CMSC 351 Fall 2023 Homework 7

Due Wednesday Nov 1, 2023 by 11:59pm on Gradescope.

Directions:

- Homework must be done on printouts of these sheets and then scanned properly, or via latex, or by downloading, writing on the PDF, and uploading.
- Do not use your own blank paper!
- The reason for this is that gradescope will be following this template to locate the answers to the problems so if your answers are organized differently they will not be recognized.
- Tagging is automatic, do not manually tag.

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1. Here is the state of the helper list (POS in Justin's notes and C in the powerpoint slides) after the **FIRST** pass:

index	0	1	2	3
value	1	3	2	1

   (a) How many elements are in the original list that we are sorting? [4 pts]

Number of elements:	7

   (b) At the end, what is the sorted list? You will not need all the blanks below so only fill in as far as you need! [6 pts]

Index	0	1	2	3	4	5	6	7	8	9
Value	0	1	1	1	2	2	3			

2. Here is the state of the helper list (POS in Justin's notes and C in the powerpoint slides) after the **SECOND** pass (cumulative version):

index	0	1	2	3	4	5	6
value	0	2	2	3	5	5	7

Answer the following questions:

(a) How many 0 s are in the original, unsorted list?	0	[4 pts]
(b) How many 3 s are in the original, unsorted list?	1	[4 pts]
(c) How many 5 s are in the original, unsorted list?	0	[4 pts]
(d) What is the largest element of the original, unsorted list?	6	[4 pts]
(e) What is the smallest element of the original, unsorted list?	1	[4 pts]

3. We are using counting sort to sort the following list of numbers

index	0	1	2	3	4	5	6
value	?	?	3	0	1	?	?

Here is the helper list after the **FIRST** (non-cumulative) step

index	0	1	2	3
value			3	

Here is the helper list after the **SECOND** (cumulative) step

index	0	1	2	3
value		2		

Fill in all the blanks below for helper list after the FIRST and SECOND steps [12 pts]

index	0	1	2	3
value	1	1	3	2

index	0	1	2	3
value	1	2	5	7

4. Running counting sort, we had a memory loss occurring right after the second pass finished. The helper list (POS in Justin's notes and C in the powerpoint slides) looks like this after the **SECOND** pass (cumulative version): Note: all the elements are integers

index	0	1	2	3	4
value	0	3	x	4	7

What value(s) could x be? [6 pts]

List all the possible values for x	3, 4

5. Suppose $b = 2$ is the fixed base. Suppose that we have a list of length n which contains integers between 0 and n inclusive. Explain why Radix Sort plus Counting Sort time complexity will be $\Theta(n \lg n)$. [10 pts]

Solution:

We are given $n = n, b = 2$. Note that

$$d = \begin{cases} \lg n + 1, & \text{if } \lg n = \lceil \lg n \rceil \\ \lceil \lg n \rceil, & \text{else} \end{cases}$$

In either case, $d \approx \lg n$ and this could vary by at most 1. For this proof, since we can factor out this constant difference, let us say $d = \lg n$. Now note that a single counting sort will take $\Theta(n + (2 - 1))$ time. Therefore, $d = \lg n$ counting sorts will take

$$\Theta(\lg n(n + 1)) = \Theta(n \lg n + \lg n)$$

Using limit theorems we compute

$$\lim_{n \to \infty} \frac{(n \lg n + \lg n)}{n \lg n} = 1 \neq 0, \infty \implies \Theta(n \lg n)$$

6. In our opening discussion of Karatsuba we observed that for $A = a_1 a_0$ and $B = b_1 b_0$ we have:

$$AB = 100 a_1 b_1 + 10\left[(a_1 + a_0)(b_1 + b_0) - a_0 b_0 - a_1 b_1\right] + a_0 b_0$$

(a) For the product AB with $A = 42$ and $B = 71$, calculate the three critical products: [6 pts]

$a_1 b_1$	28
$(a_1 + a_0)(b_1 + b_0)$	48
$a_0 b_0$	2

(b) Fill in the final calculation of AB: [4 pts]

$$AB = 100\,\boxed{28} + 10\,\boxed{18} + \boxed{2} = \boxed{2982}$$

7. In multiplying AB where A and B are two n-digit numbers it's possible to construct a variation on Karatsuba which breaks each number into three pieces instead of two. If this is done then it is possible to perform just seven multiplications (instead of nine) of one-third the size. The extra work is still $\Theta(n)$.

(a) What would the corresponding recurrence relation be? [3 pts]

$T(n) =$	$7T(\frac{n}{3}) + \Theta(n)$

(b) What would the resulting time complexity be? [3 pts]

$T(n) = \Theta$ of what?	$n^{\log_3 7}$

(c) Would this be better than, the same as, or worse than Karatsuba? [4 pts]
Explain in one or two sentences at most.

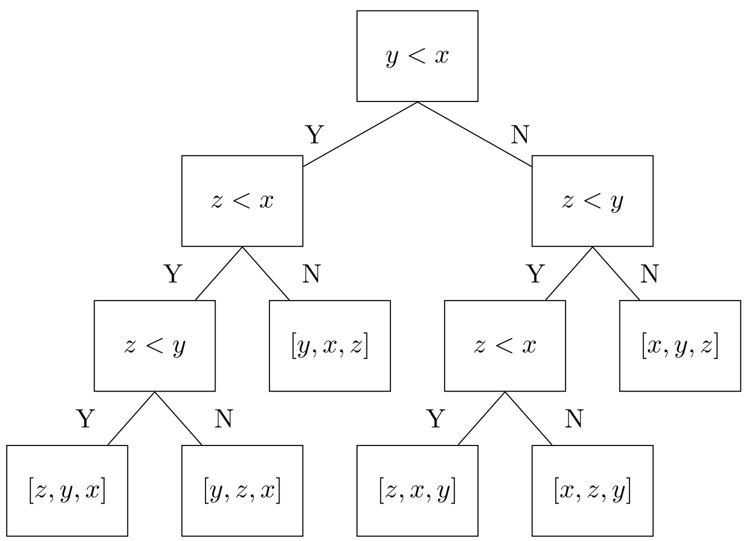
Answer; Graded:

This is worse than Karatsuba's algorithm because the time complexity of this, using Master's Theorem, is $n^{1.77} > n^{1.58}$.

8. Here is the pseudocode for Insertion Sort: [22 pts]

```
for i = 1 to n-1
    key = A[i]
    j = i-1
    while j >= 0 and key < A[j]
        A[j+1] = A[j]
        j = j - 1
    end
    A[j+1] = key
end
```

For a list $[x, y, z]$ of distinct elements, fill in the following decision tree.



Scratch Work; Not Graded: