TERM STRUCTURE OF CHARACTERISTIC-SORTED PORTFOLIOS AND MULTI-HORIZON INVESTMENT

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Motivation

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- Empirical AP uses characteristics as pricing signals. How fast are these signals dying out?

- Equivalently, what is the term-structure of ER on characteristic-sorted portfolios, i.e., ER for different holding periods, $\mathbb{E}\left[r_{t,i}|\boldsymbol{C}_{t-l}\right] = \mathbb{E}\left[r_{t+l,i}|\boldsymbol{C}_{t}\right]$, for some lags (horizons) $l = 1..L$?

- How to construct MVE portfolios for multiple investment horizons with no rebalancing (e.g., a month, quarter, year)?

Approach

- Each lag is a new characteristic. A high-dimensional problem: many characteristics and lags.
- New approach: a tensor factor model — generalization of APT-like factor models with an extra lag (horizon) dimension.
- Collect *contemporaneous* stock returns at various char. lags, $r_{t,i,l} = r_{t,i}|\boldsymbol{C}_{t-l}$, into a 3-dimensional tensor $\mathcal{R}$; find its low-rank decomposition as in PCA:

$$r_{t,i,l} = \sum_{k=1}^{K} f_{t,k} \cdot \underbrace{b_{i,k} \cdot w_{l,k}}_{\beta_{i,l,k}},$$

where $f_{t,k}$ are time-series factors (portfolios), $\beta_{i,l,k}$ are loadings, further decomposed into $b_{i,k}$ (across stocks), and $w_{l,k}$ (across lags).

Advantages of the tensor factor model approach

- Parsimonious representation of *contemporaneous* returns sorted on the entire history of characteristics, for many lags
- Use to construct predictions of one-period returns H periods ahead. Aggregating these predictions delivers estimates of multi-period returns on buy-and-hold portfolios
- It thus converts a fundamentally time-series multi-horizon predictability problem into a cross-sectional single-horizon one
- Can use high-frequency data and the model structure for multi-horizon investment, estimating means and covariances

Findings

- Massive dimensionality reduction: info in 43 chars $\times$ 60 lags can be summarized well by ≈ 20 tensor factor portfolios
- Tensor factor model (TFM) performs better than naive PCA in terms of IS and OOS average alpha, unexplained var, and reconstruction error
- TFM delivers robust investment portfolios across multiple horizons
- IS/OOS MVE Sharpe exhibit decay as horizon increases
- TFM outperforms a naive model-free approach based on long-horizon estimates of means and cov

Methodology

The setup: notation

- $t = 1..T$ time periods, $s = 1..S$ stocks, $i = 1..N$ portfolios, $l = 1..L$ lags, $k = 1..K$ factors
- Notation: $\mathcal{R}$ for 3D tensors ($T \times N \times L$), $\boldsymbol{R}_t$ for 2D slices at t ($N \times L$), $\boldsymbol{r}_{t,i}$ for 1D vectors, and $r_{t,i,l}$ for scalars
- Define monthly excess return of a char. portfolio i at time t sorted on lagged char. $\boldsymbol{C}_{t-l}$ ($S \times N$) by $r_{t,i,l}$
- The data is a 3D ($T \times N \times L$) *tensor*, denoted as $\mathcal{R}$.

The setup: a tensor factor model

- We introduce a tensor factor model of excess returns:

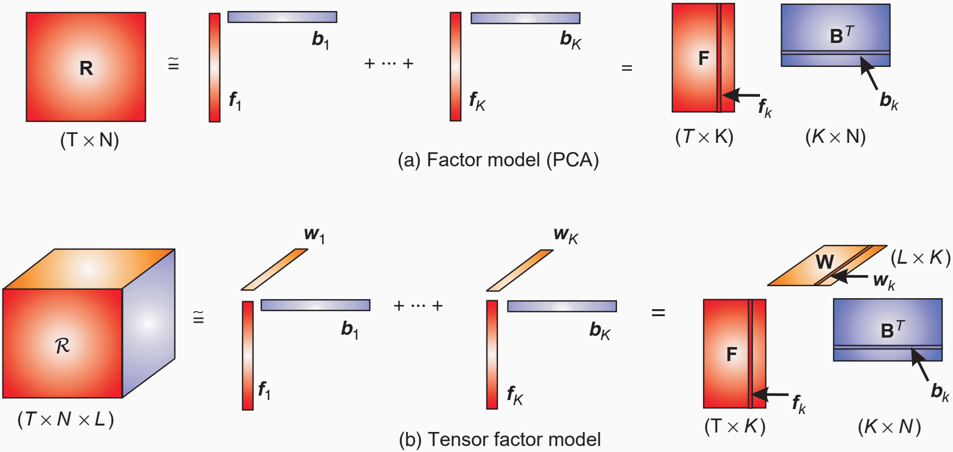
$$r_{t,i,l} = \sum_{k=1}^{K} f_{t,k} \cdot \underbrace{b_{i,k} \cdot w_{l,k}}_{\beta_{i,l,k}},$$

where $f_{t,k}$ are time-series factors, $\beta_{i,l,k}$ are loadings of a portfolio i sorted on l-lagged chars on the k-th factor, further decomposed into $b_{i,k}$ (across stocks), and $w_{l,k}$ (across lags).

