CMSC 351 Fall 2023 Homework 10

Due Wednesday Dec 6, 2023 by 11:59pm on Gradescope.

Directions:

- Homework must be done on printouts of these sheets and then scanned properly, or via latex, or by downloading, writing on the PDF, and uploading. DO NOT add pages.
- Do not use your own blank paper!
- The reason for this is that Gradescope will be following this template to locate the answers to the problems so if your answers are organized differently they will not be recognized.
- Tagging is automatic, do not manually tag. DO NOT add pages.

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1. (a) Suppose G is a complete (fully connected), unweighted, un-directed graph with four vertices labeled A, B, C, and D forming a square. [10 pts]

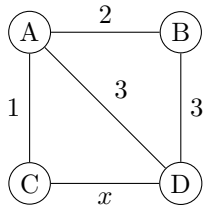
How many possible spanning trees are there that include exactly one "diagonal" edge ($A-D$ or $B-C$)?	8
How many possible spanning trees are there (total)?	16

(b) Suppose G is a complete (fully connected), weighted, un-directed graph with four vertices A, B, C, and D. The weights of the edges are as follows: $A-B$ and $C-D$ each have weight 4 while $A-C$, $A-D$, and $B-D$ each have weight 3, and $B-C$ has weight 2. [5 pts]

How many possible MINIMUM spanning trees are there?	3

Scratch Work for Both; Not Graded:

2. Consider the following graph where $x \in \mathbb{Z}^+$:



Assuming that there is at least one minimum spanning tree that includes the edge $C - D$, what must be true about x?	$x \leq 3$	[5 pts]
Assuming that any minimum spanning tree MUST include the edge $C - D$, what must be true about x?	$x \leq 2$	[5 pts]

3. An undirected, weighted graph on five vertices A, B, C, D, E, F has the following adjacency matrix (INF means that there is no edge):

	A	B	C	D	E	F
A	0	12	INF	7	INF	INF
B	12	0	3	INF	5	8
C	INF	3	0	INF	10	6
D	7	INF	INF	0	INF	11
E	INF	5	10	INF	0	4
F	INF	8	6	11	4	0

(a) How many steps will each if Prim's and Kruskal's Algorithm take? [5 pts]

Number of Steps =	5

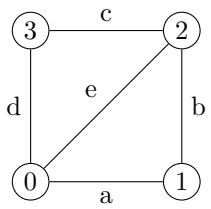
(b) In the table below, show the edges being picked at each step as we run each algorithm. There might be more columns than you need in the table below. Assume Prim's Algorithm starts at vertex A. [10 pts]

Step ⇒	1	2	3	4	5	6	7	8
Prim	A-D	D-F	F-E	E-B	B-C			
Kruskal	B-C	E-F	B-E	A-D	F-D			

(c) What is the total cost of the resulting minimum spanning tree for each? [5 pts]

Total Cost =	30

4. Suppose Kruskal's Algorithm is run on the following graph. Assume all the weights are distinct. [20 pts]



Assuming the weights are all distinct there are $5! = 120$ possible orderings, for example $a < b < c < d < e$, $e < d < c < b < a$, and so on. Assuming each of these is equally likely, what is the probability that e is included in the final tree?

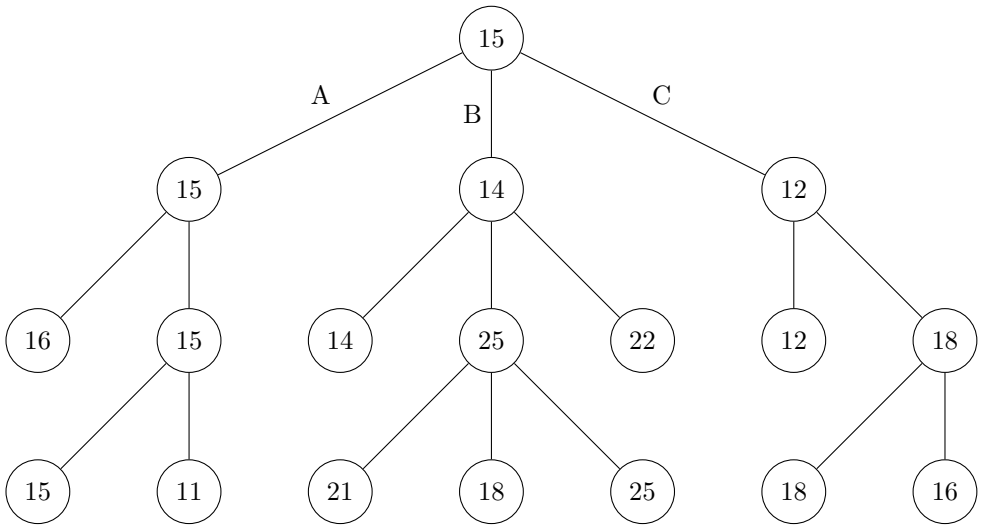
Note: Do not do this by listing all possible orderings!

Solution: Note that if e is the least weighted edge or the second least weighted edge, then it will automatically be included in the minimum spanning tree when using Kruskal's algorithm because it is not possible to create a cycle with only 2 edges. This already yields $24 + 24 = 48$ valid orderings. Now suppose e is the third element in the inequality. We have to be careful not to create a cycle here. If the first two elements were cd, dc, ab, ba then this would create a cycle. Thus, there are $4P2 - 4 = 12 - 4 = 8 \rightarrow 8 * 2$ valid orderings with e in the middle. We multiply by 2 because of the 2 variants created by the ordering of the 4th and 5th elements. Note that it is not possible to create a cycle of length 3 without e, so if e is the 4th or 5th elements, we immediately know this is not a valid ordering. Thus, our final answer is

$$(24 + 24 + 16)/120 = \frac{8}{15}$$

5. We are running the Minimax algorithm and developed the following search tree. You are MAX and it is your turn to play.

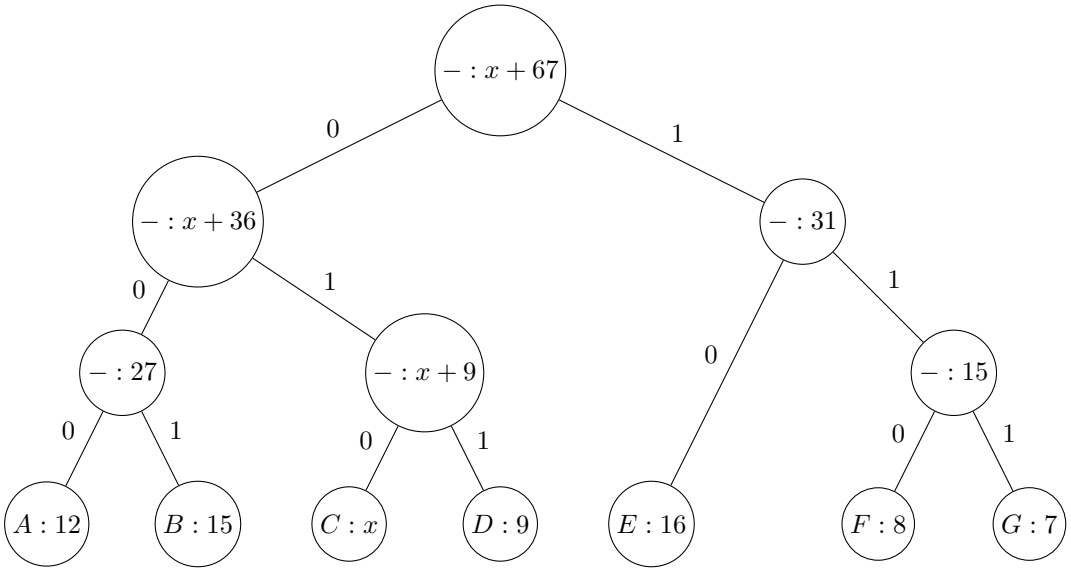
   (a) Fill in the values in the search tree below: [14 pts]



   (b) You are MAX and you are trying to win; what is your move? [6 pts]

A, B, or C?	A

6. The tree below is the result of running Huffman's algorithm, where x is unknown.



Is the code defined by the tree above a prefix code?	Yes	[3 pts]
What can you say about x (use $=$, $<$, $>$, etc.)?	$8 \leq x \leq 12$	[7 pts]
What is the encoding for ABE?	00000110	[5 pts]