Random
- repeat readings
- longer L / more λ, then divide
- adjust |s|
- apparatus precision

Systematic
- differs from true value by set amount (y-intercept)

VAPRR

Uncertainty
- + or − : add absolute uncertainties
- × or ÷ : add % uncertainties
- x^n: multiply % uncertainty n times

VALIDATE UNCONFIRMED
- hypothesis tested by experiment
- experiment needs to be repeatable

TEST THEORY'S PREDICTION
- reliable experiment doesn't support hypothesis
- hypothesis must be changed

validated by peer review; theory makes prediction (testable, repeatable), scientific community

$C_{H_2O} = 4.18\ J.g^{-1}.K^{-1}$ $R = 8.314\ J.K^{-1}mol^{-1}$

absolute zero: $-273.1°C$

CONVERSIONS
- $1m^3 = 1000L \Rightarrow 1cm^3 = 1mL$
- $1L_{H_2O} = 1kg$

Elastic & Inelastic Collisions
- → conservation of momentum
- → KE only conserved in elastic collisions

SI Units

kg, m, s, A, K, cd, mol

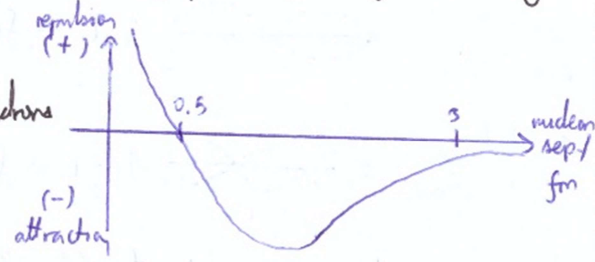
- kg → N $(kg.m.s^{-2})$ → J $(kg.m^2.s^{-2})$
- m → v (ms^{-1}), a (ms^{-2}); → Pa $(kg.m^{-1}s^{-2})$
- s → Hz (s^{-1}); → T $(A^{-1}kg.s^{-2})$
- A → C (As) → V $(A^{-1}kg.m^2s^{-3})$ → F $(A^2kg^{-1}m^{-2}s^4)$

Nuclide Notation

$^{A}_{Z}X$ — A: # of nucleons, Z: # of protons

(use % of ^{14}C for C-dating)

Strong Nuclear Force
- → repulsion closer than 0.5fm from other hadrons
- → attraction up to 3fm
- → no effect beyond 3fm



Nuclear Decay

$\alpha: {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$
- → in v. big atoms
- → nucleus too big for strong force stability
- → v. short range (~few cms in air); paper

$\beta: {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta^- + \bar{\nu}_e$
- conserves E and p in β^- decay
- → neutron-rich nuclei
- → neutron → proton
- → moderate range (~few ms in air); tin foil

Isotopes: atoms w/ different mass number but same atomic number

Specific Charge: $= \frac{charge/Q}{mass/m}$ in $C.kg^{-1}$
- ⚠ + true or −ve symbol
- → nucleus, ion or atom??

Conserved: LBSEMF
- → energy, momentum, force → charge
- → baryon # → lepton # (separately)
- → strangeness - only strong (∴ produced in pairs)

LEPTONS - collection of fundamental particles, all affected by weak force/gravity

decays: e^- electron	e^+ positron	ν_e electron neutrino, $\bar{\nu}_e$
μ^- muon	μ^+ antimuon	ν_μ muon neutrino, $\bar{\nu}_\mu$ (anti)
τ^- tau	τ^+ antitau	ν_τ tau neutrino, $\bar{\nu}_\tau$

QUARKS

q	ch.	S	B#		q	ch.	S	B#
u	+2/3	0	1/3		$\bar{u}$	−2/3	0	−1/3
d	−1/3	0	1/3		$\bar{d}$	−1/3	0	−1/3
s	−1/3	−1	1/3		$\bar{s}$	+1/3	+1	−1/3

FUNDAMENTAL FORCES

		strong (< 3fm)	weak	E.M (∞)	gravity
exchange particle		pions, gluons	W^+/W^-, Z^0 bosons	virtual photon (γ)	graviton
		produces strange particles	decays strange particles (by −1, 0, +1)		
leptons	e, μ, τ	×	✓	✓	✓
	ν_e, ν_μ, ν_τ	×	✓	×	✓
Hadrons	mesons	✓	✓	charged	✓
	baryons	✓	✓	charged	✓

Hadrons: particles w/ quarks experience strong force

Stablest baryon: p

		B#			B#
p	u u d	+1	n	d d u	+1
$\bar{p}$	$\bar{u}\bar{u}\bar{d}$	−1	$\bar{n}$	$\bar{d}\bar{d}\bar{u}$	−1

n decays. Baryons have 3 quarks.

		B#			S	B#
π^+	$u\bar{d}$	0	K^+	$u\bar{s}$	+1	0
π^-	$\bar{u}d$		K^-	$\bar{u}s$	−1	
π^0	$u\bar{u}$ / $d\bar{d}$		K^0	$d\bar{s}$	+1	
			$\bar{K}^0$	$\bar{d}s$	−1	

Mesons are a quark-antiquark pair.

Separate Lepton Numbers ($L_e/L_\mu/L_\tau$) must be conserved.
- → E.M waves release E in discrete packets of 'quanta' (or photons).
- → Every particle's corresponding antiparticle has:
  - → same mass/rest mass E → opposite charge

$E_{min} = 2E_0$

Pair Production:
- → γ photons of sufficient E interact w/ nucleus
- → converts to particle-antiparticle pair
- $\gamma \rightarrow e^- + e^+$ → excess E converted to KE of particles

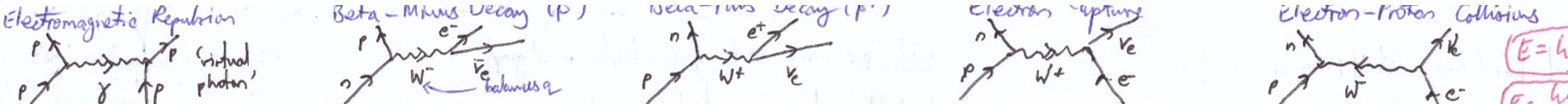
Annihilation:
- → particle meets corresponding antiparticle
- → mass converted back to E
- → two γ photons in opposite directions of same E, conserve p

$E_{min} = E_0$

Exchange Particles - change quark 'flavour', transfers E, p, sometimes q

FEYNMAN DIAGRAMS

Electromagnetic Repulsion — Beta-Minus Decay (β^-) — Beta-Plus Decay (β^+) — Electron Capture — Electron-Proton Collisions



- $n \rightarrow p + e^- + \bar{\nu}_e$ — WEAK quarks: udd → uud
- $p \rightarrow n + e^+ + \nu_e$ — WEAK quarks: uud → udd
- $p + e^- \rightarrow n + \nu_e$ — WEAK proton-rich nuclei capture e^-
- $e^- + p \rightarrow n + \nu_e$ — WEAK high speed e^- collides w/ proton

$E = hf$

$E = \frac{hc}{\lambda}$

PHOTOELECTRIC EFFECT

Work function (φ) - minimum E required to eject a photoelectron from the metal surface

Photoelectric Effect → no photo e^- emitted below threshold freq. (f_0) or φ
- ∴ can't be wave, o/w would take longer for ↓f to gain suff. E
- intensity ∝ photo e^- emitted per second

$hf = \phi + E_{k\,max}$

- → e^- do not build up over time
- ∴ photons carry quanta of energy
- max KE determined by threshold freq. (f_0)