- This implies that the term-structure of ER should equal

$$\mathbb{E}\left[r_{t,i,l}\right] = \sum_{k=1}^{K} \mathbb{E}\left[f_{t,k}\right] \cdot b_{i,k} \cdot w_{l,k}.$$

Visualizing tensor decomposition and PCA



Multi-horizon perspective

- Returns over H time periods sorted on $\boldsymbol{C}_{i,t-1}$ are

$$\sum_{l=1}^{H} r_{t-1+l,i} | \boldsymbol{C}_{t-1} =: \sum_{l=1}^{H} r_{t-1+l,i,l} =: \boldsymbol{r}_{t-1+H}^{H}.$$

- Assume stationarity:

$$\mu_{i,l} = \mathbb{E}\left[r_{t+l,i} | \boldsymbol{C}_t\right] = \mathbb{E}\left[r_{t,i} | \boldsymbol{C}_{t-l}\right]$$

- The H period ER for asset i from the tensor model is:

$$\mu_i^H = \sum_{l=1}^{H} \mu_{i,l} = \sum_{k=1}^{K} \left(\mathbb{E}[f_{t,k}] \cdot b_{i,k} \cdot \sum_{l=1}^{H} w_{l,k} \right).$$

Multi-horizon returns: in sample (IS)

- Consider the set of K basis assets such that $\boldsymbol{B} = \boldsymbol{I}_k$, i.e., they have unitary exposure to a single factor
- An H-period ER and var of basis assets are:

$$\boldsymbol{\mu}^{B,H} = \left(\sum_{l=1}^{H} \boldsymbol{w}_l\right) \odot \mathbb{E}[\boldsymbol{f}_t],$$

$$\boldsymbol{\Sigma}^{B,H} = \left(\sum_{l=1}^{H} \boldsymbol{w}_l \boldsymbol{w}_l^\top\right) \odot \text{var}(\boldsymbol{f}_t),$$

assuming returns are uncorrelated in the time series; and $\odot$ denotes element-wise matrix product.

- MVE portfolio weights in terms of basis assets are:

$$\boldsymbol{\theta}^{B,H} = \left(\boldsymbol{\Sigma}^{B,H}\right)^{-1} \boldsymbol{\mu}^{B,H}$$

- IS squared Sharpe ratio is $\left(\boldsymbol{\mu}^{B,H}\right)^\top \left(\boldsymbol{\Sigma}^{B,H}\right)^{-1} \boldsymbol{\mu}^{B,H}$.

Multi-horizon returns: out of sample (OOS)

- $\boldsymbol{B}$ $(N \times K)$ maps basis assets into char.-sorted portfolios

- Therefore, MVE weights in terms of char. portfolios are:

$$\boldsymbol{\theta}^H = \boldsymbol{B}\left(\boldsymbol{B}^\top \boldsymbol{B}\right)^{-1} \boldsymbol{\theta}^{B,H}$$

- We can now use these weights to compute OOS MVE returns on an H-period buy-and-hold portfolio and their Sharpe ratio:

$$r_{t-1+H}^{\text{mve},H} = \left(\boldsymbol{\theta}^H\right)^\top \boldsymbol{r}_{t-1+H}^H.$$

Tensor decomposition: the algorithm

- Fitting by alternating least squares (ALS): assume two modes are fixed and fit the other mode by least-squares. Iterate until convergence.

- ALS guarantees improvement of fitting in every iteration .

- Factors F are portfolios because they are linear combination of (lagged) char. portfolio returns.

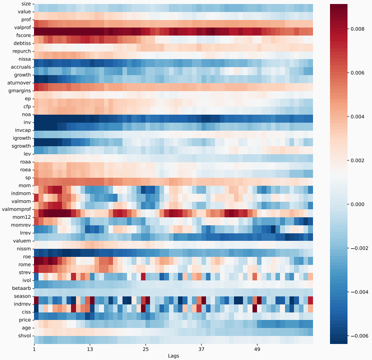
- Normalize columns of F, W, and B to have unit norm. Move scaling to S, and order factors by scalers in S.

