



EDEXCEL/AQA

# MATHS/FM

## Edexcel Maths Exams

1	Non-Calculator Paper	<b>33%</b> 105 MINS 80 MARKS
2	Calculator Paper	<b>33%</b> 105 MINS 80 MARKS
3	Calculator Paper	<b>33%</b> 105 MINS 80 MARKS

## AQA Further Maths Exams

1	Non-Calculator Paper	<b>50%</b> 90 MINS 70 MARKS
2	Calculator Paper	<b>50%</b> 90 MINS 70 MARKS

# 1 NUMBER

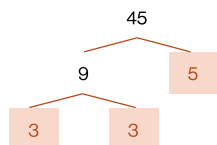
## Types of Numbers

<b>Integers</b>	whole numbers, positive, negative and zero
<b>Rational</b>	can be expressed as a fraction ( <i>integers as numerator and denominator</i> )
<b>Irrational</b>	numbers which cannot be written as fractions
<b>Surds</b>	numbers left in the form of $\sqrt{n}$
<b>Prime</b>	numbers with exactly two factors

## Prime Factors

A number can be decomposed into its prime factors.

Manual Method:



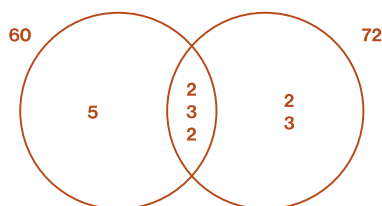
$$45 = 3^2 \times 5$$

## LCM and HCF

To find the *lowest common multiple* and *highest common factor* of two numbers, split the numbers into its prime factors, then sort it into a venn diagram.

$$72 = 2^3 \times 3^2$$

$$60 = 2^2 \times 3 \times 5$$



### Highest Common Factor

Multiply the numbers in the overlap of the venn diagram.

$$\text{HCF of 60 and 72} = 2 \times 3 \times 2 = 12$$

### Lowest Common Multiple

Multiply all numbers in the venn diagram.

$$\text{LCM of 60 and 72} = 5 \times 2 \times 3 \times 2 \times 2 \times 3 = 360$$

## Standard Form

Standard form is a way to describe very large numbers in the form of  $a \times 10^n$ , where  $a$  is a number between 1-10, and  $n$  is an integer.

An easy method to use standard form is to move the **decimal place n times to the right**.  
 $1.6 \times 10^{-4} = 0.16 \times 10^{-3} = 0.016 \times 10^{-2} = 0.0016 \times 10^{-1} = 0.00016$

## Operators

=	equal to
≈	approximately equal to
±	plus or minus
<	less than
≤	less than or equal to
>	greater than
≥	greater than or equal to

## Surds

### Simplifying Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

### Rationalising the Denominator

$$\frac{14}{\sqrt{7}} = \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{14\sqrt{7}}{7} = 2\sqrt{7}$$

$$\frac{2}{3\sqrt{20}} = \frac{2}{3\sqrt{20}} \times \frac{\sqrt{20}}{\sqrt{20}} = \frac{2\sqrt{20}}{3 \times 20} = \frac{2 \times \sqrt{4} \times \sqrt{5}}{60} = \frac{4\sqrt{5}}{60} = \frac{\sqrt{5}}{15}$$

As we know that  $(a + b)(a - b) = a^2 - b^2$ ,

$$\frac{1}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}} = \frac{2 - \sqrt{5}}{4 - 5} = \sqrt{5} - 2$$

## Fractions, Decimals and Percentages

To convert decimals to percentages, multiply the decimal by 100.  
 To convert fractions to decimals, divide the numerator by the denominator.

### Recurring Decimals to Fractions

$$a = 0.004\dot{5}$$

$$100a = 0.4\dot{5}$$

$$99a = 0.45$$

$$a = \frac{0.45}{99} = \frac{45}{9900} = \frac{1}{220}$$

## Significant Figures and Decimal Places

### Significant Figures

Every digit of a number, regardless of its positioning.  
 In a question, if the degrees of accuracy is not specified, work to *three significant figures*.

### Decimal Places

The digits of a number that come after the decimal place.

# 2 ALGEBRA

## Indices

### Laws of Indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$\frac{1}{a^n} = a^{-n}$$

$$\frac{a^m}{a^n} = a^{\frac{m}{n}}$$

$$a^0 = 1$$

## Graphs

### Equation of a Graph

$$y = mx + c$$

Where  $m$  is the gradient, and  $c$  is the y-intercept.

For the line  $y = mx + c$ ,

$y = mx + d$  is a parallel line.

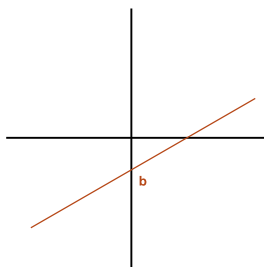
A perpendicular line has the gradient  $-\frac{1}{m}$ .

For the co-ordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ , the gradient is  $\frac{y_2 - y_1}{x_2 - x_1}$ .

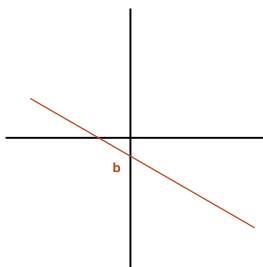
## Identifying Graphs

### Linear Graphs

$$y = ax + b, \text{ when } a > 0$$

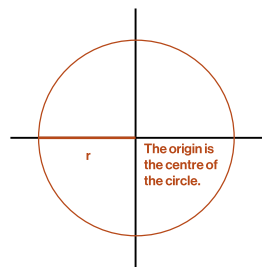


$$y = ax + b, \text{ when } a < 0$$

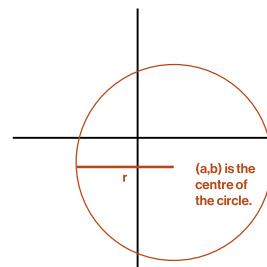


### Circle Graphs

$$x^2 + y^2 = r^2$$

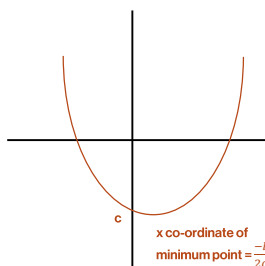


$$(x - a)^2 + (y - b)^2 = r^2, \text{ centre } (a, b)$$

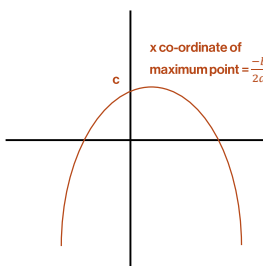


### Quadratic Graphs

$$y = ax^2 + bx + c, \text{ when } a > 0$$

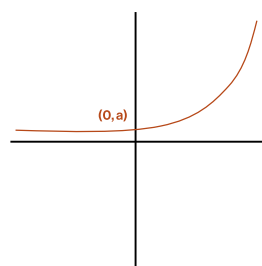


$$y = ax^2 + bx + c, \text{ when } a < 0$$



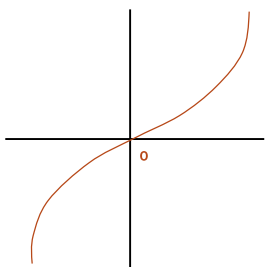
### Exponential Graphs

$$y = a \times b^x, \text{ where } b \text{ is a constant}$$

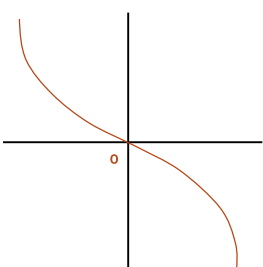


### Cubic Graphs

$$y = ax^3, \text{ when } a > 0$$



$$y = ax^3, \text{ when } a < 0$$



## Turning Points

A turning point in a graph is where the gradient changes sign (*i.e. gradient = 0*).

When the gradient goes from positive to negative, this is a **local maximum**.

When the gradient goes from negative to positive, this is a **local minimum**.

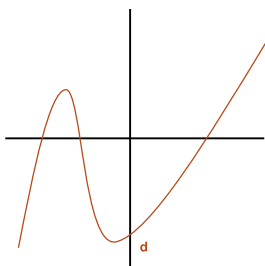
When the gradient goes from positive to positive or negative to negative, this is known as a **point of inflection**.

Find the minimum point of  $x^2 + 7x + 10$  by completing the square

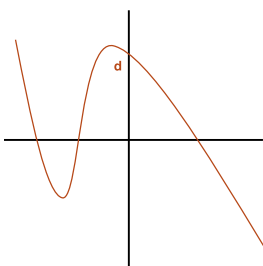
$$\left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + 10 = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$$

The smallest value of  $x$  would be  $-\frac{3}{2}$  as the value in brackets would then equal 0. Consequently, the minimum point is  $\left(-\frac{3}{2}, -\frac{9}{4}\right)$ .

$$y = ax^3 + bx^2 + cx + d, \text{ when } a > 0$$



$$y = ax^3 + bx^2 + cx + d, \text{ when } a < 0$$



## Transformations of Graphs

For the graph  $f(x)$ ,

$f(x + 2)$  would be a translation of  $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$  i.e. shifted two to the left

$f(x) + 2$  would be a translation of  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$  i.e. shifted two up

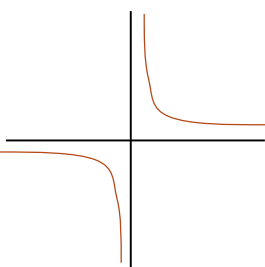
$f(2x)$  would be an enlargement of scale factor  $\frac{1}{2}$  parallel to the x axis.  
 $2f(x)$  would be an enlargement of scale factor 2 parallel to the y axis.