$hf = E_1 - E_2$

- → photo e^- emitted w/ variety of KE from 0 → max
- ∴ deeper e^- have less KE than surface e^- (require E > φ)

$E_{k\,max} = eV_s$

- 1 - to - 1 interaction of photons and e^-

Ground State - when e^- are in their minimum E states (n=1)

Electronvolt (eV) - KE carried by e^- after being accelerated through 1V

Stopping Potential - P.D needed to stop fastest moving e^- in a photocell

- **Excitation** - e^- moves up from one E level to another
- **Ionisation** - e^- is removed from atom

Emission Spectra (vapor gas)
- → using diffraction prism
- → atoms only emit photons of $E = \Delta E_1 - \Delta E_2$

Absorption Spectra (cooled gas)
- → using white light through cooled gas (so e^- n=1)
- → wavelengths missing absorbed by e^- (discrete E)

IONISATION
1. absorb suff. E from incident e^- by collision, e^- have remaining KE
2. exact amount of E absorbed by photon
3. loses discrete amounts of E, e^- return by different routes to lower levels
4. emits photons of discrete characteristic f
5. Atoms have discrete E levels

e^- DIFFRACTION PATTERN concentric circles becoming more spread out
1. particles w/ ↑mass (e.g. n) travel at same KE as e^- (i.e. v)
2. have greater momentum ∴ shorter λ
3. more tightly packed diffraction pattern than e^-

LIGHT-BULBS / FLUORESCENT TUBES
1. Mercury vapor emits UV radiation
2. Phosphorous coating absorbs γ from Hg
3. Re-emits lower λ in visible light region

- low pressure Hg vapor allows large distance between collisions so e^- gain enough E
- Hg atoms excited by e^- impact

WAVE-PARTICLE DUALITY: Light is wave - e^- diffraction/superposition; particle - photoelectric effect

$\lambda = \frac{h}{mv}$

WAVES

Transverse - E propagation ⊥ dir (displacement of particles)
e.g. S-waves, E.M waves, water surface waves, waves in string

Longitudinal - E propagation // dir (displacement of particles)
e.g. P-waves, sound waves, ultrasound, infrasound

Progressive Wave - carries E from 1 place to another w/o transferring material

Stationary Wave: (a.k.a standing wave)
- → superposition of 2 progressive waves
- → same f and A travelling in opposite directions
- → no overall E transfer happens

Superposition - vector sum of individual displacements
- → constructive (in phase λ = 0°, 360°, 2π) |s| ↑
- → destructive (antiphase ½λ = 180°, π) |s| ↓
- → out of phase = combination of both

$c = f\lambda$

Reflection - wave bounces back when hits boundary

Refraction - wave changes speed/direction entering different medium

Diffraction - wave spreads out passing through gap/round object
- rarer → denser v ↓ λ ↓ f = bends towards normal
- denser → rarer v ↑ λ ↑ f = bends away from normal

Larger λ refracts less ∴ violet (shorter λ) refracts more than red.

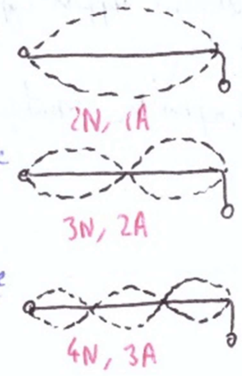
Coherent Sources: Same λ or f in a constant phase relationship

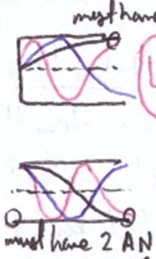
STATIONARY SOUND WAVES
1. powder in tube of air, loudspeaker produces stationary waves in tube
2. powder shaken along bottom away from anti-nodes to nodes
3. distance between powder piles = λ/2
∴ c = f × 2d = 2df

STATIONARY MICROWAVES

uses: microwave transmitter / probe + metal plate
1. move probe finding areas of none / max signal

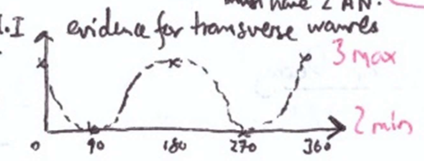
Resonant Frequency – f where standing wave is formed {node – total destr. interference |s| cancel; antinode – total constr. interference max |s|}

First Harmonic / fundamental  2N, 1A — $\lambda = 2L \Rightarrow f_1 = \frac{v}{2L}$ and $\boxed{f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}}$ T ← tension / N; μ ← mass per unit length / kg m⁻¹; length / m

Second Harmonic / first overtone — 3N, 2A — $\lambda = L \Rightarrow f_1 = \frac{v}{L} = 2f_1$ Pipe Open 1 End  must have A.N. $\boxed{L = \frac{2n-1}{4}\lambda}$

Third Harmonic / second overtone — 4N, 3A — $\lambda = \frac{2}{3}L \Rightarrow f_1 = \frac{3v}{2L} = 3f_1$ Pipe Open Both Ends — must have 2 AN. $\boxed{L = \frac{n\lambda}{2}}$

Polarised Wave – only oscillates 1 direction/plane/axis. Only light w/ oscillations // to polariser's transmission axis pass through.



Uses of Polarisation → glare restriction: removes partially-polarised reflected light — sunglasses, Polaroid photos
→ cameras/LCD displays: v. low power usage ones
→ aerial alignment: TV signals polarised by orientation on transmitting aerial ∴ to receive strongest signal, aerials must be aligned

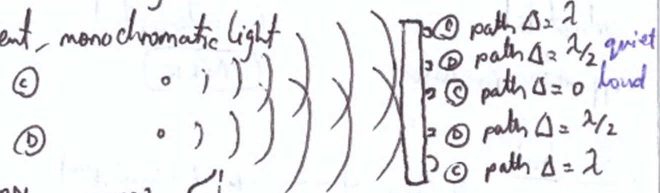
Total Internal Refraction (TIR): angle of incidence > critical angle; from denser → rarer medium
$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$ $\boxed{\frac{n_1}{v_1} = \frac{n_2}{v_2}}$

Optical (step-index) fibres: v. thin flexible tube of plastic glass carrying visible light
→ optically dense core: higher refractive index → less optically: lower refractive index — surrounded by… dense cladding for T.I.R.
Cladding prevents: scratches of core, adds tensile strength, prevents crossover of information w/ neighbouring cores, maintains quality
→ narrower cable: better quality signal, less refraction, more T.I.R. ⇒ better data transfer ↓t
→ better than Cu: no heat E lost, no electrical interference, cheaper

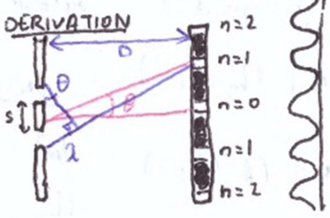
Path Difference – path travelled by 1 wave longer than another (2 coherent sources)

Diffraction in Gaps: no diffraction: gap >> λ; max diffraction: gap ≃ λ; most reflection: gap << λ
Around an object, 'diffraction shadows' may occur.

Two-Source Interference → use coherent, monochromatic light
→ Constructive: path Δ = nλ ⓒ
→ Destructive: path Δ = (n+½)λ ⓓ

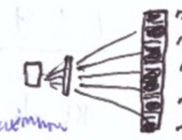


Young's Double Slit — fringe spacing → $\boxed{W = \frac{\lambda D}{s}}$ ← distance; s ← slit separation

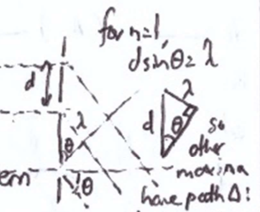


→ same fringe spacing
≈ same intensity
∴ $\frac{\lambda}{s} = \frac{w}{D}$

Diffraction Grating — grating spacing → $\boxed{d\sin\theta = n\lambda}$ ← order number for maximum



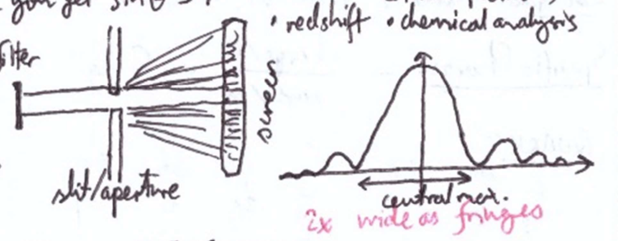
equally spread out slits very close together produces very sharp pattern



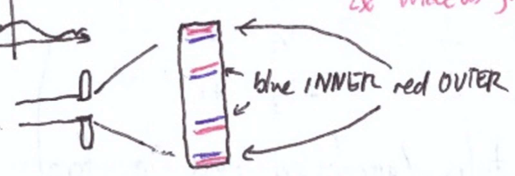
∘ Pattern more spread out: λ is bigger ∴ θ is bigger
∘ Pattern less spread out: d is bigger, sin θ is smaller so coarser
∘ Last Order: sin θ ≤ 1, so for certain n, you get sin θ > 1

USES: ∘ line absorption spectrum (star composition) ∘ redshift ∘ chemical analysis

(Single) Monochromatic Light Diffraction filter



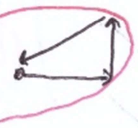
∘ Increasing slit distance (s) — diffraction ↓ central maximum NARROW, but HIGH
∘ Increasing λ — diffraction ↑ central maximum WIDER, but SHALLOW



(Single) White Light Diffraction full rainbow

SIGNAL DEGREDATION
→ absorption by fibre material ∴ reduced amplitude
→ dispersion causes pulse broadening ∴ overlaps ⇒ info loss
{modal: light rays enter diff ∠ ∴ some quicker ⇒ use single-mode fibre; material: diff refract by diff λ ∴ longer t by some ⇒ use monochromatic light}

FORCES — scalar – only magnitude: speed, dist, t, E, T; vector – magnitude w/ associated direction: $\vec{v}, \vec{s}, \vec{p}, \vec{F}, \vec{a}, \vec{J}$

Forces on object form a closed loop (equilibrium) ⇒ vector triangle 

Equilibrium – no resultant force, ∑ turning moments = 0

$\boxed{v = u + at}$ $\boxed{s = \frac{1}{2}(u+v)t}$ $\boxed{s = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2}$ $\boxed{v^2 = u^2 + 2as}$

NEWTON'S LAWS 1 velocity of object doesn't change unless resultant force acts upon it 2 F = ma 3 1 object exerts a force on another, other object exerts equal/opposite force on former

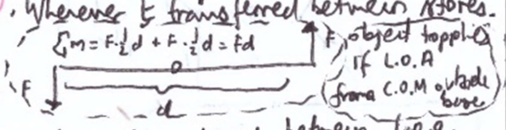
TERMINAL VELOCITY

Power: rate of work done = rate of E transfer $\boxed{P = \frac{\Delta W}{\Delta t} = \frac{Fs}{t} = Fv(\cos\theta)}$

Work Done – area under force-displacement graph $\boxed{W = Fs(\cos\theta)}$, wherever E transferred between stores.

Moments: $\boxed{M = \text{force} \times \text{perpendicular distance to line of action}}$

Principle of Moments – for a body in equilibrium, ∑ c.w = ∑ a.c.w.



Free fall: when only gravity acts on object (i.e. only force = weight)

Couple: pair of opposite, equal, coplanar forces — Moment of Couple = force × ⊥ dist. between l.o.a. of forces

Friction: force that opposes motion, KE → heat → Contact: between solid surfaces → Fluid (Drag): depends on → viscosity of fluid → force increases ∝ speed → S.A (shape)

Lift: upwards force on object through fluid, ⊥ to fluid flow. Caused by equal & opposite force on wing. Occurs when shape of an object causes fluid flowing over to change direction.

Momentum: $\boxed{p = mv}$ $\boxed{\sum p_{before} = \sum p_{after}}$ (linear) $\boxed{F = m\frac{\Delta v}{\Delta t} = \frac{m\Delta v}{t} = \frac{\Delta p}{t}}$

→ Football – less t foot in contact w/ ball = greater F
→ Packaging – more t to deform on impact, reduces |J|
→ Crumple Zones – car crumples deforming on impact ↑t ↓F (seatbelts/airbags)

Impulse: (Ns) change in momentum, area under F-t graph $\boxed{J = m(v-u) = F\Delta t}$

COLLISIONS
+ Elastic – conserves p/KE, no E dissipated
+ Inelastic – only conserves p, some KE → heat/sound
+ Perfectly Inelastic – conserves p, no KE (parts stick together)

MATERIALS

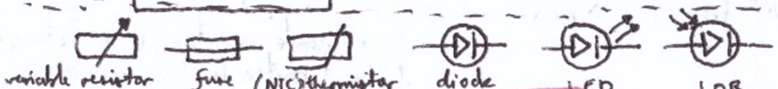
ELECTRICITY

$\boxed{\text{Density}(\rho) = \frac{\text{mass}}{\text{volume}}}$ $\boxed{\text{Hooke's Law: } F = k\Delta L}$ $\boxed{\text{Young's Modulus (E)} = \frac{\sigma}{\varepsilon} = \frac{FL}{A\Delta L}}$

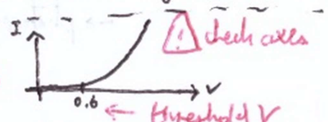
Tensile Stress (σ) + F applied ⊥ to cross-sectional area $\boxed{\sigma = F/A \ (\text{Pa})}$

Tensile Strain (ε) changing in length due to applied stress $\boxed{\varepsilon = \Delta L/L}$

→ Brittle: obeys Hooke's Law until fracture point (B), doesn't deform plastically
→ Ductile: deforms plastically → Brittle Fracture: rigid structure, cracks grow as atoms unable to move
→ Strength: deals w/ more σ until B



DIODE I-V CHARACTERISTICS — very high Ω, reverse bias



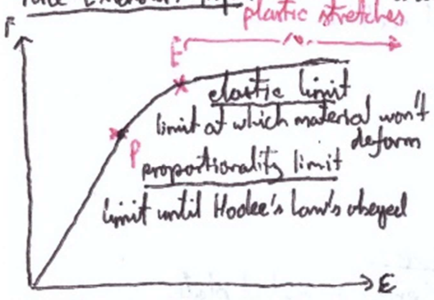
Electrical Current (I) – rate of flowed charge $\boxed{I = \frac{\Delta Q}{\Delta t}}$

Potential Difference (V) – work done per unit charge (1V = 1J.C⁻¹) $\boxed{V = \frac{W}{Q} = IR}$

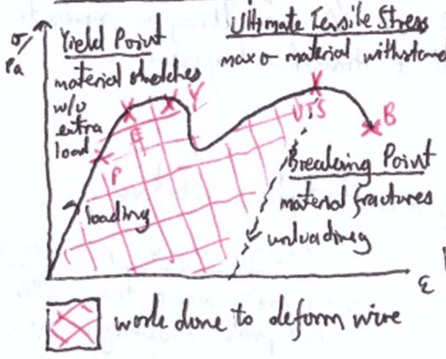
Superconductors – material w/ 0 resistance below critical temp (T_c); dependent on material
Power Cables: transmit electricity w/o power loss. V. Strong Electromagnets: used in Maglev, medicine, less heat/E dissipated

Resistivity (Ωm) $\boxed{\rho = \frac{RA}{L}}$ ← m², m
→ as T ↑, ions in material vibrate more
→ e⁻ collide w/ positive ions, transferring E ⇒ heats up wire causing more collisions
→ collisions slow rate of flow of charge and E dissipated increases

Force-Extension Graph — crumple zones: work done to deform, ↑E dissipated — plastic stretches



Stress-Strain Graphs



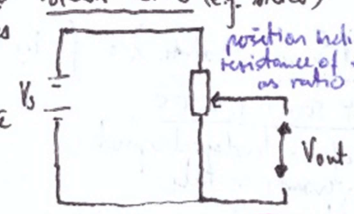
$\boxed{\text{Energy Stored} = \frac{1}{2}F\Delta L = \frac{1}{2}kx^2}$

Power (P) – rate of energy transfer $\boxed{P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}}$

Electromotive Force (e.m.f) – electrical E to each C of charge $\boxed{E = V/t}$ $\boxed{\mathcal{E} = \frac{E}{Q} = I(R+r)}$ ← load resistance, internal resistance; $\boxed{V = \mathcal{E} - Ir}$ ← gradient = −r

Potential Divider – used to supply constant/variable P.D from (e.g. LDR, thermistor)

Potentiometers (e.g. stereos)

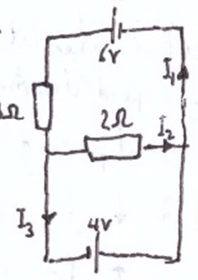


$\boxed{V_{out} = V_{in} \times \frac{R_1}{R_1 + R_2}}$

Kirchhoff's Laws
1 total current entering junction = total current leaving junction
2 total e.m.f in any closed loop = total p.d in any closed loop

$\boxed{\sum_{n=1}^{t} R_n = R_1 + R_2 + R_3 + \ldots + R_t}$
$\boxed{\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_t}}$

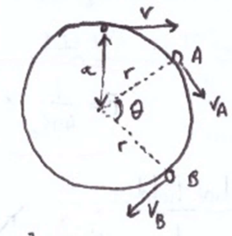
e.g.



6V = 3I₁ + 2I₂ ⇒ 10V = 3I₁ ∴ $I_1 = \frac{10}{3}$ A
4V = −2I₂ ⇒ $I_2 = -2$ A
$I_3 = I_1 - I_2 = \frac{10}{3} + 2 = \frac{16}{3}$ A

Circular Motion

Linear Speed (v): $\boxed{v = \frac{s}{t} = \frac{2\pi r}{T}}$ $\Rightarrow \frac{v}{r} = \frac{2\pi}{T} \therefore \boxed{\omega = \frac{v}{r}}$



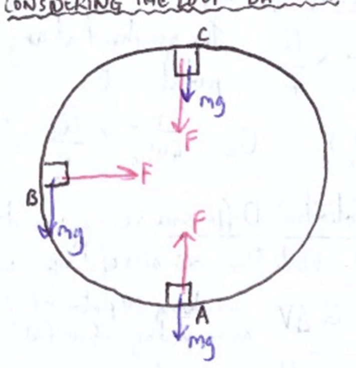
Angular Velocity (ω): $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = \boxed{2\pi f}$ $rads^{-1}$

Centripetal Acceleration (a): $\widehat{AB} = v \cdot \Delta t$

$\therefore \frac{\widehat{AB}}{r} = \frac{\Delta v}{v} \Rightarrow \frac{v\Delta t}{r} = \frac{\Delta v}{v} \therefore \frac{\Delta v}{\Delta t} = \boxed{a = \frac{v^2}{r}} = \frac{(\omega r)^2}{r} = \boxed{\omega^2 r}$

Centripetal Force (F): $F = ma = m\left(\frac{v^2}{r}\right) = \boxed{m\omega^2 r}$ ⚠ always acts towards the centre of the circle

CONSIDERING THE LOOP 'DA' LOOP



- → Circular Motion not in equilibrium; F_{res} as object constantly accelerates
- → Velocity is not constant; speed is constant but direction constantly changes
- → Geosynchronous orbits have a period $T = 24$ hours $= 86400$s
- → No work done on particle by force at $\pi/2$ radians $\boxed{W = Fs\cos\theta = F \times 0}$

THERMAL PHYSICS

$\uparrow KE = \uparrow T$ $\quad TPE = \uparrow$ entropy (arrangement) of particles

Internal Energy - sum of a body's particles' randomly-distributed KEs and PEs

Specific Heat Capacity (c) - amount of E needed to raise T of 1kg of substance by 1K

$J \cdot kg^{-1} \cdot K^{-1}$ ⚠ Assume conservation of E $\quad J \rightarrow \boxed{Q = mc\Delta\theta}$ ← kg

Specific Latent Heat (L) - the quantity of thermal E needed to be gained/lost to change the state of 1kg of a substance $\boxed{Q = mL}$

$J \cdot kg^{-1}$ {of Fusion, of Vaporisation}

GAS LAWS

Boyle's Law $\quad pV = $ constant $\Rightarrow p \propto \frac{1}{V}$ $\quad \boxed{p_1V_1 = p_2V_2}$

Charles' Law $\quad \frac{V}{T} = $ constant $\Rightarrow V \propto T$ $\quad \boxed{V_1T_2 = T_1V_2}$

Gay-Lussac's Law $\quad \frac{p}{T} = $ constant $\Rightarrow p \propto T$ $\quad \boxed{p_1T_2 = T_1p_2}$

combined $\boxed{\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}}$

Boltzmann's Constant

Ideal Gas Law: $\frac{pV}{T} = n \times$ constant $\Rightarrow \boxed{pV = nRT}$ $\quad$ Gas constant for 1 molecule $k_B = R/N_A$

depends on amount of gas; # of molecules $\therefore \boxed{pV = Nk_BT}$ $\quad \boxed{\text{Work Done} = pV}$

DERIVATION

Any particle has momentum mu, and assuming elastic collisions, $\Delta p = 2mu$.

Time between collisions $\frac{2L}{u}$. Hence, $F = \frac{\Delta p}{t} = \frac{2mu}{2L/u} = \frac{mu^2}{L}$ ⟸ negligible i.m. forces

Assume identical molecules travelling at different velocities: $F = \frac{m(u_1^2 + u_2^2 + u_3^2 + \dots)}{L}$

Let $\bar{u}^2$ (mean square speed) be: $\frac{u_1^2 + u_2^2 + u_3^2 + \dots}{N}$, then $F = \frac{mN\bar{u}^2}{L}$

Since $p = \frac{F}{A}$, we have: $p = \frac{mN\bar{u}^2/L}{L^2} = \frac{mN\bar{u}^2}{V}$ since $L^3 = V$.

