CMSC351-2023-10-30

Monday, October 30, 2023 8:10 AM

Karatsuba - Continued

① Reminder: Abstractly, to compute $(\overset{n\text{ digits}}{A_1}\overset{n\text{ digits}}{A_0})(B_1B_0)$ we instead compute three smaller products:

A_1B_1 $(A_1+A_0)(B_1+B_0)$ A_0B_0

Then we piece them back together via:

$$(A_1A_0)(B_1B_0) = 10^n A_1B_1 + 10^{n/2}\left[(A_1+A_0)(B_1+B_0) - A_1B_1 - A_0B_0\right] + A_0B_0$$

② Pseudocode:

```
\\ A,B are the list representations of numbers.
function karatsuba(A,B)
    if either A or B is single-digit
        return(A*B)
    else
        sp = floor((minimum number of digits in A,B)/2)
        A1,A0 = split A, sp digits from the right
        B1,B0 = split B, sp digits from the right
        k1 = karatsuba(A1,B1)
        k2 = karatsuba(A1+A0,B1+B0)
        k3 = karatsuba(A0,B0)
        // The powers of 10 should be thought of as shifts.
        r = 10^(2*sp)*k1 + 10^(sp)*(k2-k3-k1) + k3
        return(r)
    end
end
```

A,B = any # of digits! Might be different!

(123)(5) would just multiply

to ensure powers of 10 work out

eg (1234)(12345)

must be $(12)10^2 + 34$

and $(123)10^2 + 45$

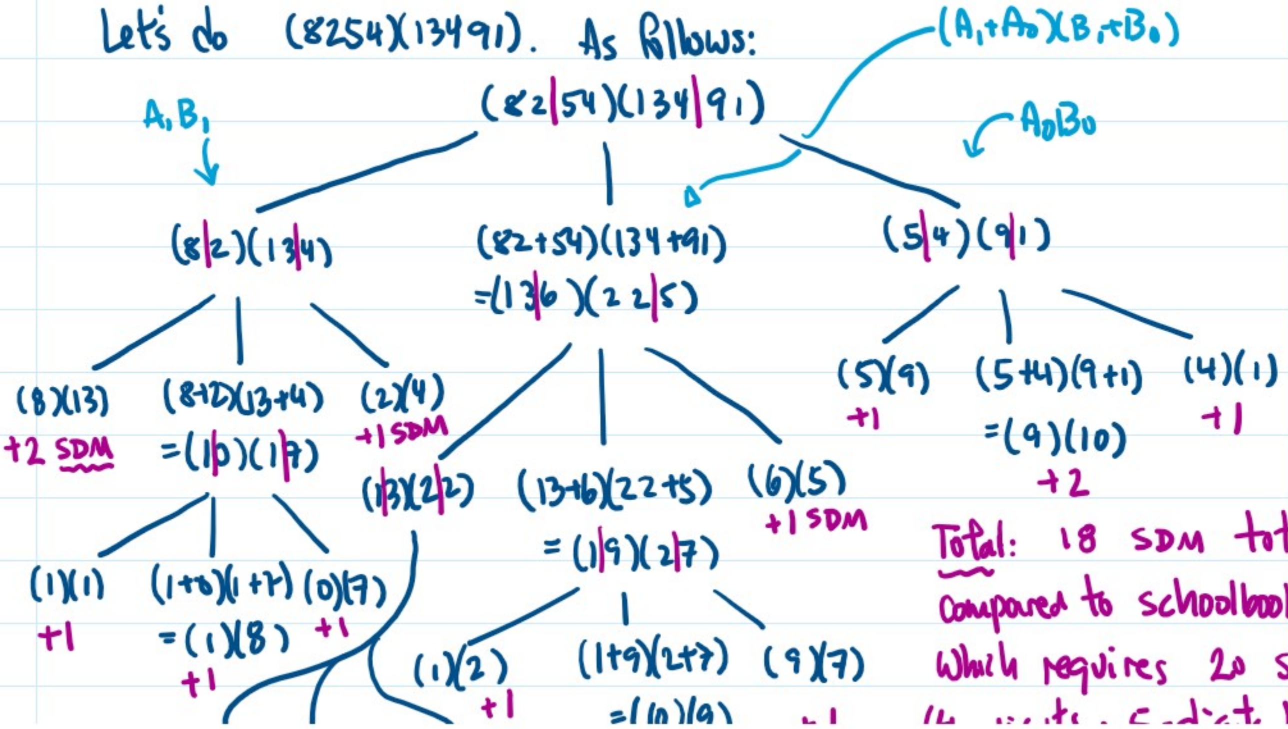
eg (12)(345678)

Recurse!

piece it back together!

