 and -1. The union of the two will create 4 points at $(-1,-1)$, $(-1,1)$, $(1,-1)$, $(1,1)$