Assume molecules of negligible volume, as compared to box, and move randomly:

$\bar{u}^2 = \bar{v}^2 = \bar{w}^2$ so by 3D Pythagoras, $\bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2 = 3\bar{u}^2 \therefore \bar{u}^2 = \frac{1}{3}\bar{c}^2$

This means $p = \frac{Nm\bar{u}^2}{V} = \frac{1}{3}Nm\frac{\bar{c}^2}{V} \Rightarrow \boxed{pV = \frac{1}{3}Nm\bar{c}^2}$

Define root mean square speed $= c_{r.m.s} = \sqrt{\bar{c}^2}$

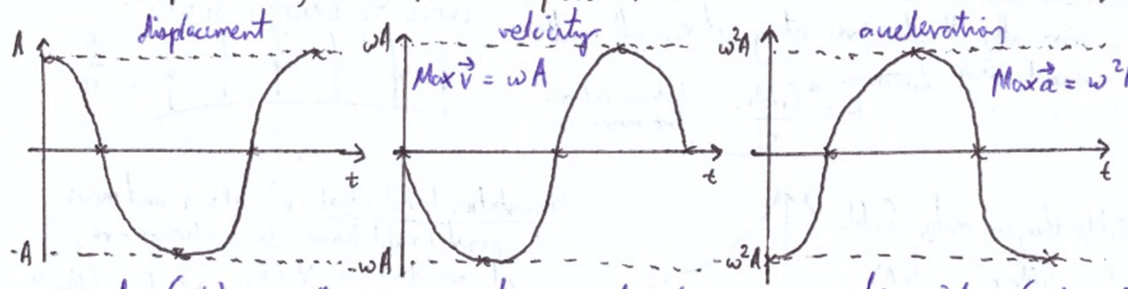
Given $pV = nRT = \frac{1}{3}Nm(c_{r.m.s})^2$ and $\frac{1}{3}m(c_{r.m.s})^2 = nRT$, we have: $\boxed{\frac{1}{2}m(c_{r.m.s})^2 = \frac{3}{2} \times \frac{nRT}{N}}$

Using the Boltzmann constant, to $\frac{3}{2} \times \frac{nRT}{N}$, $\frac{3}{2} \times \frac{N \times R \times T}{N_A \times N} = \frac{3}{2}k_BT \therefore \boxed{\frac{1}{2}m(c_{r.m.s})^2 = \frac{3}{2}k_BT = \frac{3}{2} \times \frac{RT}{N_A}}$

Development of Theories: → gas laws are empirical; kinetic theory model is theoretical. → Robert Brown: pollen grains in water zigzag randomly. Large heavy particles (eg. smoke) moved randomly by smaller quicker ones. Supports KT model and existence of atoms.

SHM

SHM - an oscillation in which the acceleration of an object is directly proportional to its displacement from its equilibrium position ($a \propto -x$) and directed towards its equilibrium



$x = A\cos(\omega t) = r\cos\theta$ $\quad v = \frac{dx}{dt} = -\omega A\sin(\omega t)$ $\quad a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t) = -\omega^2 x$

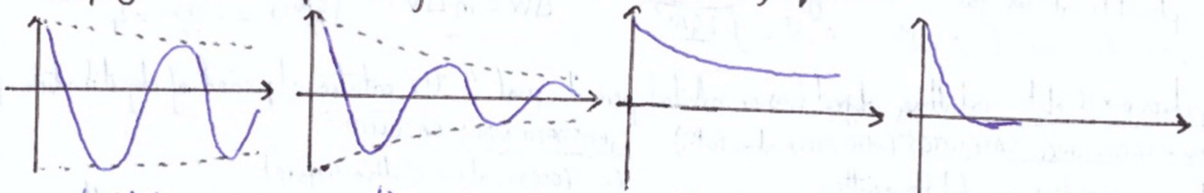
$v = \pm\omega\sqrt{A^2 - x^2}$

Conditions for SHM:

→ force $\propto$ distance from mean position, directed towards mean position

MASS-SPRING SYSTEM $\quad$ period → $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$ (mass; spring constant) $\quad$ restoring force → $\boxed{F = k\Delta L}$ ← displacement

- Restoring Force - resultant force acting on an object towards its equilibrium position
- Resonance - occurs when frequency of the driving force equals natural (resonant) frequency of oscillation of the system, causes a sudden spike in amplitude of driving force
  - → Forced Vibration: system acted upon by a periodic driving force
  - → Free Vibration: no transfer of E w/ surroundings, will oscillate indefinitely
- Damping - reduces amplitude of vibrations at resonant frequencies



Light Damping	Heavy Damping	Over damping	Critical Damping
amplitude reduces small amount per period. Longer time to stop oscillations	less time to stop. amplitude reduces by large amounts.	very heavy damping. takes longer than critically damped.	reduces the amplitude in shortest time possible.

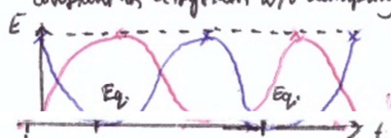
DERIVATION

From Hooke's Law: $F = -kx = -m\omega^2 x \Rightarrow m(2\pi f)^2 = k \Rightarrow 2\pi f = \sqrt{\frac{k}{m}} \therefore f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

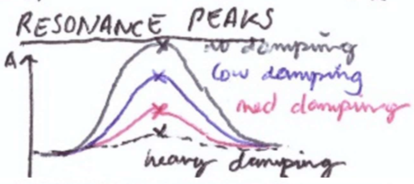
SIMPLE PENDULUM

period $\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

The mechanical energy stays constant in a system w/o damping.



RESONANCE PEAKS



DERIVATION

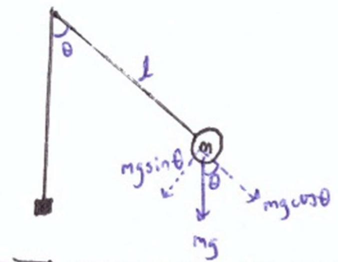
We know that restoring force acts ⊥: $F = ma = -mg\sin\theta$ $\Rightarrow a = -g\sin\theta$

When $\theta < 10°$, $\sin\theta \approx \theta$;

$s = r\theta \Rightarrow \theta = \frac{s}{r} \quad \therefore a = \frac{-gs}{l}$

We know that $a = -\omega^2 x = -(2\pi f)^2 \cdot x \Rightarrow -\frac{a}{x} = (2\pi f)^2$

$-\frac{a}{x} = -\frac{a}{s} \Rightarrow (2\pi f)^2 = \frac{g}{l} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \therefore T = 2\pi\sqrt{\frac{l}{g}}$



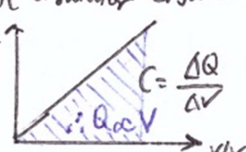
CAPACITORS

Capacitors (⊣⊢) - electrical component that can store electrical charge, composed of 2 electrical conducting plates, separated by an electrical insulator (i.e. a dielectric).

- Capacitance (of object): amount of charge it is able to store per unit p.d. across it $\quad \boxed{C = Q/V}$ (Farads; Coulombs; volts)

When a capacitor is connected to a D.C. power source, one plate becomes +vely charged and another -vely charged. Electrical insulator ensures no charge can move between plates, so P.D. $Q/_{RC}$

Voltage Rating - max P.D that can be put safely across



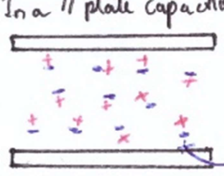
Energy Stored by Capacitors ▨ = total energy stored = area under a Q-V graph

Like charges repel, so as a capacitor charges, more work is needed to charge it.

$\boxed{E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}}$ $\quad$ capacitance (Farads) $\boxed{C = \frac{A\,\varepsilon_0\,\varepsilon_r}{d}}$ effective plate area (m^2); distance between capacitor plates (m)

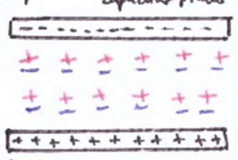
Dielectrics

In a // plate capacitor, they increase max. P.D. between plates.



