Collection of Results

April 5, 2022

1 Setup

- Sampling periods: 1972-07-01 (i.e. $\min(t - L)$) to 2020-08-01 (i.e. $\max t$). Max lag L is set to 60 months.
- 43 single-sorted rank-weighted portfolios. The resulting tensor in the full-sample is of dimension (519, 60, 43).
- Tensor X contains excess returns (simple, not log, return) $R \in \mathbb{R}^{T \times L \times C}$. X is not return in excess of the lag-one return.
- Full-sample setup
  - Run PARAFAC on 3D tensor $X \in \mathbb{R}^{T \times L \times C}$, which is decomposed into factors $F \in \mathbb{R}^{T \times K}$, lag loadings $W \in \mathbb{R}^{L \times K}$, character loadings $B \in \mathbb{R}^{C \times K}$, and scalers $S \in \mathbb{R}^{K}$. K is the pre-specified number of factors. X is not centered.
  - Unconstrained specification: the first column of W is not constrained to be constant.
  - Normalize columns of F, W, and B to have norm 1. Normalize each column in F to have positive time-series mean. Move sign and scaling to S. Reorder factors by magnitude of the corresponding scaling factors in S.
- Out-of-sample setup
  - Estimate a model using data in a rolling window of size 60 or 120 months. Results are calculated in the common out-of-sample period 1987-06-01 to 2020-08-01.
  - For the tensor model, obtain loadings W and B and scalars S in the rolling window. Regress X_{t+1} at the next period on W, B, and S to get factors F_{t+1} for the out-of-sample period. Combine factors in F_{t+1} using mean-variance weights on F.
  - For the PCA model, obtain loadings L in the rolling window. Regress X_{t+1} on L to get factors F_{t+1}.
  - For the one-lag PCA model, for which only portfolio returns based on the most recent characteristic values are included, factors are obtained in the same way as PCA.
  - For all of the competing models, calculate out-of-sample SR based on the mean-variance combination of factors. We also record fitted $\hat{X}_{t+1}$ to evaluate out-of-sample fitting quality.
- Full-sample multi-period return setup
  - X is converted into log return

  - Horizons up to 3 years. Rolling window of size 60 or 120 months.
  - Models:
    * Tensor: See Markus' notes for the setup.
    * Model-free: For each horizon, calculate overlapping multi-period returns for each of the 43 portfolios. Use mean-variance weights to combine these multi-period returns. Newey-west covariance estimator is used.
    * PCA: For each horizon, use PCA to extract the first K factors from overlapping multi-period return of the 43 portfolios. Use mean-variance weights to combine these factors. Newey-west covariance estimator is used.

- Out-of-sample multi-period return setup
  - X is converted into log return
  - Horizons up to 3 years. Rolling window of size 60 or 120 months. Results are calculated in the common out-of-sample period 1987-06-01 to 2017-09-01.
  - Tensor: Fit a tensor model in the rolling window. For horizon h and factor k, calculate the approximate multi-period return starting at time t as $FW_t = F_{k,t} \sum_{s=1}^{h} W_{k,s}$ in the rolling window. For each horizon, calculate the mean-variance weight for combining these multi-period factors. To get out-of-sample results, for each horizon h, calculate one-period out-of-sample factor F_{t+h} by regressing X_{t+h} on loadings W and B and scalars S estimated in the rolling window. Then, calculate out-of-sample multi-period returns as $\sum_{s=1}^{h} F_{k,t+s} W_{k,s}$ and use mean-variance weights to combine them.
  - Model-free: For each horizon h, calculate multi-period portfolio returns in the rolling window as $\sum_{s=1}^{h} X_{t+s,s}$. $X_{t+s,s}$ is the one-period return at $t+s$ using characteristic values with lag s. For horizon h, calculate mean-variance weights for combining multi-period returns. Newey-west covariance estimator is used. Then, calculate out-of-sample h-period return starting at the first out-of-sample period. For each h separately, combine multi-period returns using the mean-variance weights estimated in the rolling window.
  - PCA: For horizon h, use PCA to extract the first K factors from multi-period portfolio returns as defined above. For each out-of-sample period following the rolling window, get factors F by regressing multi-period returns on PCA loadings. Use mean-variance weights to combine these PCA factors out-of-sample. Newey-west covariance estimator is used.

2 Collection of results

- Evaluation (in-sample):
  - SR: Figure 40
  - Fitting RMSE (time, lag, and portfolio dimensions): Figures 41, 42, 43
  - Reconstruction error as a function of the number of factors for PCA and the tensor model: Figure 39
  - Averaged alpha and unexplained variance (XS-α and σ_ϵ as in Markus' notes)
    * Normalized averaged alpha XS-α: Figure 44
    * Averaged alpha for the lag dimension: Figures 47 and 48

    - Averaged alpha for the portfolio dimension: Figures 45 and 46
    - Unexplained variance: Figure 49
  - Term structure of (fitted) mean returns: Figures 51 to 55 for fitted return. Figure 50 for mean returns (model free).
  - Term structure of alpha: Figures 56 to 60
  - Decomposition of alpha into stale and dynamic components: Figures 61 to 65 for the stale component. Figures 66 to 70 for the dynamic component.

- Evaluation (out-of-sample):
  - SR: Figures 4 (window size: 60 months) and 5 (window size: 120 months)
  - Averaged alpha and unexplained variance (XS-α and σ_ϵ as in Markus' notes)
    - Normalized averaged alpha XS-α: Figure 6
    - Averaged alpha for the lag dimension: Figures 12 and 14
    - Averaged alpha for the portfolio dimension: Figures 8 and 10
    - Unexplained variance: Figure 16
  - Term structure of (fitted) mean returns: Figures 19 to 23 for fitted return. Figure 18 for mean returns (model free).
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- Multi-period evaluation (in-sample and out-of-sample): SR as a function of horizon and number of factors: Figures 2 and 1

- Illustrations for the tensor model (in-sample, up to 20 factors):
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- Illustration for PCA (in-sample):
  - PCA loadings: Figure 3

List of Figures

Figure 1: Out-of-sample SR of multi-period returns

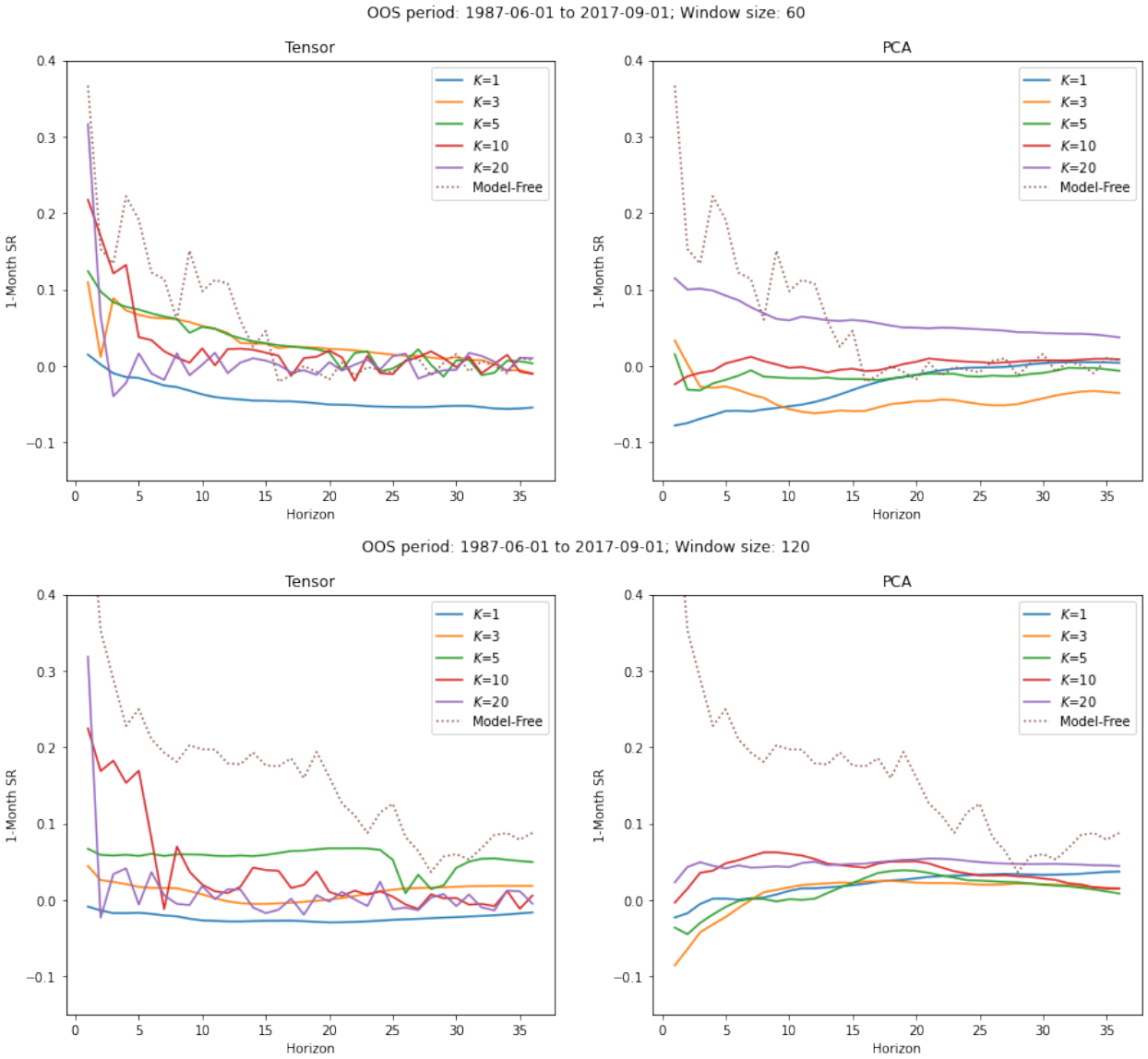

Figure 2: In-sample SR of multi-period returns

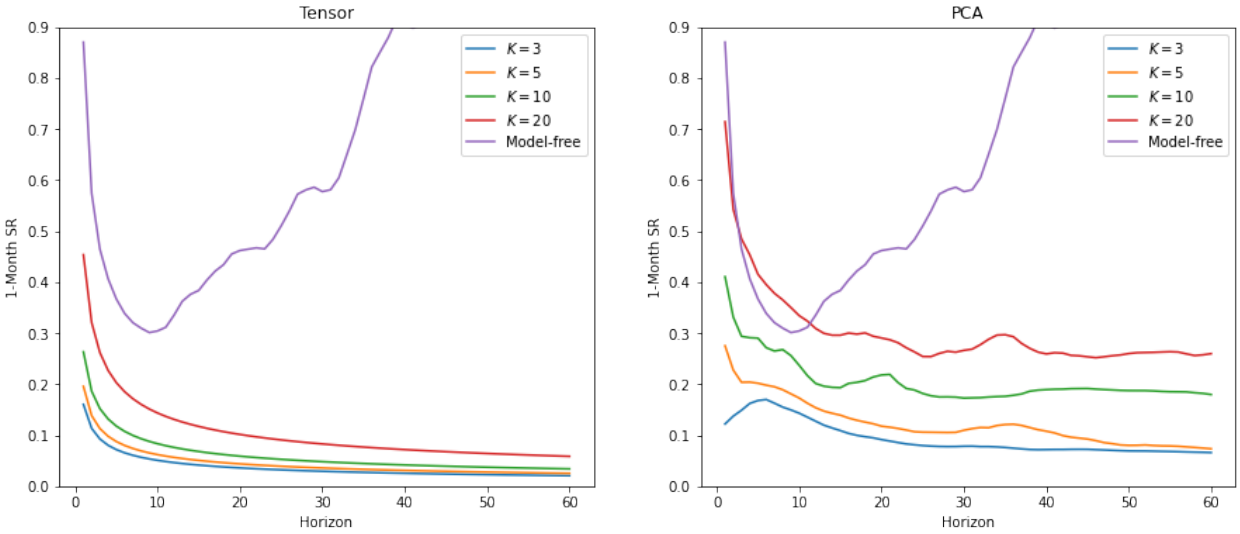

Figure 3: PCA loadings

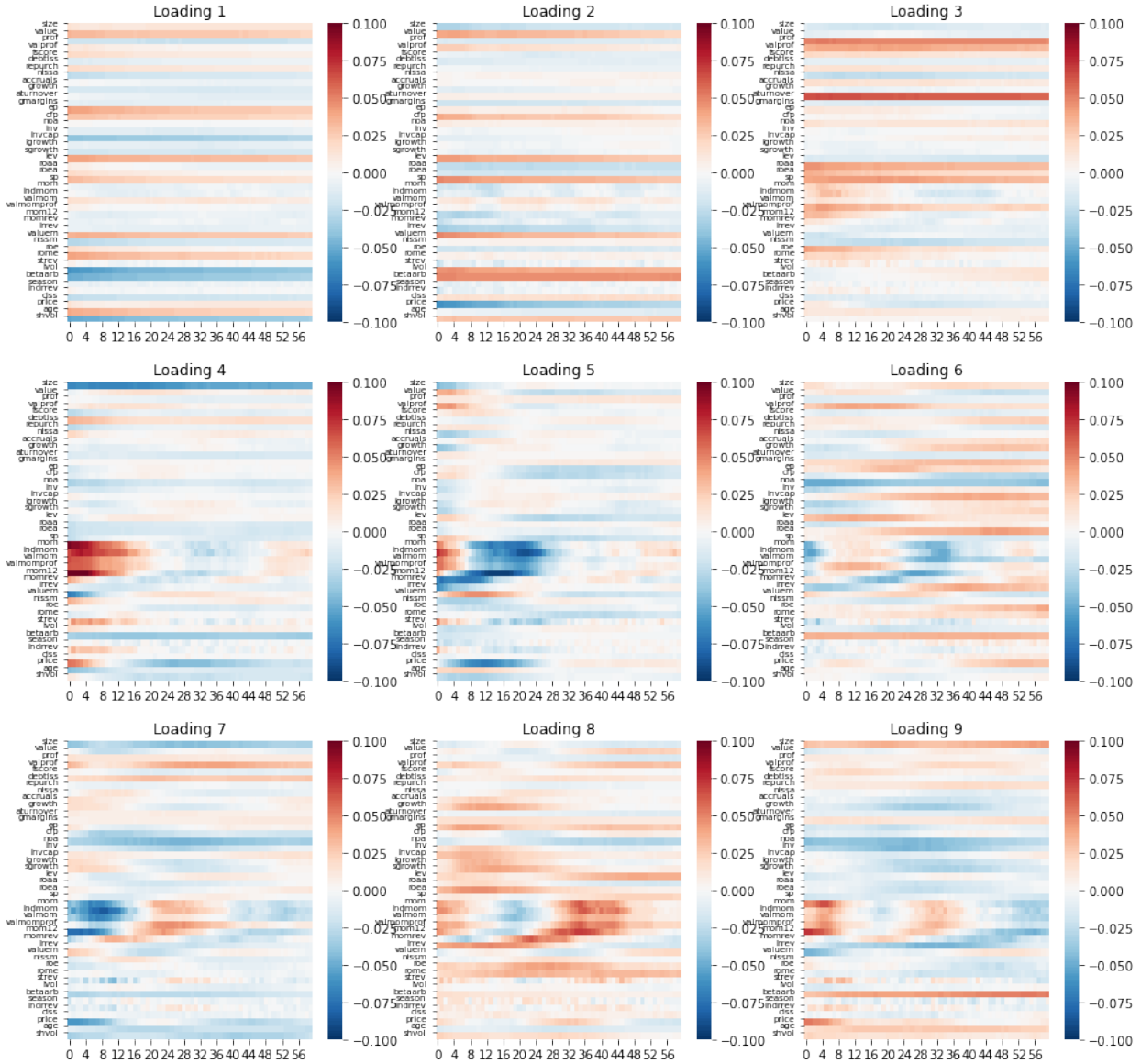

Figure 4: Out-of-sample SR; Window size: 60 months

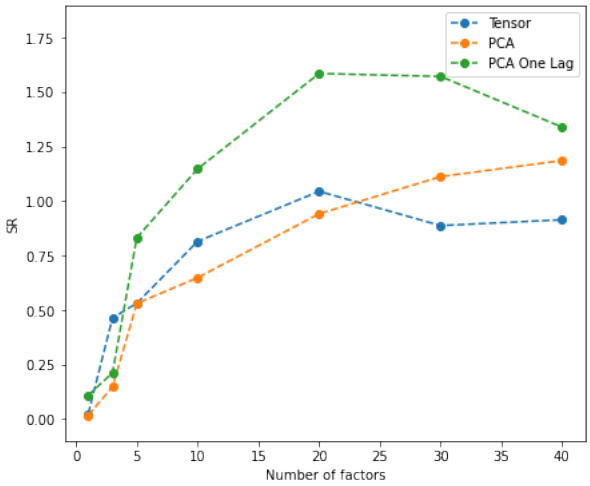

Figure 5: Out-of-sample SR; Window size: 120 months

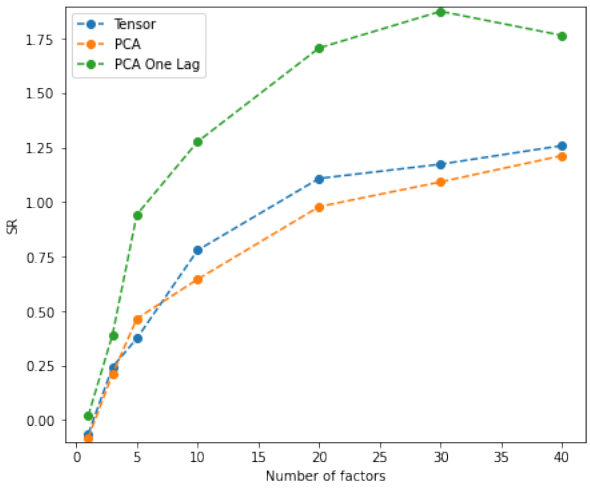

Figure 6: Normalized averaged alpha (OOS); Window size: 60 months

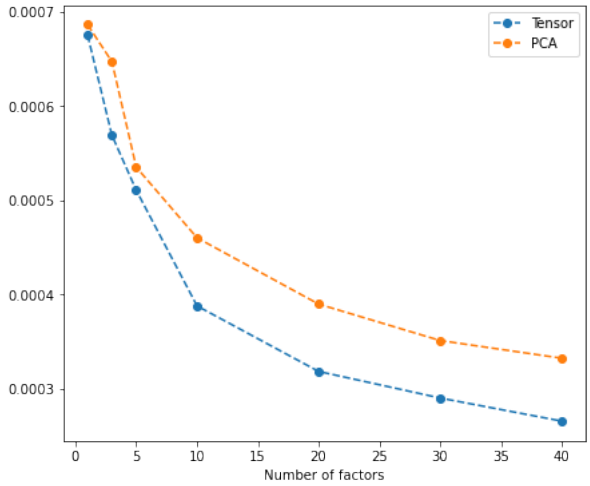

Figure 7: Normalized averaged alpha (OOS); Window size: 120 months

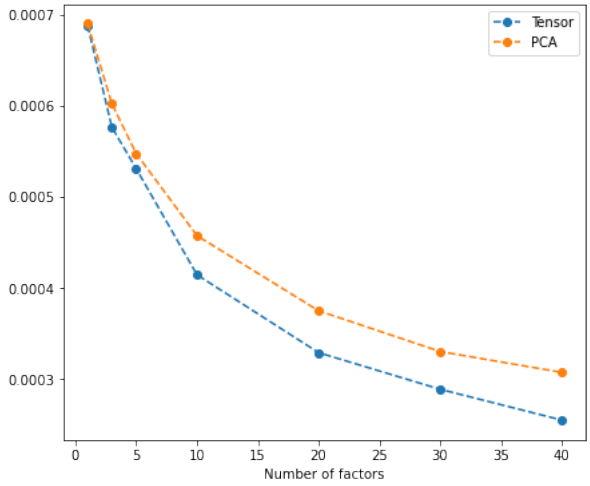

Figure 8: Cross-sectional averaged alpha for portfolios (OOS); Tensor model; Window size: 60 months

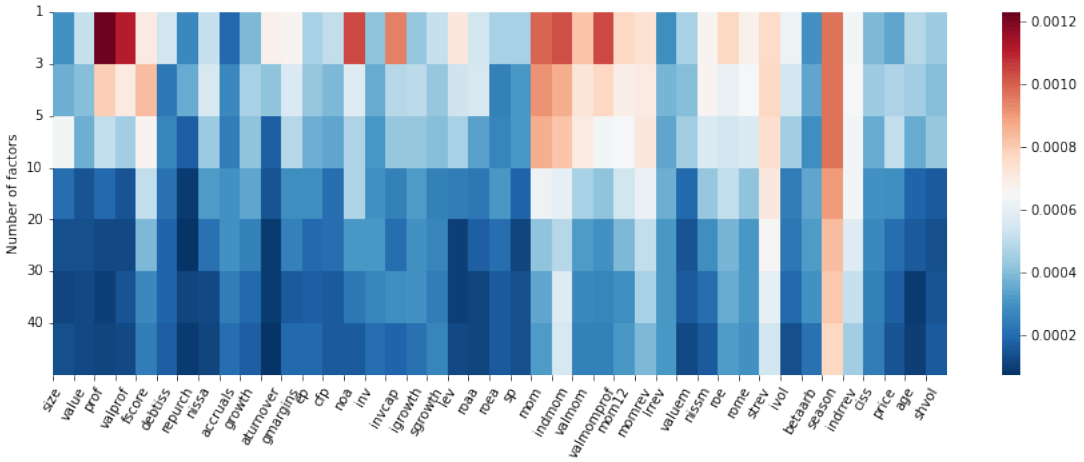

Figure 9: Cross-sectional averaged alpha for portfolios (OOS); Tensor model; Window size: 120 months

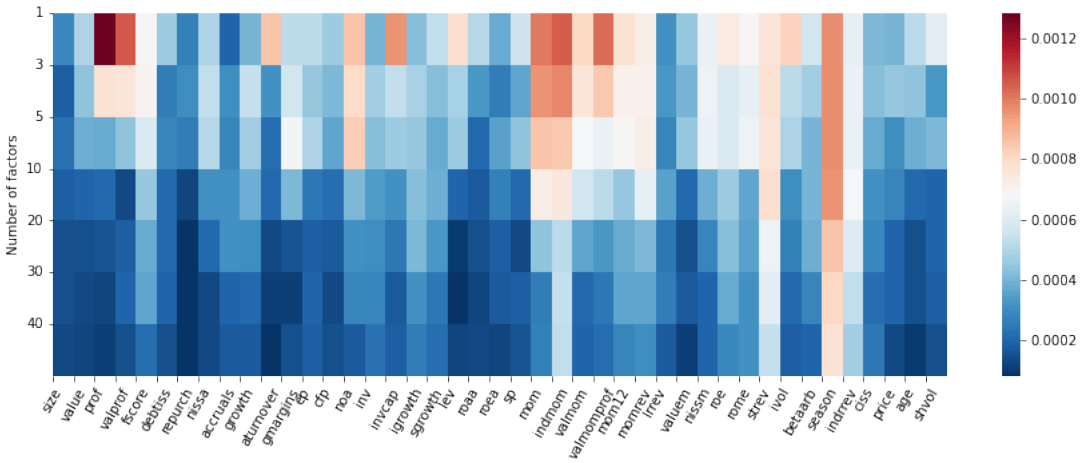

Figure 10: Cross-sectional averaged alpha for portfolios (OOS); PCA model; Window size: 60 months

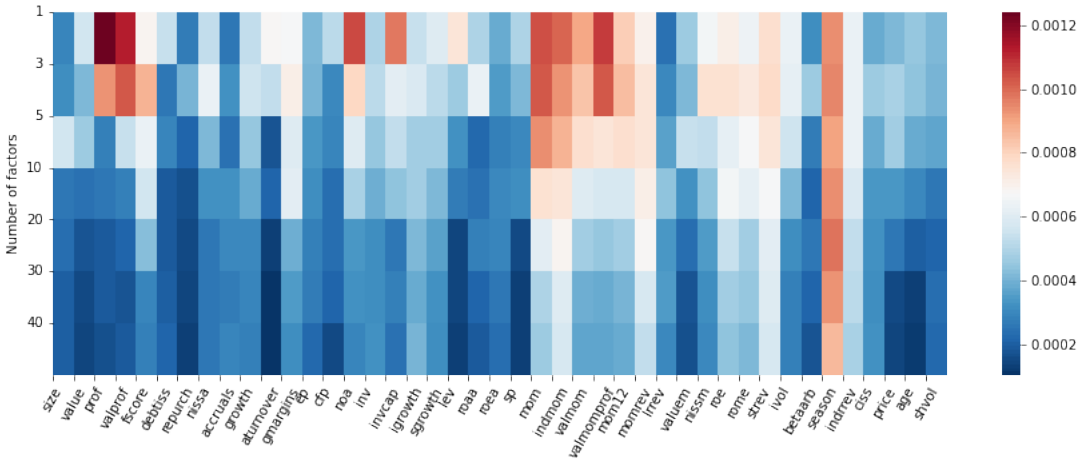

Figure 11: Cross-sectional averaged alpha for portfolios (OOS); PCA model; Window size: 120 months

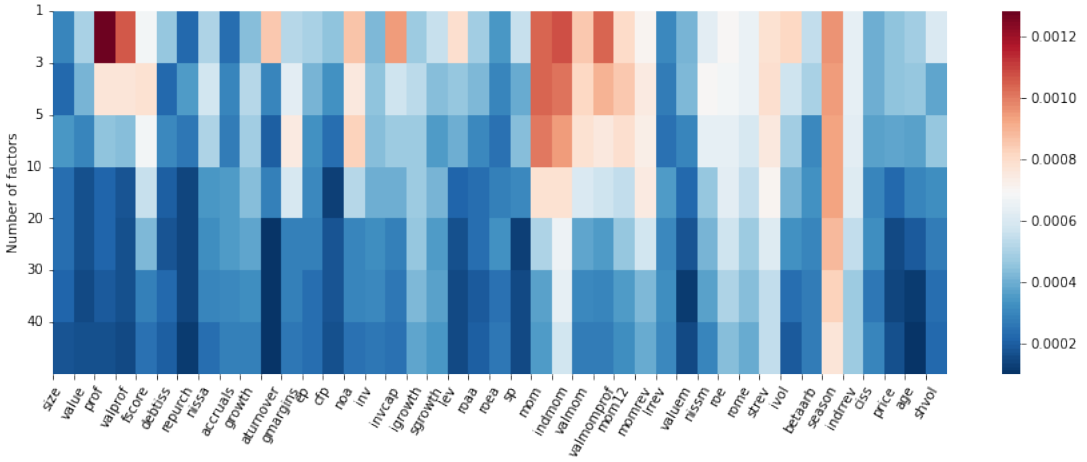

Figure 12: Averaged alpha for the lag dimension (OOS); Tensor model; Window size: 60 months

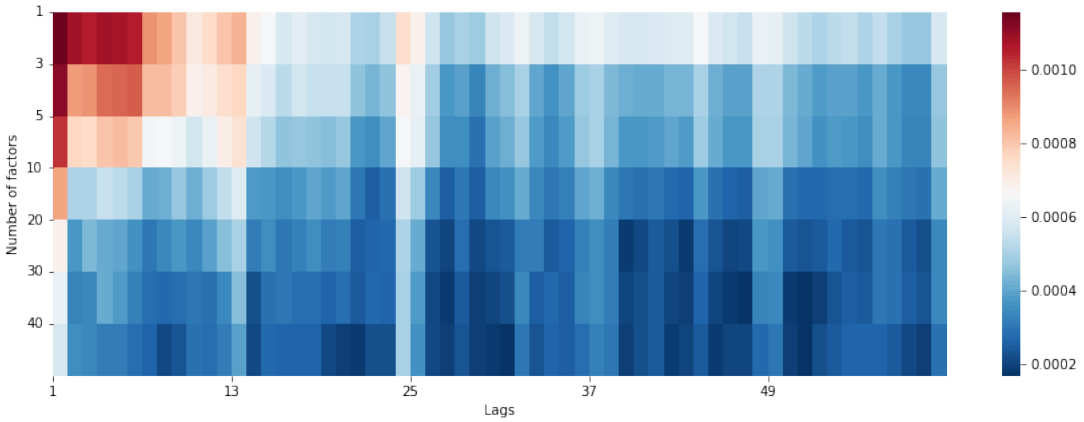

Figure 13: Averaged alpha for the lag dimension (OOS); Tensor model; Window size: 120 months

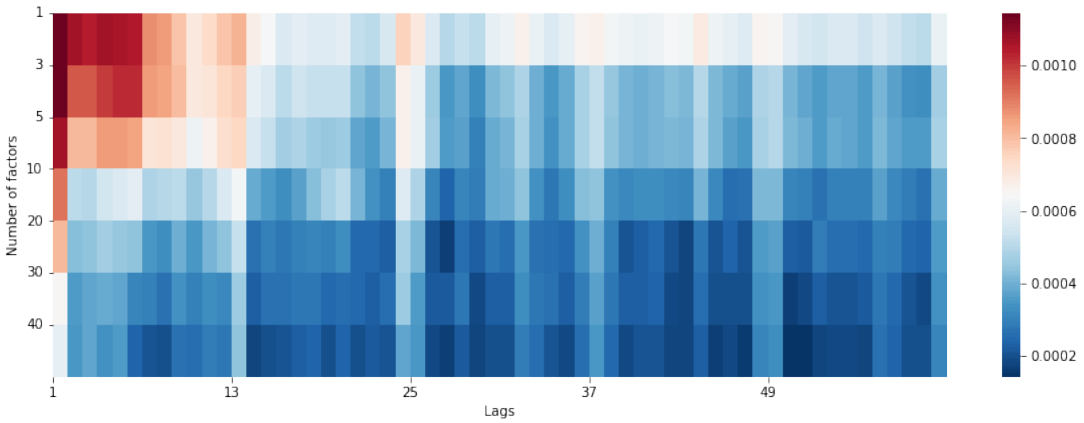

Figure 14: Averaged alpha for the lag dimension (OOS); PCA; Window size: 60 months

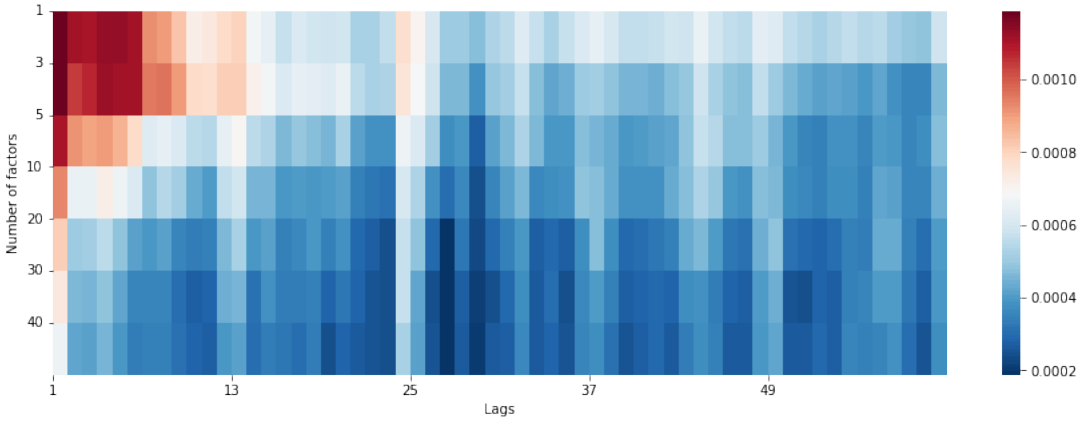

Figure 15: Averaged alpha for the lag dimension (OOS); PCA; Window size: 120 months

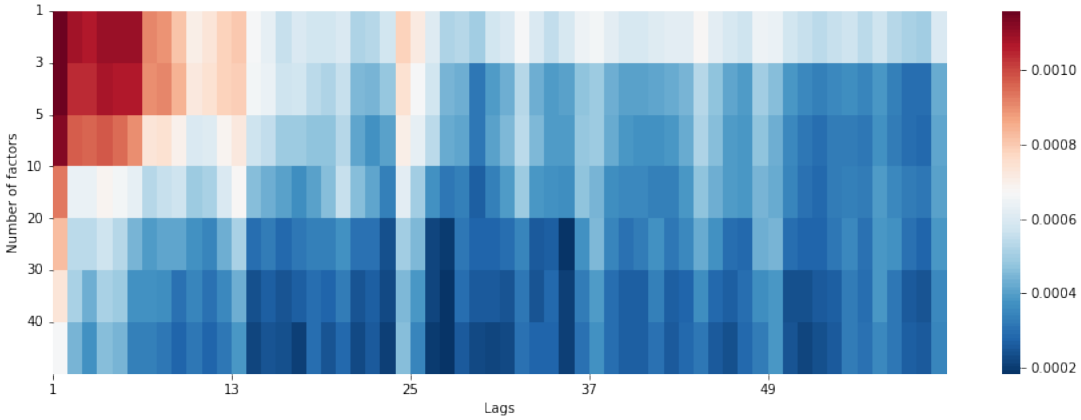

Figure 16: Normalized unexplained variance σ_ϵ (OOS): Window size: 60 months

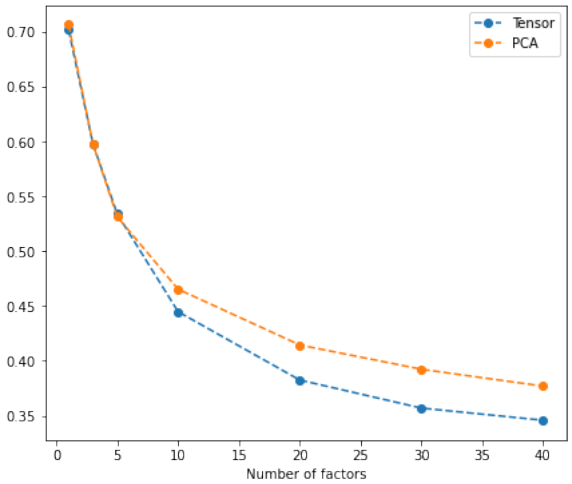

Figure 17: Normalized unexplained variance σ_ϵ (OOS): Window size: 120 months

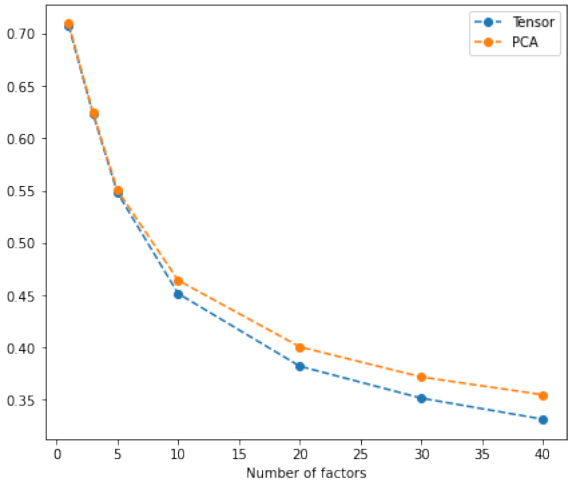

Figure 18: Term structure of mean returns (OOS)

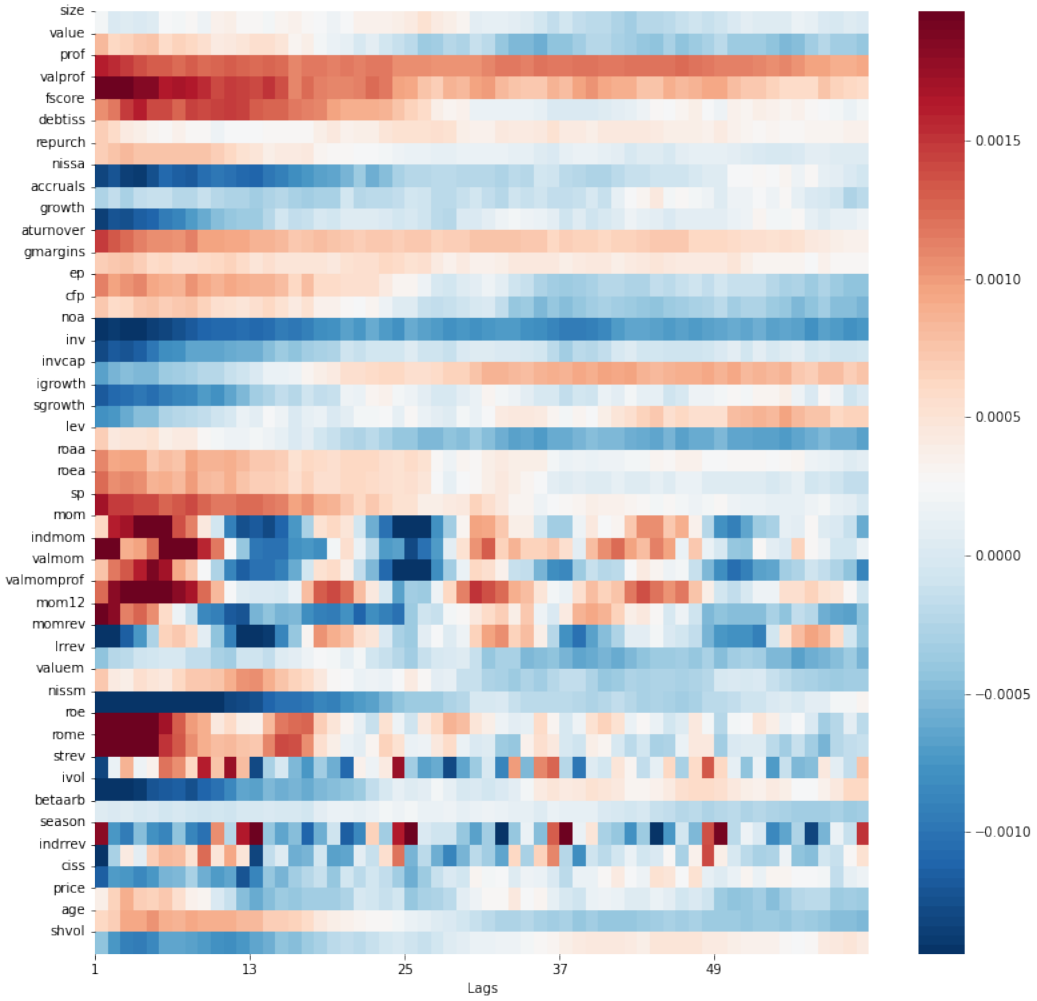

Figure 19: Term structure of fitted mean returns with rank 1 (OOS); Window size: 60 months

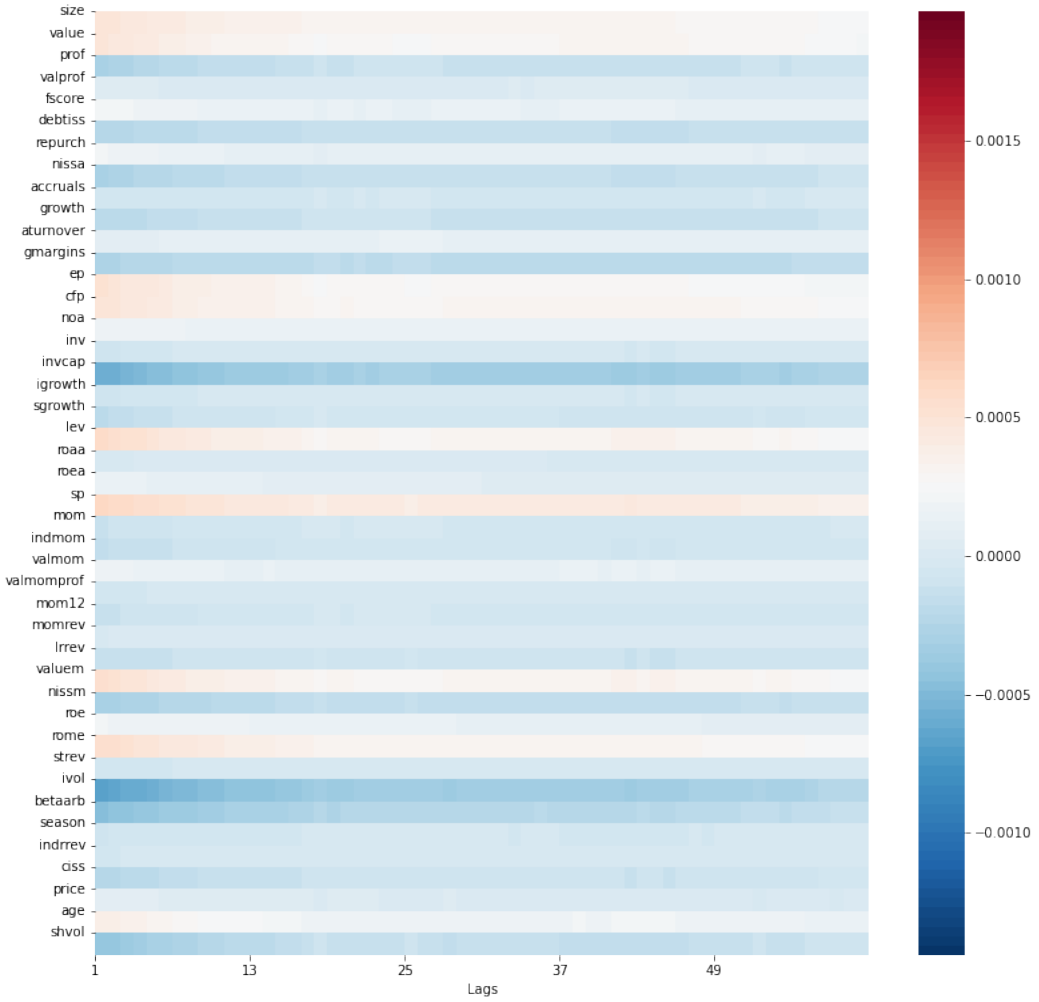

Figure 20: Term structure of fitted mean returns with rank 3 (OOS); Window size: 60 months

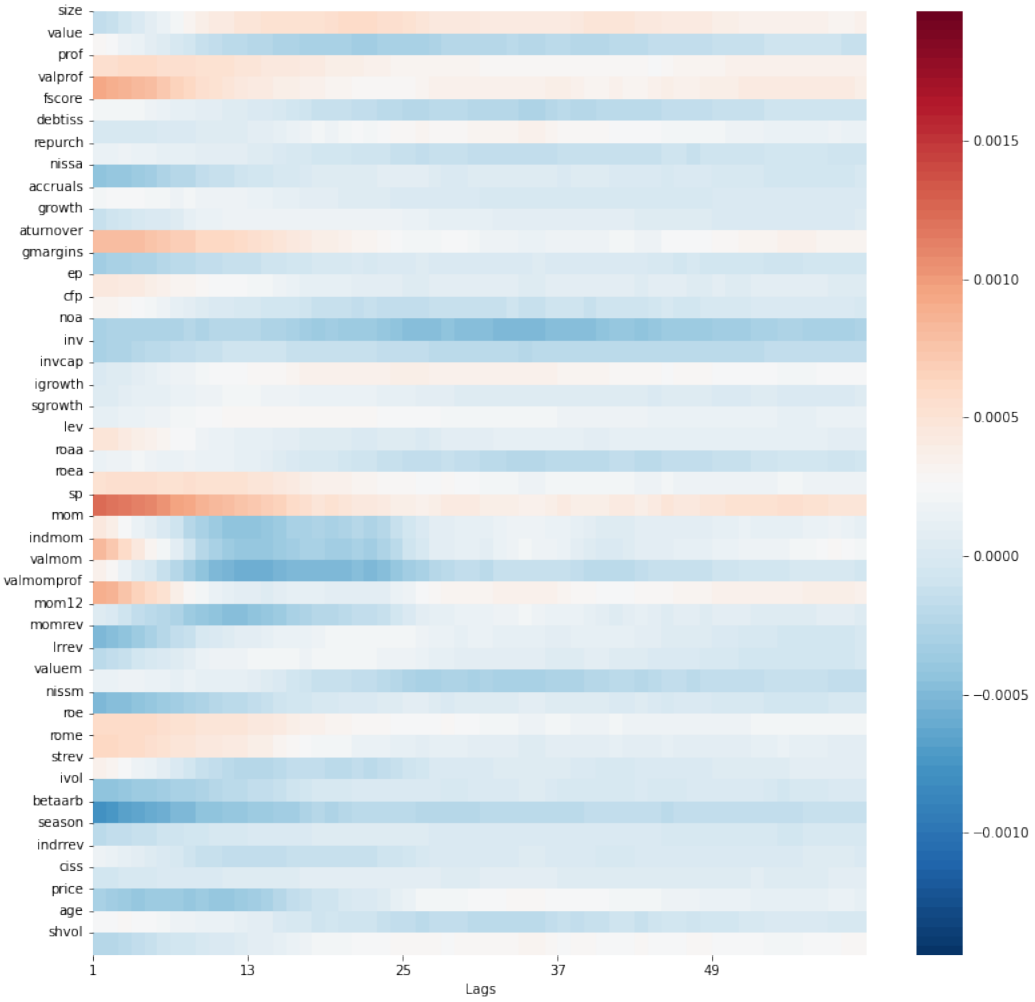

Figure 21: Term structure of fitted mean returns with rank 5 (OOS); Window size: 60 months

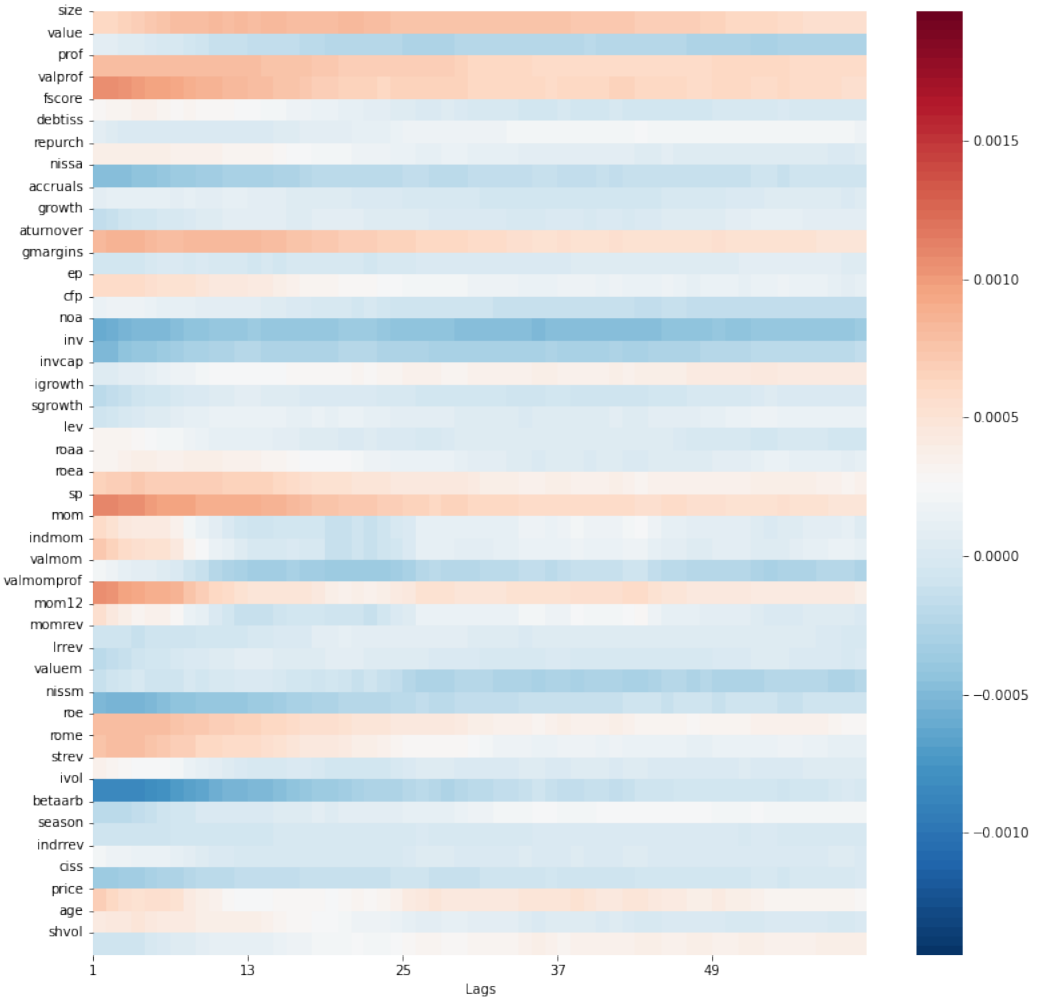

Figure 22: Term structure of fitted mean returns with rank 10 (OOS); Window size: 60 months

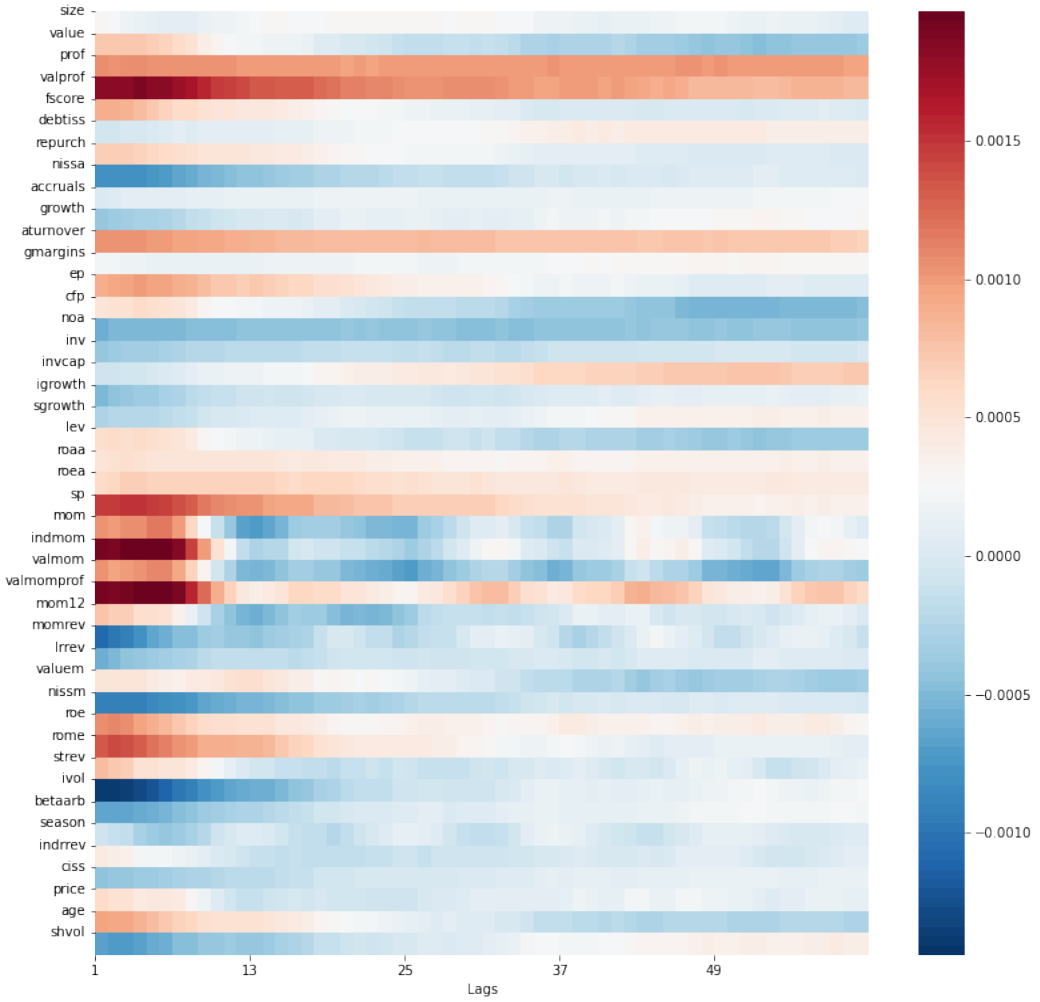

Figure 23: Term structure of fitted mean returns with rank 20 (OOS); Window size: 60 months

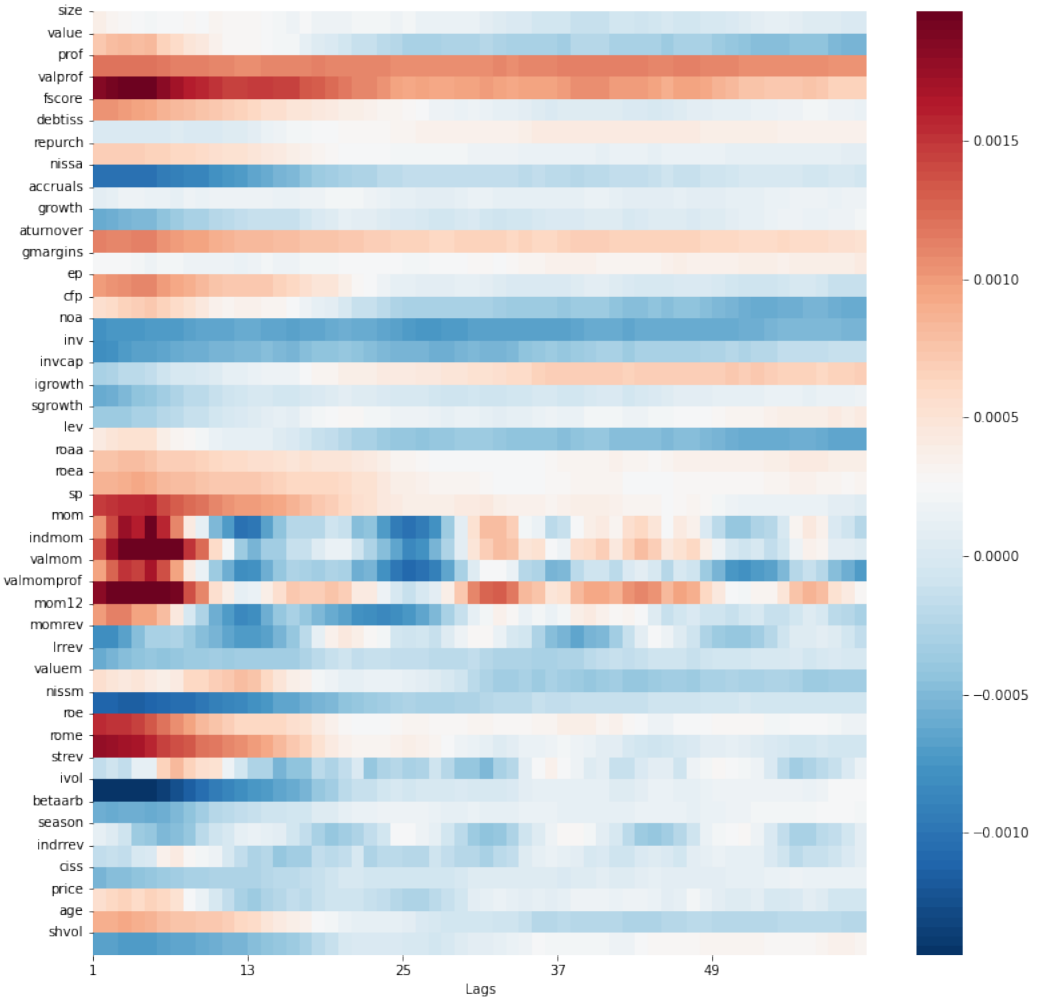

Figure 24: Term structure of alpha with rank 1 (OOS); Window size: 60 months

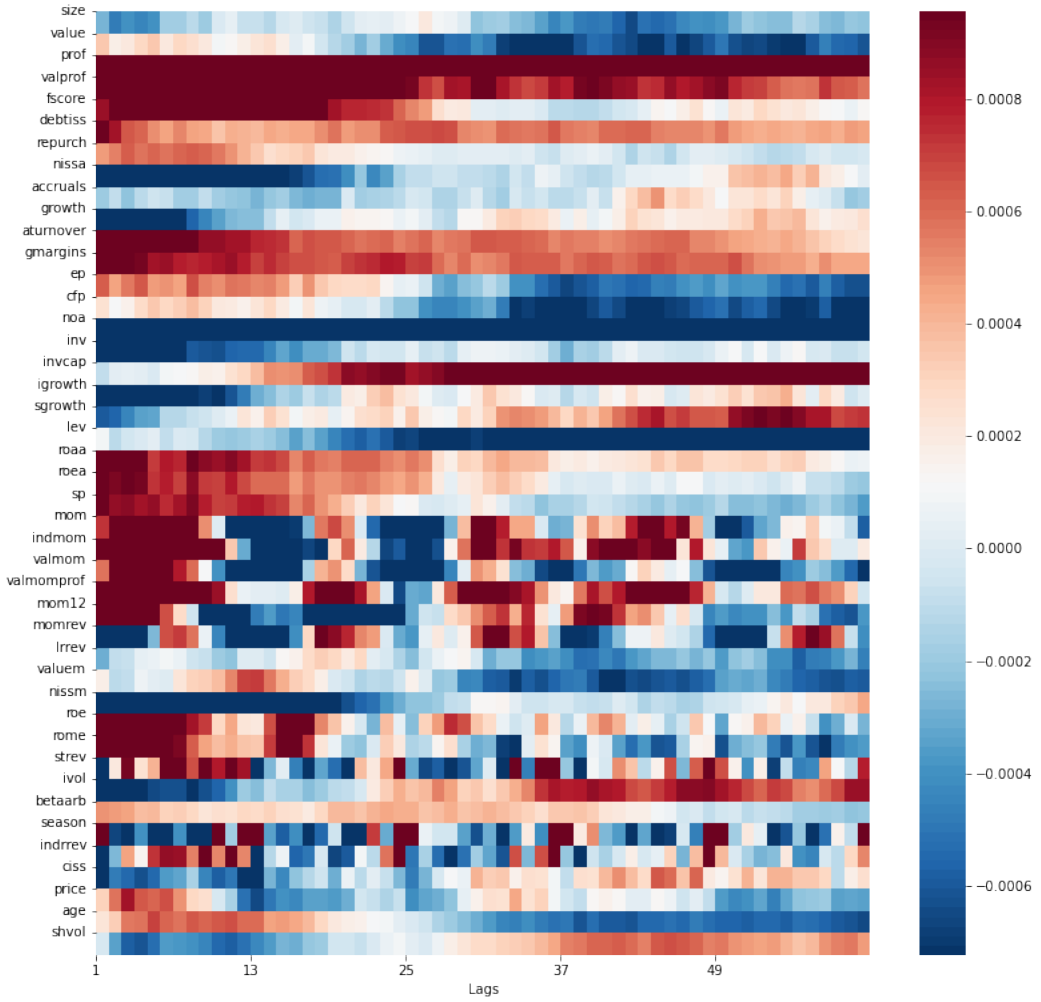

Figure 25: Term structure of alpha with rank 3 (OOS); Window size: 60 months

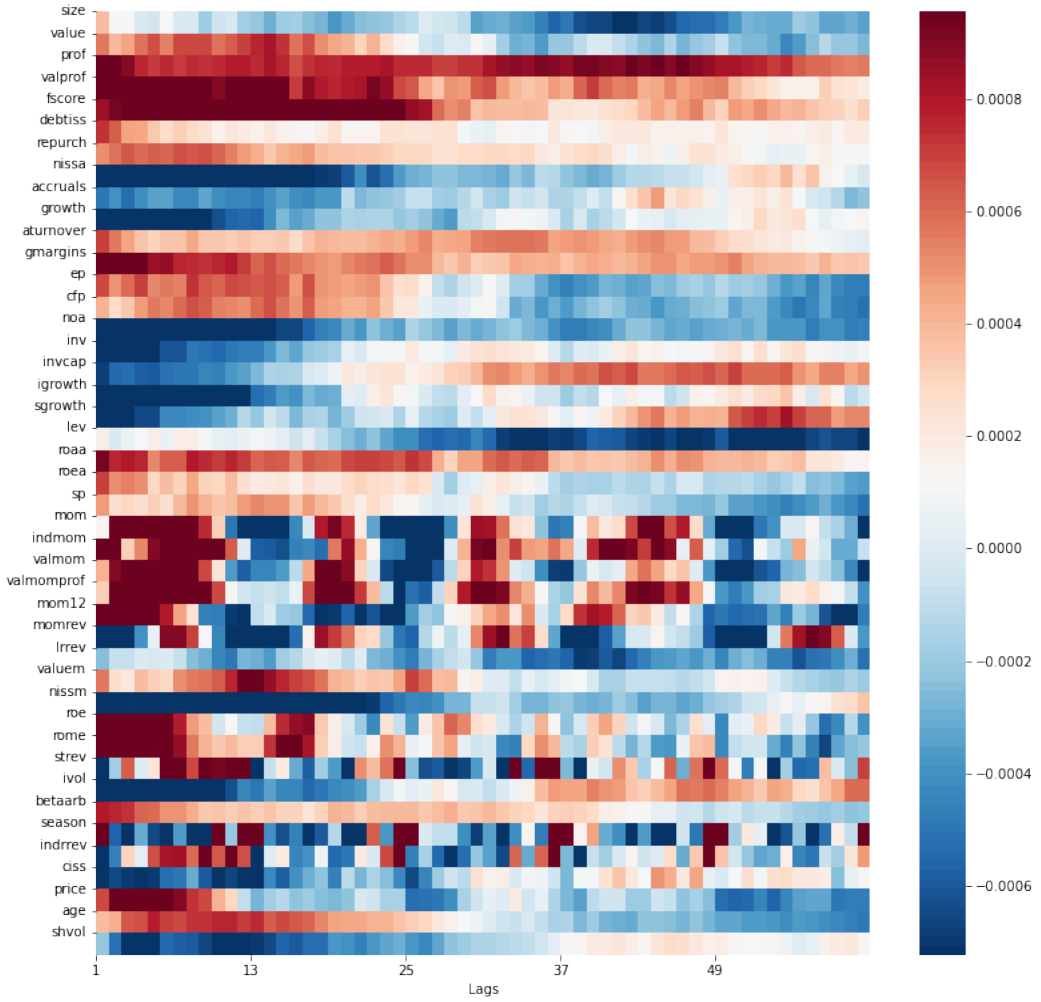

Figure 26: Term structure of alpha with rank 5 (OOS); Window size: 60 months

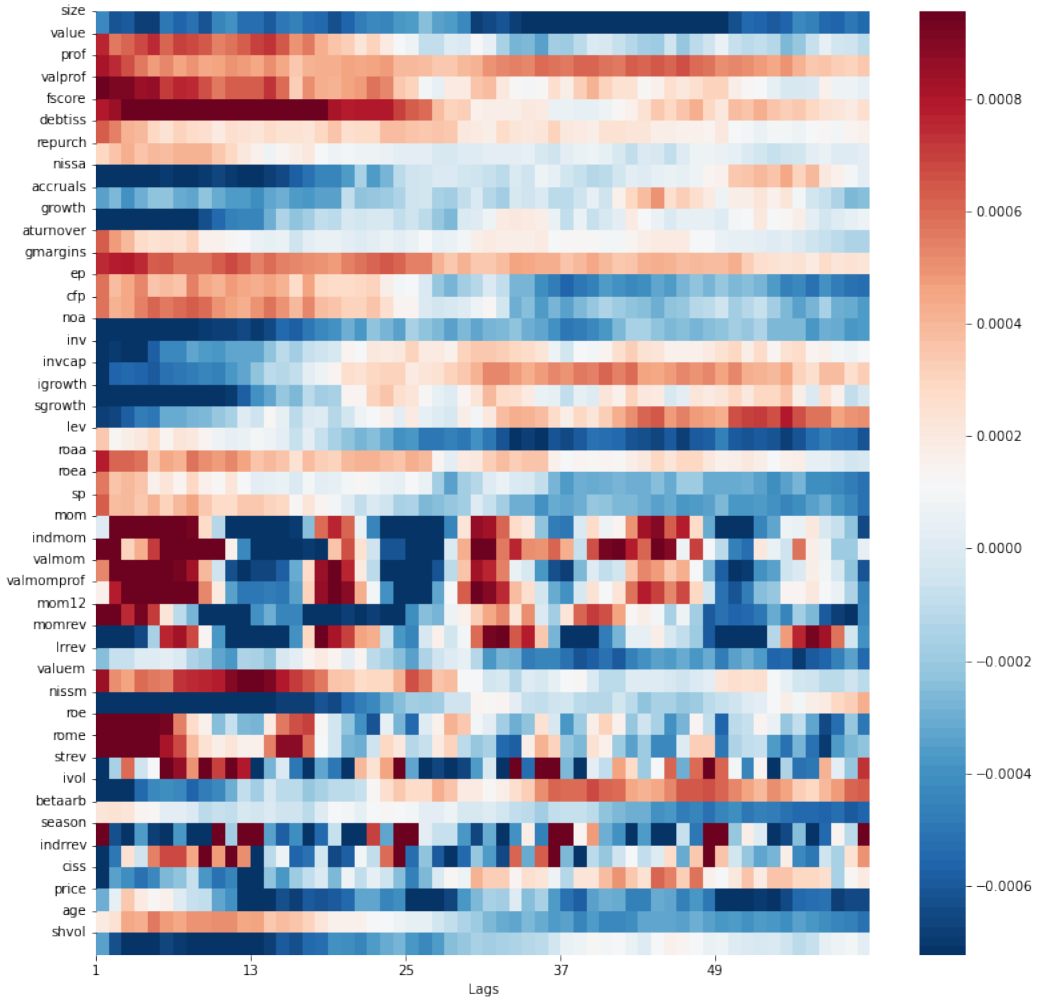

Figure 27: Term structure of alpha with rank 10 (OOS); Window size: 60 months

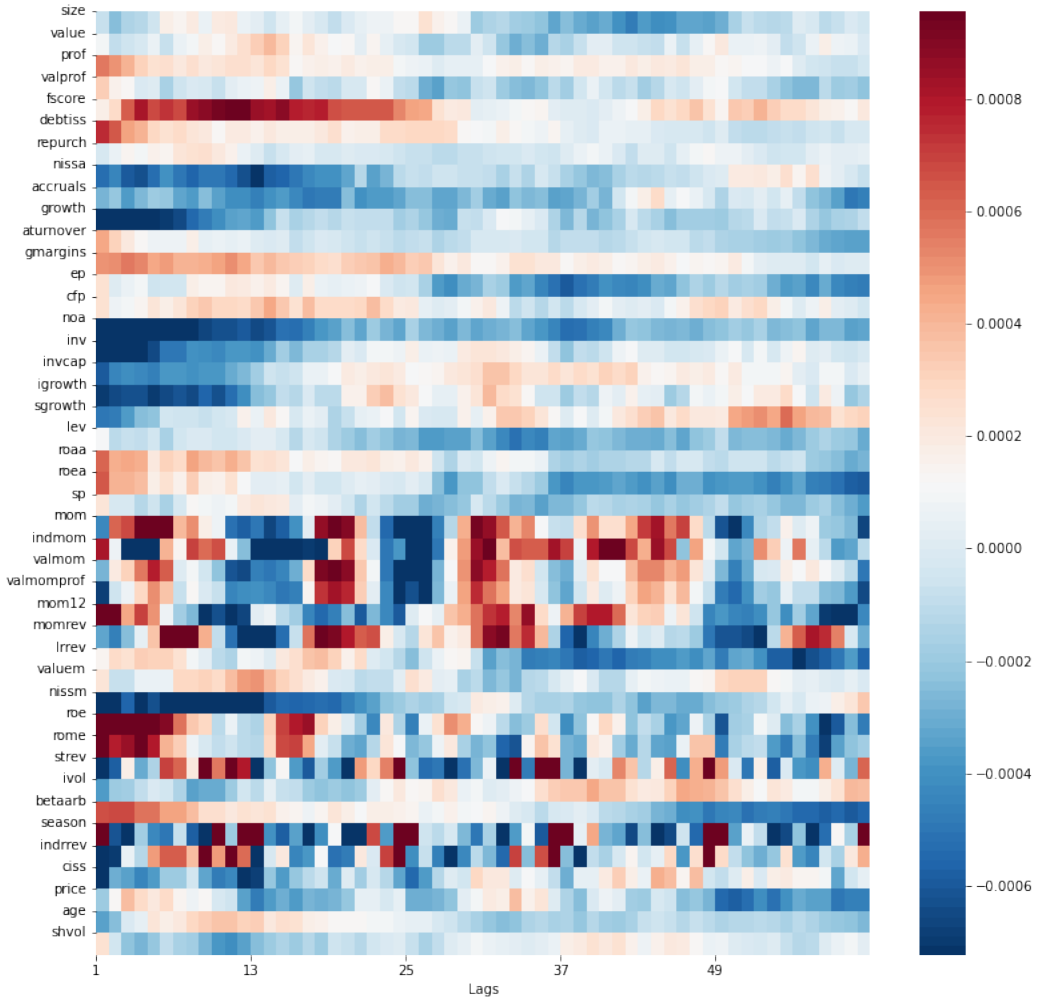

Figure 28: Term structure of alpha with rank 20 (OOS); Window size: 60 months

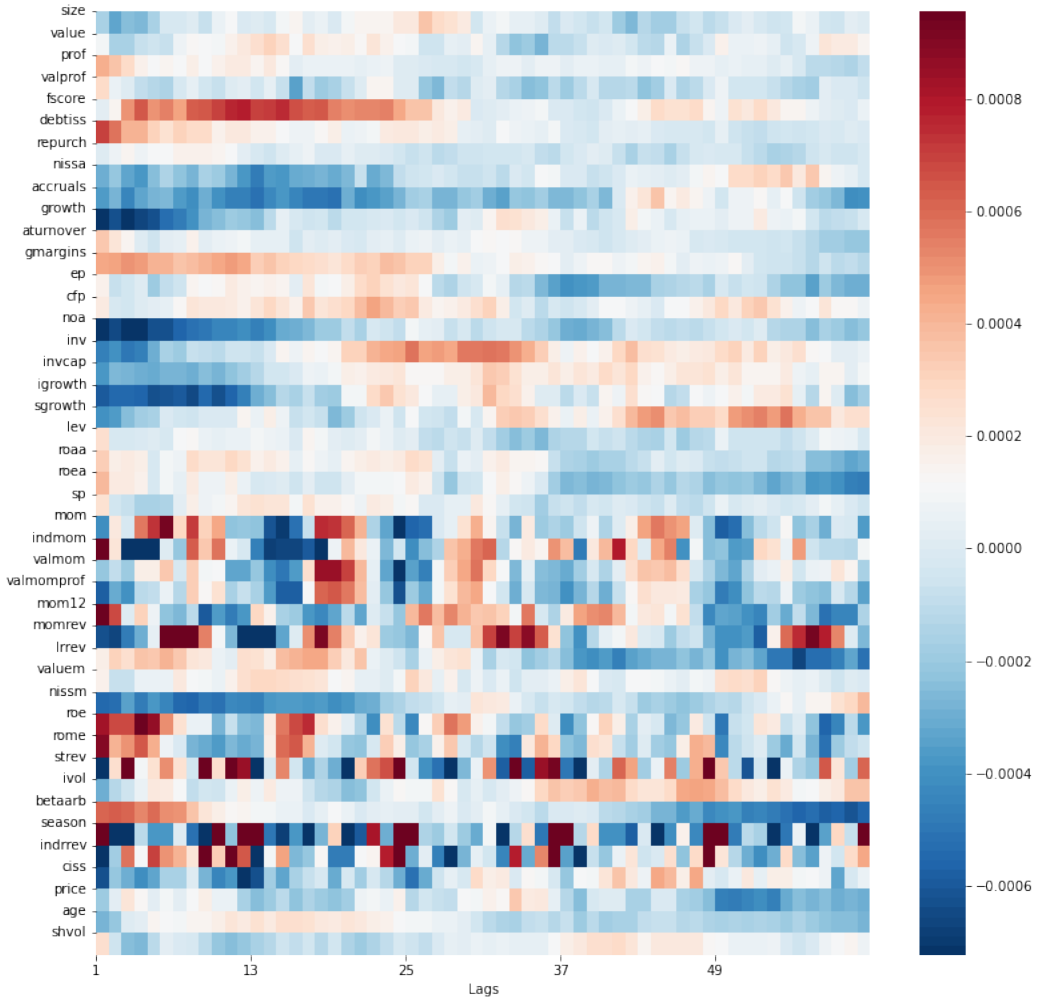

Figure 29: Term structure of CAPM alpha with rank 1 (OOS); Window size: 60 months

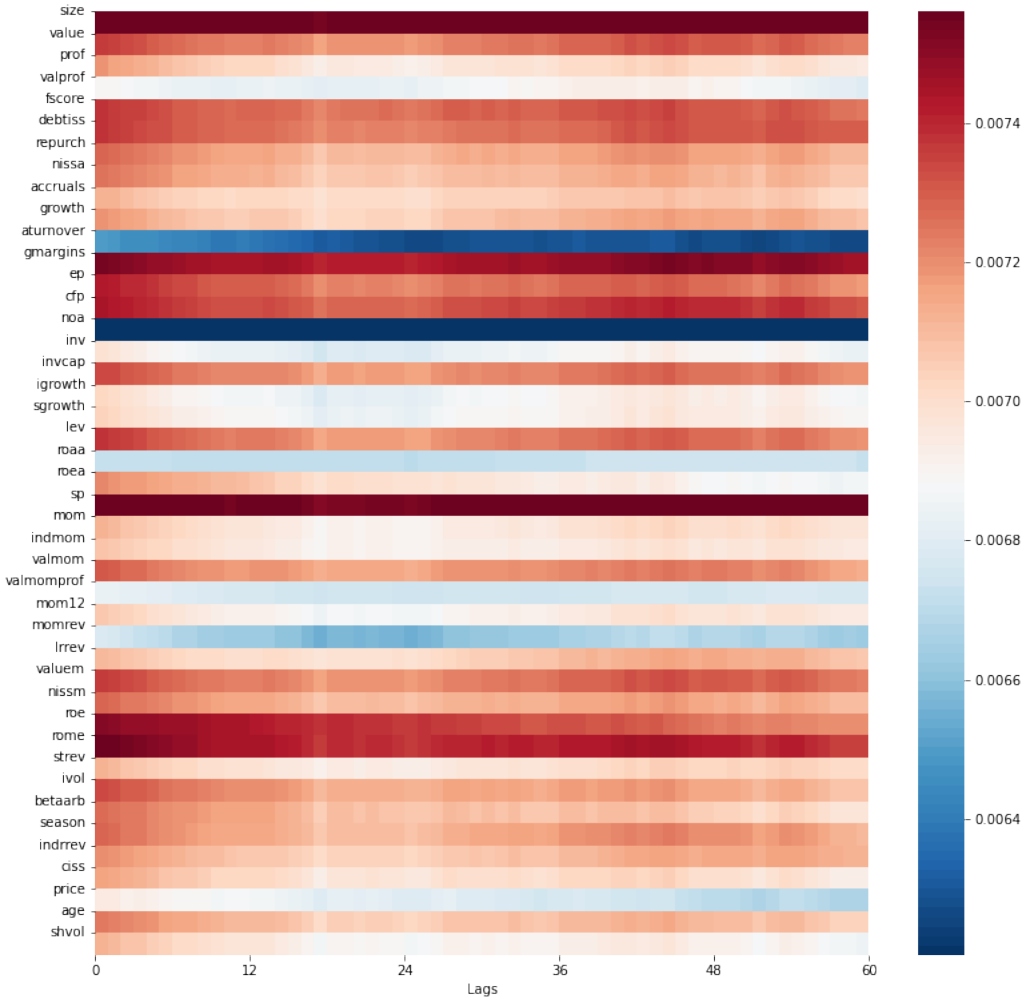

Figure 30: Term structure of CAPM alpha with rank 3 (OOS); Window size: 60 months

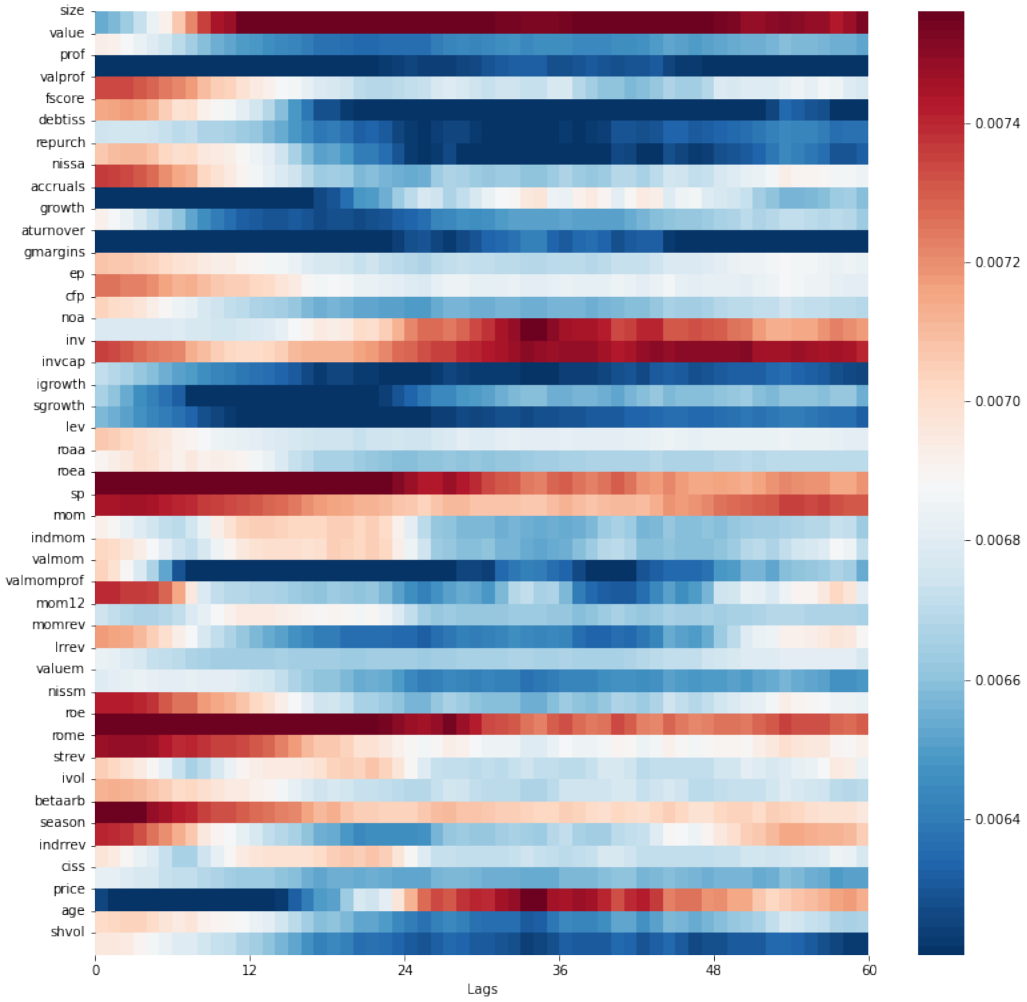

Figure 31: Term structure of CAPM alpha with rank 5 (OOS); Window size: 60 months

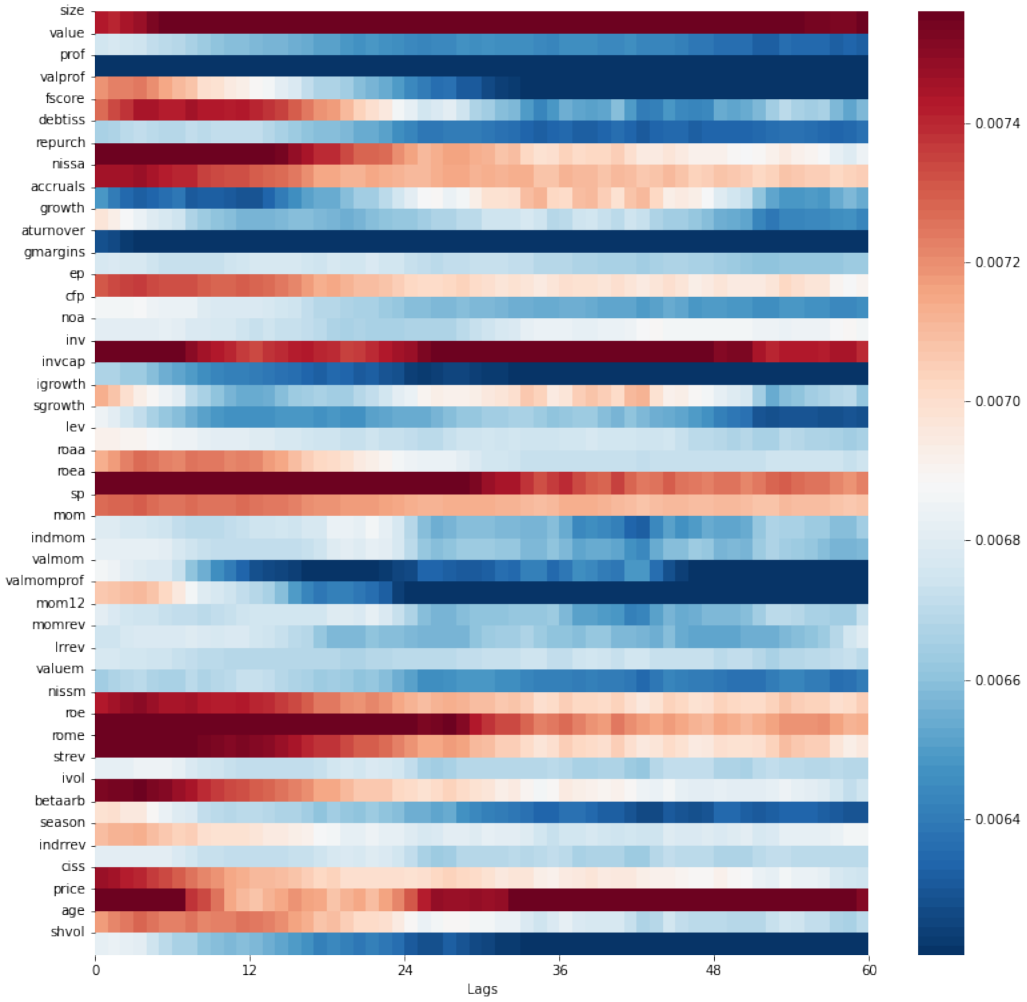

Figure 32: Term structure of CAPM alpha with rank 10 (OOS); Window size: 60 months

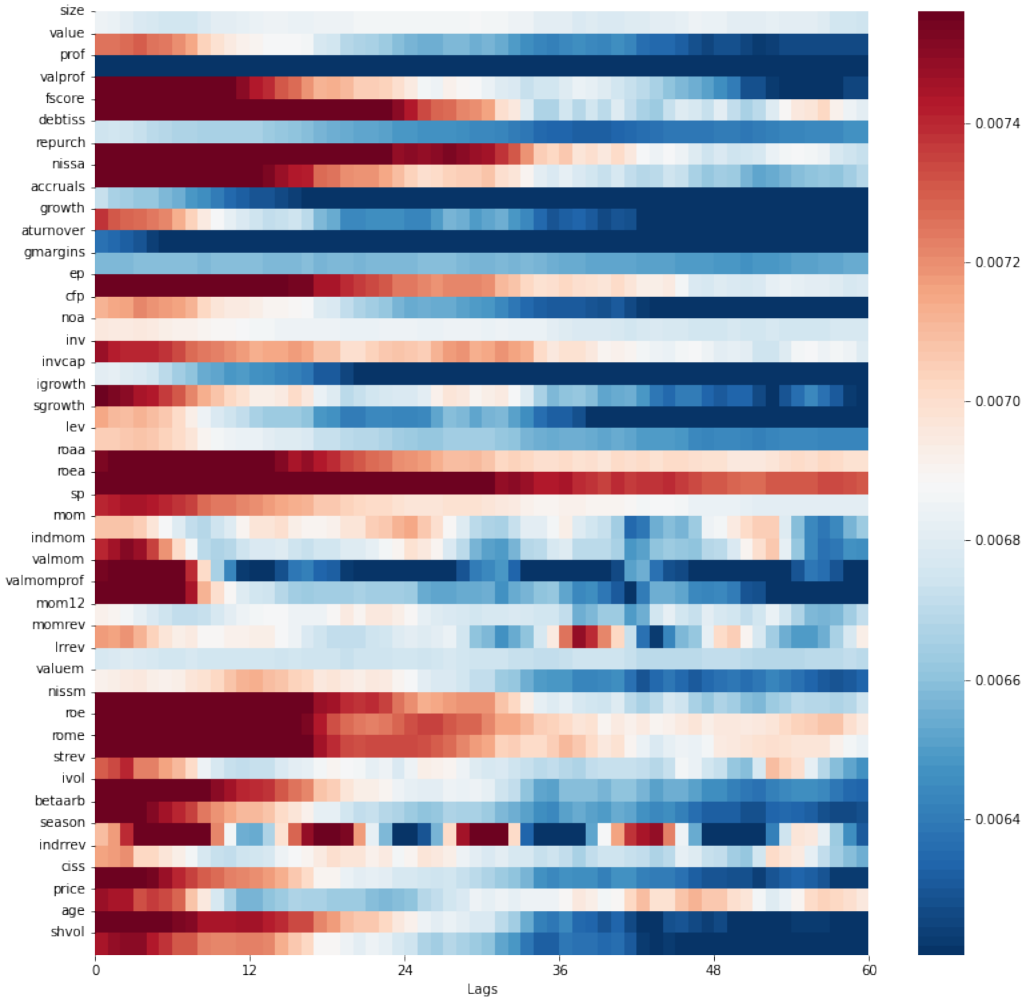

Figure 33: Term structure of CAPM alpha with rank 20 (OOS); Window size: 60 months

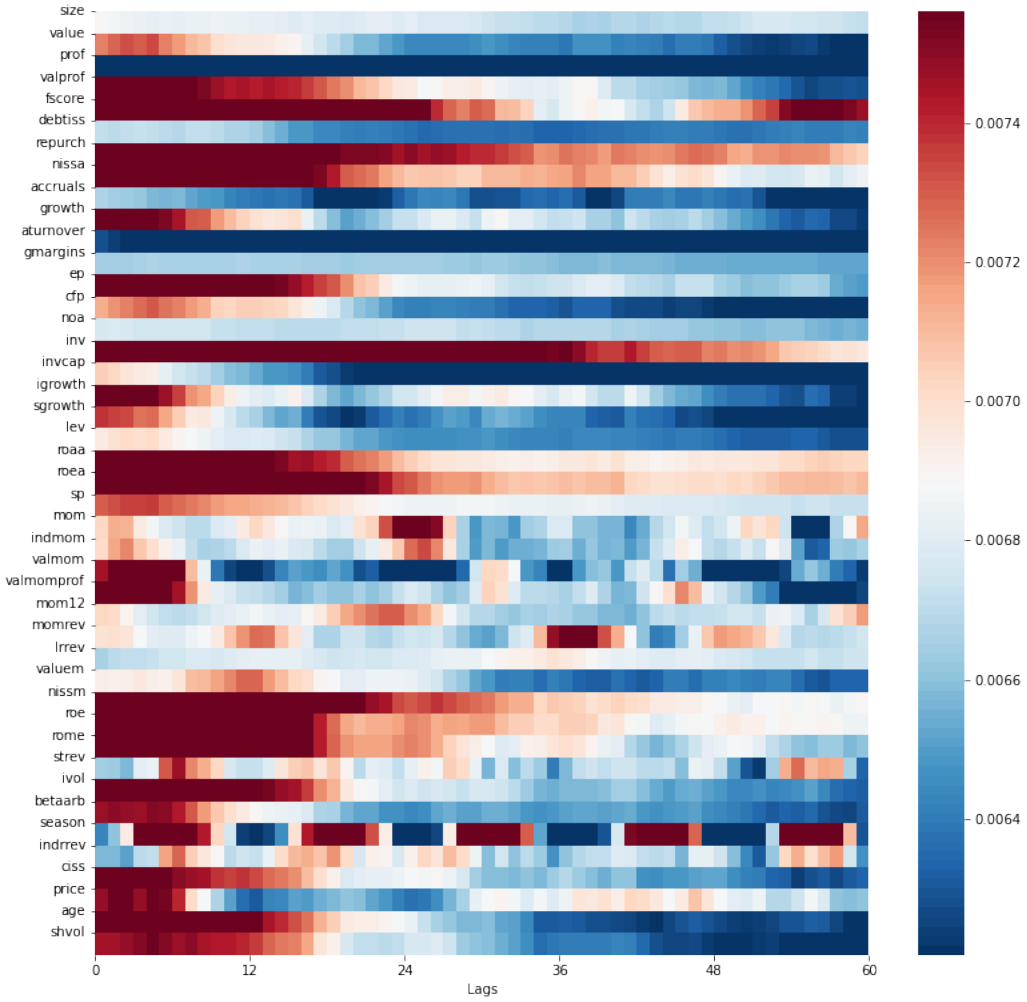

Figure 34: Time pattern

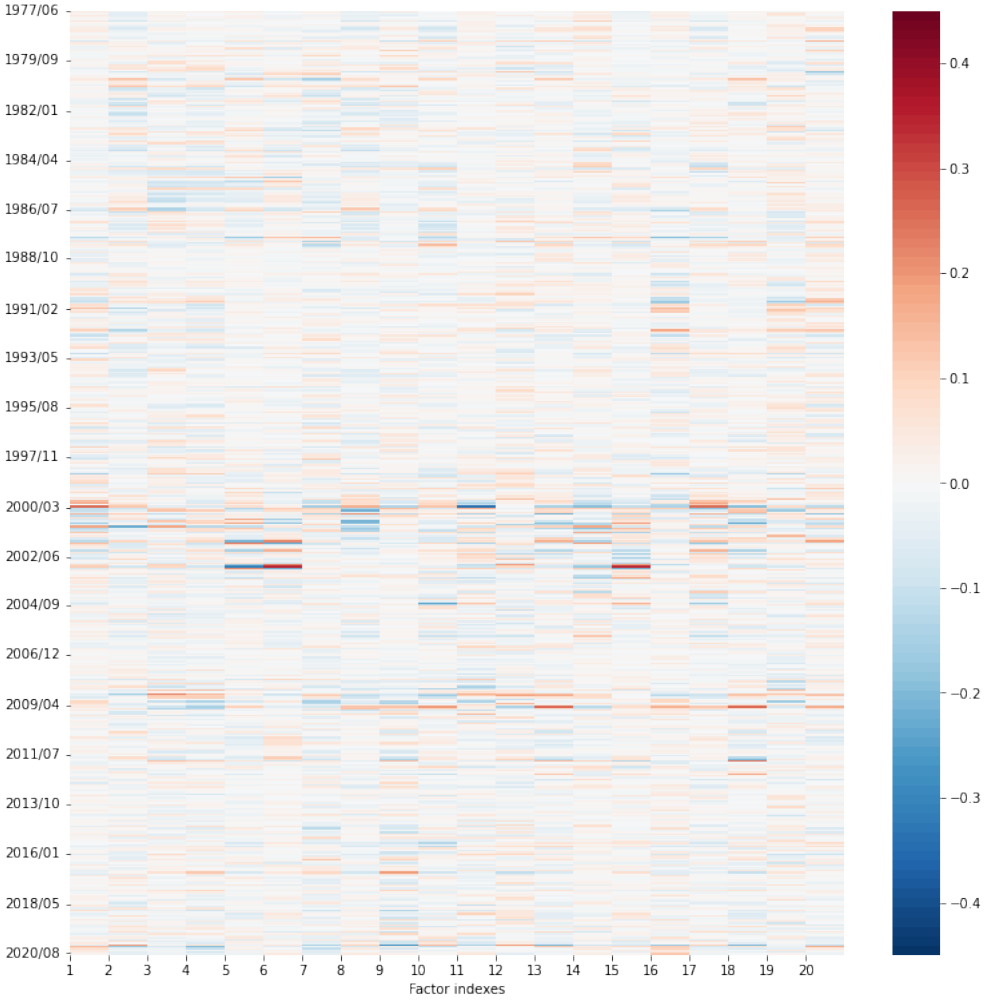

Figure 35: Cumulative excess returns of factors

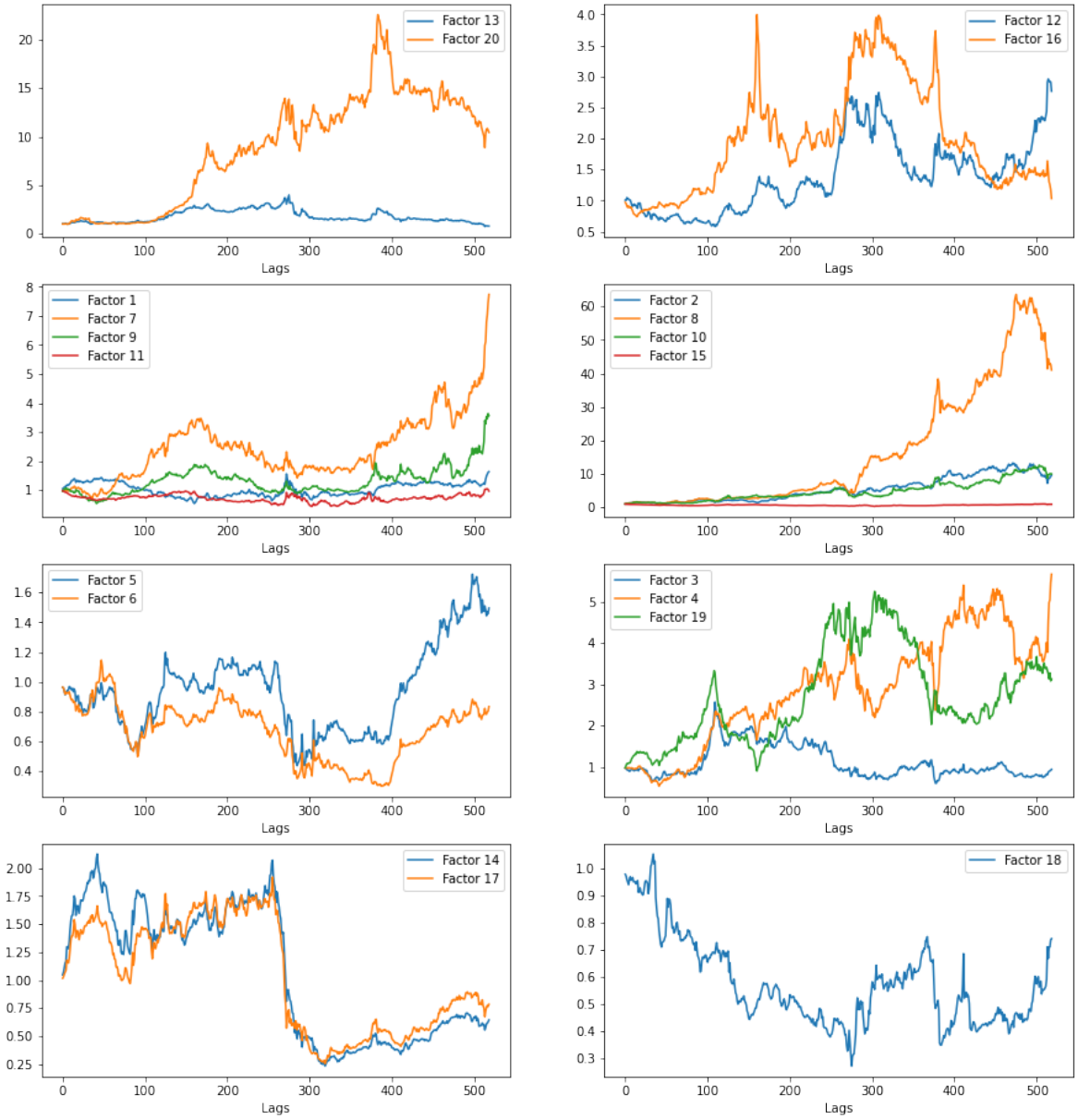

Figure 36: Portfolio pattern

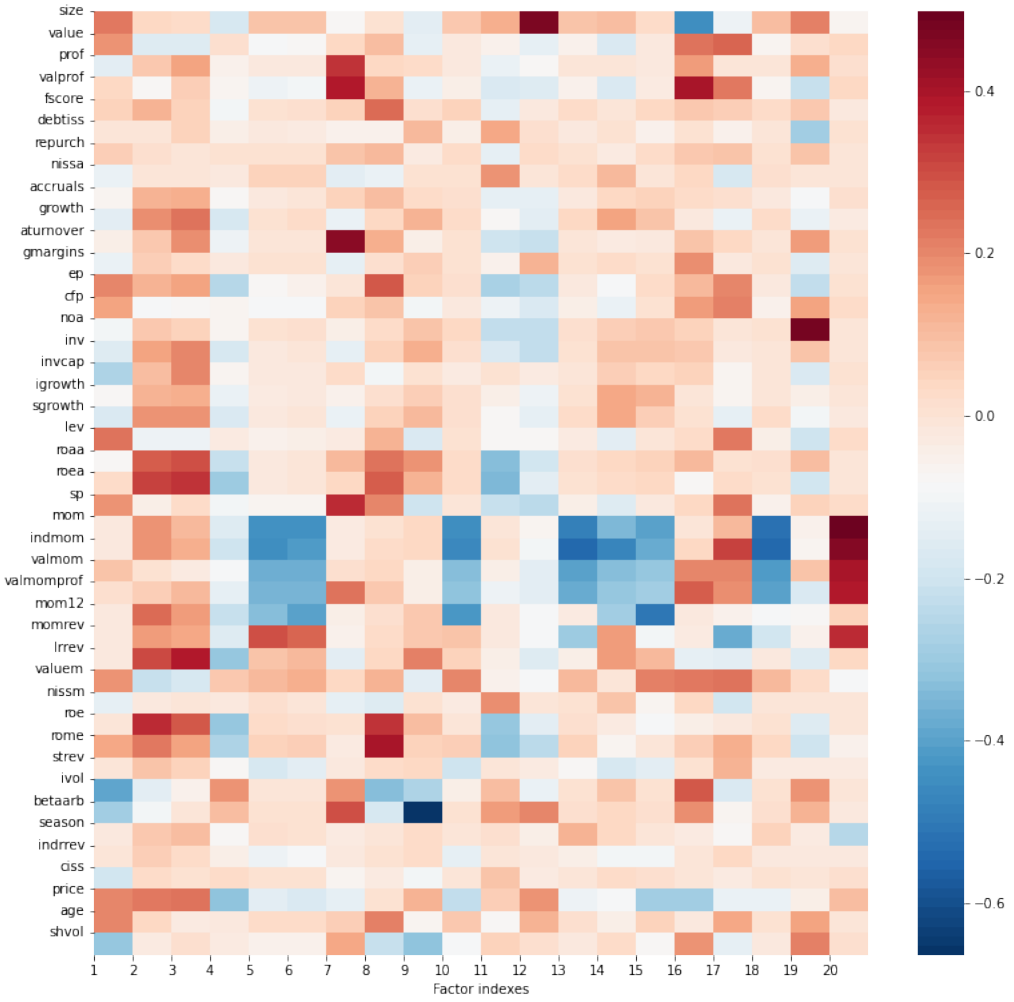

Figure 37: Lag pattern

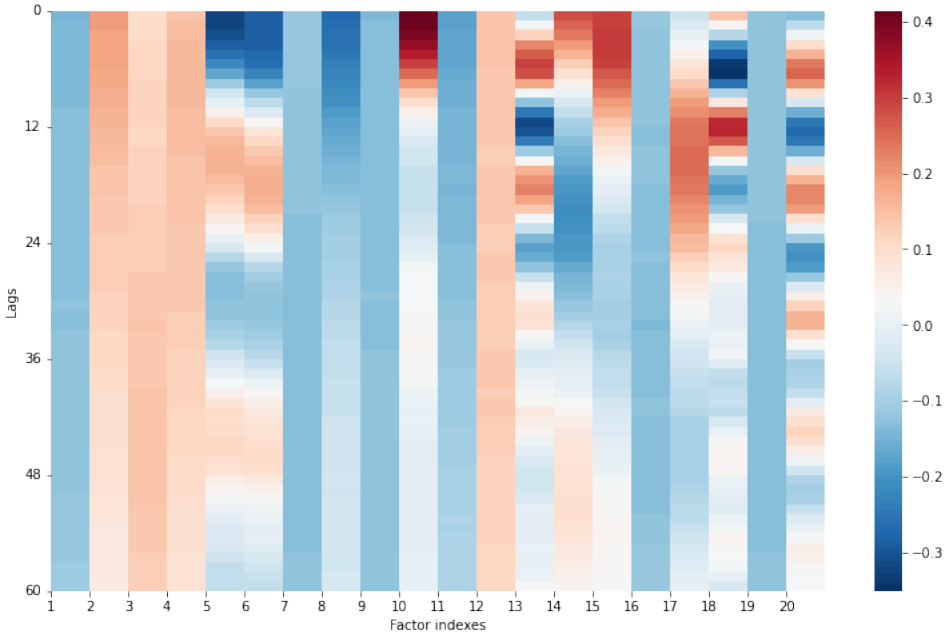

Figure 38: Lag pattern (line plot)

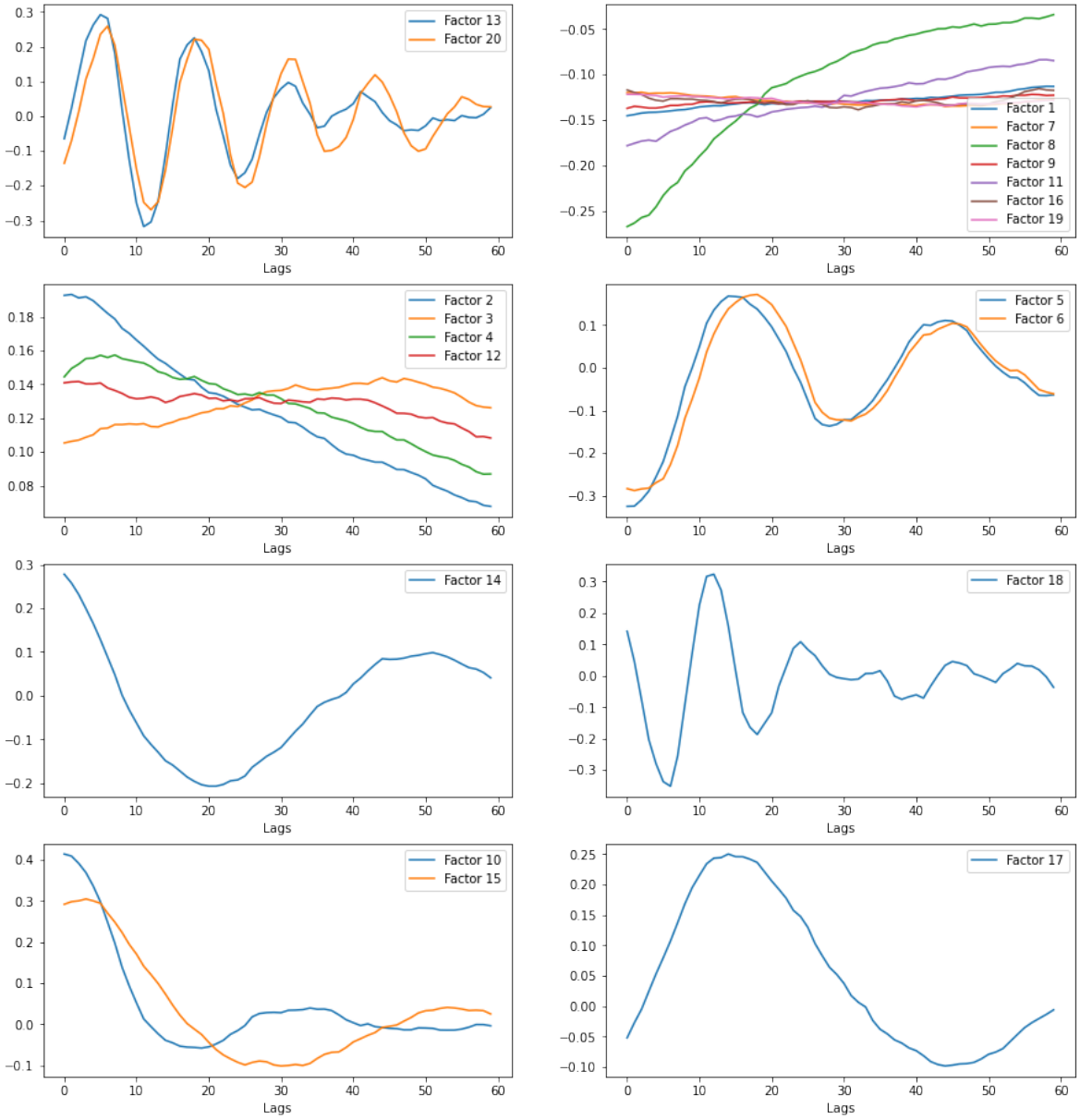

Figure 39: Reconstruction error with refit as a function of rank

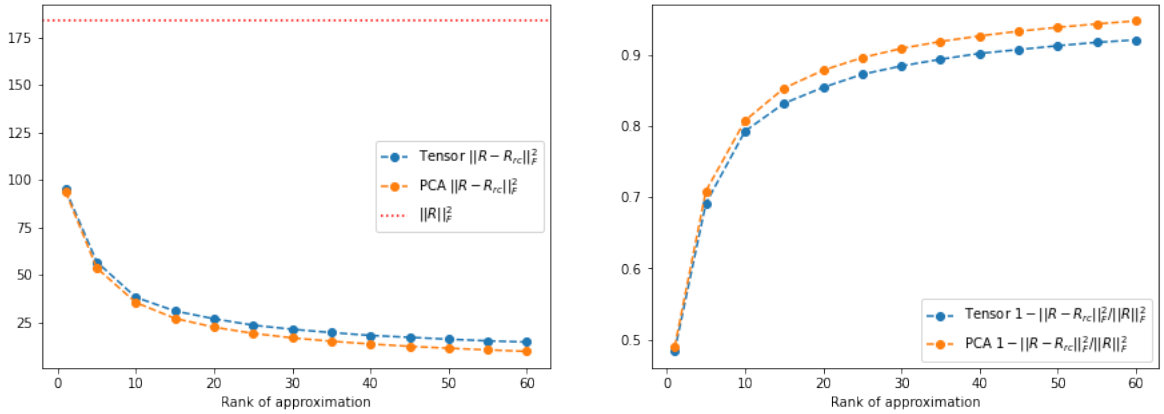

Figure 40: In-sample SR

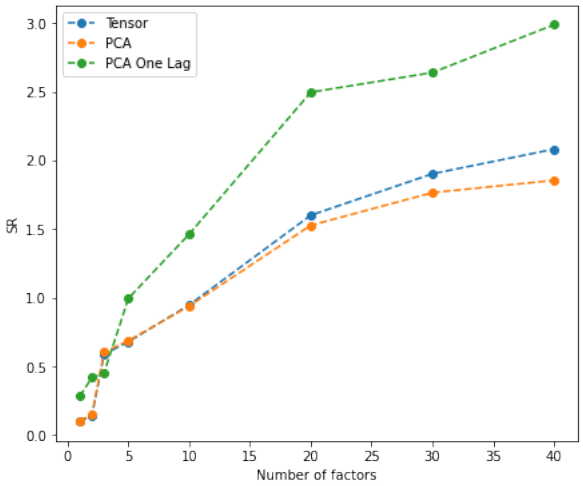

Figure 41: RMSE of time pattern fitting

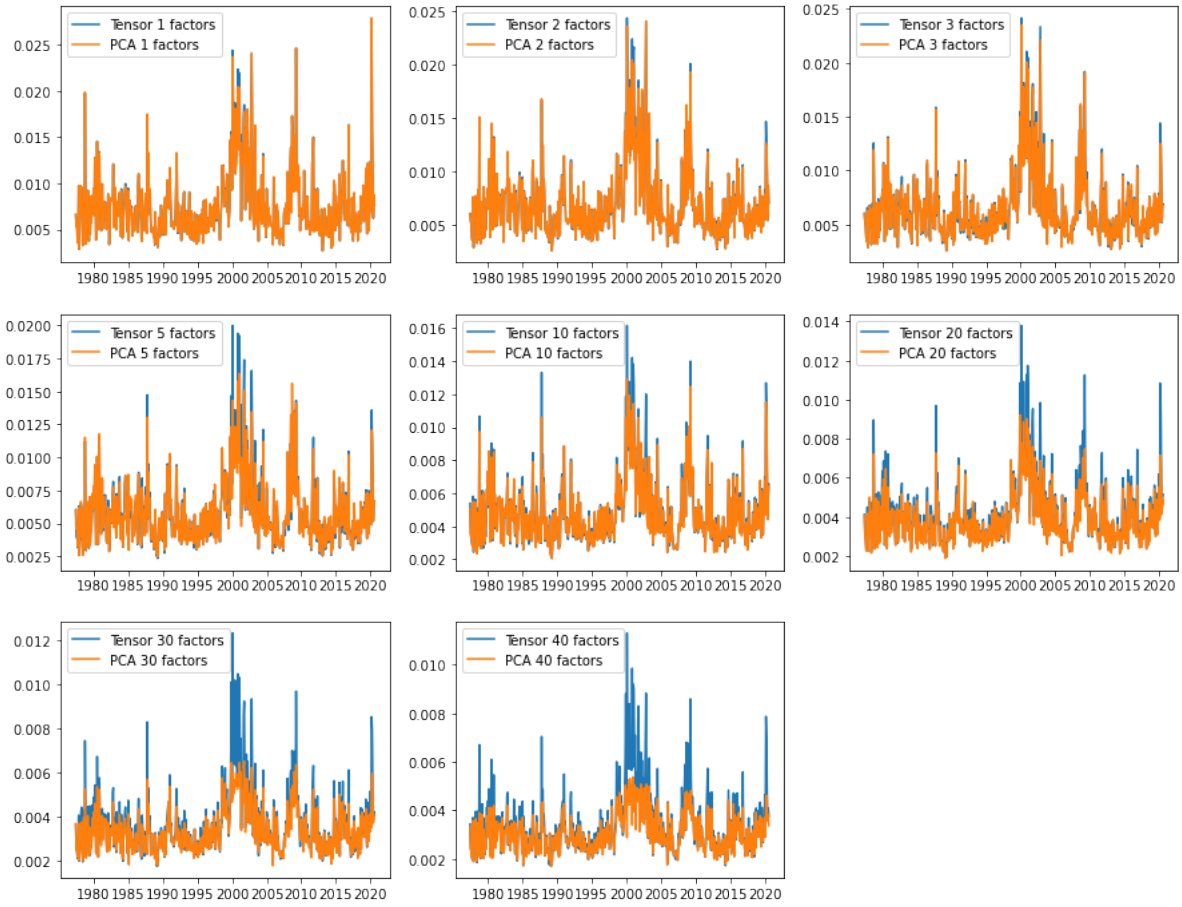

Figure 42: RMSE of lag pattern fitting

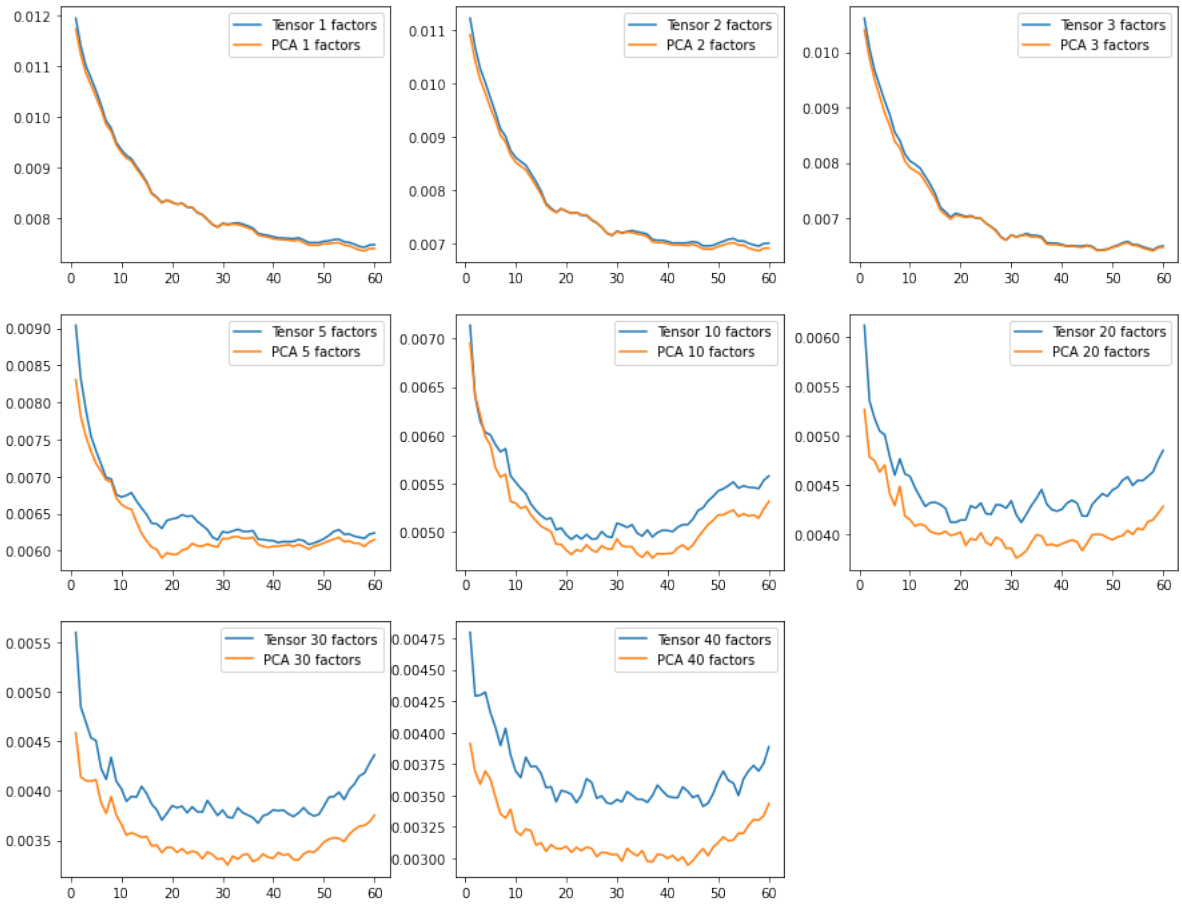

Figure 43: RMSE of portfolio pattern fitting

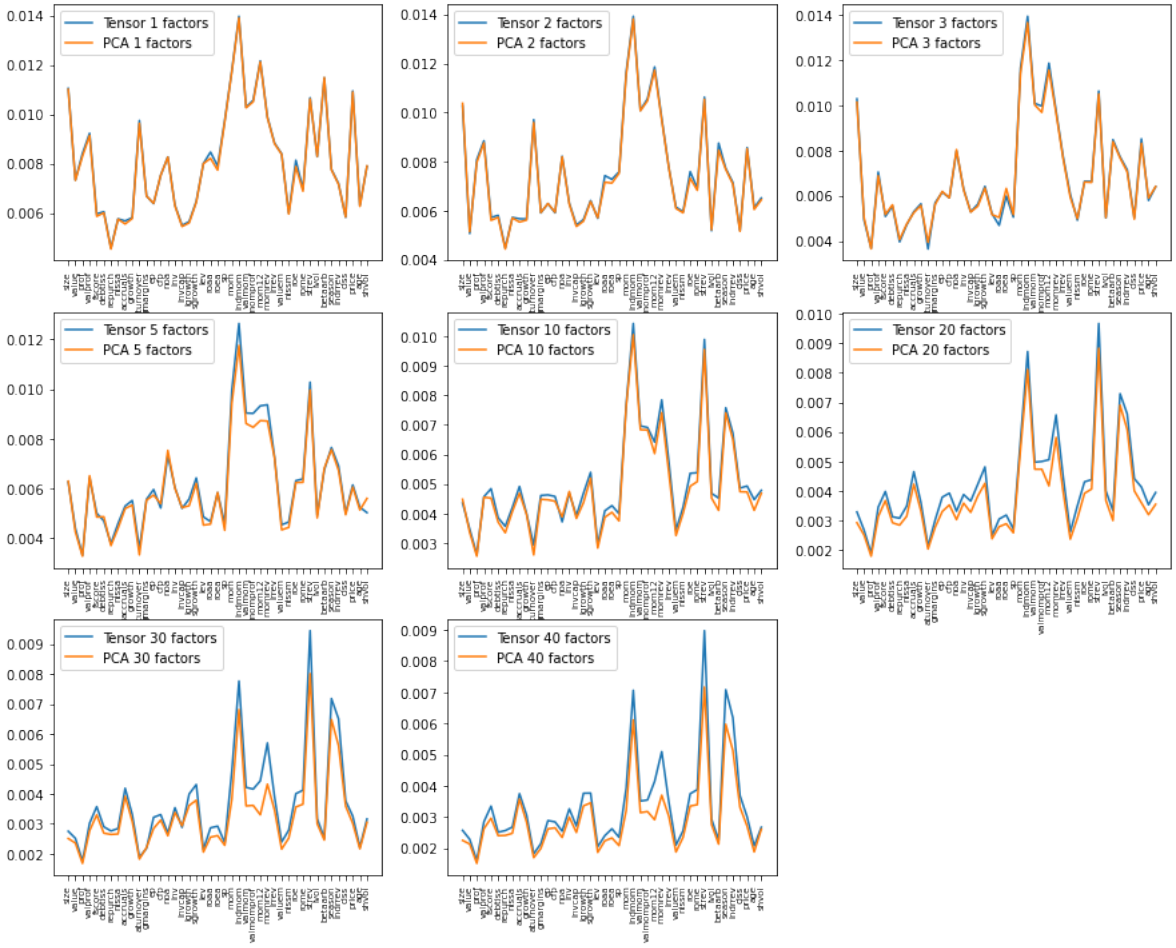

Figure 44: Normalized averaged alpha

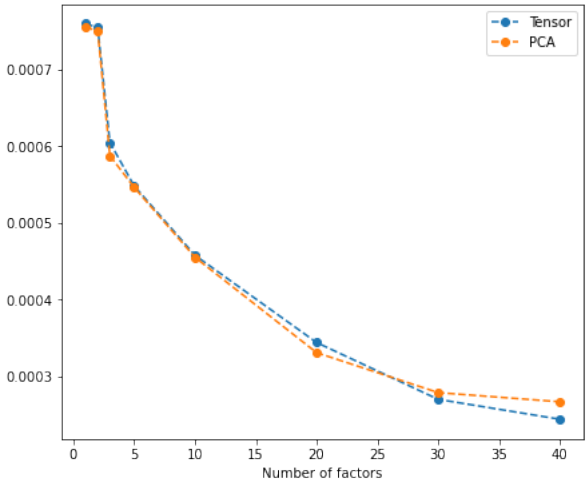

Figure 45: Cross-sectional averaged alpha for portfolios; Tensor

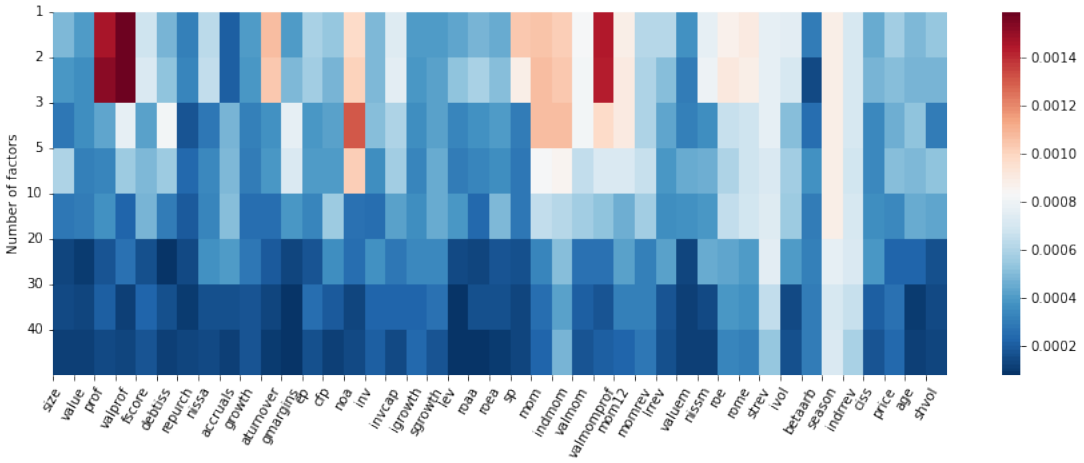

Figure 46: Cross-sectional averaged alpha for portfolios; PCA

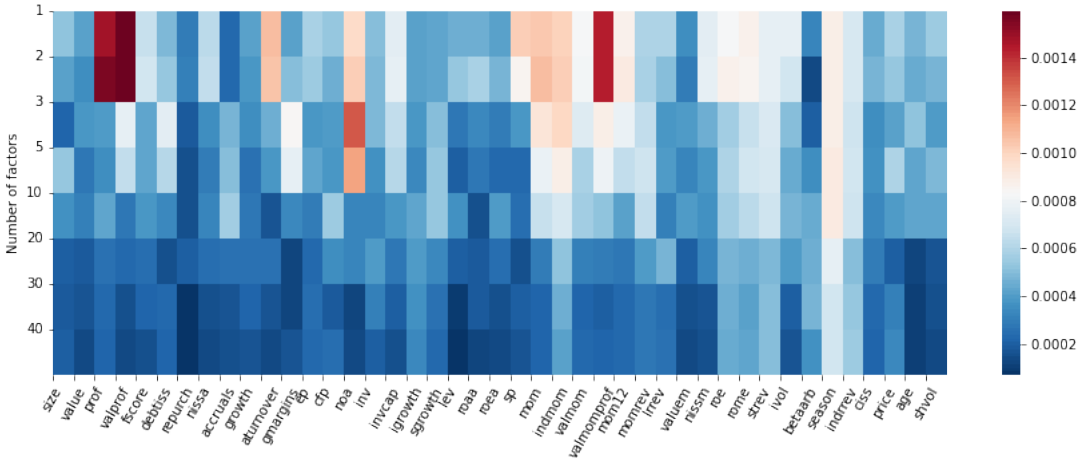

Figure 47: Averaged alpha for the lag dimension; Tensor

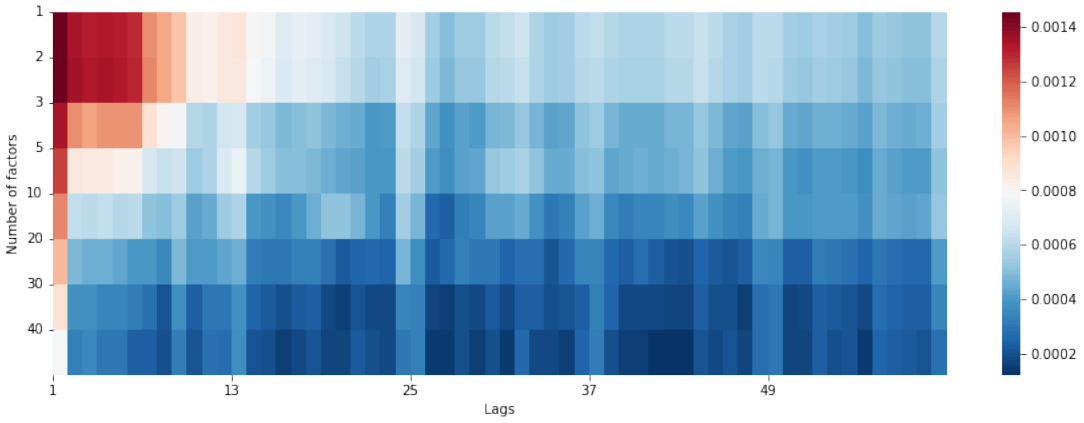

Figure 48: Averaged alpha for the lag dimension; PCA

