

MATH403: Homework 2

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- 12.** Suppose that an element X of a dihedral group is the product of m rotations and n reflections. Complete the following statement: X is a rotation if and only if _____.

Proof.

□

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18. Consider an infinitely long strip of equally spaced H's:

$$\dots H H H H \dots$$

Describe the symmetries of this strip. Is the group of symmetries of the strip Abelian?

Proof.

□

- 24.** If F is a reflection in the dihedral group D_n find all elements X in D_n such that $X^2 = F$ and all elements X in D_n such that $X^3 = F$.

Proof.

□

6. In each case, perform the indicated operation.

a. In \mathbf{C}^* , $(7 + 5i)(-3 + 2i)$

b. In $GL(2, Z_{13})$, $\det \begin{bmatrix} 7 & 4 \\ 1 & 5 \end{bmatrix}$

c. In $GL(2, \mathbf{R})$, $\begin{bmatrix} 6 & 3 \\ 8 & 2 \end{bmatrix}^{-1}$

d. In $GL(2, Z_7)$, $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}^{-1}$

Proof.

□

10. List the elements of $U(20)$ and find the inverse of each one.

Proof.

□

- 36.** Prove that in a group, $(ab)^2 = a^2b^2$ if and only if $ab = ba$.
Prove that in a group, $(ab)^{-2} = b^{-2}a^{-2}$ if and only if $ab = ba$.

Proof.

□