③ Tree Diagram Showing the products

Let's do (8254)(13491). As follows:

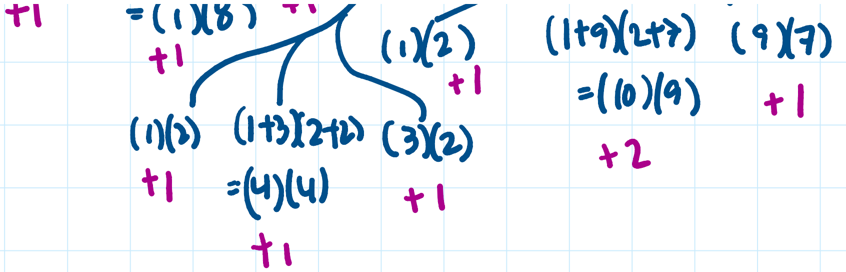


Total: 18 SDM total

compared to schoolbook mult.

Which requires 20 SDM total

(4 digits · 5 digits)

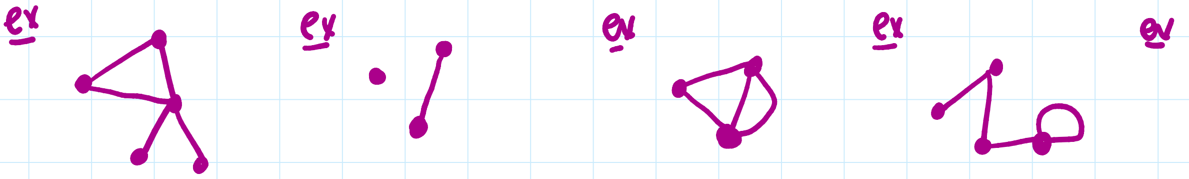


Which requires 20 SDM total (4-digits · 5-digits)

Closing note: As # digits increases, the savings in terms of SDM goes up!

Intro to Graphs!

① defn: A graph consists of a collection of vertizes or nodes whereby we may have edges (lines) joining vertizes.



5 vertizes
5 edges

3 vertices
1 edge

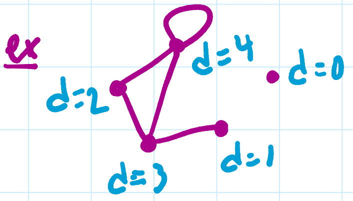
3 V.
4 E.

4 V
4 E

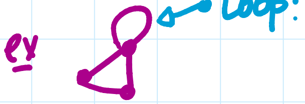
0 V.
0 E.

② defn: Two vertizes are adjacent if they are joined by an edge.

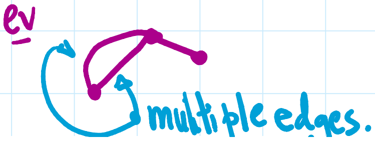
③ defn: The degree of a vertex equals the # of edge connections.

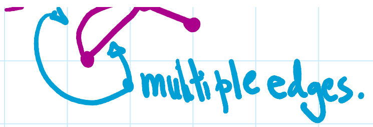


④ defn A loop is an edge joining a vertex to itself.



⑤ defn Multiple edges means two or more edges joining the same two vertizes.



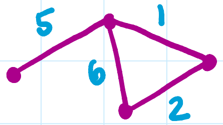


⑥ defn: A graph is simple if it has no loops or multiple edges.

⑦ defn: A graph is weighted if each edge has a positive value (a weight) assigned.

think: distance or cost!

ex



⑧ defn: A graph is directed if each edge has an associated direction.

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