CMSC 351 Fall 2023 Homework 8

Due Wednesday Nov 8, 2023 by 11:59pm on Gradescope.

Directions:

- Homework must be done on printouts of these sheets and then scanned properly, or via latex, or by downloading, writing on the PDF, and uploading.
- Do not use your own blank paper!
- The reason for this is that gradescope will be following this template to locate the answers to the problems so if your answers are organized differently they will not be recognized.
- Tagging is automatic, do not manually tag.

---

1. Suppose we are sorting a list of names using radix sort. Show the list contents at each step in the table below: [9 pts]

Words	BEE	COM	BOM	BOA	SIM	EVE
Step 1	BOA	BEE	EVE	COM	BOM	SIM
Step 2	BEE	SIM	BOA	COM	BOM	EVE
Step 3	BEE	BOA	BOM	COM	EVE	SIM

2. Using radix sort, we have generated this CORRECT sorting sequence where x is unknown: [10 pts]

Start	246	x	852	367
Step 1	852	246	367	x
Step 2	246	852	x	367
Step 3	246	x	367	852

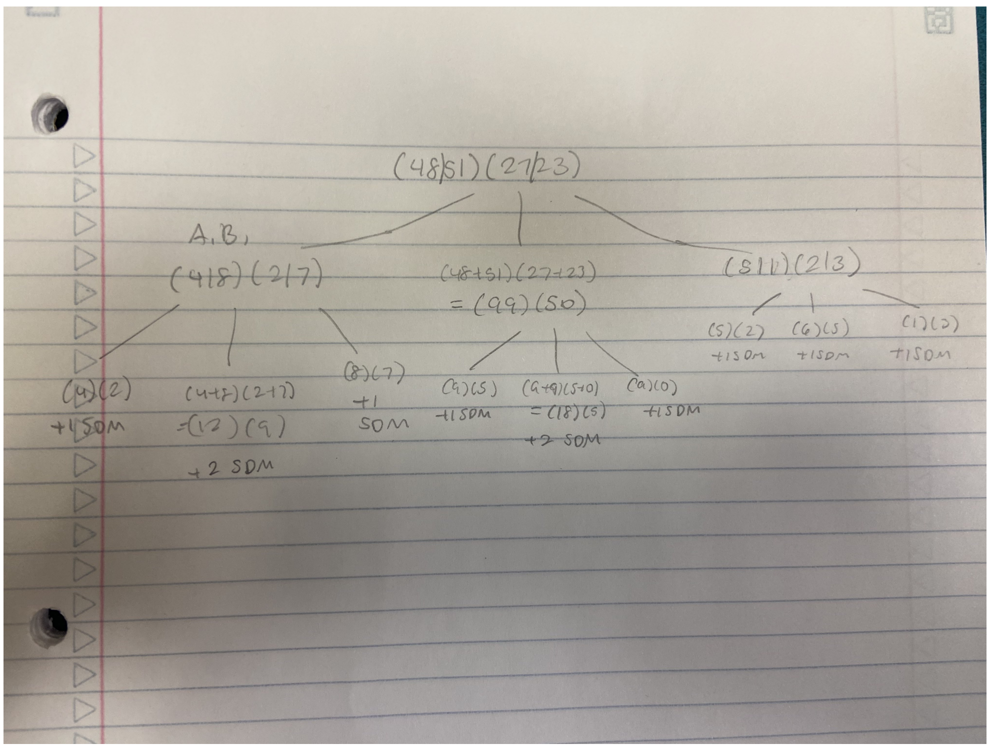
What are all the possible values for x?
258, 259, 358, 359

3. We are using Karatsuba's algorithm to calculate (4851)(2723). In what follows, SDM means single-digit multiplication.

(a) Using schoolbook multiplication how many SDMs will be performed?	16	[2 pts]
(b) How many times will the base case be reached?	9	[4 pts]
(c) How many SDMs will eventually be performed?	11	[4 pts]

(d) Draw the corresponding tree. [13 pts]

Solution:



4. We can write the recurrence formula for Karatsuba's algorithm is $T(n) = 3T(n/2) + \Theta(n)$. We proved, using the Master theorem, that it runs in $\Theta(n^{\lg 2}) \approx \Theta(n^{1.5849..})$. To show that, we decomposed two n-digit numbers A and B as a_1a_0, and b_1b_0 where a_0, a_1, b_0, and b_1 all have $n/2$ digits. By modifying the way we calculate the number, we ended up with 3 multiplications (of $n/2$ digits numbers) instead of 4 (and a few additions, subtractions and shifts), resulting in the recurrence relation above.

