CMSC 351 Spring 2022 Exam 2

Version Tue/Thu

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1. Mark each of the following as true or false. Write the word TRUE or FALSE in the box to the right. No justification is required.   [10 pts]

   (a) MergeSort is stable.

   (b) A list with three elements can be sorted with just two comparisons.

   (c) The Master Theorem applies to $T(n) = T(n/2) + f(n)$ with $f(n) = \mathcal{O}(n)$.

   (d) In a max heap the smallest element must be a leaf.

   (e) If a list is $[5, 8, 3, 7, 10, 11]$ after the first iteration of QuickSort, the pivot value could have been 7.

2. Suppose a list A with n elements contains integers between 0 and $2^{(n^2)} - 1$ inclusive. Using base 2 to represent these numbers and RadixSort + CountingSort, calculate the Θ time complexity.   [15 pts]

   Solution:

3. Apply the Master Theorem to solve the following recurrence relation: [15 pts]

$$T(n) = 2\,T(n/4) + \sqrt{n} + 5$$

Solution:

4. Apply the Master Theorem to solve the following recurrence relation: [15 pts]

$$T(n) = 5\,T(n/4) + \lg n$$

Solution:

5. Consider the recurrence relation:

$$T(n) = 2T(n/3) + 3n + 2 \quad \text{with} \quad T(1) = 7$$

(a) Draw the complete recursion tree for $T(9)$ with its values filled in. [10 pts]

Solution:

(b) From (a) what is the value of $T(9)$? [5 pts]

Solution:

6. Here is the pseudocode for the partition function from QuickSort in case you need it: [10 pts]

```
function partition(A,L,R)
    pivotvalue = A[R]
    t = L
    for i = L to R-1
        if A[i] <= pivotvalue
            A[t] <-> A[i]
            t = t + 1
        end
    end
    A[t] <-> A[R]
    return t
end
```

Consider the list:

$$[5, 8, 3, 4, 10, 7]$$

If partition(A,0,6) is run on this list, show the result after each swap which actually exchanges different elements.

Solution:

7. Suppose you have a heap which has N layers and is full on every layer, thus it contains $n = 2^N - 1$ nodes. You select a node uniformly at random and run `maxheapify` on it. If a swap takes 2 seconds and nothing else takes any time at all, what is the expected value for the time it takes `maxheapify` to run in the worst case? You can leave your answer as a summation including n but it should not include N. [20 pts]

Solution: