Name: Taylor Heilman **Due: Wed. Oct 7**

Math 210: Homework 4 - Spring 2015

1. (Problem 4.36) Prove that for every positive real number x that $1 + \frac{1}{x^4} \geq \frac{1}{x} + \frac{1}{x^3}$.

   PF

   $x^4(1 + \frac{1}{x^4} \geq \frac{1}{x} + \frac{1}{x^3})$

   $\Rightarrow x^4 + 1 \geq x^3 + x$

   $\Rightarrow x^4 + 1 - x^3 - x \geq 0$

   $\Rightarrow x^3(x + \frac{1}{x^3}) - (x^3 - x) \geq 0$

   $\Rightarrow (x^3 - 1)(x - 1) \geq 0$

   Since $(x^3-1)(x-1) \geq 0$, it follows that $x^4+1-x^3-x \geq 0$ and so $x^4+1-x^3-x \geq 0$

   Dividing this by x^4 we get the desired result.

   $1 + \frac{1}{x^4} \geq \frac{1}{x} + \frac{1}{x^3}$.

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2. (Problem 4.46) Let A and B be sets. Prove that $A \cup B = A \cap B$ if and only if $A = B$.

   PF

   ($\Rightarrow$) By Contrapositive

   Assume $A \neq B$

   Let $a \in A$ but $a \notin B$

   Then $a \in A \cup B$

   and $a \notin A \cap B$

   So $A \cup B \neq A \cap B$

   ($\Leftarrow$)

   Given A = B

So for every $a \in A$ then there is also $a \in B$

Then $A \cup B = A = B$

and $A \cap B = A = B$

Therefore $A \cup B = A \cap B$

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3. (Problem 4.56) Let A, B and C be sets. Prove that $(A-B) \cup (A-C) = A-(B \cap C)$.

   PF

   $(\subseteq)$

   Let $x \in (A-B) \cup (A-C)$

   wlog Assume $x \in A \times B$

   By definition $x \in A$ and $x \notin B$

   Similarly $x \in A$ and $x \notin C$

   Therefore $x \in (A-B) \cup (A-C)$

   $(\supseteq)$

   Let $x \in A - (B \cap C)$

   By definition $x \notin B \cap C$

   Then $x \in A$ and either $x \notin B$ and $x \in C$, or $x \in B$ and $x \notin C$, or $x \notin B$ and $x \notin C$

   Case 1: $x \notin B$ and $x \in C$

   Since $x \notin B$

   By definition $x \notin B \cap C$

   Case 2: $x \in B$ and $x \notin C$

   Since $x \notin C$

   By definition $x \notin B \cap C$

Case 3: $x \notin B$ and $x \notin C$

Since $x \notin C$ and $x \notin C$

By definition $x \notin B \cap C$

Therefore $x \in A - (B \cup C)$

So $(A - B) \cup (A - C) = A - (B \cup C)$

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4. (Problem 4.58) Let A, B and C be sets. Prove that $A \cap \overline{(B \cap \overline{C})} = \overline{(\overline{A} \cup B) \cap (\overline{A} \cup \overline{C})}$.

   PF

   $A \cap \overline{(B \cap \overline{C})}$

   $= A \cap (\overline{B} \cup \overline{\overline{C}})$

   $= (A \cap \overline{B}) \cup (A \cap \overline{\overline{C}})$

   $= \overline{(A \cup \overline{B})} \cap \overline{(A \cup \overline{\overline{C}})}$

   $= \overline{(\overline{A} \cup B) \cap (\overline{A} \cup \overline{C})}$.

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5. (Problem 4.68) Let A, B, C and D be sets. Prove that
   $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

   PF

   $(\subseteq)$

   Let $(x, y) \in (A \times B) \cap (B \times D)$

   Then $(x, y) \in A \times B$ and $(x, y) \in B \times D$

   Thus $(x, y) \in A \times B$ and $(x, y) \in C \times D$

   By definition $(x, y) \in (A \times B) \cap (C \times D)$

   $(\supseteq)$

   Let $(x, y) \in (A \cap C) \times (B \cap D)$

thus $x \in (A \cap C)$ and $y \in (B \cap D)$

Then $x \in A$, $x \in C$

Also $y \in B$ and $y \in D$

By definition $(x, y) \in (A \cap C) \times (B \cap D)$

So $(A \times B) \cap (B \times D) = (A \cap C) \times (B \cap D)$

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