$y = f(-x)$  is a reflection of the graph in the y axis.

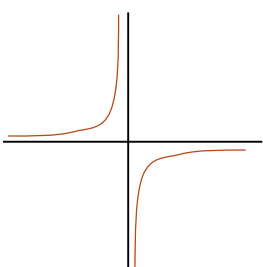
$y = -f(x)$  is a reflection of the graph in the x axis.

### Reciprocal Graphs

$$y = \frac{a}{x}, \text{ when } a > 0$$



$$y = \frac{a}{x}, \text{ when } a < 0$$



## Iteration

### Using Iteration

$$x_1 = 3 \text{ and } x_{n+1} = x_n + 2$$

What is the fourth term in the sequence?

$$x_2 = 3 + 2 = 5$$

$$x_3 = 5 + 2 = 7$$

$$x_4 = 7 + 2 = 9$$

### Forming Iterative Formulae

Solve  $x = \sqrt{x} + 1$  using the quadratic formula and iteration.

$$x = \sqrt{x} + 1$$

$$x^2 = x + 1 \quad \therefore x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{5}}{2} = 1.62 \text{ (3.s.f.)}$$

$$x_{n+1} = \sqrt{x_n} + 1$$

$$x_0 = 1$$

$$x_1 = 1.41 \text{ (3.s.f.)}$$

$$x_2 = 1.55 \text{ (3.s.f.)}$$

$$x_3 = 1.60 \text{ (3.s.f.)}$$

$$x_4 = 1.61 \text{ (3.s.f.)}$$

## Sequences

### Linear Sequences

Linear sequences have the same first differences.

$$\begin{array}{ccccccc} & 1 & & 4 & & 7 & & 10 & & 13 \\ \Delta_1 & & 3 & & 3 & & 3 & & 3 \end{array}$$

$$u_n = 3n - 2$$

### Quadratic Sequences

Quadratic sequences have the same second differences.

$$\begin{array}{ccccccc} & 1 & & 3 & & 6 & & 10 & & 15 \\ \Delta_1 & & 2 & & 3 & & 4 & & 5 \\ \Delta_2 & & & 1 & & 1 & & 1 \end{array}$$

For a quadratic sequence  $an^2 + bn + c$

$$\begin{array}{ccccccc} & a+b+c & & 4a+2b+c & & 9a+3b+c & & 16a+4b+c & & 25a+5b+c \\ \Delta_1 & & 3a+b & & 5a+b & & 7a+b & & 9a+b \\ \Delta_2 & & & 2a & & 2a & & 2a \end{array}$$

$$\therefore 2a = 1, a = \frac{1}{2}$$

$$\therefore 3a + b = 2, b = \frac{1}{2}$$

$$\therefore a + b + c = 1, c = 0$$

$$u_n = \frac{1}{2}n^2 + \frac{1}{2}n$$

### Limiting Values

As  $n \rightarrow \infty$  in a sequence, there is a limiting value.

$$\text{As } n \rightarrow \infty, \frac{n-3}{n+1} = \frac{n}{n} = 1$$

## Functions

### Inputting Values into Functions

$$f(x) = 2x + 5$$

$$f(3) = 2(3) + 5 = 11$$

$$f(x+2) = 2(x+2) + 5 = 2x + 9$$

### Inverse Functions

$$f(x) = \frac{x}{5} + 1$$

$$y = \frac{x}{5} + 1$$

$$x = 5y - 5$$

$$f^{-1}(x) = 5x - 5$$

### Composite Functions

$$fg(2) = f[g(2)]$$

$$f(x) = 3x \quad g(x) = x + 1$$

$$fg(x) = f(x+1) = 3x + 3$$

$$g^2(x) = (x+1) + 1 = x + 2$$

### Piecewise Functions

Piecewise functions define different functions over different ranges of  $x$  values.

$$f(x) = \begin{cases} 5 & -3 \leq x < 0 \\ 5 + 4x - x^2 & 0 \leq x < 1 \\ 0 & 1 \leq x < 2 \end{cases}$$

### Domain and Range

The domain is the set of possible inputs (*values in the  $x$  axis*).