Suppose instead that we create a new version called Karatsuba-5 in which we decompose the two n-digit numbers A and B as $a_4a_3a_2a_1a_0$ and $b_4b_3b_2b_1b_0$ where a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4 all have $n/5$ digits as follows:

$$A = a_4a_3a_2a_1a_0 = 10^{4n/5}a_4 + 10^{3n/5}a_3 + 10^{2n/5}a_2 + 10^{n/5}a_1 + a_0$$
$$B = b_4b_3b_2b_1b_0 = 10^{4n/5}b_4 + 10^{3n/5}b_3 + 10^{2n/5}b_2 + 10^{n/5}b_1 + b_0$$

To multiply A by B, we group and use a similar trick and end up with 15 multiplications instead of x (not given) of numbers that are $n/5$ digits wide (and additions, subtractions and shifts, overall taking $\Theta(n)$.

(a) What is the value of x above?	25	[4 pts]
(b) What is the recurrence relation for Karatsuba-5?	$T(n) = 15T(\frac{n}{5}) + \Theta(n)$	[6 pts]
(c) What is the exact Θ of Karatsuba-5?	$\Theta(n^{\log_5(15)})$	[4 pts]

(d) If we wanted this to be faster (measured by Θ) than Karatsuba, what is the maximum number of multiplications we could have, rather than x? Explain. [6 pts]

Solution; Graded:

We seek x such that $\Theta(n^{\log_5 x}) < \Theta(n^{\lg 3})$. Therefore, this just boils down to

$$\log_5 x < \lg 3$$

$$x < \frac{\log 3 * \log 5}{\log 2} \implies x < 12.82 \implies$$

The maximum number of multiplcations we could have rather than x is 12.

5. A directed, unweighted graph on 4 vertices A, B, C, D has the following adjacency list:

$$A \longrightarrow B, C$$

$$B \longrightarrow D$$

$$C \longrightarrow A, D$$

$$D \longrightarrow B$$

What is its adjacency matrix? [8 pts]

	A	B	C	D
A	0	1	1	0
B	0	0	0	1
C	1	0	0	1
D	0	1	0	0

6. Suppose an unweighted graph with 9 vertices has the following adjacency matrix. The values of x and y are unknown, they are either 0 or 1. [10 pts]

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & x & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & y \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & y & 1 & 0 \end{bmatrix}$$

Suppose the shortest path algorithm is run on this graph starting at $source = 0$ and ending at $target = 8$. The length of that shortest path was found to be 4. We want to know the possible pairs (x, y).

What are the possible pairs (x, y)?
$(1, 0), (1, 1)$

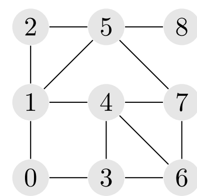
7. Suppose the shortest path algorithm is run a graph with six vertices labeled 0 to 5, starting at the vertex 0. No target is specified so it runs until the queue is empty. The progression of the queue is shown here: [10 pts]

Starting Queue	0
Then	3,4
Then	4,2,5
Then	2,5
Then	5,1
Then	1
Then	Empty

What will the predecessor list look like?

Index	0	1	2	3	4	5
Entry	NULL	2	3	0	0	3

8. Suppose the shortest path algorithm is run on the following graph starting at the vertex 3. From a vertex, adjacent vertices are visited in increasing order. No target is specified so it runs until the queue is empty. [10 pts]



Show each possible state of the queue starting with the first nonempty queue. You will not need all of the spaces!

Starting Queue	3								
Then	0	4	6						
Then	4	6	1						
Then	6	1	7						
Then	1	7							
Then	7	2	5						
Then	2	5							
Then	5								
Then	8								
Ending Queue									