Collection of Results

April 5, 2022

1 Setup

- Sampling periods: 1972-07-01 (i.e. $\min(t-L)$) to 2020-08-01 (i.e. $\max t$). Max lag L is set to 60 months.
- 43 single-sorted rank-weighted portfolios. The resulting tensor in the full-sample is of dimension (519, 60, 43).
- Tensor X contains excess returns (simple, not log, return) $R \in \mathbb{R}^{T \times L \times C}$. X is not return in excess of the lag-one return.
- Full-sample setup
 - Run PARAFAC on 3D tensor $X \in \mathbb{R}^{T \times L \times C}$, which is decomposed into factors $F \in \mathbb{R}^{T \times K}$, lag loadings $W \in \mathbb{R}^{L \times K}$, character loadings $B \in \mathbb{R}^{C \times K}$, and scalers $S \in \mathbb{R}^{K}$. K is the pre-specified number of factors. X is not centered.
 - Unconstrained specification: the first column of W is not constrained to be constant.
 - Normalize columns of F, W, and B to have norm 1. Normalize each column in F to have positive time-series mean. Move sign and scaling to S. Reorder factors by magnitude of the corresponding scaling factors in S.

• Out-of-sample setup

- Estimate a model using data in a rolling window of size 60 or 120 months. Results are calculated in the common out-of-sample period 1987-06-01 to 2020-08-01.
- For the tensor model, obtain loadings W and B and scalars S in the rolling window. Regress X_{t+1} at the next period on W, B, and S to get factors F_{t+1} for the out-of-sample period. Combine factors in F_{t+1} using mean-variance weights on F.
- For the PCA model, obtain loadings L in the rolling window. Regress X_{t+1} on L to get factors F_{t+1} .
- For the one-lag PCA model, for which only portfolio returns based on the most recent characteristic values are included, factors are obtained in the same way as PCA.
- For all of the competing models, calculate out-of-sample SR based on the mean-variance combination of factors. We also record fitted \hat{X}_{t+1} to evaluate out-of-sample fitting quality.
- Full-sample multi-period return setup
 - -X is converted into log return

- Horizons up to 3 years. Rolling window of size 60 or 120 months.
- Models:
 - * Tensor: See Markus' notes for the setup.
 - * Model-free: For each horizon, calculate overlapping multi-period returns for each of the 43 portfolios. Use mean-variance weights to combine these multi-period returns. Newey-west covariance estimator is used.
 - * PCA: For each horizon, use PCA to extract the first K factors from overlapping multi-period return of the 43 portfolios. Use mean-variance weights to combine these factors. Newey-west covariance estimator is used.
- Out-of-sample multi-period return setup
 - X is converted into log return
 - Horizons up to 3 years. Rolling window of size 60 or 120 months. Results are calculated in the common out-of-sample period 1987-06-01 to 2017-09-01.
 - Tensor: Fit a tensor model in the rolling window. For horizon h and factor k, calculate the approximate multi-period return starting at time t as $FW_t = F_{k,t} \sum_{s=1}^h W_{k,s}$ in the rolling window. For each horizon, calculate the mean-variance weight for combining these multi-period factors. To get out-of-sample results, for each horizon h, calculate one-period out-of-sample factor F_{t+h} by regressing X_{t+h} on loadings W and B and scalars S estimated in the rolling window. Then, calculate out-of-sample multi-period returns as $\sum_{s=1}^h F_{k,t+s}W_{k,s}$ and use mean-variance weights to combine them.
 - Model-free: For each horizon h, calculate multi-period portfolio returns in the rolling window as $\sum_{s=1}^{h} X_{t+s,s}$. $X_{t+s,s}$ is the one-period return at t+s using characteristic values with lag s. For horizon h, calculate mean-variance weights for combining multi-period returns. Newey-west covariance estimator is used. Then, calculate out-of-sample h-period return starting at the first out-of-sample period. For each h separately, combine multi-period returns using the mean-variance weights estimated in the rolling window.
 - PCA: For horizon h, use PCA to extract the first K factors from multi-period portfolio returns as defined above. For each out-of-sample period following the rolling window, get factors F by regressing multi-period returns on PCA loadings. Use mean-variance weights to combine these PCA factors out-of-sample. Newey-west covariance estimator is used.

2 Collection of results

- Evaluation (in-sample):
 - SR: Figure 40
 - Fitting RMSE (time, lag, and portfolio dimensions): Figures 41, 42, 43
 - Reconstruction error as a function of the number of factors for PCA and the tensor model: Figure 39
 - Averaged alpha and unexplained variance (XS- α and σ_{ϵ} as in Markus' notes)
 - * Normalized averaged alpha XS- α : Figure 44
 - * Averaged alpha for the lag dimension: Figures 47 and 48

- * Averaged alpha for the portfolio dimension: Figures 45 and 46
- * Unexplained variance: Figure 49
- Term structure of (fitted) mean returns: Figures 51 to 55 for fitted return. Figure 50 for mean returns (model free).
- Term structure of alpha: Figures 56 to 60
- Decomposition of alpha into stale and dynamic components: Figures 61 to 65 for the stale component. Figures 66 to 70 for the dynamic component.
- Evaluation (out-of-sample):
 - SR: Figures 4 (window size: 60 months) and 5 (window size: 120 months)
 - Averaged alpha and unexplained variance (XS- α and σ_{ϵ} as in Markus' notes)
 - * Normalized averaged alpha XS- α : Figure 6
 - * Averaged alpha for the lag dimension: Figures 12 and 14
 - * Averaged alpha for the portfolio dimension: Figures 8 and 10
 - * Unexplained variance: Figure 16
 - Term structure of (fitted) mean returns: Figures 19 to 23 for fitted return. Figure 18 for mean returns (model free).
 - Term structure of alpha: Figures 24 to 28
 - Term structure of CAPM alpha: Figures 29 to 33
- Multi-period evaluation (in-sample and out-of-sample): SR as a function of horizon and number of factors: Figures 2 and 1
- Illustrations for the tensor model (in-sample, up to 20 factors):
 - Plots for the tensor components
 - * Time pattern: Figure 34
 - * Portfolio pattern: Figure 36
 - * Lag pattern: Figures 37 and 37
 - Cumulative excess returns of factors: Figure 35
- Illustration for PCA (in-sample):
 - PCA loadings: Figure 3

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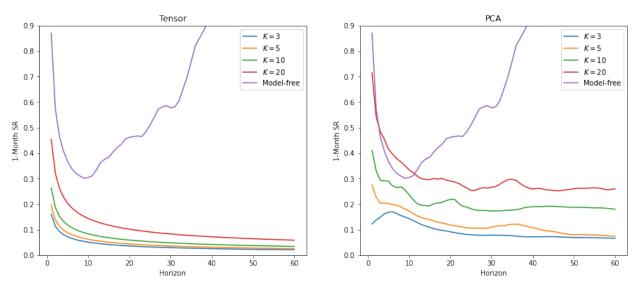
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Figure 1: Out-of-sample SR of multi-period returns

OOS period: 1987-06-01 to 2017-09-01; Window size: 60 Tensor PCA 0.4 0.4 K=1 K=1 K=3 K=3 K=5 K=5 0.3 0.3 K=10 K=10 K=20 K=20 ···· Model-Free ····· Model-Free 0.2 0.2 1-Month SR 1-Month SR 0.1 0.1 0.0 0.0 -0.1 -0.1 20 Horizon Ó 10 15 25 30 35 10 15 20 25 30 35 Horizon OOS period: 1987-06-01 to 2017-09-01; Window size: 120 Tensor PCA 0.4 0.4 K=1 K=1 K=3 K=3 K=5 K=5 0.3 0.3 - K=10 K=10 K=20 K=20 Model-Free Model-Free 0.2 0.2 1-Month SR 0.1 0.1 0.0 0.0 -0.1 -0.1 20 Horizon 10 15 25 30 35 10 20 Horizon 25 35 30

Figure 2: In-sample SR of multi-period returns





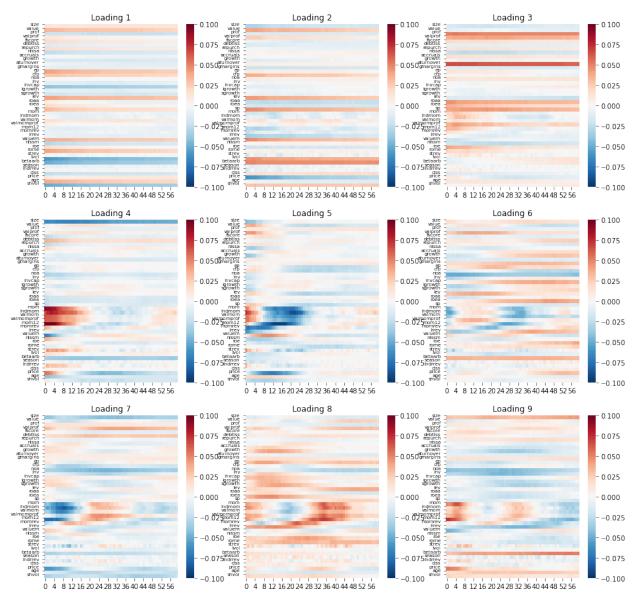


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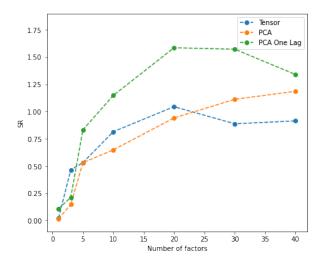


Figure 5: Out-of-sample SR; Window size: 120 months

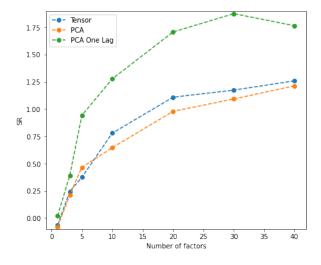


Figure 6: Normalized averaged alpha (OOS); Window size: 60 months

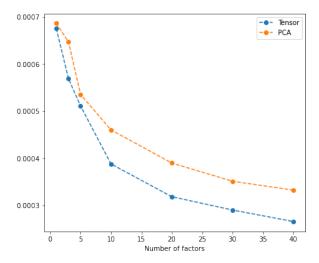


Figure 7: Normalized averaged alpha (OOS); Window size: 120 months

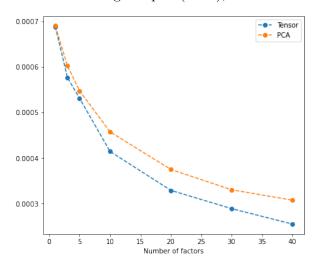


Figure 8: Cross-sectional averaged alpha for portfolios (OOS); Tensor model; Window size: 60 months

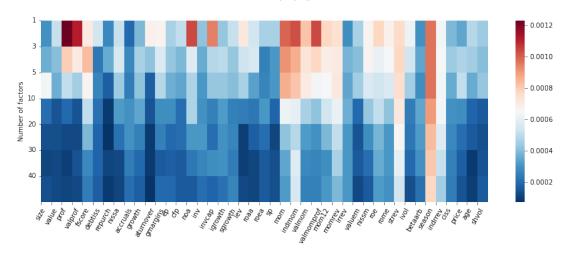


Figure 9: Cross-sectional averaged alpha for portfolios (OOS); Tensor model; Window size: 120 months

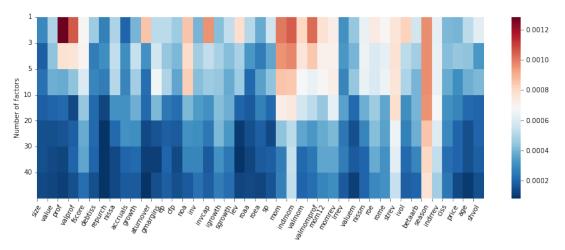


Figure 10: Cross-sectional averaged alpha for portfolios (OOS); PCA model; Window size: 60 months

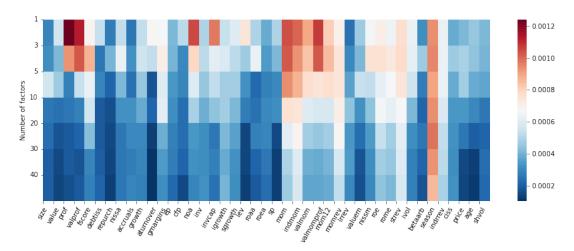


Figure 11: Cross-sectional averaged alpha for portfolios (OOS); PCA model; Window size: 120 months

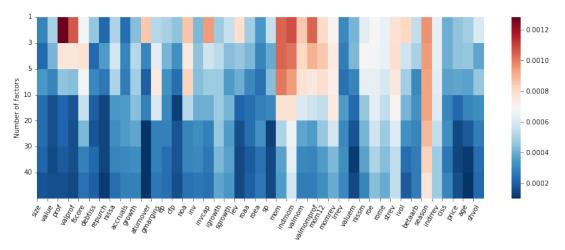


Figure 12: Averaged alpha for the lag dimension (OOS); Tensor model; Window size: 60 months

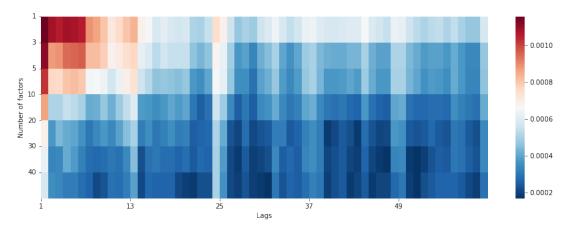


Figure 13: Averaged alpha for the lag dimension (OOS); Tensor model; Window size: 120 months

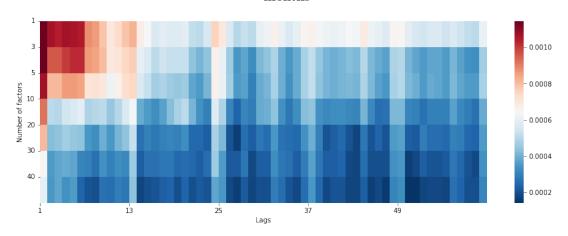


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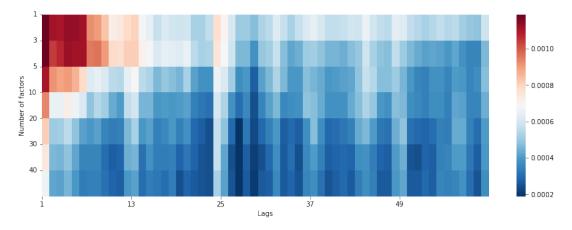


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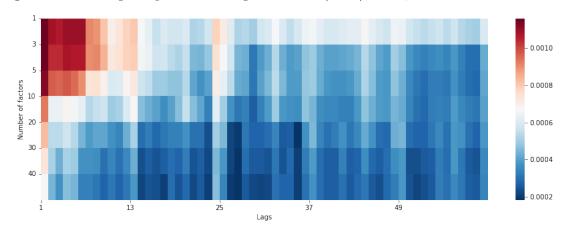


Figure 16: Normalized unexplained variance σ_{ϵ} (OOS): Window size: 60 months

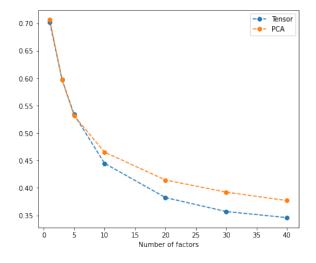


Figure 17: Normalized unexplained variance σ_{ϵ} (OOS): Window size: 120 months

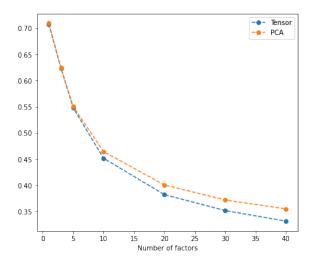


Figure 18: Term structure of mean returns (OOS)