The range is the set of possible outputs (*values in the  $y$  axis*).

### Constructing Functions from Domain and Range

$y = f(x)$  is a straight line, and an increasing function.

The domain of  $f(x)$  is  $1 \leq x \leq 5$ .

The range of  $f(x)$  is  $3 \leq f(x) \leq 11$ .

The two co-ordinates are (5,11) and (1,3).

$$m = \frac{\Delta f(x)}{\Delta x} = \frac{8}{4} = 2$$

$$y = 2x + c$$

$$11 = 10 + c$$

$$y = 2x + 1$$

$$\therefore c = 1$$

# 3 RATIO, PROPORTION, RATES OF CHANGE

## Direct and Indirect Proportion

### Direct Proportion

$$y = kx \quad y = kx^2$$

where  $k$  is a constant

### Indirect Proportion

$$y = \frac{k}{x}$$

$$y = \frac{k}{x^2}$$

## Ratios of Area and Volume

$$\text{Volume Scale Factor} = \text{Length Scale Factor}^3$$

$$\text{Area Scale Factor} = \text{Length Scale Factor}^2$$

The dimensions of Shape A are 3 times longer than Shape B. If the area of Shape B is  $9\text{cm}^2$ , calculate the area of Shape A.

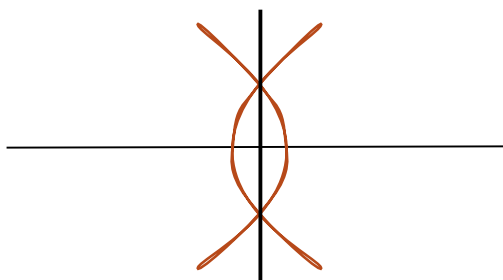
$$\text{Area Scale Factor} = \text{Length Scale Factor}^2 = 3^2 = 9$$

$$\text{Area of Shape A} = \text{Area of Shape B} \times 9 = 81\text{cm}^2$$

# 4 GEOMETRY AND MEASURES

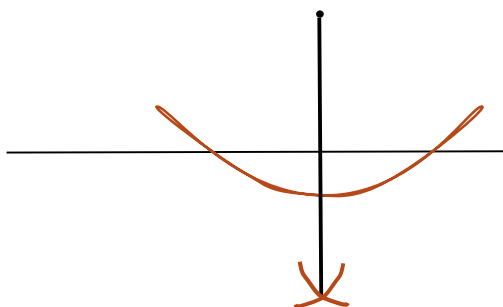
## Constructions

### Perpendicular Bisector of Line



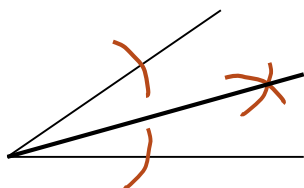
1. Draw an arc with your compass with a radius larger than half the line segment length.
2. Draw another arc of same radius from the other side of the line segment.
3. Draw a line connecting the two intersection points of the arcs.

### Perpendicular Bisector from Point



1. Draw an arc from the point, with the compass distance longer than the line segment.
2. Draw arcs from each point of intersection, with the same compass distance.
3. Where the arcs intersect, connect the point with this point of intersection.

### Angle Bisector



1. Draw an arc to cross both lines.
2. At the points of intersection, draw two more arcs of the same compass distance.
3. Where the arcs intersect, connect this point.

## Angles

### Total Internal Angles in a Polygon

$$(n - 2) \times 180^\circ \quad \text{where } n \text{ is the number of sides}$$

### Interior Angles

$$\frac{(n - 2) \times 180^\circ}{n} \quad \text{where } n \text{ is the number of sides}$$

$$\text{interior angle} = 180^\circ - \text{exterior angle}$$

## Angles (con.)

### Parallel Lines

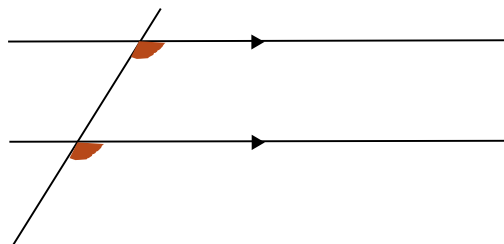
Parallel lines are straight lines which never meet. They are marked with an arrow.



Parallel lines can form sets of equal angles.

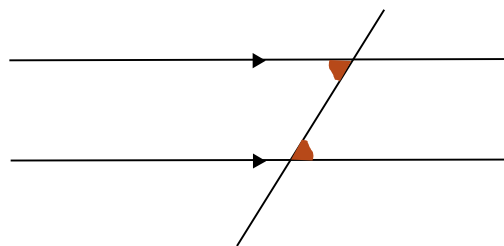
### Corresponding Angles

Sometimes called "F" angles.



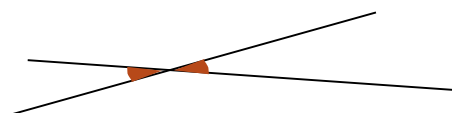
### Alternate Angles

Sometimes called "Z" angles.



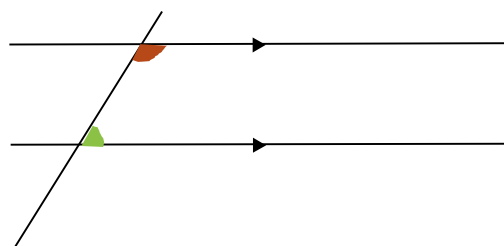
### Vertically Opposite Angles

They are opposite each other in a vertex.



### Co-Interior Angles

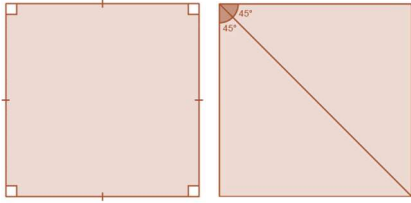
These angles add up to 180 degrees.





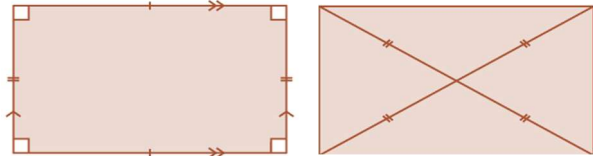
## Quadrilaterals

### Square



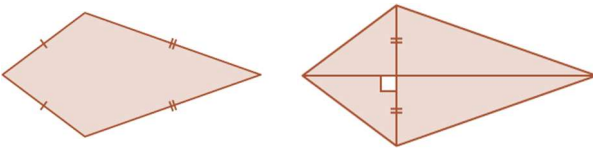
A square has four equal sides and four right angles. Each diagonal splits a corner into two angles of  $45^\circ$ .

### Rectangle



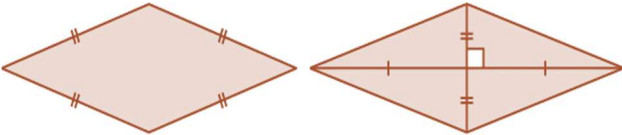
A rectangle has opposite sides which are equal and parallel. It has four right angles. The diagonals are equal and split the rectangle into four isosceles triangles.

### Kite



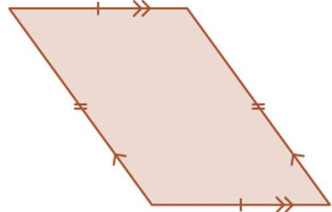
A kite is made up of two isosceles triangles joined base to base. Its diagonals are perpendicular to each other. The longer diagonal is a line of symmetry.

### Rhombus



A rhombus has four equal sides and its opposite sides are parallel. Its diagonals are not equal, but bisect each other at right angles. Both diagonals are lines of symmetry.

### Parallelogram



Opposite sides are equal and parallel. Opposite angles are equal.

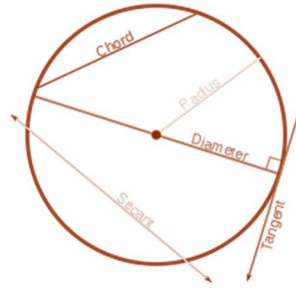
## Similarity

Two triangles are congruent if they share:

- Three sides (SSS)
- Two sides and the angle between them (SAS)
- Two angles and the side between them (ASA)
- A right angle, hypotenuse and side (RHS)

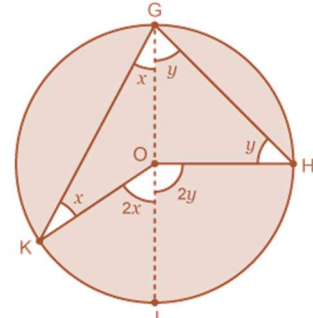
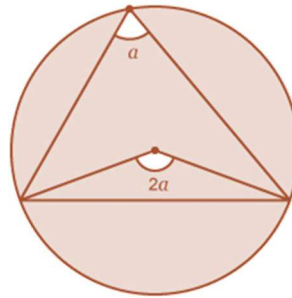
## Circles

