CMSC 351 Fall 2023 Homework 9

Due Wednesday Nov 15, 2023 by 11:59pm on Gradescope.

Directions:

- Homework must be done on printouts of these sheets and then scanned properly, or via latex, or by downloading, writing on the PDF, and uploading. DO NOT add pages.
- Do not use your own blank paper!
- The reason for this is that gradescope will be following this template to locate the answers to the problems so if your answers are organized differently they will not be recognized.
- Tagging is automatic, do not manually tag. DO NOT add pages.

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1. Suppose we have a graph with 6 vertices, A, B, C, D, E, F. Dijkstra's algorithm has generated the following path recovery information using B as the source vertex: [10 pts]

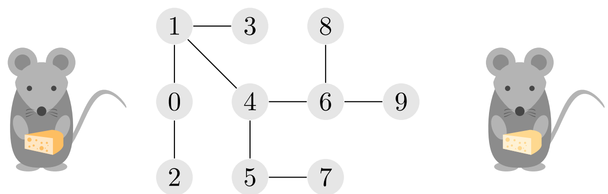
Vertex	Predecessor(Vertex)
A	C
B	N/A
C	B
D	B
E	A
F	E

What is the path from B to A?
B, C, A

What is the path from B to F?
B, C, A, E, F

2. Two very hungry mouse brothers both start at vertex 0 in the following graph. One is named Daniel Frederick Turing (DFT) and the other is named Bruce Frederick Turing (BFT). Both can teleport from one node to another but Daniel always does depth-first traverse while Bruce always does breadth-first traverse. For consistency when vertex choices may be made, both use increasing numerical order.

   Note: Don't think of any particular pseudocode algorithms, just follow the vertices as instructed. If you're not sure what this means, ask.



   (a) List the order in which Daniel traverses the nodes. [5 pts]

0	1	3	4	5	7	6	8	9	2

   (b) List the order in which Bruce traverses the nodes. [5 pts]

0	1	2	3	4	5	6	7	8	9

   (c) Suppose a piece of cheese is located at some node t. Which values of t would result in Daniel finding it first? In Bruce finding it first? In it being a tie? [5 pts]

Daniel	3, 4, 5, 7, 8, 9
Bruce	2
Tie	0, 1, 6

   (d) Suppose this happens every day and every day the cheese is located at some random node. If the brothers tie then they split the cheese in half. What is the expected value of the daily amount of cheese that each brother gets? You should have one value for each brother. [5 pts]

Daniel	$\frac{3}{4}$
Bruce	$\frac{1}{4}$

   Scratch Work; Not Graded:

   For Daniel,

$$E(X) = \frac{3}{5}(1) + \frac{3}{10}(\frac{1}{2}) = \frac{3}{5} + \frac{3}{20} = \frac{3}{4}$$

   For Bruce,

$$E(X) = \frac{1}{10}(1) + \frac{3}{10}(\frac{1}{2}) = \frac{1}{4}$$

3. Suppose we have a graph with 6 vertices, A, B, C, D, E, F. We ran Dijkstra's algorithm on it using C as the source vertex: Generating the paths from the recovery table, we get: [10 pts]

$$C \longrightarrow E \longrightarrow D \longrightarrow B$$
$$C \longrightarrow E \longrightarrow F \longrightarrow A$$

Fill in the blanks below for the path recovery table.

Vertex	Predecessor(Vertex)
A	F
B	D
C	n/a
D	E
E	C
F	E

4. Assume that we do NOT know in which order vertices are processed (alphabetical or not). [15 pts] Here is the adjacency matrix of an undirected weighted graph. As you can see, one edge has an unknown, positive integer value x:

	A	B	C	D	E
A	0	8	INF	5	7
B	8	0	x	17	INF
C	INF	x	0	10	9
D	5	17	10	0	6
E	7	INF	9	6	0

We know that Dijkstra's algorithm has generated the following path recovery information using A as the source vertex on the left and B as the source vertex on the right (note: x is an integer):

Node	Pred(Node)
A	n/a
B	A
C	B
D	A
E	A

Node	Pred(Node)
A	B
B	n/a
C	B
D	A
E	C

What can you tell about the value of x (be as precise as possible)
$3 \leq x \leq 6$

Note: Your answer above could be something like $x \geq 45$, or $x < 32$, or $x = 16$, or $60 \leq x \leq 62$.

Scratch Work; Not Graded:

5. Here is the adjacency matrix of a directed weighted graph. [20 pts]

	A	B	C
A	0	6	14
B	INF	0	3
C	5	INF	0

Run Floyd's algorithm and fill the distance (left) and recovery path (right) tables below:

"Pass by A" (fill in distance matrix left below and recovery path matrix right below)

	A	B	C
A	0	6	14
B	INF	0	3
C	5	11	0

	A	B	C
A	A	A	A
B	NULL	B	B
C	C	A	C

"Pass by B" (fill in distance matrix left below and recovery path matrix right below)

	A	B	C
A	0	6	9
B	INF	0	3
C	5	11	0

	A	B	C
A	A	A	B
B	NULL	B	B
C	C	A	C

"Pass by C" (fill in distance matrix left below and recovery path matrix right below)

	A	B	C
A	0	6	9
B	8	0	3
C	5	11	0

	A	B	C
A	A	A	B
B	C	B	B
C	C	A	C

6. Suppose that we modified Floyd's algorithm as follows: [10 pts]

We replace `p[i][j]=p[k][j]` with `p[i][j]=k`

We use this modified version of Floyd's algorithm on a graph with 6 vertices (A, B, C, D, E, F) to compute the shortest distances between each pair of vertices. We have generated the following recovery matrix.

	A	B	C	D	E	F
A				B		
B			A			
C		E				
D					F	
E		F	B			
F	E					

What is the path from A to D?

A, B, D

What is the path from E to C?

E, F, B, A, C

7. As in the previous question, suppose that we modified Floyd's algorithm as follows: [15 pts]

We replace `p[i][j]=p[k][j]` with `p[i][j]=k`

We used this modified version of Floyd's algorithm (in the usual order: pass by A, pass by B, pass by C, pass by D) on a directed graph with 4 vertices (A, B, C, D) to compute the shortest distances between each pair of vertices. We have also generated the corresponding recovery matrix.

Using that recovery matrix, we have generated the following shortest paths:

$$C \longrightarrow D \longrightarrow B \longrightarrow A$$

$$A \longrightarrow D \longrightarrow B \longrightarrow C$$

Furthermore, we know that the shortest path between B and D is neither direct nor

$$B \longrightarrow C \longrightarrow D$$

What was the path recovery matrix (for the modified algorithm described above)? Fill in the blanks below (**leave it blank from a vertex to itself and also blank if the path is direct**)

	A	B	C	D
A		D	D	
B				A
C	D	D		
D	B		B	