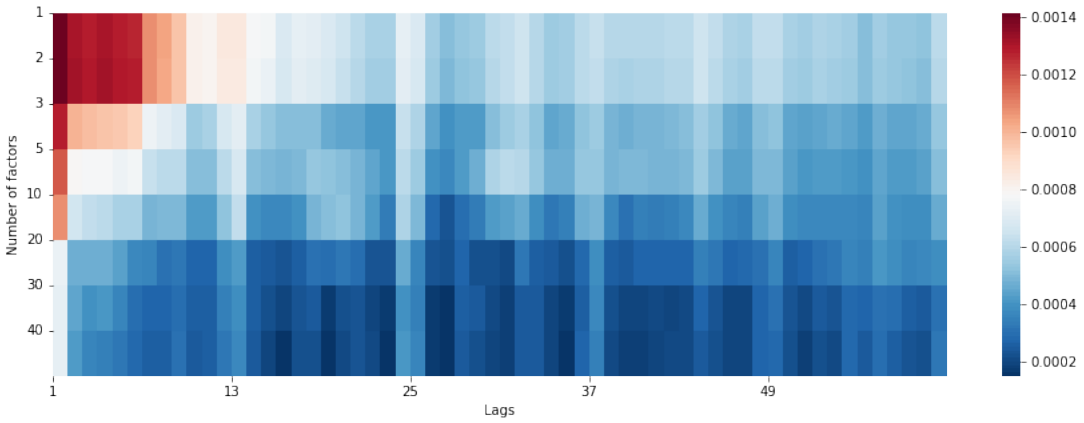

Figure 49: Normalized unexplained variance σ_ϵ

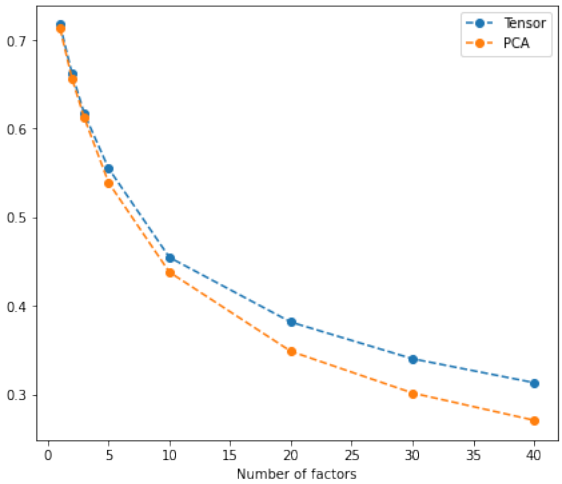

Figure 50: Term structure of mean returns

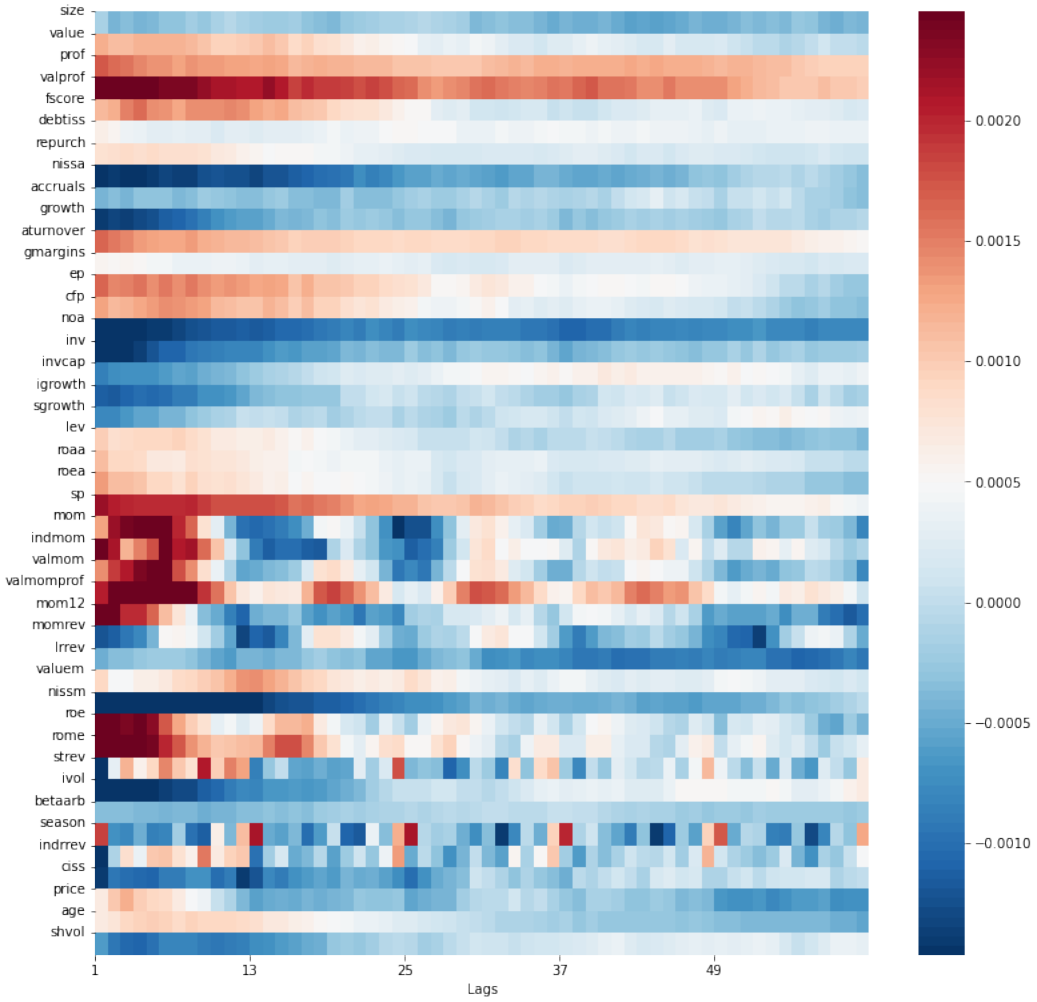

Figure 51: Term structure of fitted mean returns with rank 1

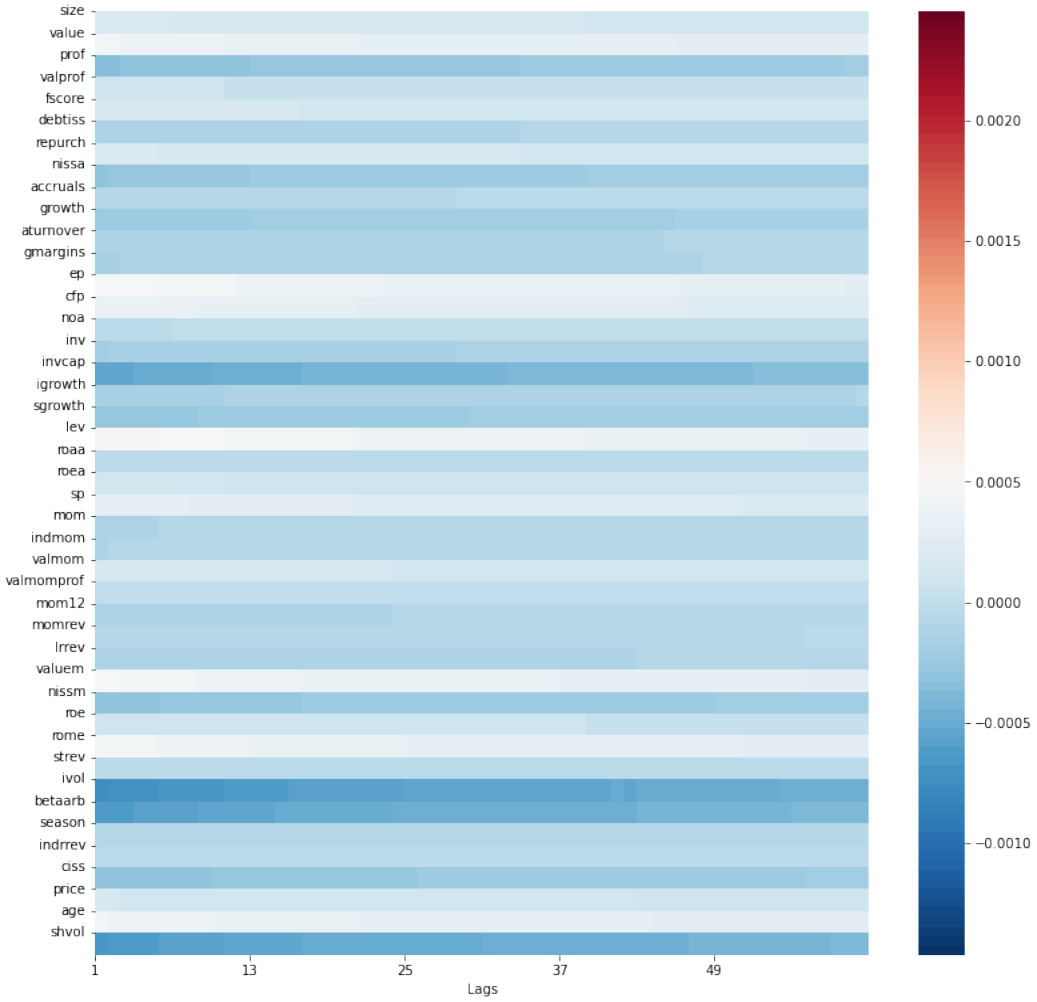

Figure 52: Term structure of fitted mean returns with rank 3

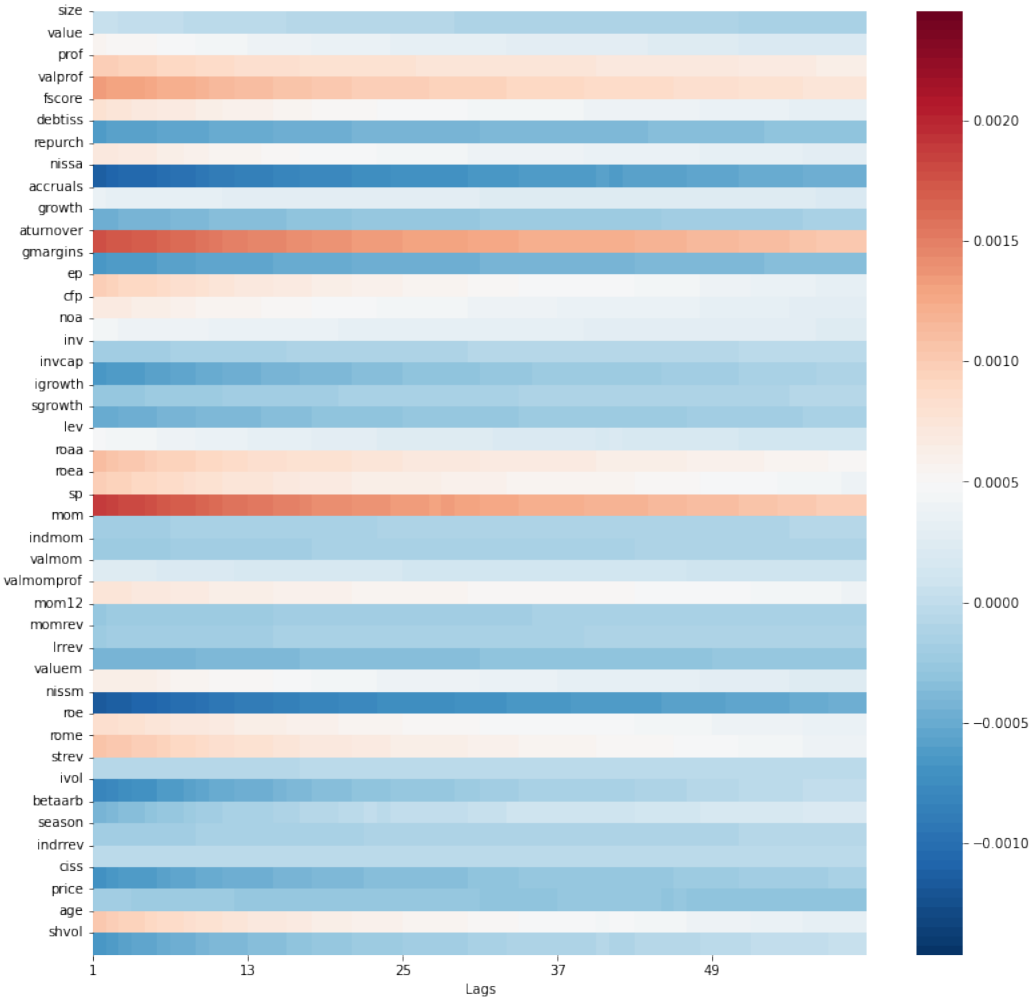

Figure 53: Term structure of fitted mean returns with rank 5

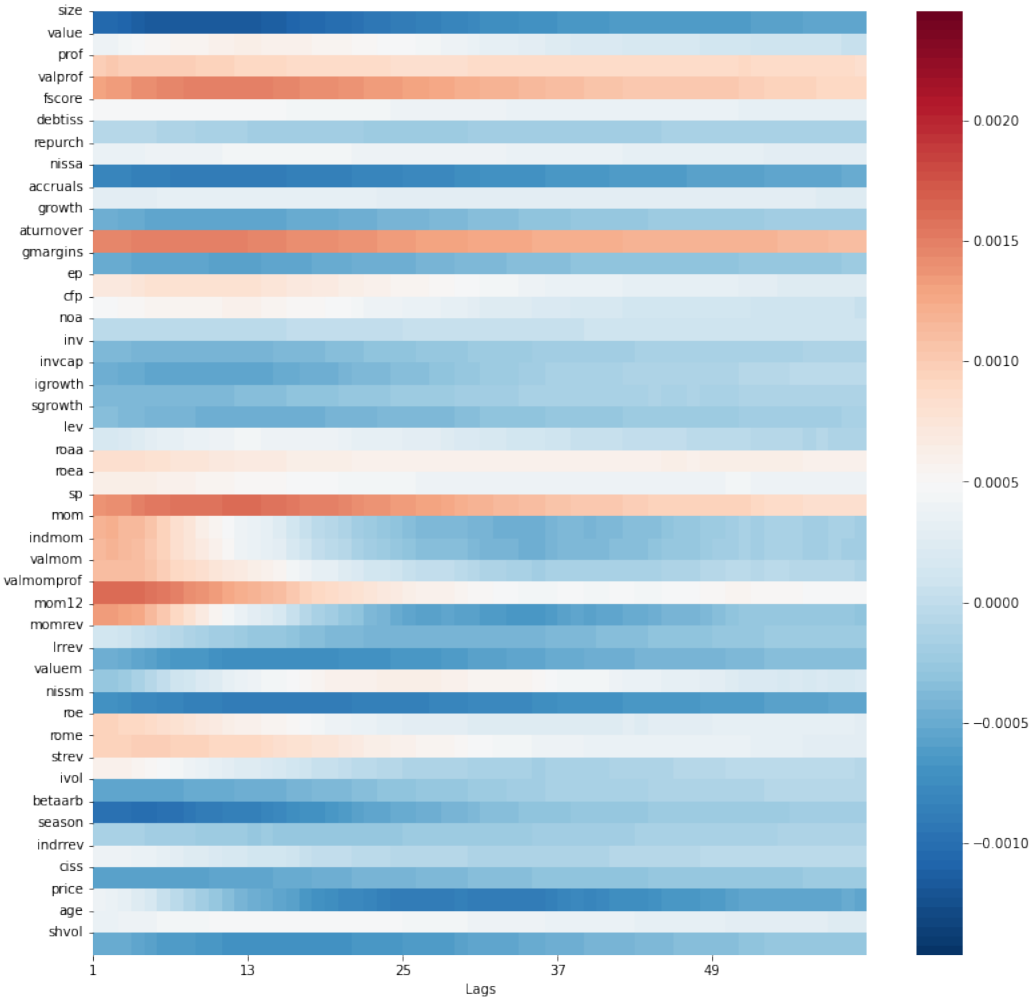

Figure 54: Term structure of fitted mean returns with rank 10

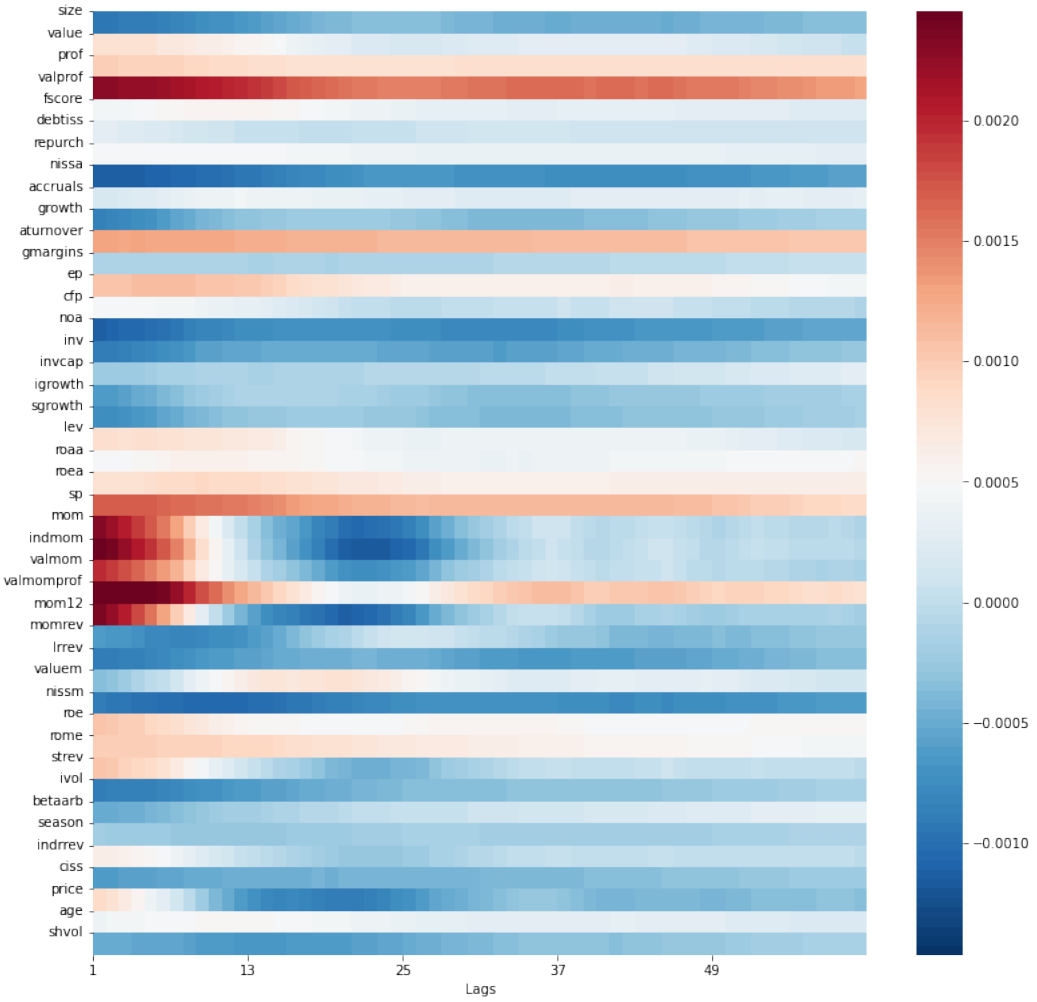

Figure 55: Term structure of fitted mean returns with rank 20

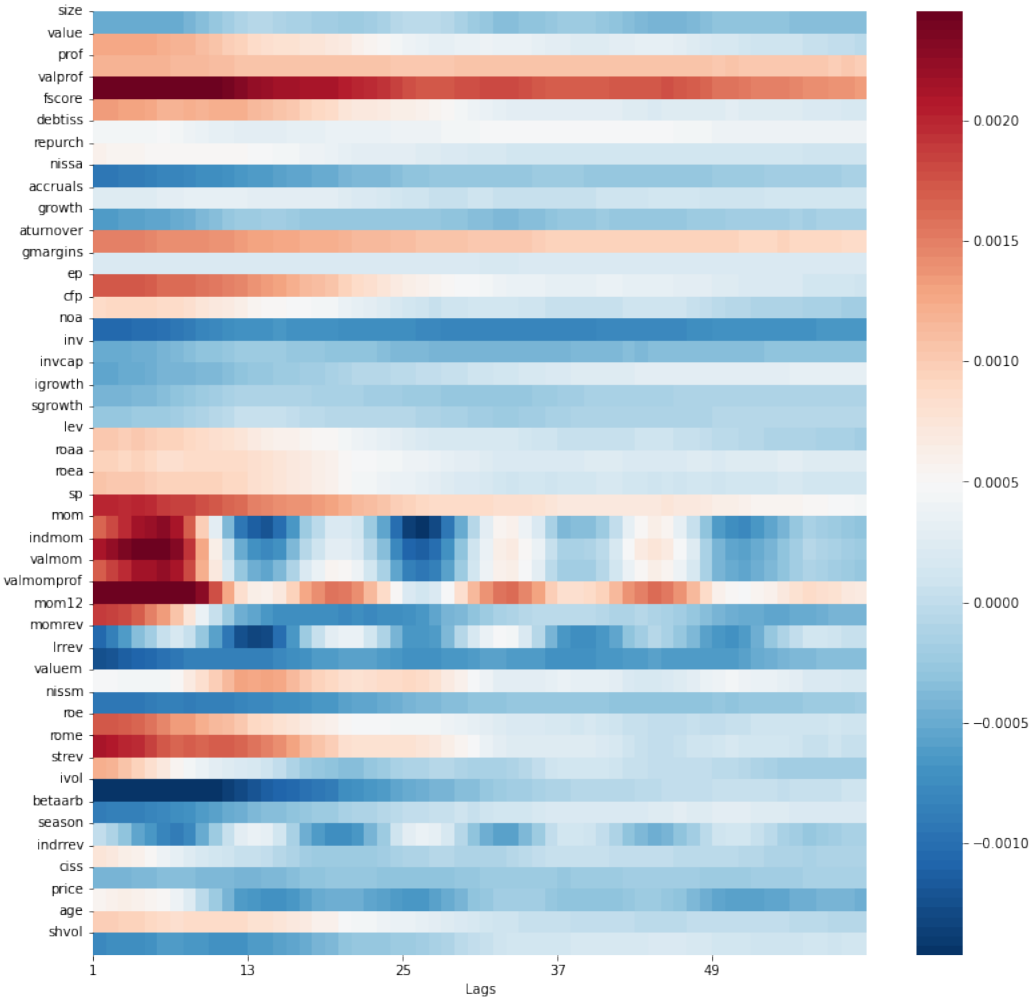

Figure 56: Term structure of alpha with rank 1

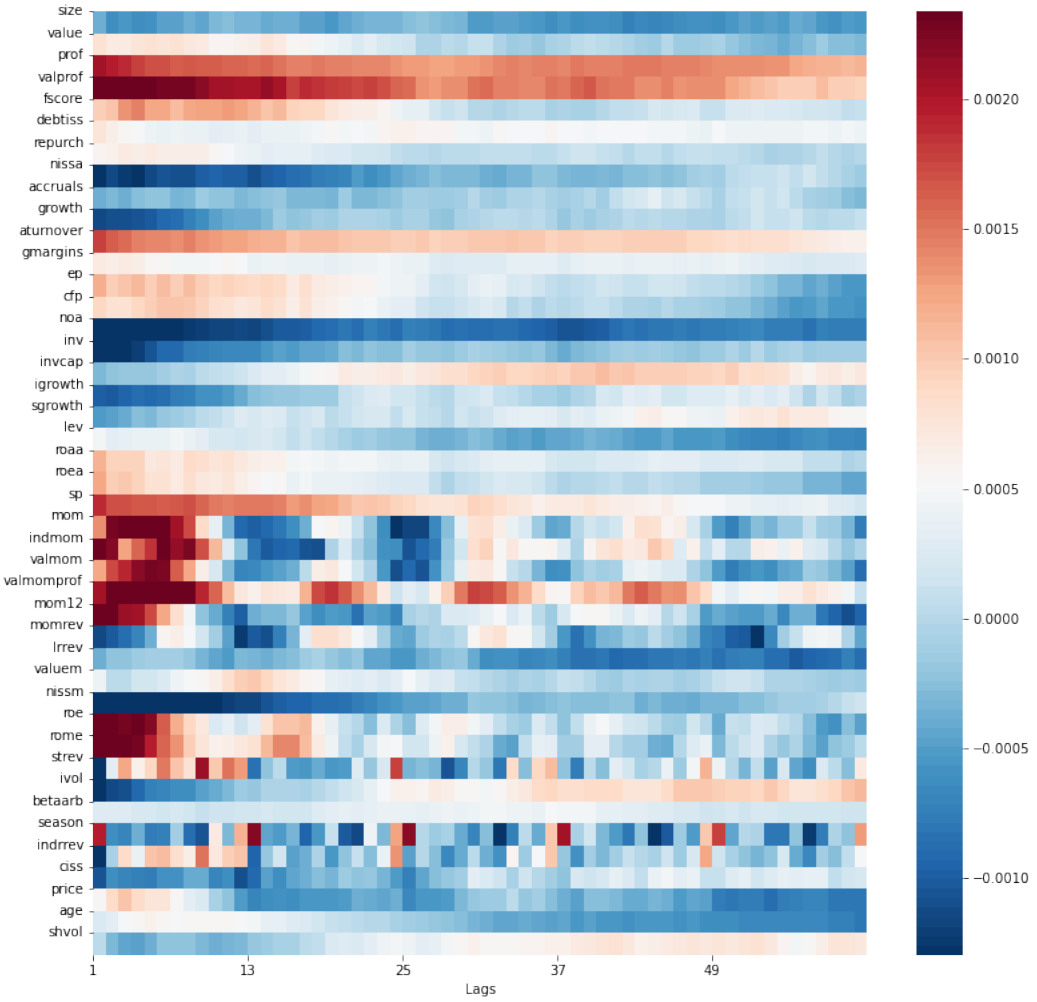

Figure 57: Term structure of alpha with rank 3

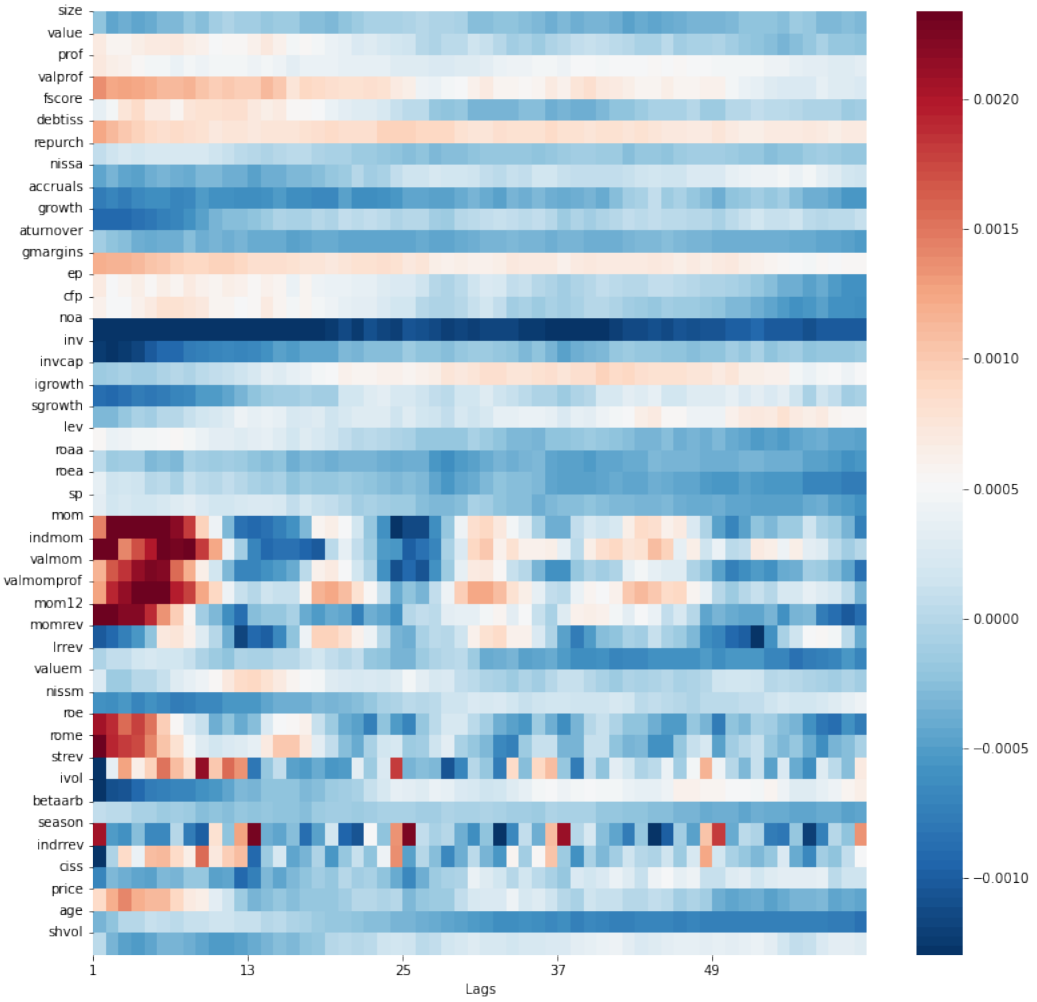