### Circle Definitions

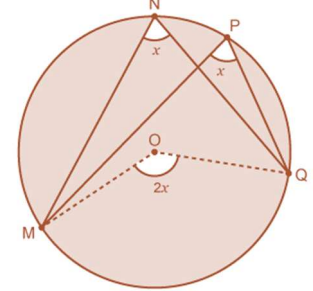
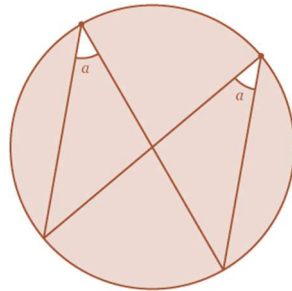


### Circle Theorems

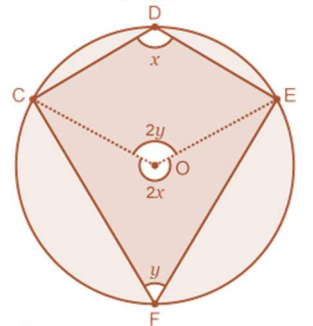
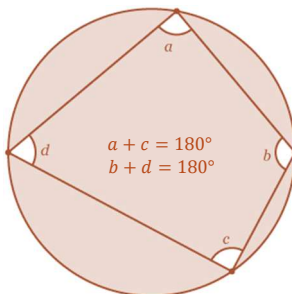
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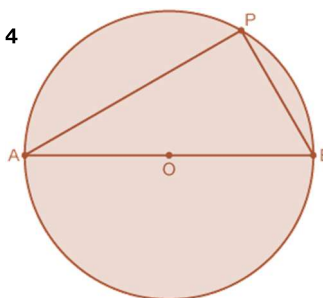
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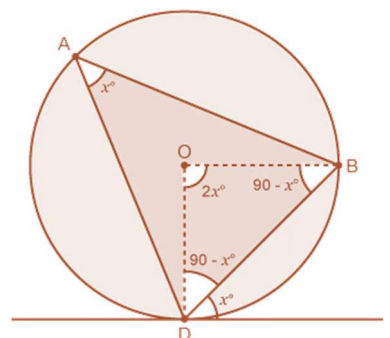
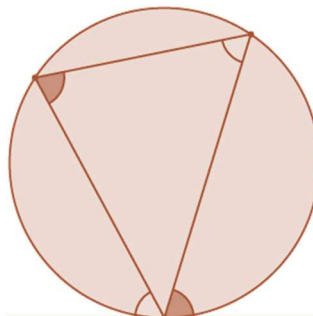
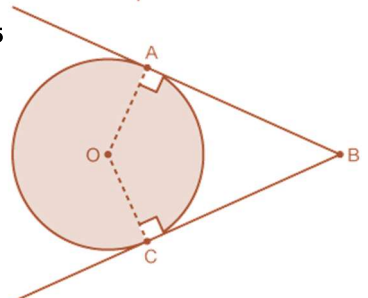
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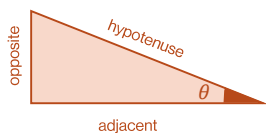


5



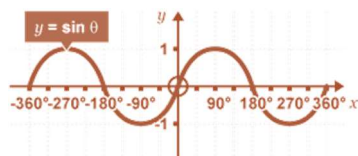
## Trigonometry

### Right Angled Triangle



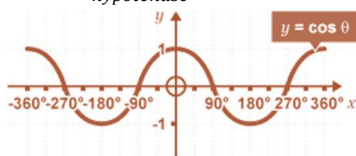
### Sine Graph

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$



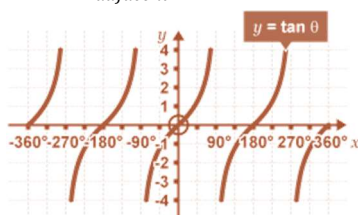
### Cosine Graph

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$



### Tan Graph

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



The tan graph is made up of asymptotes repeating every 180 degrees.

## Units

Metric		Imperial
1.61 km	=	1 mile
0.914 m	=	1 yard
30.5 cm	=	1 foot
25.4 mm	=	1 inch

1000 m in 1 km  
100 cm in 1 m  
10 mm in 1 cm

2760 yards in 1 mile  
3 feet in 1 yard  
12 inches in 1 foot

Metric		Imperial
1016 kg	=	1 imperial ton
6.35 kg	=	1 stone
434 g	=	1 pound
26.3 g	=	1 ounce

1000 g in 1 kg  
1000 g in 1 mg

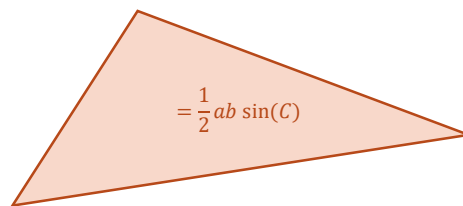
160 stone in 1 imperial ton  
14 pounds in 1 stone  
16 ounces in 1 pound

