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Math 210: Homework 8 - Fall 2015

1. Problem 9.12(c):

   PF

   Let $x \in f(C) - f(D)$

   Therefore $x \in f(C)$ and $x \notin f(D)$

   So $x = f(y)$, where $y \in C$ and $x \notin f(D)$

   Then if $y \in D, x = f(y)$ then $x \in f(D)$ this is a contradiction because we know $x \notin f(D)$

   So $y \notin D$

   Hence $x = f(y)$, where $y \in C$ and $y \notin D$

   Therefore $x = f(y)$ where $y \in C - D$

   So $x \in f(C - D)$

   Thus $x \in f(C) - f(D)$

   $= x \in f(C - D)$

   So $f(C) - f(D) \subseteq f(C - D)$

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2. Problem 9.12(f):

   PF

   (WTS: $f^{-1}(E - F) \subseteq f^{-1}(E) - f^{-1}(F)$ and $f^{-1}(E) - f^{-1}(F) \subseteq f^{-1}(E - F)$)

   $\Rightarrow$

   Let $x \in f^{-1}(E - F)$

   So $x = f^{-1}(y) \exists y \in E - F$ such that

   $y \in E, Y \notin F$

$\Rightarrow x = f^{-1}(y)$ where $y \in E$, $y \notin F$

$\Rightarrow x = f^{-1}(y)$ where $y \in E$, $x = f^{-1}(y)$ where $y \notin F$

$\Rightarrow x = f^{-1}(E)$, $x \notin f^{-1}(F)$

$\Rightarrow x = f^{-1}(E) - f^{-1}(E)$

So $x \in f^{-1}(E - F)$

$\Rightarrow x \in f^{-1}(E) - x \in f^{-1}(F)$ for every x

Hence $f^{-1}(E - F) \subseteq f^{-1}(E) - f^{-1}(F)$

$\Leftarrow$

Let $x \in f^{-1}(E) - f^{-1}(F)$

So $\exists$ and $\notin$ $y \in E$ and $y \notin F$ such that $x = f^{-1}(y)$

$\Rightarrow x = f^{-1}(y)$ where $y \in E - F$

$\Rightarrow x \in f^{-1}(E - F)$

Hence $x \in f^{-1}(E) - f^{-1}(F)$

$\subseteq f^{-1}(E - F)$

Therefore $f^{-1}(E - F) = f^{-1}(E) - f^{-1}(F)$

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3. Problem 9.40:

   PF

   Let $x \in A$

   Then $(f \text{ o } i_A)(a) = f(i_A(a))$

   $= f(a)$

   Similarly $(i_B \text{ o } f)(a) = i_B(f(a))$

   $= f(a)$

   Therefore $(f \text{ o } i_A) = f$ and $(i_B \text{ o } f) = f$

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4. Problem 9.46:

PF

(a.)

$(g \text{ o } f)(3,8) = g(f(3,8))$

$= g(3(3) - 8, 3 + 8)$

$= g(1, 11)$

$=(1 - 11, 2(1) + 11)$

$=(-10, 13)$

Hence $(g \text{ o } f)(3,8) = (-10, 13)$

(b.)

Let $(w, x), (y, z) \in AxB$

Assume $(g \text{ o } f)(w, x) = (g \text{ o } f)(y, z)$

So $g(f(w, x)) = g(f(y, z))$

Then $g(3w - x, w + x) = g(3y - z, y + z)$

Thus $(3w - x - w - x, 6w - 2x + w + x) = (3y - z - y - z, 6y - 2z + y + z)$

So $(2w - 2x, 5w - x) = (2y - 2z, 5y - z)$

Hence $2w - 2x = 2y - 2z$ and $5w - x = 5y - z$

Therefore $w - y = x - z$ and $5(w - y) = x - z$

So $w - y = 5(w - y)$

$4(w - y) = 0$

$w = y$

Plugging this into the first equation 4 lines above

$0 = x - z$

$x = z$

Hence $(w, x) = (y, z)$

So the function is one to one

(c.)

Let $(x, y) \in BxA$

So x, y are odd and even integers

Hence $a = \frac{2y-x}{12} \in A$ and $b = \frac{2y-7x}{12} \in B$

Therefore $g(\text{ o } f)(a, b) = g(f(a, b))$

$= g(f(\frac{2y-x}{12}, \frac{2y-7x}{12}))$

$= g(3(\frac{2y-x}{12}) - \frac{2y-7x}{12}, \frac{2y-x}{12} + \frac{2y-7x}{12})$

$= g(\frac{4y+4x}{12}, \frac{4y-8x}{12})$

$= g(\frac{y+x}{3}, \frac{y-2x}{3})$

$= (\frac{y+x}{3} - \frac{y-2x}{3}, 2(\frac{y+x}{3}) + \frac{y-2x}{3})$

$= (x, y)$

So the function is onto

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5. Problem 9.58:

PF

Let $g : B \to A$ be surjective

Then $g(B) = A$

So $g(y) = x$

Consider $(f \text{ o } g)(y) = f(g(y))$

$= f(x)$

$= y$

$= I_B(y)$

Also $(g \text{ o } f)(x) = g(f(x))$

$= g(y)$

$= x$

$= I_A(x)$

Therefore $(g \text{ o } f)(x) = I_A(x))$

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