Collection of Results

Prof. Serhiy Kozak & James Zhang

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All results in this document pertain to the SCS dataset. All window sizes are 120 months. The out-of-sample start date is January 1, 2005. Recall that we study a three-dimensional tensor $\mathcal{R}$ of excess returns ie. returns minus the risk-free rate, where

$$\mathcal{R}_{t,l,i} \quad t = 1, \ldots, T, l = 1, \ldots, L, i = 1, \ldots, N, \tag{1}$$

Residuals are defined as differences between returns and the factor model's implied returns. For the tensor model, this is

$$\epsilon_{t,l,i} = \mathcal{R}_{t,l,i} - \sum_{k=1}^{K} \lambda_k F_{t,k} W_{l,k} B_{i,k} \tag{2}$$

NOTE: there is a slight issue with our statistics related to our PCA estimator. We're doing it in a PooledPCA fashion, which we eliminated from the multiperiod section of our results, so this would be inconsistent.

1. In-sample evaluations

   - Reconstruction error as a function of K and L for unconstrained, orthogonal tensor models and also as a function of rank for PCA model.

   $$\epsilon_{rc} = 1 - \frac{\|\mathcal{R} - \mathcal{R}_{rc}\|_F^2}{\|\mathcal{R}\|_F^2} \tag{3}$$

   - Excess alpha. First we define pricing errors for a single asset across time as

   $$\alpha_{l,i} = \frac{1}{T} \sum_{t=1}^{T} \epsilon_{t,l,i} \tag{4}$$

   Then it follows that excess alpha for all assets and lags is

   $$\text{XS-}\alpha = \frac{1}{NL} \sqrt{\sum_{i=1}^{N} \sum_{l=1}^{L} \alpha_{l,i}^2} \tag{5}$$

- Similarly, for a single asset, the unexplained variance over time is

$$\sigma^2_{\epsilon,l,i} = \frac{1}{T}\sum_{t=1}^{T} \epsilon^2_{t,l,i} - \alpha^2_{l,i} \tag{6}$$

  It follows that the normalized averaged unexplained variance is

$$\sigma_\epsilon = \sqrt{\frac{1}{NL}\sum_{i=1}^{N}\sum_{l=1}^{L} \sigma^2_{\epsilon,l,i}} \Bigg/ \sqrt{\frac{1}{NL}\sum_{i=1}^{N}\sum_{l=1}^{L} \left[\mathrm{Var}_t(\mathcal{R})\right]_{l,i}} \tag{7}$$

- Pricing errors
- Term structure of fitted mean returns for the models?

2. Out-of-sample evals
   - Reconstruction errors
   - Excess alphas
   - Normalized unepxlained variances
3. Multiperiod results - unconstrained TFM, Naive, Hor1, Orthogonal, PCA, and RP-PCA (note to self: make the axis the same for all plots in this 2x3)
4. Tensor model visualizations and economic content