

Quicksort

① Intro: Quicksort is a fast divide-and-conquer sorting algorithm

② Idea: The idea is we pick a value in the list called the pivot value (pv) then we rearrange the list so all values \leq pv are to the left of the pv and all values $>$ are to the right of the pv. we then do the same thing to the left and right sublists. At the bottom of the recursion a list of length 1 is sorted. DONE!

note: pivot value is an entry in the list, often called the pivot.
 note: * is called the partitioning process.

③ Partitioning

(A) Intuition: Sps we have a list
 Choose the pv (don't worry about how for now)
 Find leftmost value $>$ pv
 Find first subsequent value \leq pv.
 Swap those
 Repeat until done

ex 2, 5, 4, 1, 0, 3 chosen pv = 3
 $\begin{matrix} & \underline{2} & \underline{5} & \underline{4} & \underline{1} & \underline{0} & \underline{3} \\ & >pv & \leq pv & & & & \end{matrix}$ swap!

2, 1, 4, 5, 0, 3
 $\begin{matrix} & \underline{2} & \underline{1} & \underline{4} & \underline{5} & \underline{0} & \underline{3} \\ & >pv & \leq pv & & & & \end{matrix}$ swap!

2, 1, 0, 5, 4, 3 swap
 $\begin{matrix} & \underline{2} & \underline{1} & \underline{0} & \underline{5} & \underline{4} & \underline{3} \\ & >pv & \leq pv & & & & \end{matrix}$

2, 1, 0, 3, 4, 5
 $\begin{matrix} & \underline{2} & \underline{1} & \underline{0} & \underline{3} & \underline{4} & \underline{5} \\ & >pv & & & & & \end{matrix}$ but no subseq. value \leq pv. STOP

2, 1, 0, 3, 4, 5

if $pv \leq 0$ subseq. value = pv . else

This is one step of the partitioning done!

(B) Pseudocode

factn partition (A, L, R)

$$pV = A[R]$$
$$t = L$$

for $i = L$ to $R-1$

if $A[i] \leq pv$

$$A[t] \leftrightarrow A[i]$$

++++

end if

end for

$$A[L] \leftrightarrow A[R]$$

return t \rightarrow returns index of where pv. ends up.

end fctn

(c) why does the pseudocode do what we discussed?!

What the alg does is:

t is "hunting for" the Leftmost value $> pv$.

i is looking for the first subsequent value $\leq pv$.

note: if t encounters values $\leq p_v$ we will do some useless swapping. That's okay.

ex 2 5 4 1 0 3 partition(A, 0, 5)

$t_i \leftarrow A[i] \leq pv \quad \uparrow \quad pv = 3$

useless swap! $t++$, $i++$

2	5	4	1	0	3
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$$t_i \mapsto A[i] \neq pv$$

no swap! i++

7 6 5 4 3 2 1

no swap! $i++$

2 5 4 1 0 3
 $t \quad i \quad \leftrightarrow A[i] \neq pv$

no swap! $i++$

2 5 4 1 0 3
 $t \quad i \quad \leftrightarrow A[i] \leq pv$

Swap! $t++, i++$

2 1 4 5 0 3
 $t \quad i \quad \leftrightarrow A[i] \leq pv$

Swap! $t++, i++$

2 1 0 5 4 3
 $t \quad i \quad \leftrightarrow$ loop ended at $i=4$ (previous index)
 exit for loop.

2 1 0 3 4 5
 $\uparrow \quad \quad \quad \uparrow$ swap after loop!

DONE!

(>) Choosing the pivot value

- in theory we could choose any value in the list.
- the pseudocode requires it to be the last value.
 to use a different value, swap that value w/ the last value right at the start of partition.
- ideally we'd use the median b/c then, at the end, the pv would essentially be in the middle which is nice for divide and conquer.
 However - finding the median is not obvious!
- in practice the pv is chosen randomly.

④ Now on to Quicksort!

At this point quicksort is easy!

fctn quicksort(A, L, R)

← Assm A = global list

if $L < R$

resultingpivotindex = partition(A, L, R)

quicksort(A, L, resultingpivotindex - 1)

quicksort(A, resultingpivotindex + 1, R)

end

end

To start the whole process:

quicksort(A, 0, length(A) - 1)

③ Time Complexity? Aux Space? In-Place? Stable