

MATH401 Homework 7

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1. Problem 6.7 from the text

a. $T = \begin{bmatrix} 0.70 & 0.05 & 0.03 \\ 0.28 & 0.85 & 0.02 \\ 0.02 & 0.10 & 0.95 \end{bmatrix}$

b.

$$Tx_0 = \begin{bmatrix} 0.0970 \\ 0.0480 \\ 0.8570 \end{bmatrix}$$

$$T^2v_0 = \begin{bmatrix} 0.0959 \\ 0.0834 \\ 0.8207 \end{bmatrix}$$

$$T^5v_0 = \begin{bmatrix} 0.0975 \\ 0.0160 \\ 0.7415 \end{bmatrix}$$

c. After normalization, the steady state is $\begin{bmatrix} 0.1081 \\ 0.2829 \\ 0.6090 \end{bmatrix}$

d. Using Matlab, the smallest value of k is $k = 66$

2. In the homework

a. $T = \begin{bmatrix} 0.95 & 0.45 \\ 0.05 & 0.55 \end{bmatrix}$

b. $T^3 = \begin{bmatrix} 0.9125 & 0.7875 \\ 0.0825 & 0.2125 \end{bmatrix} \Rightarrow 91.25\% \text{ three days from now}$

c. $\lim T^k = \begin{bmatrix} 0.9 & 0.9 \\ 0.1 & 0.1 \end{bmatrix} \Rightarrow 10\%$

d. Using Matlab once more, we obtain 5 days

3. In the homework

a) $T = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & 0.5 & 0 & 0 \end{bmatrix}$

b) T is not regular because there are entries that are 0 and for example, a mouse in room 2 cannot get to mouse 3.

c) $P(\text{room } 5 = 0)$ and $P(\text{room } 2 = 1)$

d) $P(\text{room } 5 = 1)$ and $P(\text{room } 2 = 0)$

e) An example probability vector is $\begin{bmatrix} 0 \\ 0.7 \\ 0 \\ 0 \\ 0.3 \end{bmatrix}$ because there will never be a steady state since mice in rooms 2 and 5 will forever alternative.

4. In the homework

$$\text{a) } T = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0.5 \\ 0 & 1 & 0.5 & 0 & 0 \end{bmatrix}$$

b) Yes, a state can reach every other state and itself within 5 or less states so it is regular.

c) The probabilities are

$$P(1) = \frac{2}{15}, P(2) = \frac{3}{15}, P(3) = \frac{2}{15}, P(4) = \frac{4}{15}, P(5) = \frac{4}{15}$$

5. In the homework

$$\text{a) } P(ANVAN|A) = 1 * 0.7 * 0.65 * 1 = 0.455$$

$$\text{b) } P(NPCVP) = 0.1 * 0.15 * 0.15 * 0.1 = 0.000225$$

c) Their difference suggests that ANVAN is more likely to occur than NVCVP

$$\text{d) } \text{Noun followed by article TWO words later is } T^2 \implies 0.605$$

$$\text{e) } \text{Noun followed by article FOUR words later is } T^4 \implies 0.6422$$

f) The associated probabilities are

$$N \approx 0.3310, V \approx 0.2703, P \approx 0.0438, A \approx 0.2481, C \approx 1.1068$$