Results

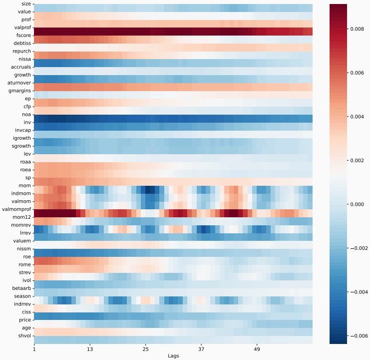
Data

- Monthly excess returns 1977–2020
- Build portfolio on characteristics lagged 1–60 months
- Work with a tensor $T \times N \times L$ ($519 \times 43 \times 60$)
- Fit all returns jointly ($43 \times 60 = 2580$) time-series jointly using either the Tensor Factor Model (TFM) or naive PCA applied to 2,580 return series
- OOS starts in 2000

Term structure of mean returns



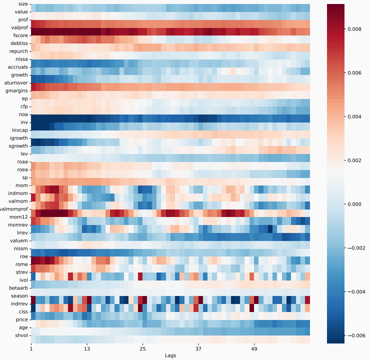
(a) Mean returns



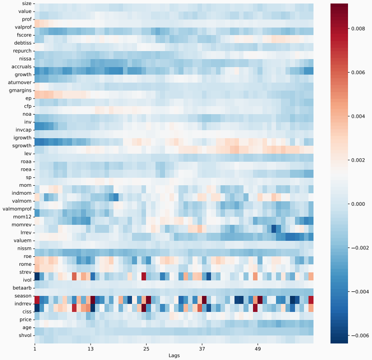
(b) Fitted mean returns with $K = 20$

- Tensor factor model (TFM) reproduces essential features of the term-structure of ER of char.-sorted portfolios

Term structure of alphas



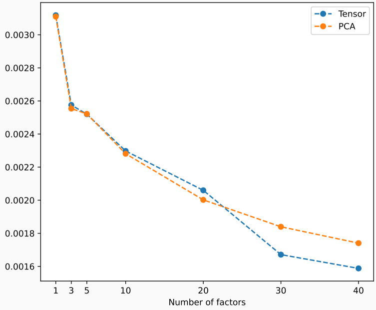
(a) $K = 1$



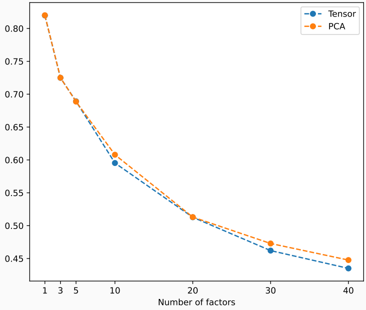
(b) $K = 20$

- TFM leaves little alpha for most char.-sorted portfolios except for strategies with strong seasonality

Normalized average alpha and unexplained variance (OOS)



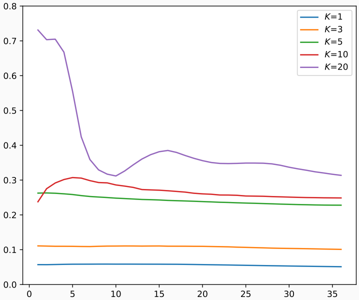
(a) Average alpha



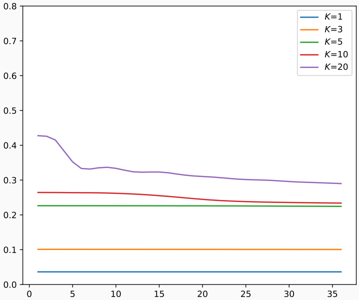
(b) Unexplained variance

- TFM explains alphas and variances as well as naive PCA

Multi-horizon approach: IS MVE Sharpe



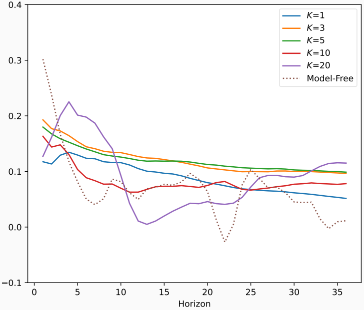
(a) Weights on char. portfolios



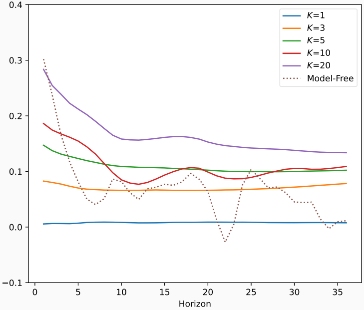
(b) Weights on basis assets

- IS MVE SR decay mildly with horizon, mostly flat

Multi-horizon approach: OOS MVE Sharpe



(a) Weights on char. portfolios



(b) Weights on basis assets

- OOS MVE SR decay a little faster with horizon but still relatively flat
- SR level is a bit lower in late sample
- Model-free (dashed) estimates long-term means and covs to form MVE weights – performs significantly worse than TFM