No charge stored. No electric field generated. Random molecular alignment. polar molecule

Charge applied to +/- plates. Electric field generated.

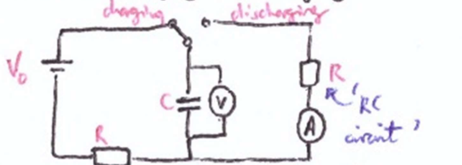


Permittivity ($F \cdot m^{-1}$) - how difficult it is to generate an electric field in a medium

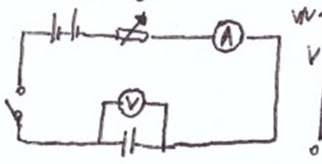
relative permittivity of a material $\boxed{\varepsilon_r = \frac{\varepsilon_1}{\varepsilon_0}}$ ← permittivity of material; ← permittivity of free space

Molecules each have their own electric field in this alignment, opposes applied electric field of the capacitor = Larger permittivity

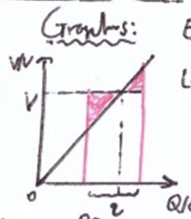
Circuits: charging & discharging



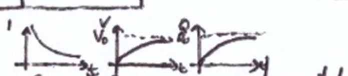
investigating capacitance



Graphs: $E = \Delta W = Q\Delta V$. Let q be small charge. Let v be the average p.d. over that step. $\therefore E = qV$

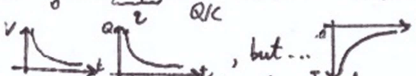


CHARGING



- → current flows from D.C. power supply until capacitor is fully charged
- → e^- flow from ⊖ to ⊕ means plate facing -ve is -ve: equal and opposite charge between plates. From repelled/attracted e^-, but no charge flow.

DISCHARGING



→ allow capacitor to discharge through resistor (given functions: $Q(t)$, $I(t)$, $V(t)$ and max values: Q_0, I_0, V_0)

DERIVATIONS

$V_0 = IR + \frac{Q}{C} \Rightarrow I = \frac{1}{R}\left(V_0 - \frac{Q}{C}\right)$

$\frac{dQ}{dt} = \frac{1}{RC}(V_0 C - Q) = -\frac{1}{RC}(Q - V_0 C) = -\frac{1}{RC}(Q - Q_0)$

$\Rightarrow \frac{dQ}{Q - Q_0} = \frac{-dt}{RC} \Rightarrow \int_0^Q \frac{dQ}{Q - Q_0} = -\frac{1}{RC}\int_0^t dt$

$[\ln|Q - Q_0|]_0^Q = -\frac{1}{RC}[t]_0^t$

$\ln|Q - Q_0| - \ln|-Q_0| = \frac{-t}{RC} \Rightarrow \frac{Q - Q_0}{-Q_0} = e^{-t/RC}$

$Q = Q_0 - Q_0 e^{-t/RC}$

$\boxed{Q(t) = Q_0(1 - e^{-t/RC})}$

as $Q \propto V$, also: $\boxed{V(t) = V_0(1 - e^{-t/RC})}$

$I = \frac{dQ}{dt} = (Q_0 - Q_0 e^{-t/RC})' = \frac{1}{RC}Q_0 e^{-t/RC} = \frac{Q_0}{RC}e^{-t/RC}$

$\frac{V_0}{R}e^{-t/RC} = I_0 e^{-t/RC}$ $\quad \boxed{I(t) = I_0 e^{-t/RC}}$

$IR = -\frac{Q}{C} \Rightarrow I = -\frac{Q}{RC}$

$\frac{dQ}{dt} = -\frac{Q}{RC} \Rightarrow \int_{Q_0}^{Q}\frac{dQ}{Q} = -\int_0^t \frac{dt}{RC}$

$\ln\left(\frac{Q}{Q_0}\right) = \frac{-t}{RC} \Rightarrow \frac{Q}{Q_0} = e^{-t/RC}$

$\boxed{Q(t) = Q_0 e^{-t/RC}}$

as $Q \propto V$, also: $\boxed{V(t) = V_0 e^{-t/RC}}$

$I = \frac{dQ}{dt} = (Q_0 e^{-t/RC})' = -\frac{Q_0}{RC}e^{-t/RC} = -I_0 e^{-t/RC}$

$\boxed{I(t) = I_0 e^{-t/RC}}$

FIELDS & THEIR CONSEQUENCES

Force field – region in which a body experiences a non-contact force. Arises from interaction of mass / static or moving charges.

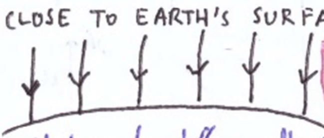
Fields { Gravitational : always attractive (–) / Electrical : charges attract (–) / repel (+) } both follow inverse-square laws, field lines & equipotentials

GRAVITATIONAL FIELDS

Gravity – universally attractive force acting between all matter

Newton's law of Gravitation: for 'point masses' $\boxed{F = \frac{GMm}{r^2}}$ (G: universal gravitational constant; r: distance between two masses /m)

CLOSE TO EARTH'S SURFACE

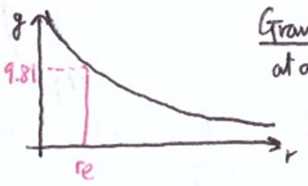


$\boxed{g = \frac{F}{m}}$

fields lines almost //, equally spaced

Radial Fields: they are vector fields

$$g = \frac{F}{m} = \frac{GMmr^{-2}}{m} \boxed{= \frac{GM}{r^2}}$$

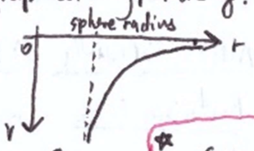


Gravitational Potential (V): GPE a unit mass at a point would have. It is always –ve; at ∞ distance; V = 0.

$$\boxed{V = -\frac{GM}{r}} \qquad V = \frac{E_p}{m} = \frac{-GMmr^{-2}}{m}$$

The gradient of the graph at any point = –g.

$$\boxed{g = -\frac{\Delta V}{\Delta r}}$$



Orbits

$$F = \frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GM}{r} \therefore v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi r\sqrt{r}}{\sqrt{GM}} = \sqrt{\frac{4\pi^2 r^3}{GM}} \therefore \boxed{T^2 \propto r^3}$$

Kepler's Third Law (plot linear: log T and log R)

∴ the greater the radius of a satellite's orbit, the longer it takes to complete 1 orbit, and slower it will travel.

In any orbiting satellite, KE + PE = constant

G.P.E (E_p): $E_p = mgh = -m \times \frac{GM}{r^2} \times r \therefore \boxed{E_p = -\frac{GMm}{r}}$ ☆

Gravitational Potential Difference: energy needed to move a unit mass. Work is done against gravity moving an object from 1 gravitational potential.

$$\boxed{\Delta W = m\Delta V}$$

Also, $W = F \cdot r \Rightarrow \int W = \int F \, dr$

$\therefore \int \frac{GMm}{r^2} dr = -\frac{GMm}{r} = E_p$

Escape Velocity – minimum speed an unpowered object needs in order to leave the gravitational field (of a planet) and not fall back due to gravitational attraction

K.E lost = G.P.E gained $\frac{1}{2}mv^2 = \frac{GMm}{r}$ $\therefore \boxed{\vec{v} = \sqrt{\frac{2GM}{r}}}$

Synchronous Orbits – orbiting object has an orbital period equal to the rotational period of object it orbits

LOW-ORBITTING SATELLITES (~180–2000km above Earth)

→ cheaper, require less powerful transmitters

→ close proximity means needs multiple satellites to maintain constant coverage (e.g. imaging, meterology)

→ usually lie in plane including N pole / S pole

GEOSTATIONARY SATELLITES – always above same point on Earth

(~ 36000km above Earth's surface)

→ useful for phone/TV signals: don't have to alter receiver angle

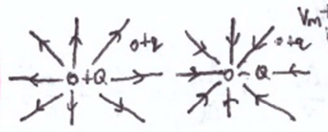
ELECTRIC FIELDS

Coulomb's Law: $\boxed{F = \frac{1}{4\pi\varepsilon_0} \times \frac{Qq}{r^2}}$

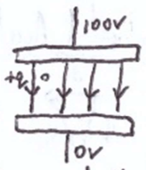
Electric Field Strength: force applied per unit +ve charge $\boxed{E = \frac{F}{Q}}$ (NC^{-1}) vector in direction a (+)ve charge would move.

Radial Fields:

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}}$$



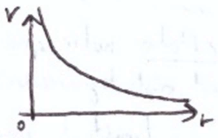
Uniform Fields: $\boxed{E = \frac{V}{d}}$ (Vm^{-1}; V: p.d. between plates / V; d: plate separation/m)



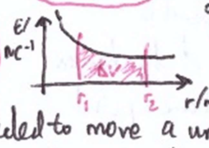
Electric Potential (V): EPE that a unit +ve charge (+1C) would have at that point. The sign of V depends on Q's charge.

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r}}$$

The gradient at any point = E. $\boxed{E = \frac{\Delta V}{\Delta r}}$



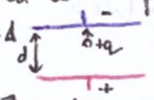
E.P.E (U_E): $\boxed{U_E = \frac{1}{4\pi\varepsilon_0} \times \frac{Qq}{r}}$



Electric Potential Difference: energy needed to move a unit charge between 2 points in a field. Work Done ∝ size of charge moved ∝ size of potential diff. moved across.

$$\boxed{\Delta W = Q\Delta V}$$

Considering 2 // plates, w/ p.d. Δ across, creating uniform field



then $E = \frac{\Delta V}{d} = \frac{F}{Q} \Rightarrow Q\Delta V = Fd$

work done = $F \cdot d$ = $\Delta W = Q\Delta V$

EQUIPOTENTIALS [line in 2D / surfaces in 3D] that join together all of the points with the same gravitational potential (V). Field lines always ⊥ to equipotentials

Moving along an equipotential, no work is done as $\Delta V = 0$.

Force at Subatomic Levels – distances between particles tiny, so expect gravitational / E.S. forces to be massive.

BUT: → gravity is negligible: particles have miniscule masses, much smaller than E.S

→ nucleus not destroyed by e.s. repulsion: strong force holds together

ELECTROMAGNETISM

NUCLEAR PHYSICS

Structure of an Atom

- → 5th century BC – Democritus, matter made of indentical, divisible 'ατομος'
- → 1804 – John Dalton: different 'Daltonspheres' for different elements
- → J.J. thompson – e^- removable (cathode rays) from atoms, so "plum pudding model"
- → Rutherford – (Geiger/Marsden: 1909) – α particle scattering through thin gold foil, flashing on circular flourescent screen
  1913 – Rutherford proposes nuclear model
- → Rutherford/Kay (1919) – p discovered, firing high E α particles at gases

Estimating Nuclear Radius

Using Coulomb's Law – α particle bounces back when $KE_{initial} = EPE$

$$\boxed{KE_\alpha = \frac{Qq_\alpha}{4\pi\varepsilon_0 \times r}}$$

initial KE of α; Q nucleus; $Q_\alpha = 2e$; r – distance of closest approach – gives upper bound

Using Electron Diffraction – Leptons don't interact w/ strong nuclear force, so e diff. gives accurate estimation

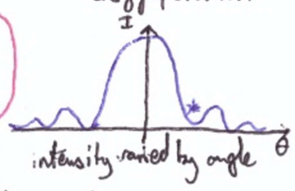
$$\boxed{\lambda \simeq \frac{hc}{E}}$$

relativistic effects of high speed; (use high E e⁻ so diffraction w/ atoms); E – energy of electron

First minimum appears where: *

$$\boxed{\sin\theta \simeq \frac{1.22\lambda}{2R}} \qquad \boxed{R \simeq \frac{0.61\lambda}{\sin\theta_{min}}}$$

θ – scattering ∠; R – radius of nucleus e⁻ scattered by



Using Experimental Data

$$\boxed{R = R_0 \times A^{\frac{1}{3}}}$$

R – nuclear radius; $R_0 = 1.4 \times 10^{-15}$ m; A – nucleon number

Nuclear Density

$$\rho = \frac{mass}{volume} = \frac{A \times m_{nucleon}}{\frac{4}{3}\pi R^3} = \frac{A \times m_{nucleon}}{\frac{4}{3}\pi (R_0 A^{\frac{1}{3}})^3} = \boxed{\frac{3 \times m_{nucleon}}{4\pi R_0^3}}$$

Nuclear Decay

radioactive decay – emission is both random and spontaneous. A large population of nuclei show overall trend, but can't predict an individual nucleus' decay.

$$\boxed{A = \lambda N = -\frac{\Delta N}{\Delta t}} \quad w/ \quad \boxed{W = A \times E}$$

A – activity/Bq; λ – decay constant/s⁻¹; N – # of unstable nuclei; W – power of source; E – photon energy/J; A – activity/Bq

Decay Constant (λ) – probability of an individual nucleus decaying in 1 second

$$\frac{dN}{dt} = -\lambda N \Rightarrow \lambda = \frac{\Delta N / N}{t} \Rightarrow \int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda \, dt$$

rate of change depends on itself ∴ uses e

$$\Rightarrow [\ln N]_{N_0}^{N} = -\lambda t \Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t \therefore \boxed{N = N_0 e^{-\lambda t}}$$

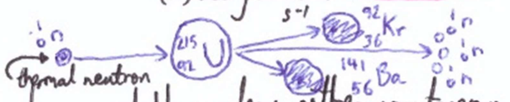
As A ∝ N, also: $A = \lambda N = \lambda N e^{-\lambda t} \Rightarrow \boxed{A = A_0 e^{-\lambda t}}$

Half Life ($T_{1/2}$) – average time taken for number of unstable nuclei to halve.

When $t = T_{1/2}$, then $N = \frac{1}{2} N_0$ ∴ $\frac{1}{2} = e^{-\lambda T_{1/2}} \Rightarrow -\ln 2 = -\lambda T_{1/2}$ ∴ $\boxed{T_{1/2} = \frac{\ln 2}{\lambda}}$

(λ) decay constant/s⁻¹

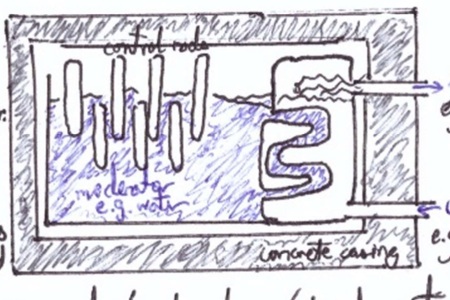
Nuclear Fission & Fusion



Nuclear Fission – splitting of a large, unstable nucleus, either spontaneous or induced. E released as new smaller nuclei have much higher BE/nucleon.

Nuclear Fusion – combining 2 lighter nuclei to create a larger nuclei. Much E released as heavier nuclei have much higher BE/nucleon. All nuclei +vely charged, so overcomes E.S attraction before strong force acts.