Metric		Imperial
4.55 litres	=	1 imperial gallon
568 ml	=	1 imperial pint

1000 ml in 1 litre  
1 ml = 1 cm<sup>3</sup>

8 pints in 1 gallon

### Area of a Triangle



### Trigonometric Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

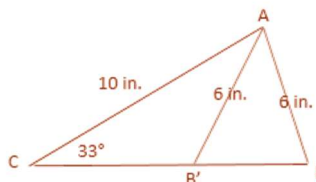
Can be proved using Pythagoras' Theorem.

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

### Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

With the sine rule, when calculating an unknown angle opposite a longer side length, this is known as the *ambiguous case*. The other angle is  $180^\circ - \theta$ .



### Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$A = \cos^{-1} \left( \frac{b^2 + c^2 - a^2}{2bc} \right)$$

## Trigonometry Values

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

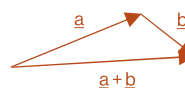
## Vectors

Vector quantities have both magnitude and direction.

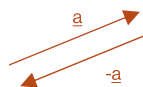
When describing a vector, remember to underline it:  $\underline{a}$

### Adding Vectors

Vectors can be added by the *Triangle Law*.



Vectors of the same magnitude but opposite directions have opposite sides.



### Parallel Vectors

Any vector parallel to  $\underline{a}$  may be written as  $\lambda \underline{a}$  where  $\lambda$  is a non-zero scalar.

If two vectors are parallel and share a common point, they are a straight line.

### Translations

Translations can be expressed as vectors.

$\begin{pmatrix} -8 \\ 1 \end{pmatrix}$  would mean 8 to the left and 1 up.

# 5 PROBABILITY

## Probability

### Theoretical Probability

0 impossible    unlikely    0.5 even    likely    1 certain

$$\text{probability} = \frac{\text{number of ways the outcome can happen}}{\text{total number of possible outcomes}}$$

**Mutually Exclusive** events that cannot happen at the same time (*heads or tails*)  
An exhaustive set of mutually exclusive events sum to one.

### Frequency Tree Diagrams

Multiply the possible conditions for an event together.

### Sample Space Diagrams

They are a visual way of recording the possible outcomes of two events.

### Conditional Probability

$$P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)}$$

# 6 STATISTICS

## Collecting Data

**Discrete Data** numerical data that can only take certain values  
**Continuous Data** numerical data that can take any value within a range  
**Primary Data** data collected from a original source  
**Secondary Data** data extrapolated from this original source (e.g. a mean)

### Using a Population

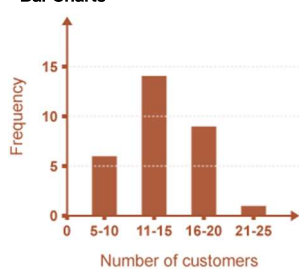
- Advantage:* all opinions are accounted for, results are more reliable
- Disadvantage:* takes a long time, expensive

### Using a Sample

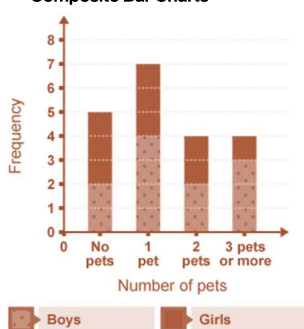
- Advantage:* quick to conduct, cost-effective
- Disadvantage:* only a selection of opinions, selection method could cause bias

## Representing Data

### Bar Charts

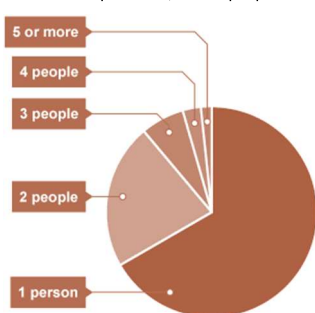


### Composite Bar Charts



### Pie Charts

To draw a pie chart, find its proportion of 360 degrees.



### Scatter Graph

Scatter graphs can have positive correlation, negative correlation or no correlation.

### Interpretation

finding data within the range of data values

### Extrapolation

determining a value outside the range of data values

## Analysing Data

### Mean

$$\text{mean} = \frac{\text{sum of all numbers}}{\text{amount of numbers}}$$

*Advantage:* takes account of all values, calculating an average

*Disadvantage:* very small/large values can affect the mean

### Median

$$\text{median} = \left( \frac{\text{number of data values} + 1}{2} \right) \text{th number}$$

*Advantage:* median not affected by very small/large values

*Disadvantage:* if there is an even number of numbers, the median is obtained through an average. This means the median may not actually be a number in the original data set.