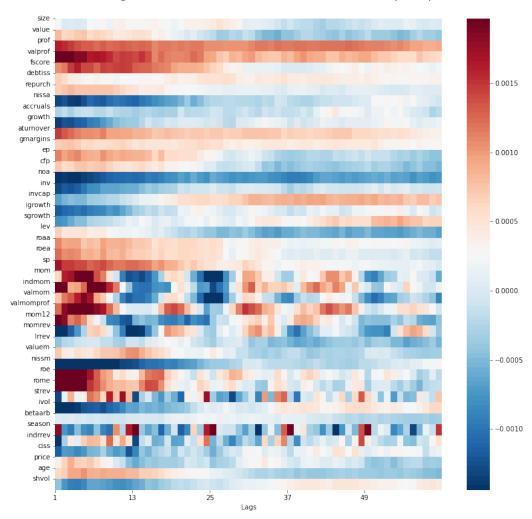


Figure 19: Term structure of fitted mean returns with rank 1 (OOS); Window size: 60 months

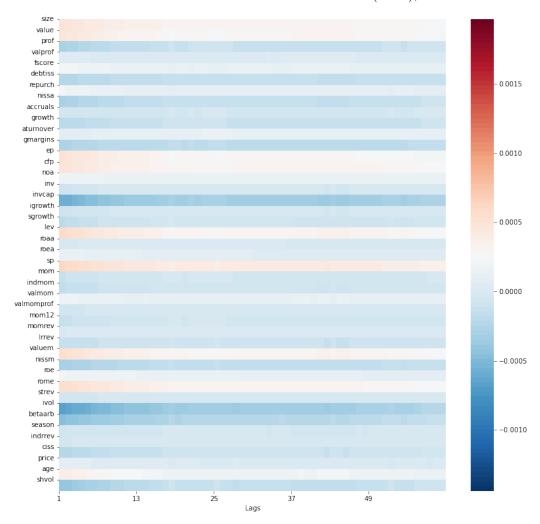


Figure 20: Term structure of fitted mean returns with rank 3 (OOS); Window size: 60 months

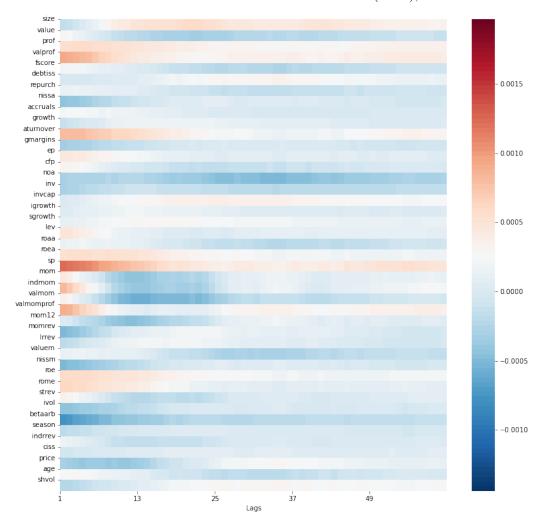


Figure 21: Term structure of fitted mean returns with rank 5 (OOS); Window size: 60 months

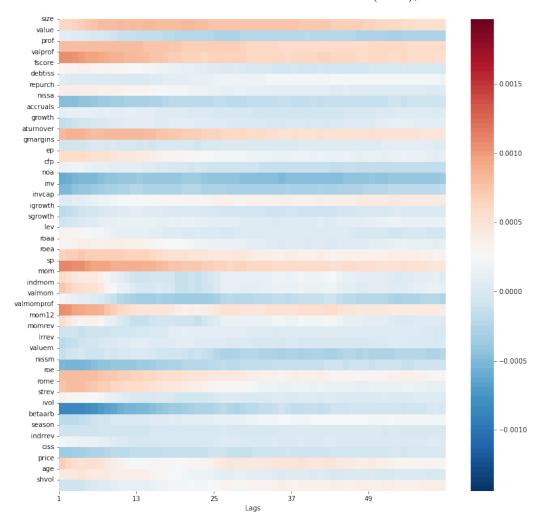


Figure 22: Term structure of fitted mean returns with rank 10 (OOS); Window size: 60 months

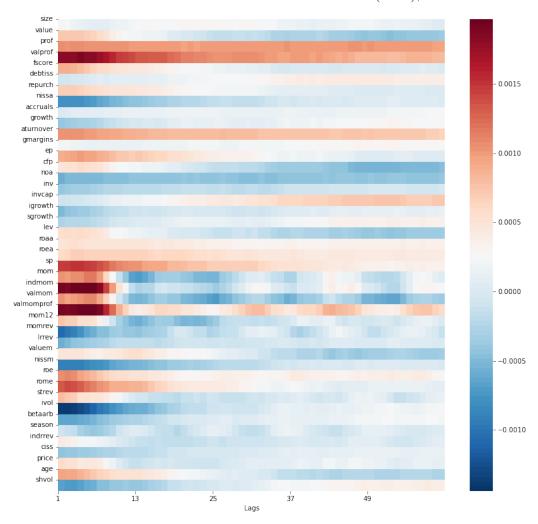
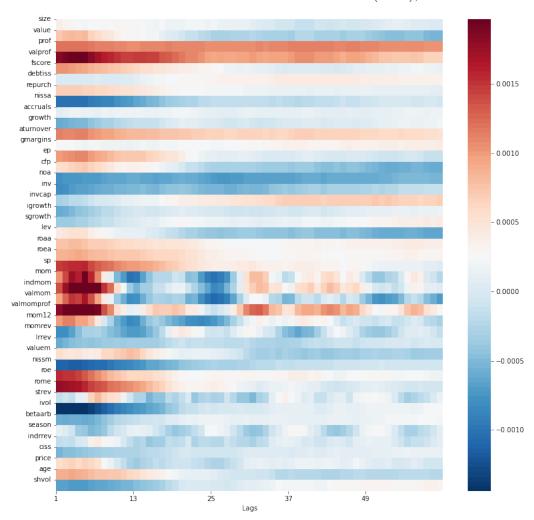
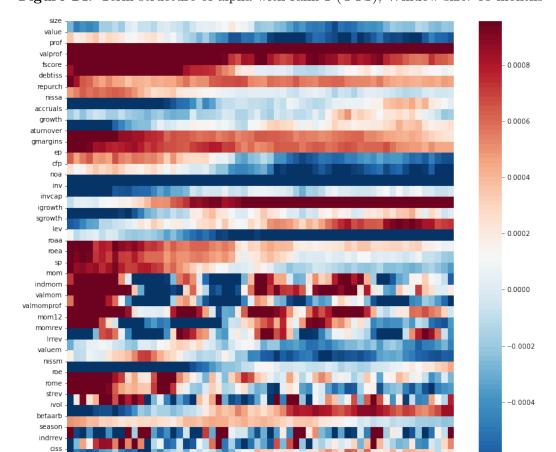


