# BUFN402: Portfolio Management

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These are my notes for UMD's BUFN402: Portfolio Management, which is an elective ("live-TEX"-ed). This course is taught by Professor Seokwoo Lee.

## Contents



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# §1 Probability Theory

See Professor Lee's comprehensive slideshow on the probability theory concepts that we need to know.

# §2 Portfolio Theory

## §2.1 Review of Two Asset Case

Suppose we have two assets denoted as random variables $R_1, R_2$ and $E(R_1) = \mu_1, E(R_2) = \mu_2$. We also have $SD(R_1) = \sigma_1, SD(R_2) = \sigma_2$. Then we have the portfolio $R_p$ as

$$R_p = w_1 R_1 + w_2 R_2 = w_1 R_1 + (1 - w_1) R_2$$

Now constucting a mean variance optimization,

$$\mathbb{E}(R_p) = w_1 \mu_1 + w_2 \mu_2$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2 + 2 w_1 (1 - w_1) \rho_{12} \sigma_1 \sigma_2$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

We wish to maximize $\max(\mathbb{E}(R_p) - \mathrm{Var}(R_p))$ which we can doing by taking the derivative with respect to $w_1$ and setting this to zero.

$$w_1 \mu_1 + (1 - w_1) \mu_2 - w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2 + 2 w_1 (1 - w_1) \rho_{12} \sigma_1 \sigma_2$$

$$w_1 \mu_1 + \mu_2 - w_1 \mu_2 - w_1^2 \sigma_1^2 + (1 - 2w_1 + w_1^2) \sigma_2^2 + 2 w_1 (1 - w_1) \rho_{12} \sigma_1 \sigma_2$$

$$w_1 \mu_1 + \mu_2 - w_1 \mu_2 - w_1^2 \sigma_1^2 + (1 - 2w_1 + w_1^2) \sigma_2^2 + (2 w_1 - 2 w_1^2) \rho_{12} \sigma_1 \sigma_2$$

Taking partial derivative we get

$$\mu_1 - \mu_2 - 2 \sigma_1^2 w_1 - 2 \sigma_2^2 + 2 w_1 \sigma_2^2 + (2 - 4 w_1) \rho_{12} \sigma_1 \sigma_2 = 0$$

$$\ldots$$

### Example 2.1

We want to maximize variance of two assets such that $X \sim N(\mu_x, 1), Y \sim N(\mu_y, 1), \rho > 0$. Maximize the variance given that $w_1^2 + w_1^2 = 1$. Note that maximizing this variance is the same thing as minimizing the negative value of this function.

$$\text{Var}(R_p) = w_1^2 + w_2^2 + 2w_1w_2\rho$$
$$-\text{Var}(R_p) = -w_1^2 - w_2^2 - 2w_1w_2\rho$$

Rearranging the constraint yields

$$1 - w_1^2 - w_1^2$$

$$\mathcal{L}(w_1, w_2, \lambda) = -w_1^2 - w_2^2 - 2w_1w_2\rho + \lambda(1 - w_1^2 - w_2^2)$$

The first order conditions are therefore

$$\frac{\partial \mathcal{L}}{\partial w_1} = -2w_1 - 2w_2\rho - 2\lambda w_1 = 0 \implies w_1 + w_2\rho + \lambda w_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = -2w_2 - 2w_1\rho - 2\lambda w_2 = 0 \implies w_2 + w_1\rho + \lambda w_2 = 0$$

We want to solve for $w_1, w_2$ in terms of $\lambda$. Recall that the constraint enforces $w_1^2 + w_2^2 = 1 \implies w_1 = \sqrt{1 - w_2^2}$ and $w_2 = \sqrt{1 - w_1^2}$. Plugging these in yield

$$w_1 + \rho\sqrt{1 - w_1^2} + \lambda w_1 = 0 \implies w_1^2 + \rho^2(1 - w_1^2) + \lambda^2 w_1^2 = 0$$

$$w_1^2 - \rho^2 w_1^2 + \lambda^2 w_1^2 = -\rho^2$$
$$w_1^2(1 - \rho^2 + \lambda^2) = -\rho^2$$
$$w_1 = \pm\sqrt{\frac{-\rho^2}{1 - \rho^2 + \lambda^2}} = w_2$$