Nuclear Fission Reactors



Fission is induced by thermal neutr.

CHAIN REACTIONS – fission reactions produce more n then induce more fission. The neutrons must be slowed down to be captured by nuclei.

MODERATORS – fuel rods placed in water to slow down/absorb neutrons through elastic collisions by conservation of p. Want $m_{moderator} \simeq m_{neutron}$.

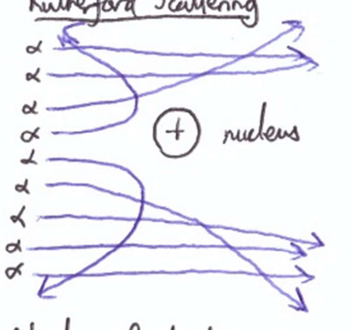
CONTROL RODS – allows chain reaction to be sustained at a steady rate by limiting # of neutrons in reactor, absorbing surplus neutrons.
Critical Mass: amount of fuel so 1 fission follows another.
Lowered/lifted accordingly to prevent meltdown.

COOLANT – removes heat produced by fission to make steam which turns turbine; cool water supplied

⚠ Safety Aspects

- → Reactor Shielding – thick concrete case to contain everything
- → Emergency Shutdown – fully lowers control rods
- → Handling/Storage – unused only emit α. Spent rods has neutron-rich products which β and γ. Products cooled until safe activity levels – use tongs.

Rutherford Scattering



Ensure that:
- → α have same speed – otherwise would have different deflections
- → container is a vacuum – α blocked by air
- → α source with long $t_{1/2}$ – rate of emission can be taken as constant

Conclusions:
- → atom is mostly empty space – majority of α undeflected
- → nucleus must be +vely charged – some α particles repelled through 180°
- → nucleus ~ $\frac{1}{10000}$th the atomic radius – v. few α bounced straight back
- → mass concentrated in nucleus – high p α particles are deflected

Nuclear Radiation

	nature	charge (Q)	mass (m)	penetrating power	ionising power	magnetic fields (affected?)
→ Alpha (α)	^4He nucleus	+2	~4u	paper/skin ~few cm in air (least)	dangerous if ingested (strongly)	✓
→ Beta (β)	e^- (or e^+ for β^+)	−1 (or +1)	~0	3mm aluminium ~few m in air	(lower m/a = less damage)	✓
→ Gamma (γ)	E.M radiation	0	0	lead, concrete (∞ range) (most)	v. weak = use in medicine (weakly)	× (as Q=0)

APPLICATIONS

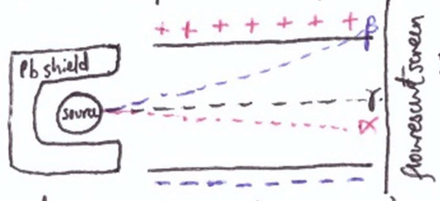
- α **smoke detectors**: allows current to flow, but when smoke is present, α particles can't reach detector. Alarm!
- β **thickness monitoring**: creating sheets – material passes through adjustable rollers. Too thick = less radiation absorbed, ∴ rollers move closer. Too thin = β⁻ detector detects too much ∴ rollers move apart.
- γ **radioactive tracers**: diagnose patients non-invasively. Source w/ short $t_{1/2}$ to prevent prolonged exposure. PET scanner then used to detect rays emitted.
- γ **cancer treatment**: fired in all directions w/ tumour at centre. Lessens damage to surrounding tissue. Side effects: tiredness, soreness to the skin. Medical staff leave room.

IDENTIFICATION

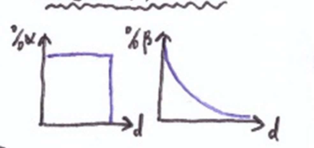
Background Radiation: reading of count rate from radiation present in the environment – take 3 Geiger counter readings, average them, subtract from each measurement

Sources:
- → the air: radioactive Rd gas released from rocks (α-emitter). Largest contributor
- → the ground/buildings: nearly all rocks contain radioactive materials
- → cosmic rays: most high E particles from space, collide w/ upper atmosphere particles to produce radiation
- → living things: during photosynthesis, plants absorb some atmospheric ^{14}C from cosmic rays
- → manmade sources: medical, industrial, nuclear power, weapons testing

1. record background count rate (w/o source)
2. place unknown source near Geiger-Muller tube record (corrected) count rate
3. place sheets of paper/aluminium and record count rates; subtract by background radiation

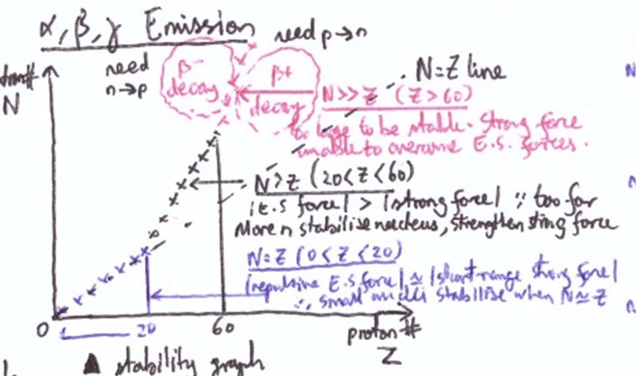


Penetration Curves



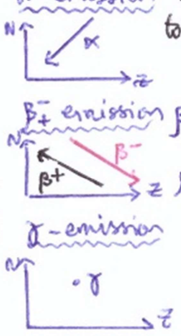
⚠ Safe Handling: hold source w/ tongs (∵ $\boxed{I = k/x^2}$), if not working directly, stay far away! Store γ in Pb box, out of box as little.

α, β, γ Emission



▲ stability graph

DECAY MODE GRAPHS



α-emission: in v. heavy atoms where atoms are too massive to be stable. 1-on-1 interaction so same p to α.

β± emission: β⁻: in neutron-rich isotopes; ejected w/ $\bar{\nu}_e$ so shares p
$n \rightarrow p + e^- + \bar{\nu}_e$
β+: in proton-rich isotopes
$p \rightarrow n + e^+ + \nu_e$

γ-emission: often after α/β decay, nucleus is in excited state and loses excess energy by γ rays.
e⁻ capture also makes nucleus unstable/emit γ.

Applications of Radioactive Isotopes

→ **Radioactive Dating**: ^{14}C taken in during photosynthesis. $T_{1/2} \approx 6000$ years. Test amount of ^{14}C remaining.

but:
- → in manmade objects: can only find age of material used, not object itself
- → contamination from other sources/high background count overpowering objects
- → uncertainty in ^{14}C thousands of years ago; small count rates are unreliable

→ **Medical Diagnosis**: Tc99m used as medical tracer to show tissue/organ function. Tracer injected/swallowed, then emitted radiation recorded: image produced.
- → emits γ with $T_{1/2}$ = 6 hrs (long enough for data recording, but short enough to limit exposure)
- → decays to much stabler isotopes

Mass Defect & Binding Energy

When 2 atoms come together to form a stable molecule { E released making a nucleus – exothermic; E required to separate a nucleus – endothermic }

Binding Energy of Nucleus: work done to separate a nucleus into its constituent nucleons

Mass Defect (Δm): the difference between mass of separated nucleons and $m_{nucleus}$

$$\boxed{\Delta m = Z m_p + (A - Z) m_n - m_{nucleus}} \qquad \boxed{B.E. = Q = \Delta mc^2}$$

unified atomic mass unit: $\boxed{1u \approx 931.5 \text{ MeV}}$

Average B.E. per nucleon is a measure of how stable a nucleus is.