Figure 23: Term structure of fitted mean returns with rank 20 (OOS); Window size: 60 months





price age shvol

13

25

Lags

-0.0006

Figure 24: Term structure of alpha with rank 1 (OOS); Window size: 60 months

37

49

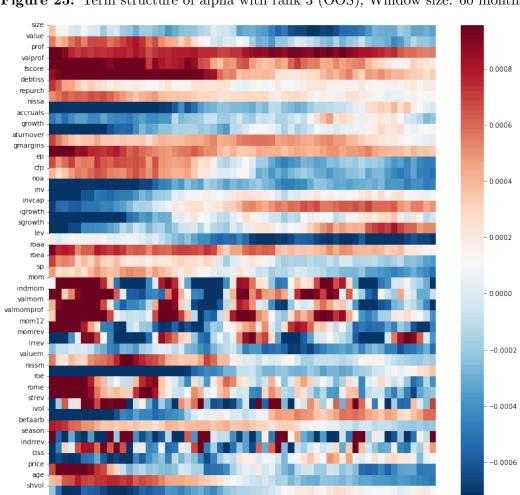


Figure 25: Term structure of alpha with rank 3 (OOS); Window size: 60 months

Lags

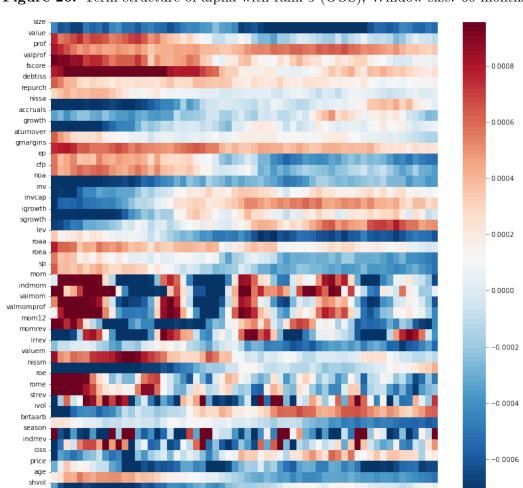
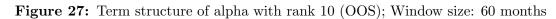
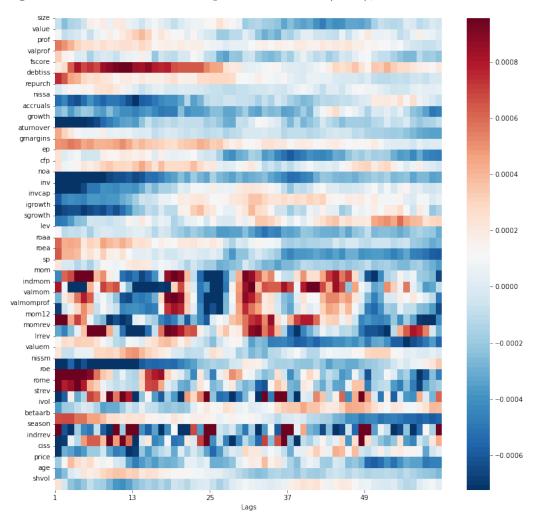
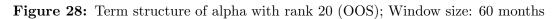


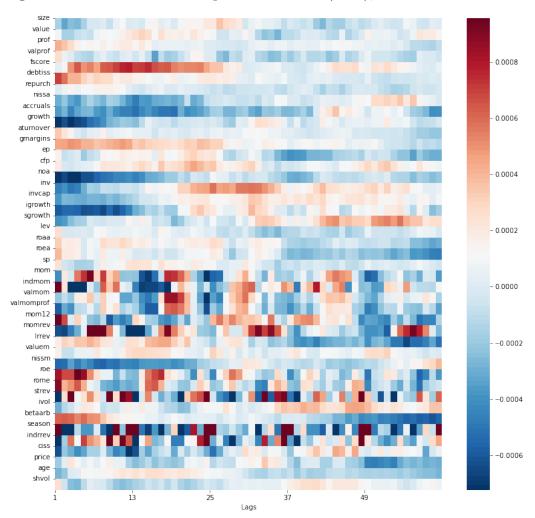
Figure 26: Term structure of alpha with rank 5 (OOS); Window size: 60 months

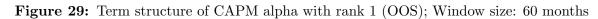
Lags

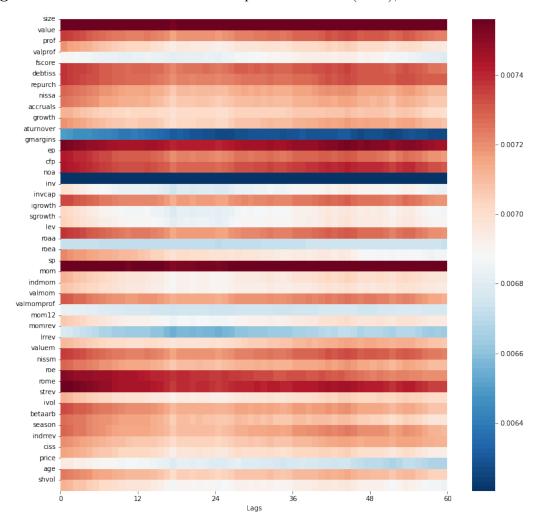


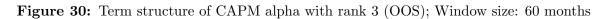


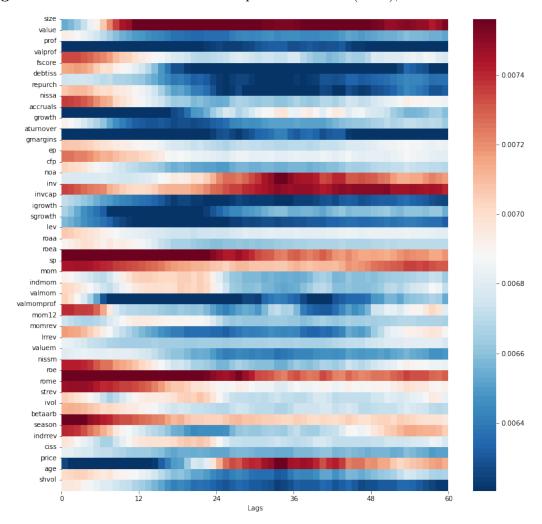


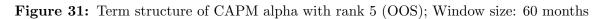


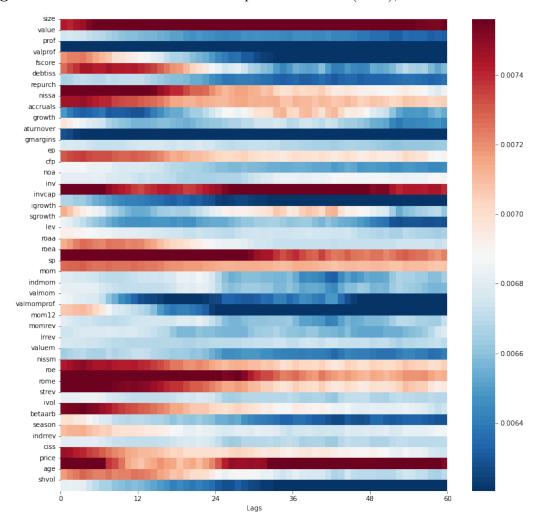


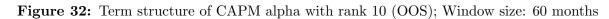












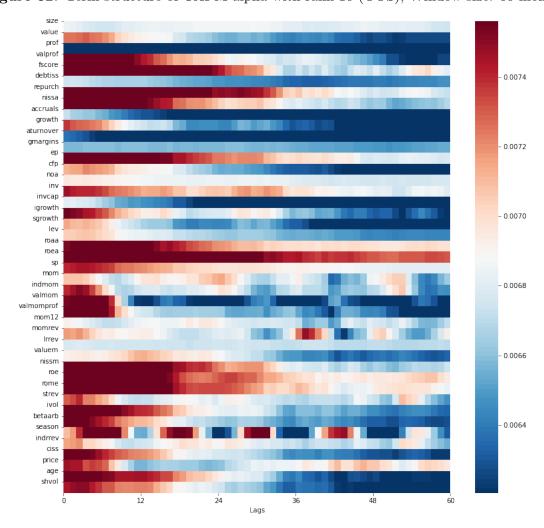
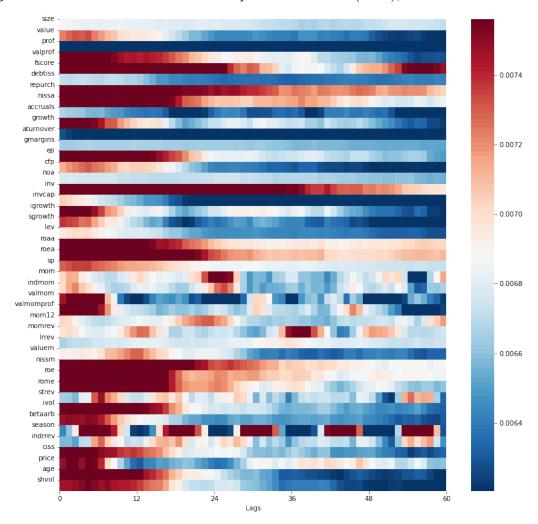


Figure 33: Term structure of CAPM alpha with rank 20 (OOS); Window size: 60 months



1977/06 1979/09 - 0.4 1982/01 1984/04 - 0.3 1986/07 1988/10 - 0.2 1991/02 1993/05 - 0.1 1995/08 1997/11 - 0.0 2000/03 2002/06 - -0.1 2004/09 2006/12 -0.2 2009/04 2011/07 -2013/10 -0.3 2016/01 -2018/05 -0.4 2020/08 10 11 12 Factor indexes

Figure 34: Time pattern

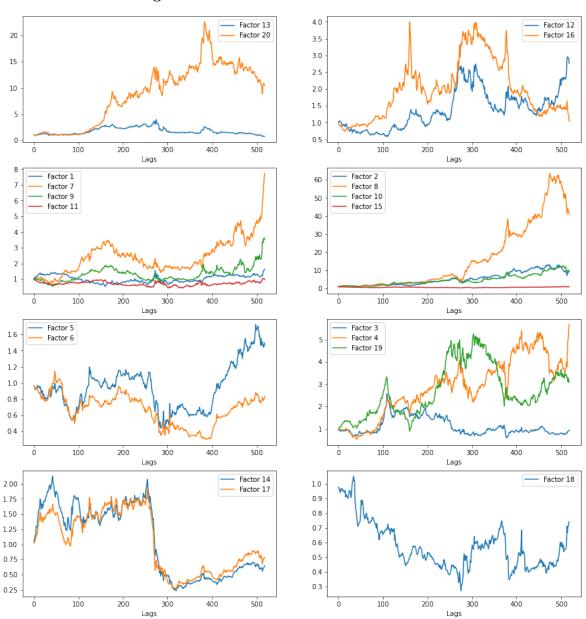


Figure 35: Cumulative excess returns of factors

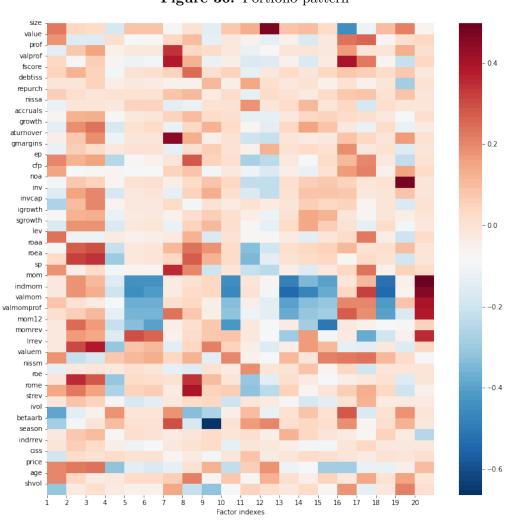


Figure 36: Portfolio pattern

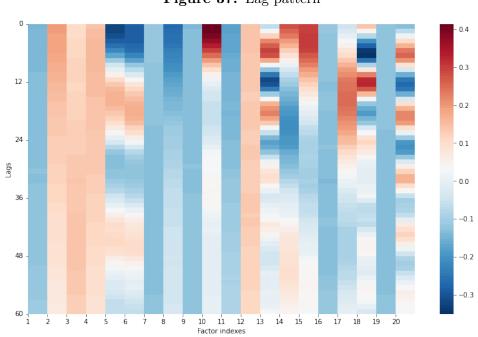


Figure 37: Lag pattern

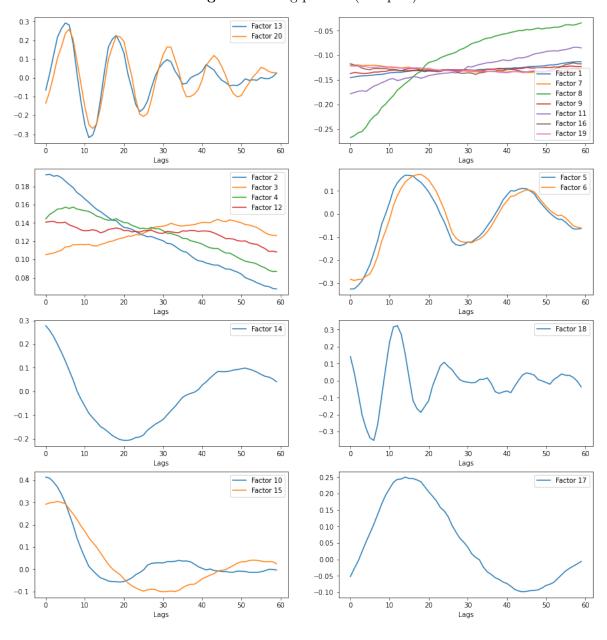


Figure 38: Lag pattern (line plot)

Figure 39: Reconstruction error with refit as a function of rank

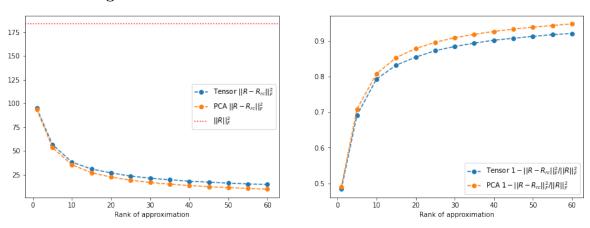
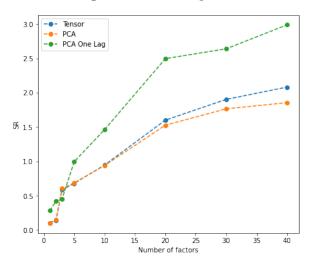


Figure 40: In-sample SR



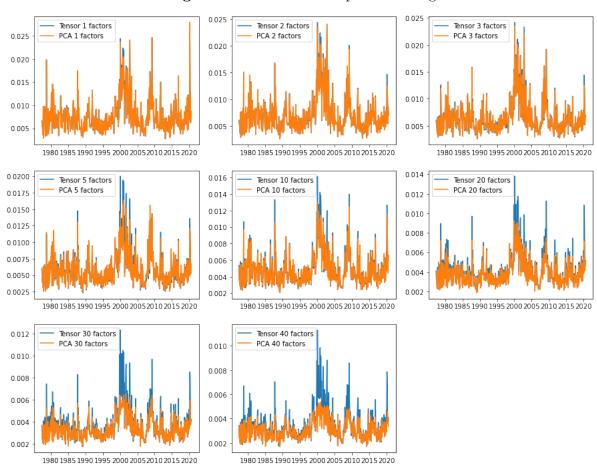


Figure 41: RMSE of time pattern fitting

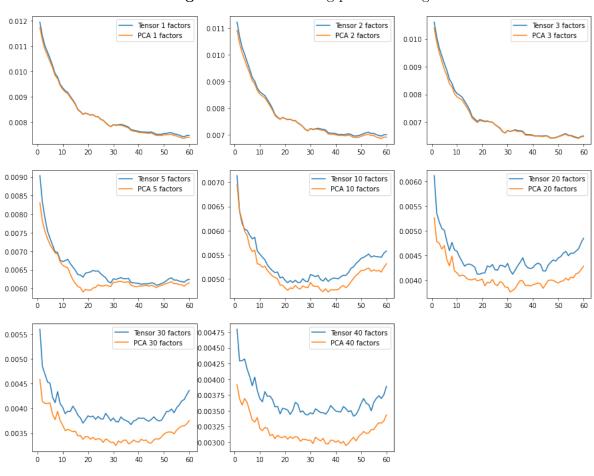


Figure 42: RMSE of lag pattern fitting

Figure 43: RMSE of portfolio pattern fitting

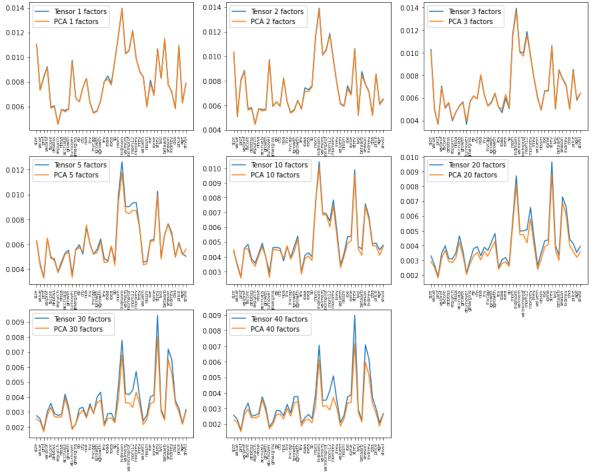


Figure 44: Normalized averaged alpha

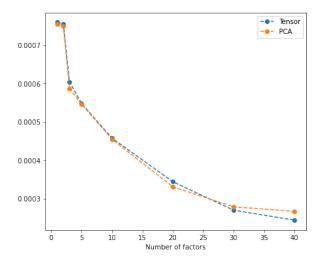


Figure 45: Cross-sectional averaged alpha for portfolios; Tensor

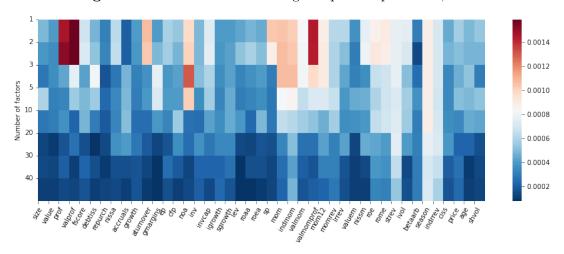


Figure 46: Cross-sectional averaged alpha for portfolios; PCA

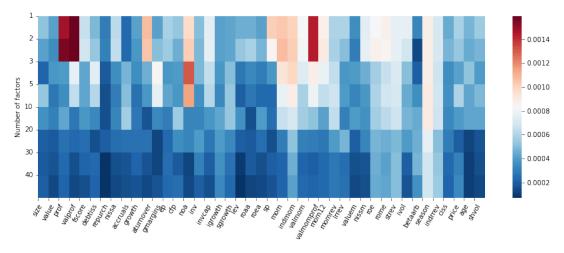


Figure 47: Averaged alpha for the lag dimension; Tensor

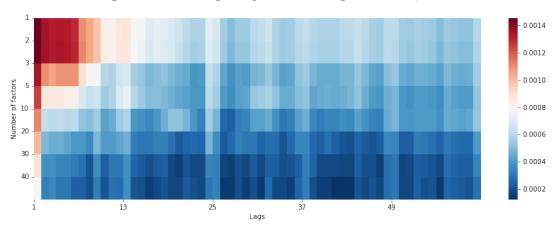


Figure 48: Averaged alpha for the lag dimension; PCA

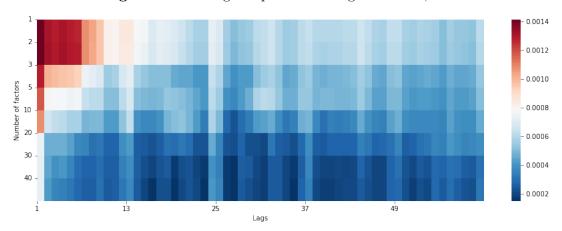
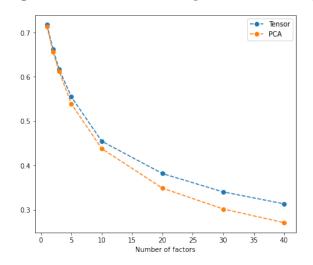


Figure 49: Normalized unexplained variance σ_{ϵ}



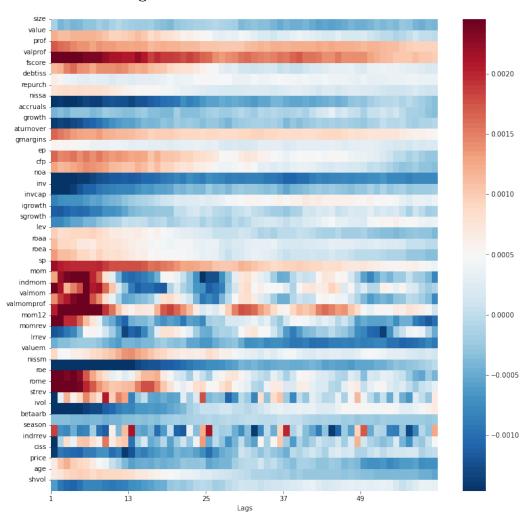


Figure 50: Term structure of mean returns

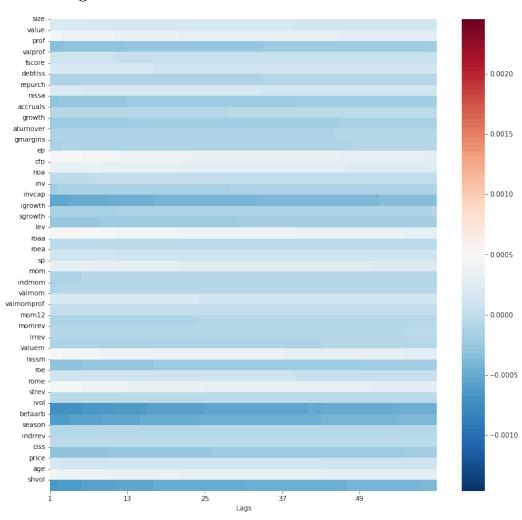


Figure 51: Term structure of fitted mean returns with rank 1

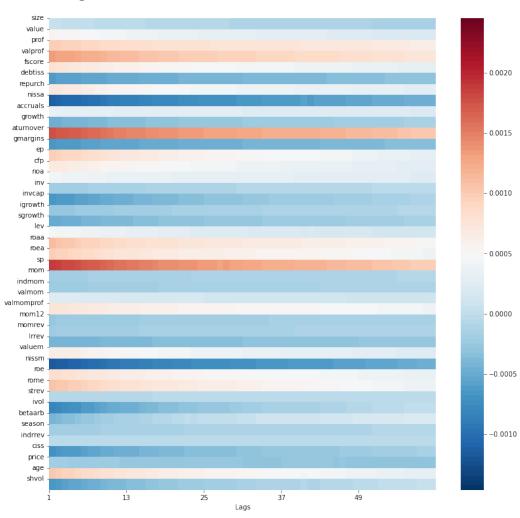


Figure 52: Term structure of fitted mean returns with rank 3

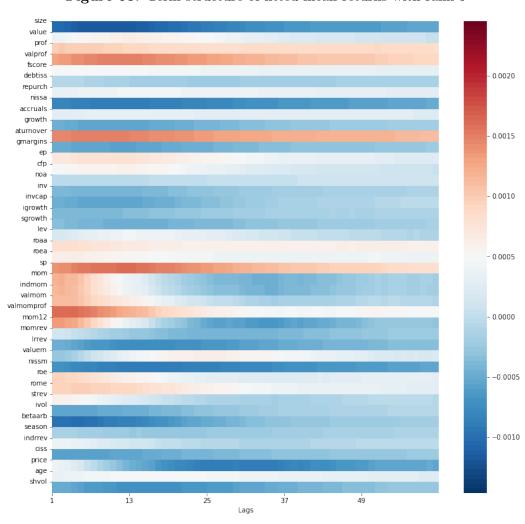


Figure 53: Term structure of fitted mean returns with rank 5

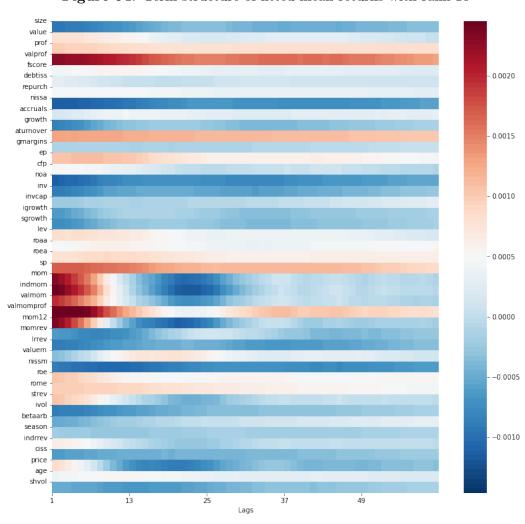


Figure 54: Term structure of fitted mean returns with rank 10

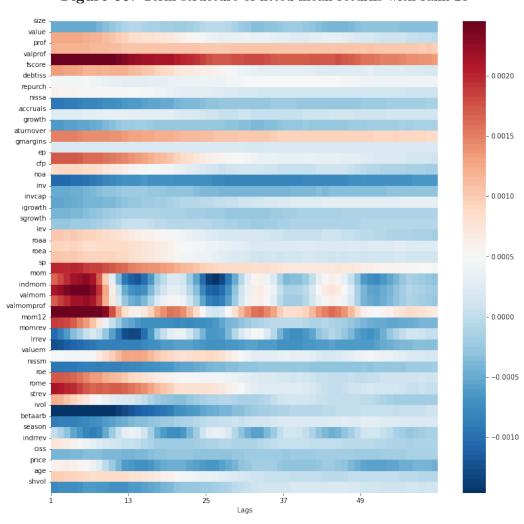


Figure 55: Term structure of fitted mean returns with rank 20

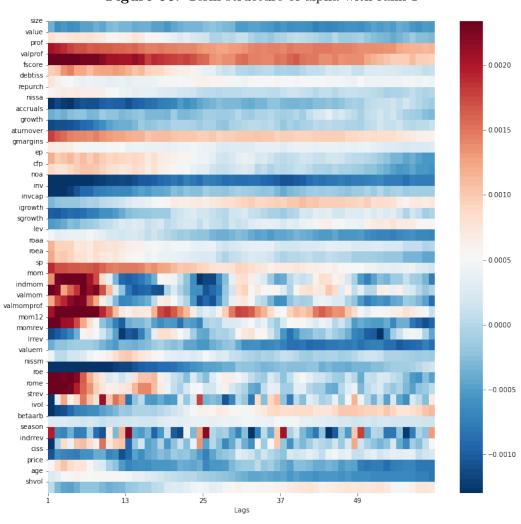


Figure 56: Term structure of alpha with rank 1

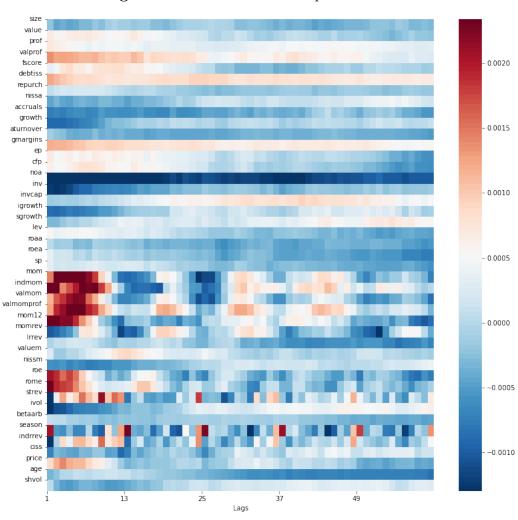


Figure 57: Term structure of alpha with rank 3

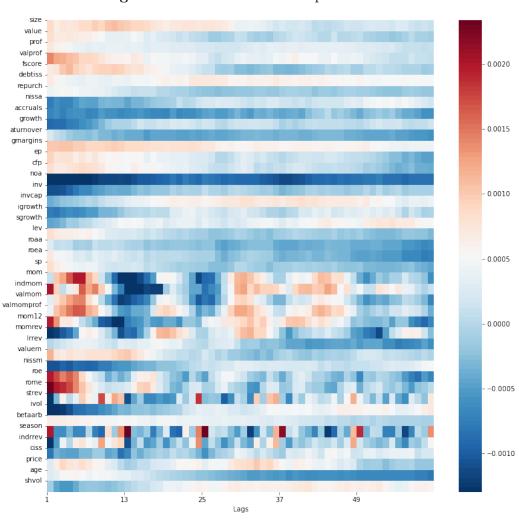


Figure 58: Term structure of alpha with rank 5

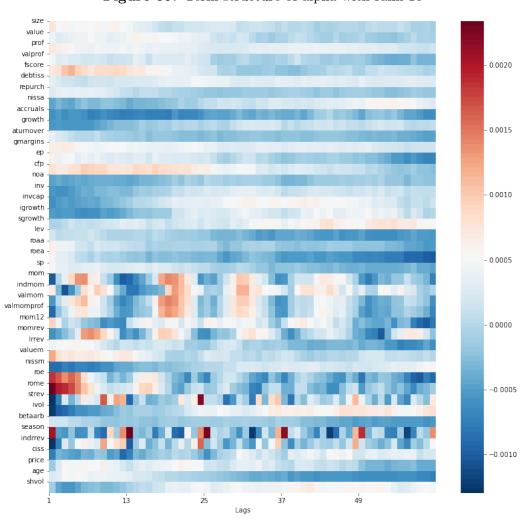


Figure 59: Term structure of alpha with rank 10

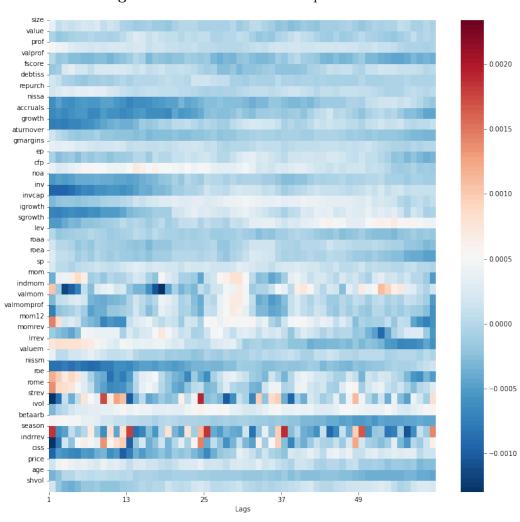


