Piano Note Recognition using FFT

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Abstract

Songs play a vital role in our day to day life. A song contains two things, vocal and background music. Where the characteristics of the voice depending on the singer and in case of background music, it involves a mixture of different musical instruments like piano, guitar, drum, etc. To extract the characteristic of a song becomes more critical for various objectives like learning, teaching, composing. This project takes a known note as an input, extracts the features and detects and identifies the notes. First, the note is recorded, and digital signal processing algorithms used to identify the characteristics. The experiment is done with the several piano songs where the notes are already known, and identified notes are compared with original notes until the detection rate goes higher.

1. Introduction

Music is a ubiquitous and vital part of the lives of billions of people worldwide. Musical creations and performances are among the most complex and intricate of our cultural artefacts, and the emotional power of music can touch us in surprising and profound ways. The music spans an enormous range of forms and styles, from simple, unaccompanied folk songs, to orchestras and other large ensembles, to a minutely constructed piece of electronic music resulting from months of work in the studio.

The recognition of music has been an area of interest to many. Much effort has been invested in the identification of sound sources, such as the type of instruments played, and the automatic transcription of musical pieces. In recent years, with the proliferation of personal computers and multimedia systems, research in these areas have gained increasing interest.

This is a small project that takes a note as an input, extract the feature and identifies the note. First, the note is recorded, and a digital signal processing algorithm used to identify the characteristics. The experiment is done with the several piano notes where the notes are already known, and identified notes are compared with original notes.

We have used **FFT(Fast Fourier Transform)** , **DTFT(Discrete Time Fourier Transform)** and **Sampling** from Signals and Systems.

2. Piano Note Identification using Fourier Analysis

In this project database, we have 88 files(.wav) each of which represents different single piano notes. This section contains 3 sub sections.

2.1 Block Diagram

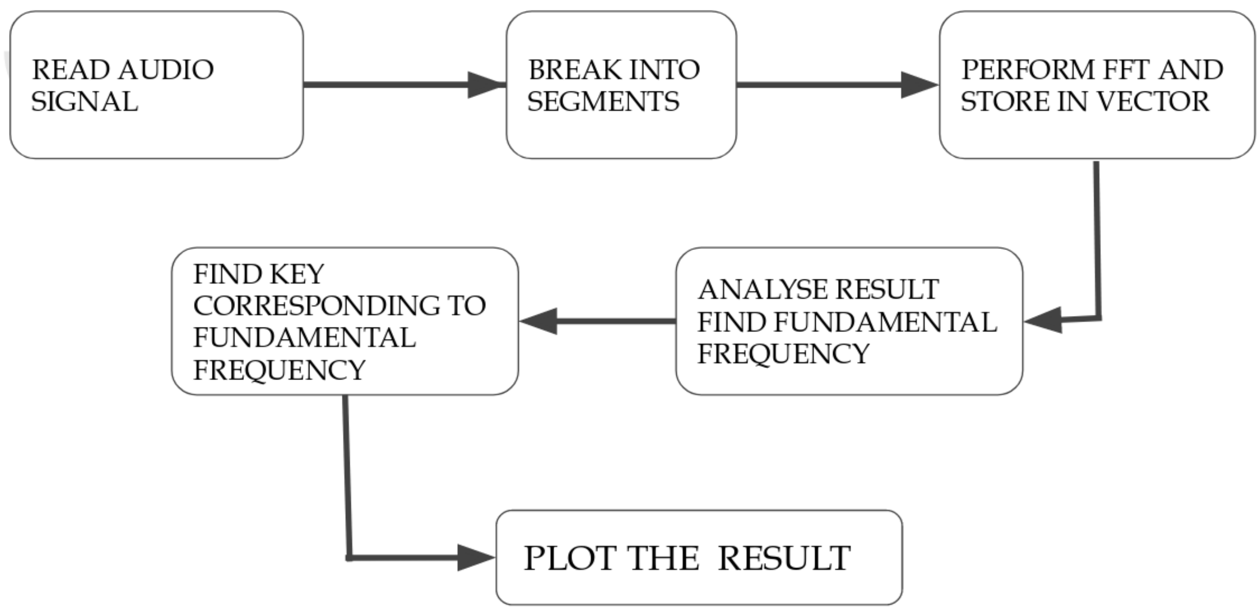


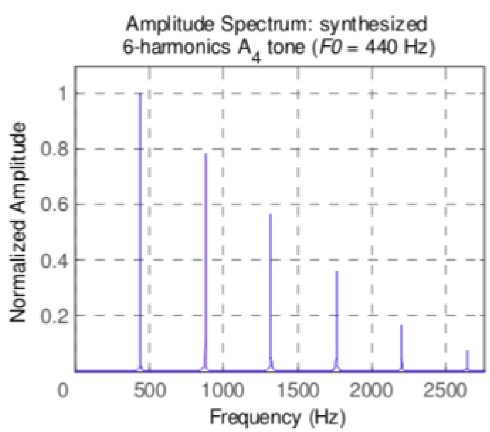
Fig 1: Flow/Block Diagram of the algorithm

2.2 Fundamental Frequency:

The fundamental frequency, often referred only as fundamental, is defined as the lowest frequency of a periodic waveform. In music, the fundamental is the musical pitch of a note that is perceived as the lowest partial present. In terms of a superposition of sinusoids, the fundamental frequency is the lowest frequency sinusoidal in the sum.

