Biomathematisches Praktikum am PC 2017

SIMULATION DES WA-TOR RÄUBER-BEUTE MODELLS UND ERWEITERUNGEN

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4.1 General

The Helmholtz equation is a partial differential equation of elliptic type.

$$\begin{aligned} \nabla^2 u + \kappa^2 u &= 0 \qquad (4.1)\\ \text{or} \quad \Delta u + \kappa^2 u &= 0 \end{aligned}$$

where Δ is the Laplace Operator with $\Delta u := \sum_{i=1}^{d} \frac{\partial^2 u}{\partial x_i^2}$.

We are interested in radial symmetric solutions to the Helmholtz equation. In order to find them it is helpful to transform equation (4.1) into polar coordinates for the two dimensional case or spherical coordinates for the three dimensional case.

Lemma 4.1. *The Laplace operator in $\mathbb{R}^2$ is given by $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. The Cartesian coordinates can be represented by polar coordinates as follows:*

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

Using these substitutions it can be shown that:

$$\Delta u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

Similarly, in $\mathbb{R}^3$ we transform to a spherical coordinate system:

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

which yields the form:

$$\Delta u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2} \qquad (4.2)$$

Helmholtz' equation is obtained by adding the term $\kappa^2 u$. Since we are solely interested in radial symmetric solutions we would like to eliminate the angular variables θ and φ from the equation. This leads us to:

Abbildungsverzeichnis

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