Figure 60: Term structure of alpha with rank 20

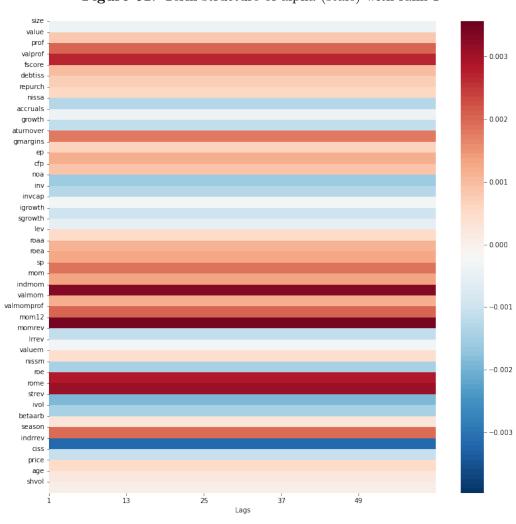


Figure 61: Term structure of alpha (stale) with rank 1

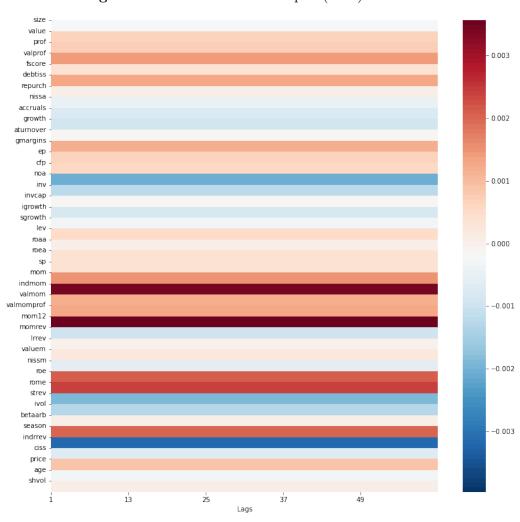


Figure 62: Term structure of alpha (stale) with rank 3

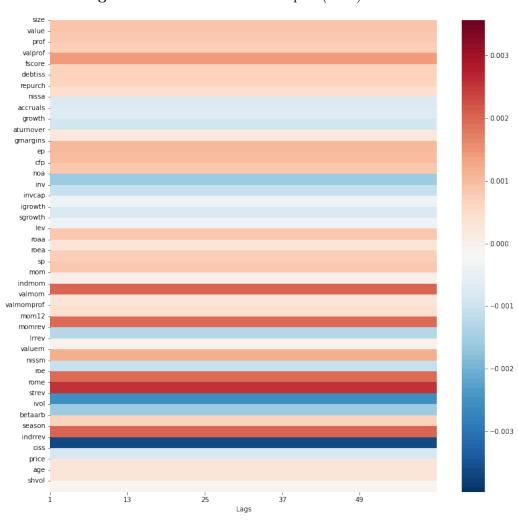


Figure 63: Term structure of alpha (stale) with rank 5

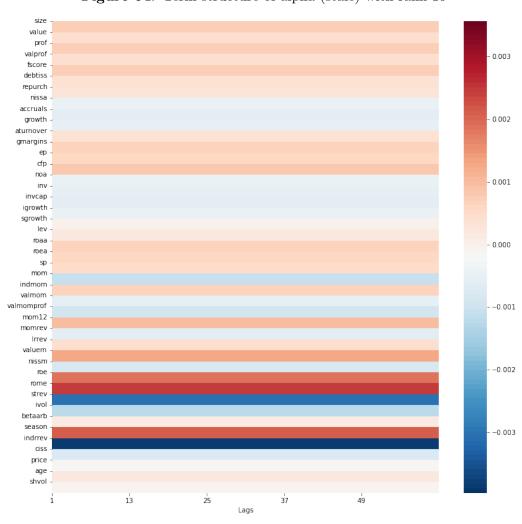


Figure 64: Term structure of alpha (stale) with rank 10

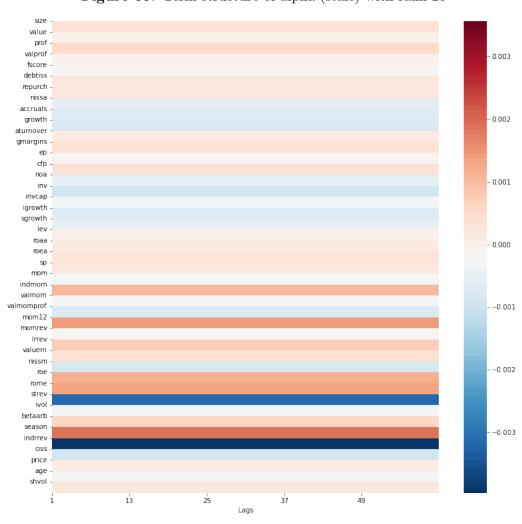


Figure 65: Term structure of alpha (stale) with rank 20

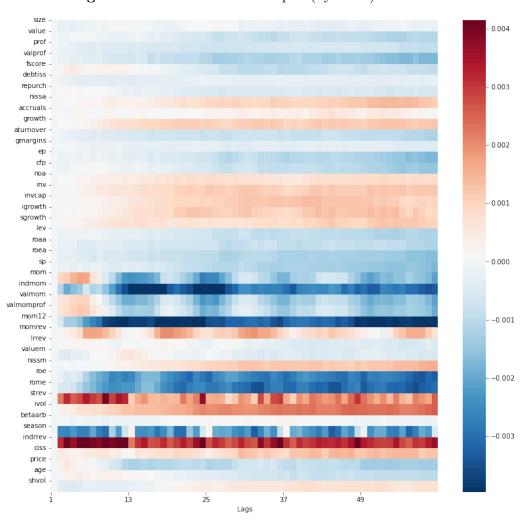


Figure 66: Term structure of alpha (dynamic) with rank 1

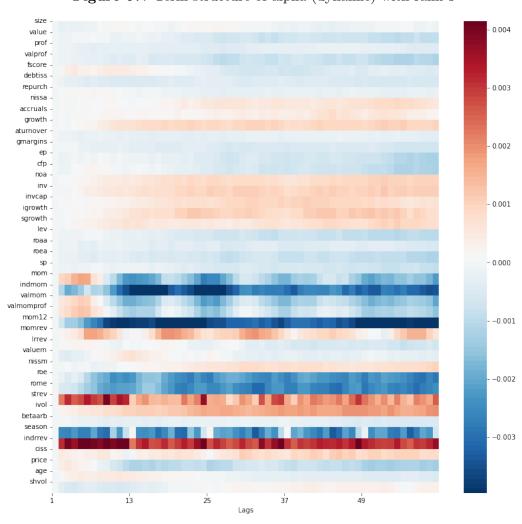


Figure 67: Term structure of alpha (dynamic) with rank 3

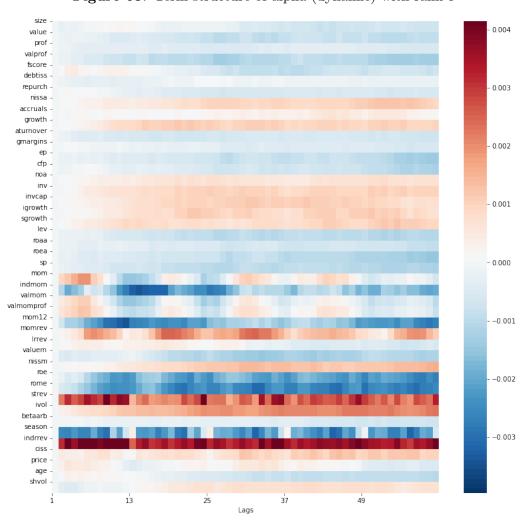


Figure 68: Term structure of alpha (dynamic) with rank 5

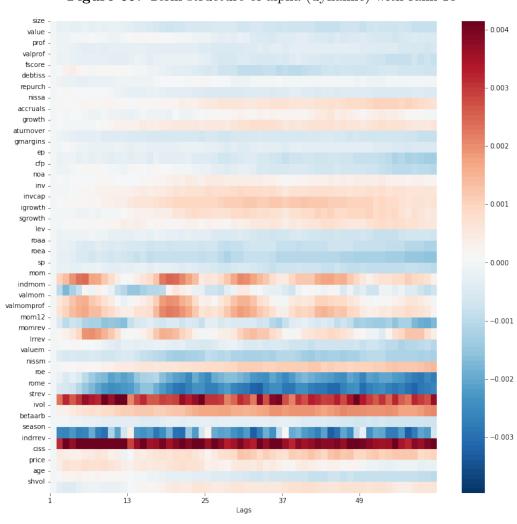


Figure 69: Term structure of alpha (dynamic) with rank 10

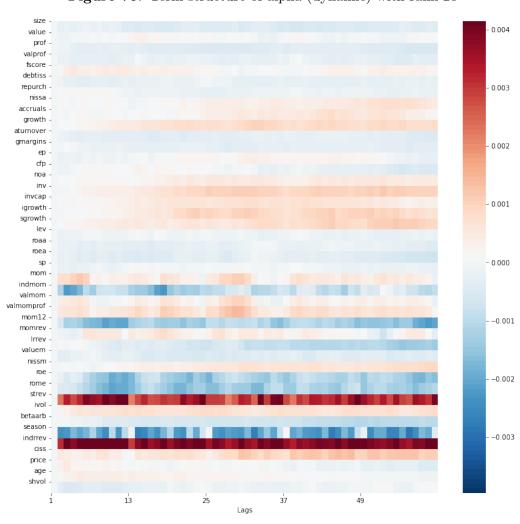


Figure 70: Term structure of alpha (dynamic) with rank 20