2.2.1 Missing Fundamentals

It is a situation when overtones suggest a fundamental frequency, but the sound lacks a component at the fundamental frequency itself. For example, when a note (that is not a pure tone) has a pitch of 100 Hz, it will consist of frequency components that are integer multiples of that value (e.g. 100, 200, 300, 400, 500.... Hz). However, smaller loudspeakers may not produce low frequencies, and so in our example, the 100 Hz component may be missing.



In a modern Piano, there are 88 keys with the 49th key as A4 440Hz.

Notice that these frequencies are all integral multiples of the smallest frequency 440Hz. In a real-world scenario, the standard frequency may vary by 0 - 0.5Hz.

Fig 2.1: Fourier Transformation of A4 piano key

The following equation gives the frequency f of the n^{th} key, as shown in the table:

$$f(n) = \left(\sqrt[12]{2}\,\right)^{n-49} \times 440\,\mathrm{Hz}$$

(a' = A_4 = A440 is the 49th key on the idealized standard piano)

Alternatively, this can be written as:

$$f(n) = 2^{\frac{n-49}{12}} \times 440\,\mathrm{Hz}$$

Fig 2.2: Relationship between fundamental frequency and node number

2.3 FFTs

Whether for good or ill, we have come to live in a digital world. The audio signals of piano notes discussed above were recorded in digital form, and their Fourier spectra were computed digitally. The method of digitally computing Fourier spectra is widely referred to as the FFT (short for fast Fourier transform). An FFT provides an extremely efficient method for computing approximations to Fourier series coefficients; these approximations are called DFTs (short for discrete Fourier transforms).

$$\begin{aligned} c_n &\approx \frac{1}{\Omega} \sum_{k=0}^{N-1} f(t_k) e^{-i2\pi n t_k/\Omega}\, \Delta t \\ &= \frac{1}{N} \sum_{k=0}^{N-1} f(t_k) e^{-i2\pi nk/N}. \end{aligned}$$

The last quantity above is the DFT of the finite sequence of numbers $\{f(t_k)\}$. That is, we define the DFT of a sequence $\{f_k\}$ of N numbers by

$$F[n] = \frac{1}{N} \sum_{k=0}^{N-1} f_k\, e^{-i2\pi nk/N}.$$

The DFT is the sequence of numbers {F[n]}, and we see from the discussion above that a DFT can approximate the Fourier coefficients of a function f.

Two unique properties of DFTs are: (1) they can be inverted, and (2) they preserve energy (up to a scale factor). The inversion formula for the DFT is

$$f_k = \sum_{n=0}^{N-1} F[n] e^{i2\pi nk/N}$$

and the conservation of energy property is

$$\frac{1}{N} \sum_{k=0}^{N-1} |f_k|^2 = \sum_{n=0}^{N-1} |F[n]|^2.$$

2.4 Algorithm

We will detect notes using FFT (Fast Fourier Transform) and featured vectors.

Let s_i be a 44100 Hz signal, where $i = 1 \ldots 4096$

The length of the signal would be $\frac{4096}{44100} = 0.0929s \approx 0.1s$

Now do FFT of s_i and get vector f_i, where $i = 1 \ldots 4096$

FFT works over complex number so even if we put real signal then result will still be complex.
Now get the absolute value of f_i :

$$a_i = |(f_i)| = \sqrt{\left(\Re(f_i)\right)^2 + \left(\Im(f_i)\right)^2}$$

Only the first half of vector makes the spectrum, the higher frequencies are of no use.

We need to find the four most loud signal from given signals. Now, we have spectrum's of notes: $a_{j,i}$, where $j = 1 - 60$ note number , $i = 1 - 2048$ frequency vector.

Let us imagine these spectrums as vectors in 2048d space. We can cut the spectrums into 327 indexes (3.5 kHz) because higher frequencies were of no use. So, now we have 327d space and 60 vectors in it.

We also have $0.0929s$ input signals, s_i $i = 1 - 4096$ that have spectrum c_i $i = 1 - 2048$. We know that the sum of several notes can be considered as the sum of spectrums of these notes. It's because spectrum changes with time and one note always have a little different structure with each case push, because we are working with naturally generated signal.

Now, cut the essential part of spectrum ($i = 1 - 327$)

We can calculate scalar product of these vectors. Say $(\overrightarrow{a_1}, \overrightarrow{a_1}) = 1$, $(\overrightarrow{a_1}, \overrightarrow{a_2}) < 1$

It is considered that vectors are already normalized:

$a_{j,i} \leftarrow \frac{a_{j,i}}{\sqrt{\sum_i a_{j,i}^2}}$, so scalar product can be used in detection.

3. Results and Observations

Frequency Detected(Hz)	Frequency Assigned(Hz)	Output Notes
525.75590	523.2511	C5
496.09542	493.88330	B4
441.54419	440	A4
701.76803	698.4564	F5
662.25688	659.2551138	E5
590.30444	587.3295358	D5
788.3985	783.9905	G5

4. Conclusions and Limitations

We have used various application of signal and systems for identifying the given piano notes. We were successful in identifying a single piano note. Our project is limited to identifying a single piano note only. In the case of two or more note played simultaneously, our project fails to identify both the notes. This project can be further improved so that it can identify two or more notes if played simultaneously.

References

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