Time series of factors f_t

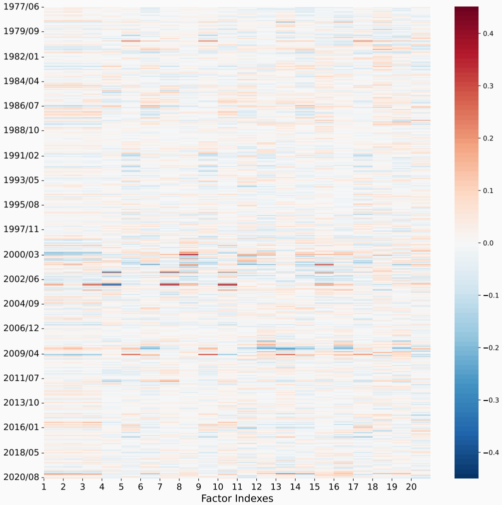


Figure 1: Time pattern, $K = 20$

Portfolio loadings b_i

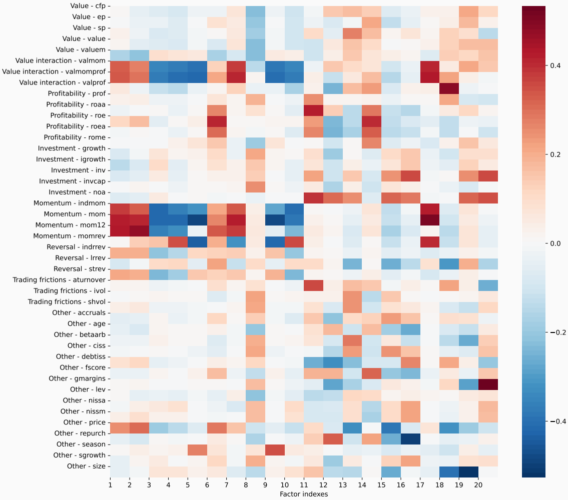


Figure 2: Portfolio pattern, $K = 20$

- Patterns of factor loadings on styles, e.g., value, momentum

Lag weights w_l

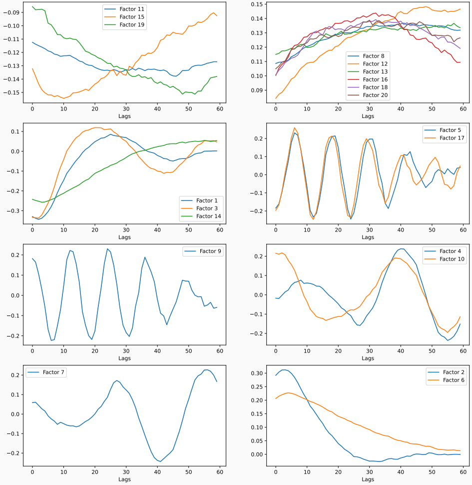
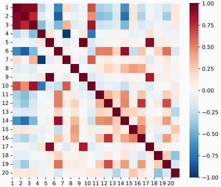


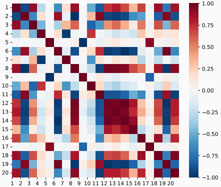
Figure 3: Clustered lag patterns, $K = 20$

- Lag loadings exhibit persistent, decaying, and cyclical patterns

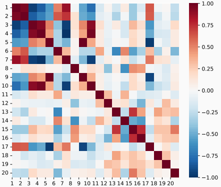
Factor correlation



(a) F



(b) W



(c) B

Figure 4: Correlation matrices of F, W, and B. $K = 20$

- Unlike PCA, factors are generally correlated

Conclusions

- We propose a novel approach that links the term-structure of ER on characteristic-sorted portfolios to characteristics decay
- To accomplish this, we find a parsimonious representation of contemporaneous returns sorted on the entire history of characteristics, for many lags
- We thus convert a fundamentally time-series multi-horizon predictability problem into a cross-sectional single-horizon one
- A new modeling approach – a tensor factor model for 3D data
- TFM fits the data well and generates robust multi-horizon predictions and MVE portfolio weights for long-horizon investment

References