### Mode

The most frequent value in a set of data.

*Advantage:* only average that can be taken with a data set not in numbers

*Disadvantage:* there can be more than one mode, which is not representative of the data

### Interquartile Range

$$\text{interquartile Range} = \text{upper quartile} - \text{lower quartile}$$

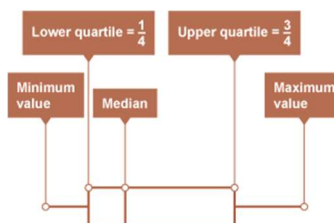
$$\text{upper quartile} = (\text{number of data values} + 1) \times \frac{3}{4} \text{th number}$$

$$\text{lower quartile} = (\text{number of data values} + 1) \times \frac{1}{4} \text{th number}$$

### Histograms

$$\text{frequency} = \text{frequency density} \times \text{class width}$$

### Box Plots





# FM CALCULUS

## Differentiation

### Standard Differentiation

$$y = kx^n \quad \rightarrow \quad \frac{dy}{dx} = nkx^{n-1}$$

Work out  $\frac{dy}{dx}$  of  $\frac{x^5+x^2}{x}$ .

$$y = \frac{x^5}{x} + \frac{x^2}{x} = x^4 + x$$

$$\frac{dy}{dx} = 4x^3 + 1$$

### Increasing and Decreasing Functions

A function is increasing if  $\frac{dy}{dx} > 0$

A function is decreasing if  $\frac{dy}{dx} < 0$

### Stationary Points

You can use the second derivative to ascertain whether a stationary point is a *local maximum*, *local minimum* or *point of inflection*.

If  $f''(x) > 0$ , this is a local minimum.

If  $f''(x) < 0$ , this is a local maximum.

If  $f''(x) = 0$ , this requires further investigation.

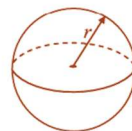
To investigate further, use the gradient to the left and to the right of this stationary point.

A **point of inflection** exists when the gradient on either side of the point is the same sign.

## A Formulae Sheet

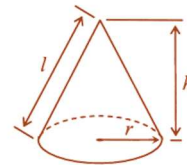
$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$



$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area of cone} = \pi r l$$



In any triangle ABC

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

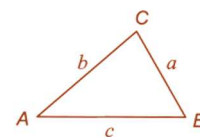
$$\text{Sine rule} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

### The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by  $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$



### Trigonometric Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

# FM MATRICES

## Matrix Multiplication

### Matrices

A matrix with  $m$  rows and  $n$  columns is called a  $m \times n$  matrix. Matrix multiplication is not commutative, meaning the order of the multiplication matters.

$$\mathbf{m}_1 \times \mathbf{n}_1 \text{ matrix} \times \mathbf{m}_2 \times \mathbf{n}_2 \text{ matrix} = \mathbf{m}_1 \times \mathbf{n}_2 \text{ matrix}$$

### Multiplication by a Scalar

$$A = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$$

$$4A = \begin{pmatrix} 8 & -12 \\ -4 & 20 \end{pmatrix}$$

### Multiplication of a $2 \times 2$ Matrix by a $2 \times 1$ Matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} ae + bf \\ ce + df \end{pmatrix}$$

$$\begin{pmatrix} -3 & -1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -11 \\ 16 \end{pmatrix}$$

### Multiplication of a $2 \times 2$ Matrix by a $2 \times 2$ Matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -13 & -5 \end{pmatrix}$$

## Matrix Transformations

### Transformation Matrix

The transformation that maps  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  to  $\begin{pmatrix} a \\ c \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  to  $\begin{pmatrix} b \\ d \end{pmatrix}$  has the matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

### Image Point

For the point P  $(x, y)$  and the image point P'  $(x', y')$ .

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

## Identity Matrix

The matrix  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  is called the identity matrix **I**.

When **I** is used as a transformation matrix, no movement occurs.

## Transformations of the Unit Square

To work out the matrix that represents a transformation, consider the images of the vectors  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ . These transformations are limited to:

- reflections in the  $x$  axis,  $y$  axis, line  $y=x$  and line  $y=-x$
- rotations of  $90^\circ$ ,  $180^\circ$  and  $270^\circ$  about the origin
- enlargements centered on the origin

## Combining Transformations

To combine transformations, multiply the transformation matrices together. A transformation using matrix **A** followed by matrix **B** has a combined transformation of matrix **BA**.