Figure 58: Term structure of alpha with rank 5

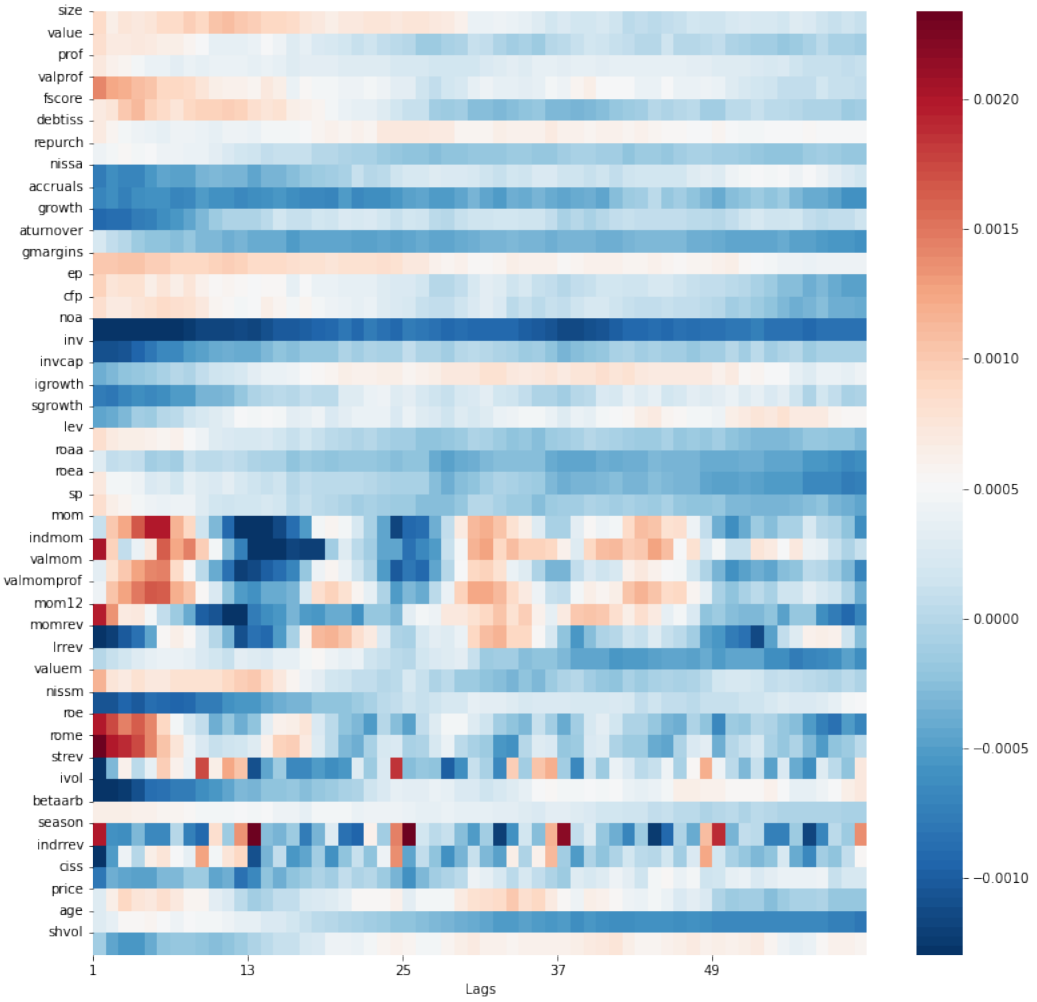

Figure 59: Term structure of alpha with rank 10

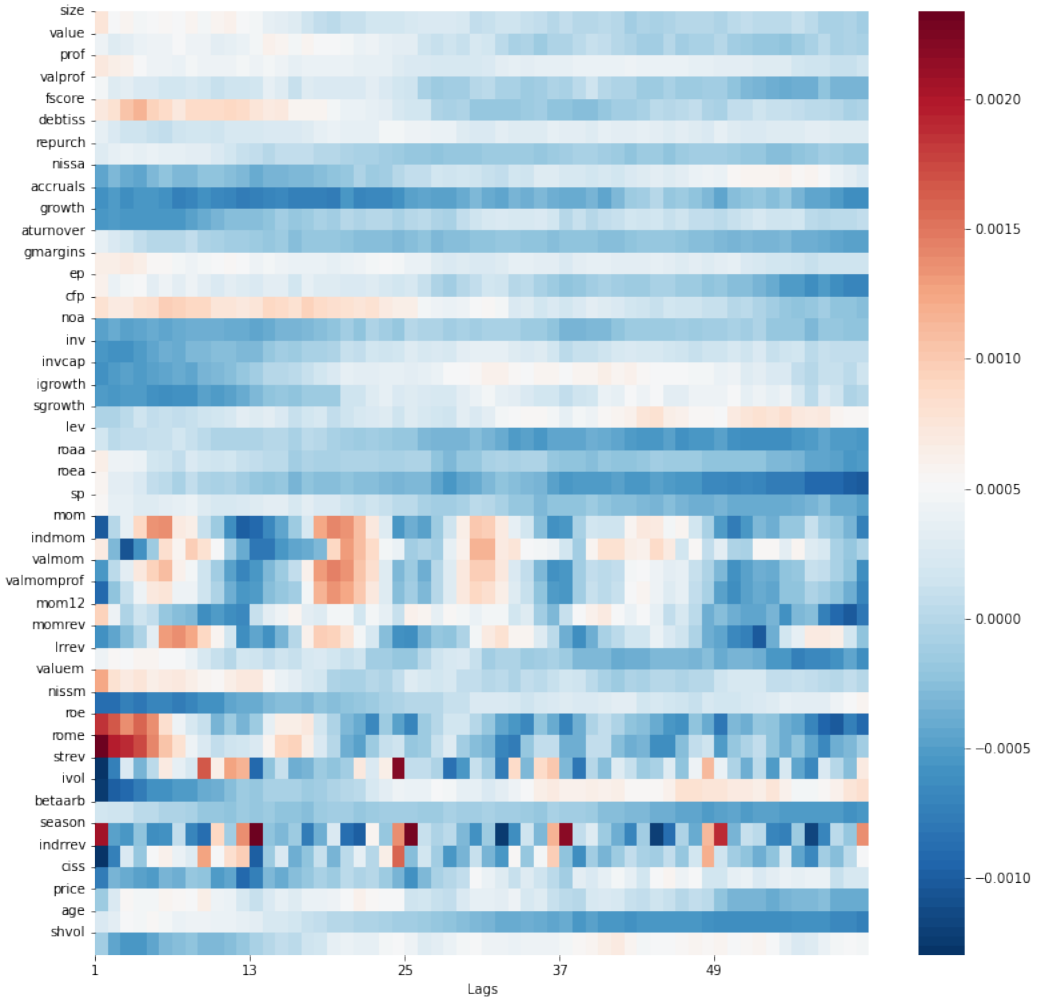

Figure 60: Term structure of alpha with rank 20

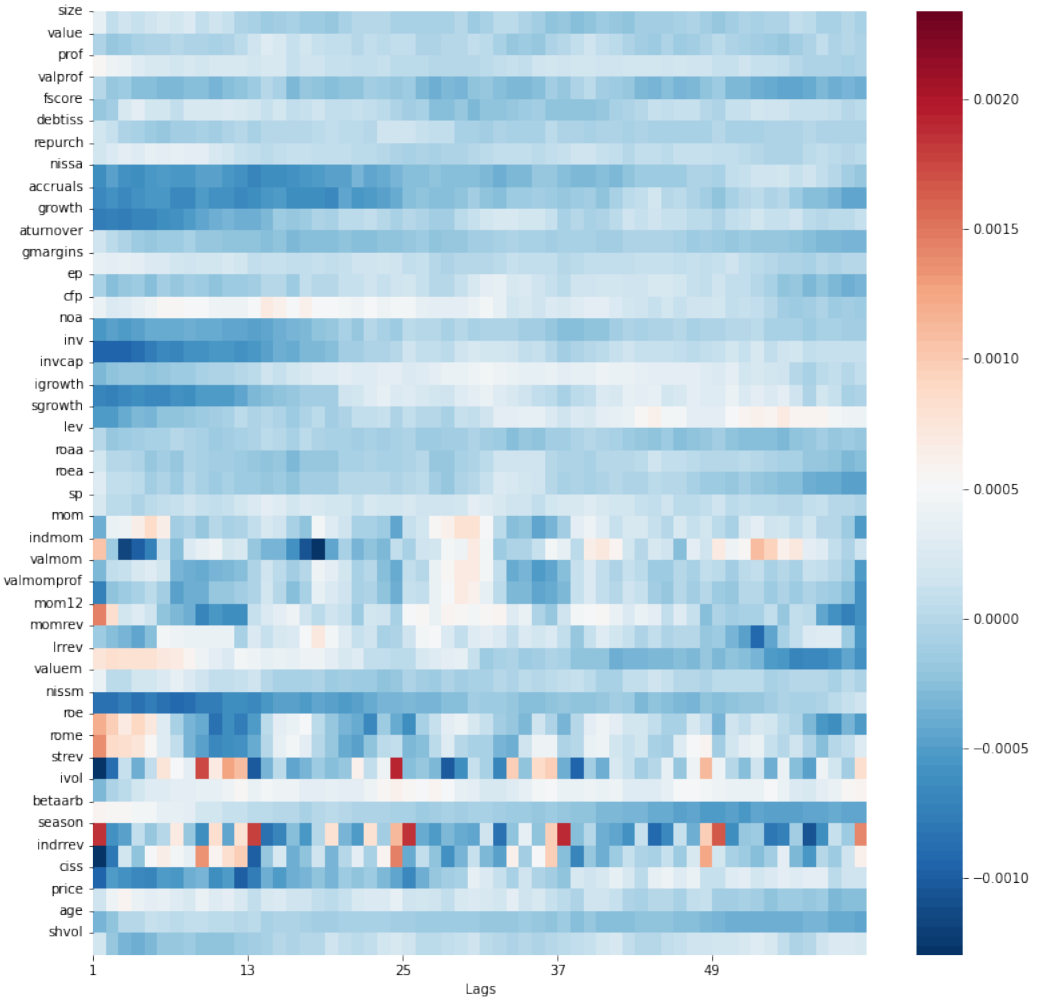

Figure 61: Term structure of alpha (stale) with rank 1

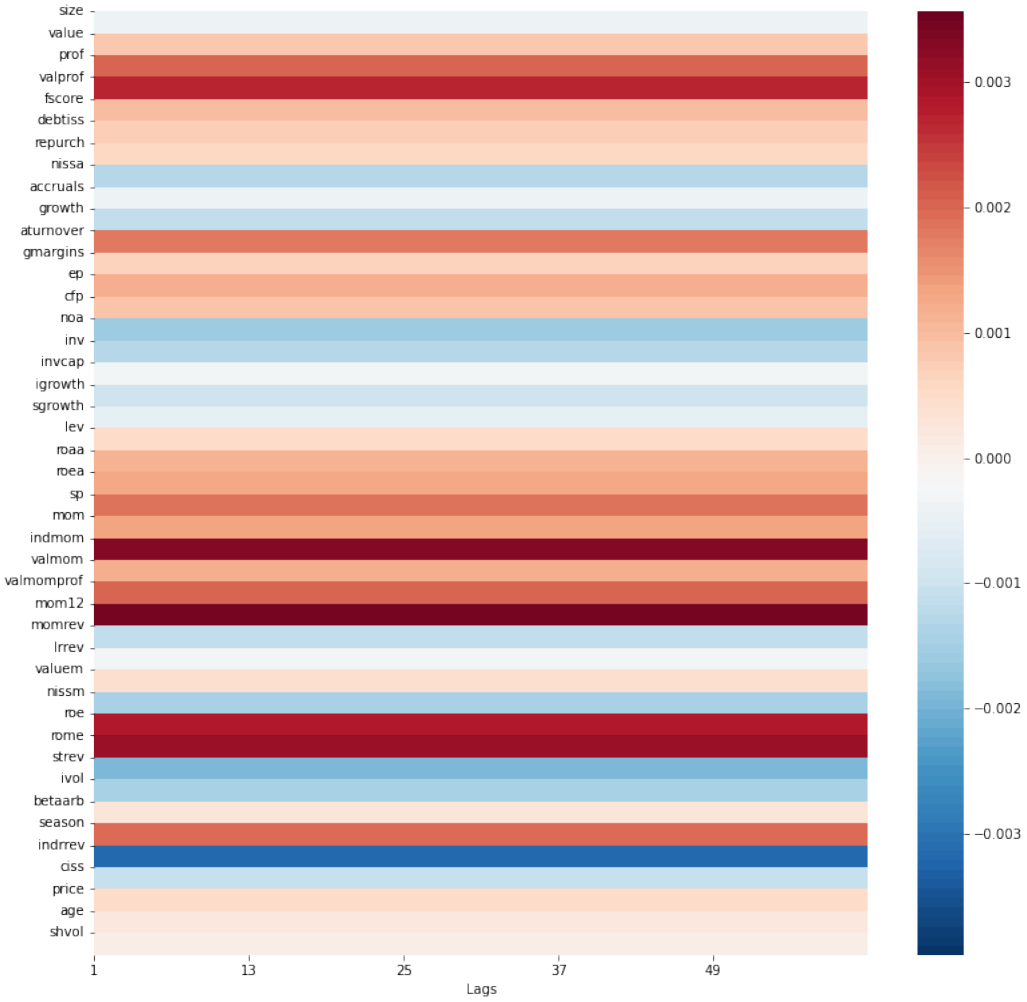

Figure 62: Term structure of alpha (stale) with rank 3

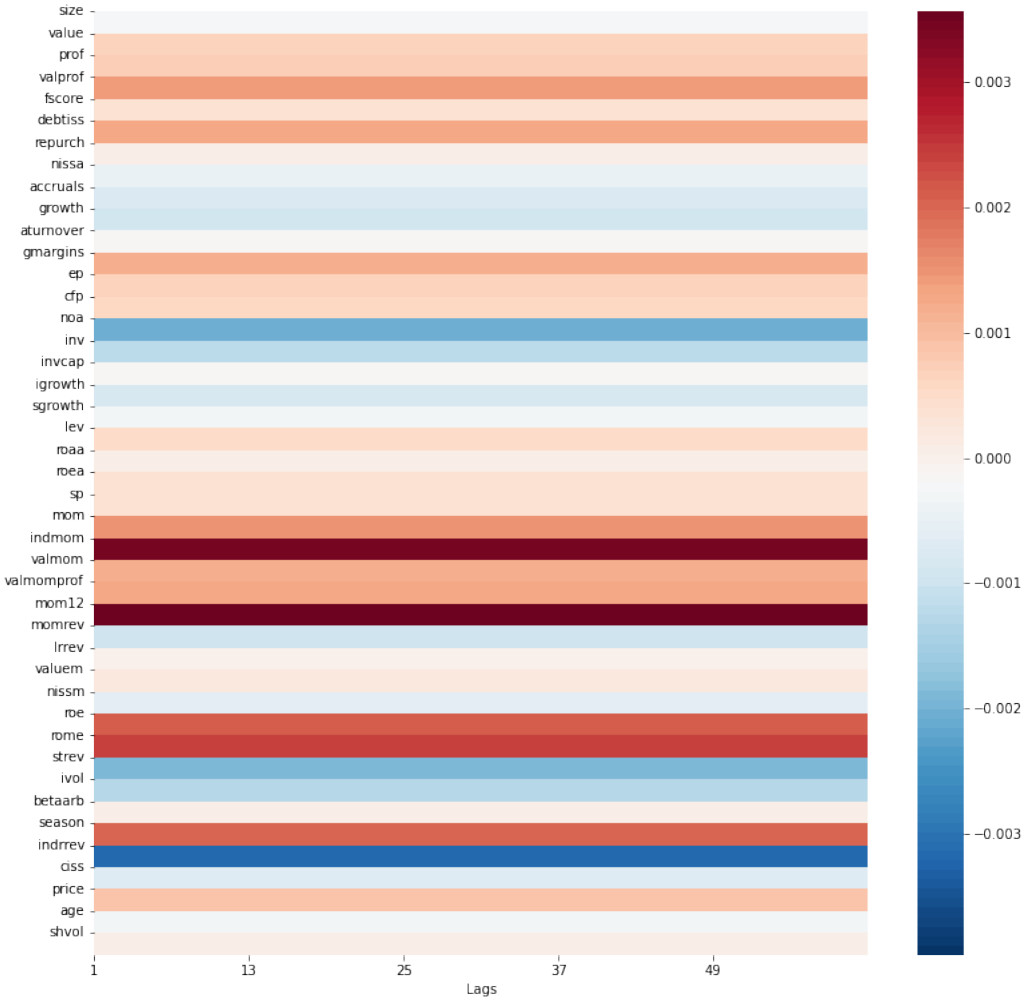

Figure 63: Term structure of alpha (stale) with rank 5

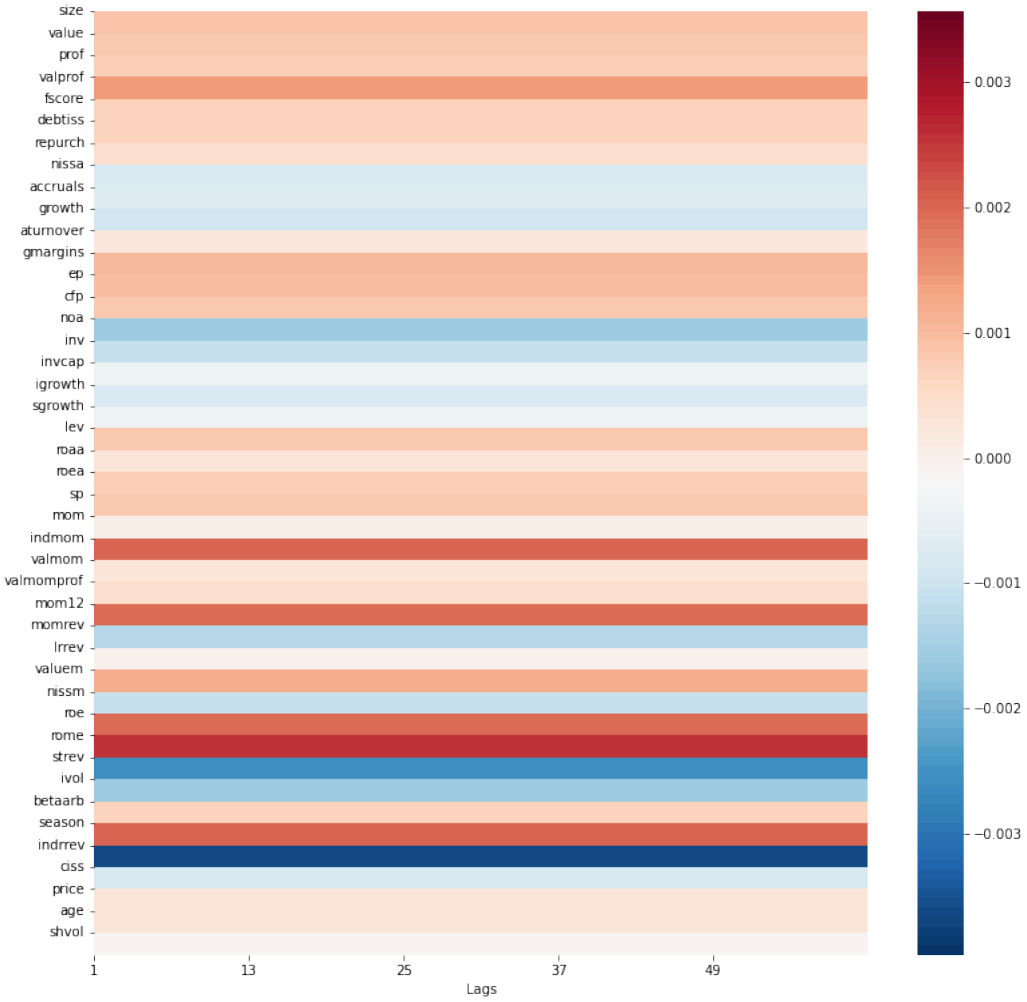

Figure 64: Term structure of alpha (stale) with rank 10

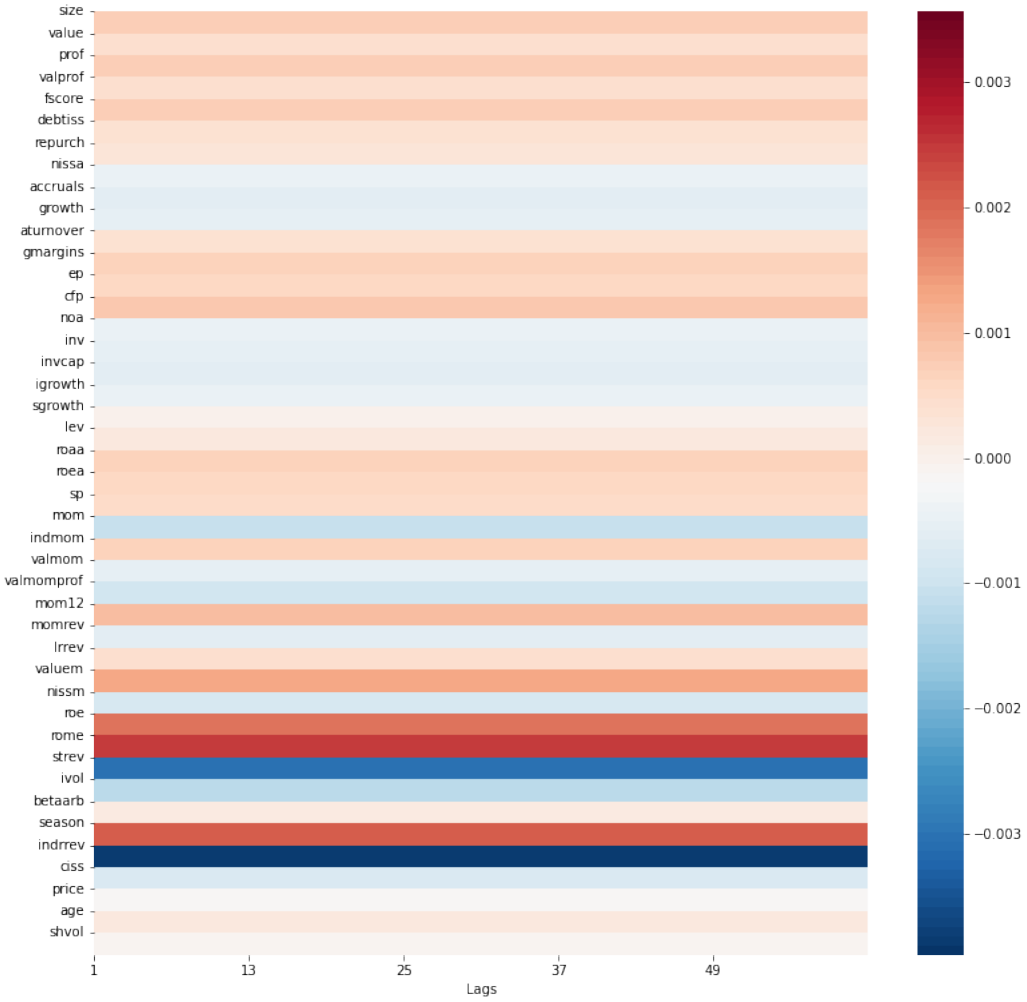

Figure 65: Term structure of alpha (stale) with rank 20

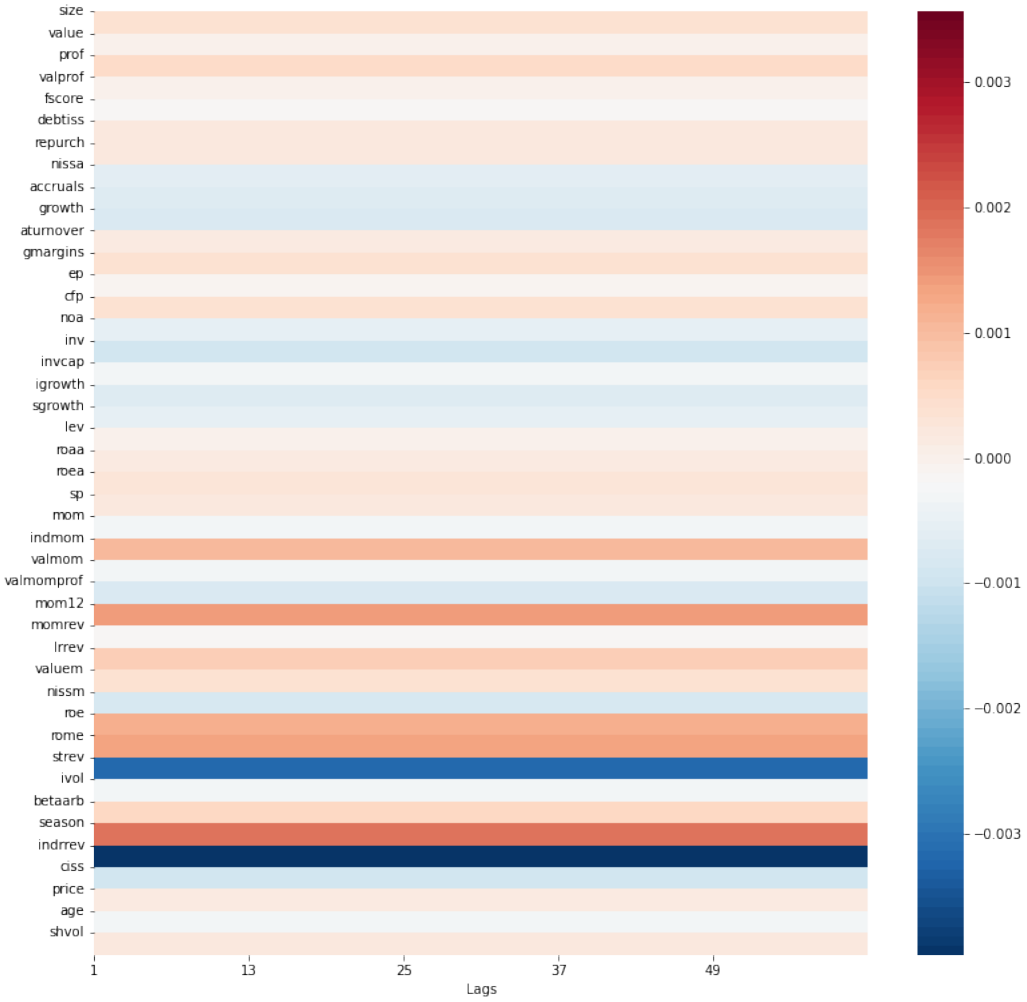

Figure 66: Term structure of alpha (dynamic) with rank 1

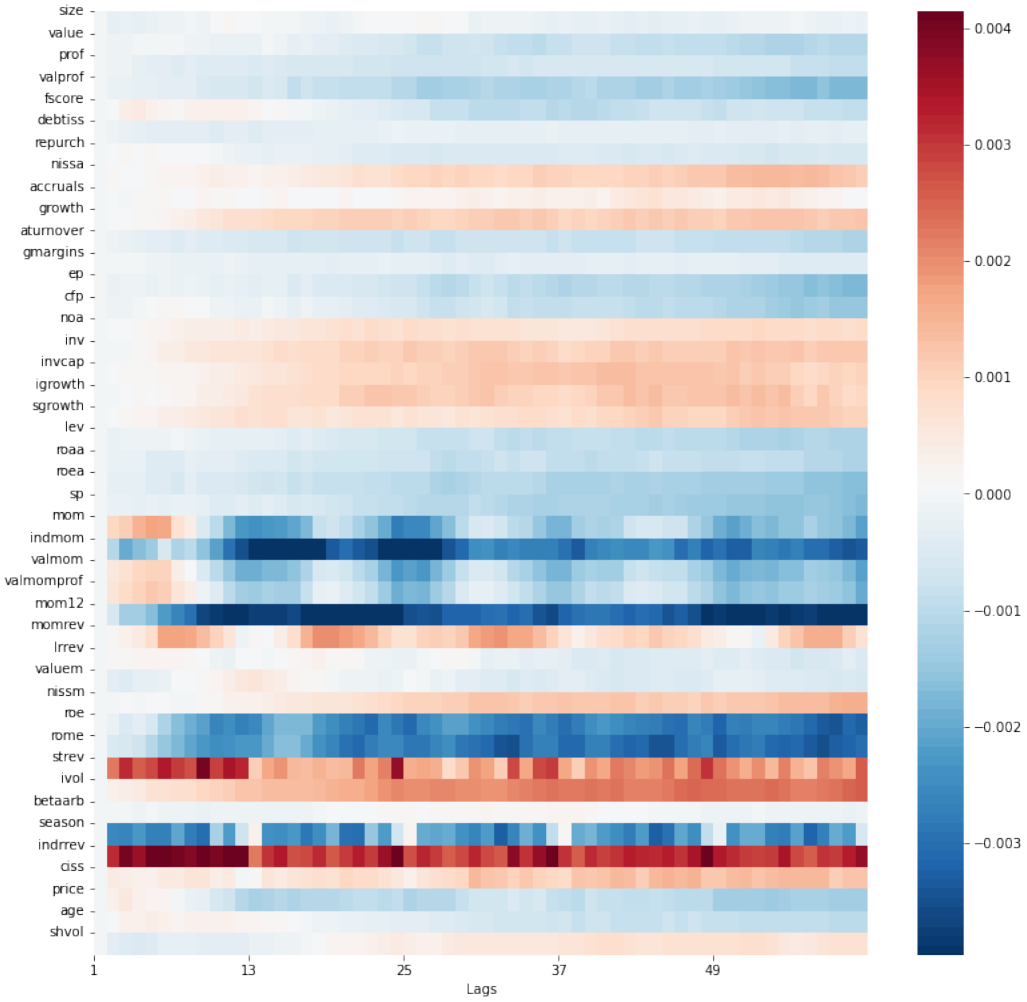

Figure 67: Term structure of alpha (dynamic) with rank 3

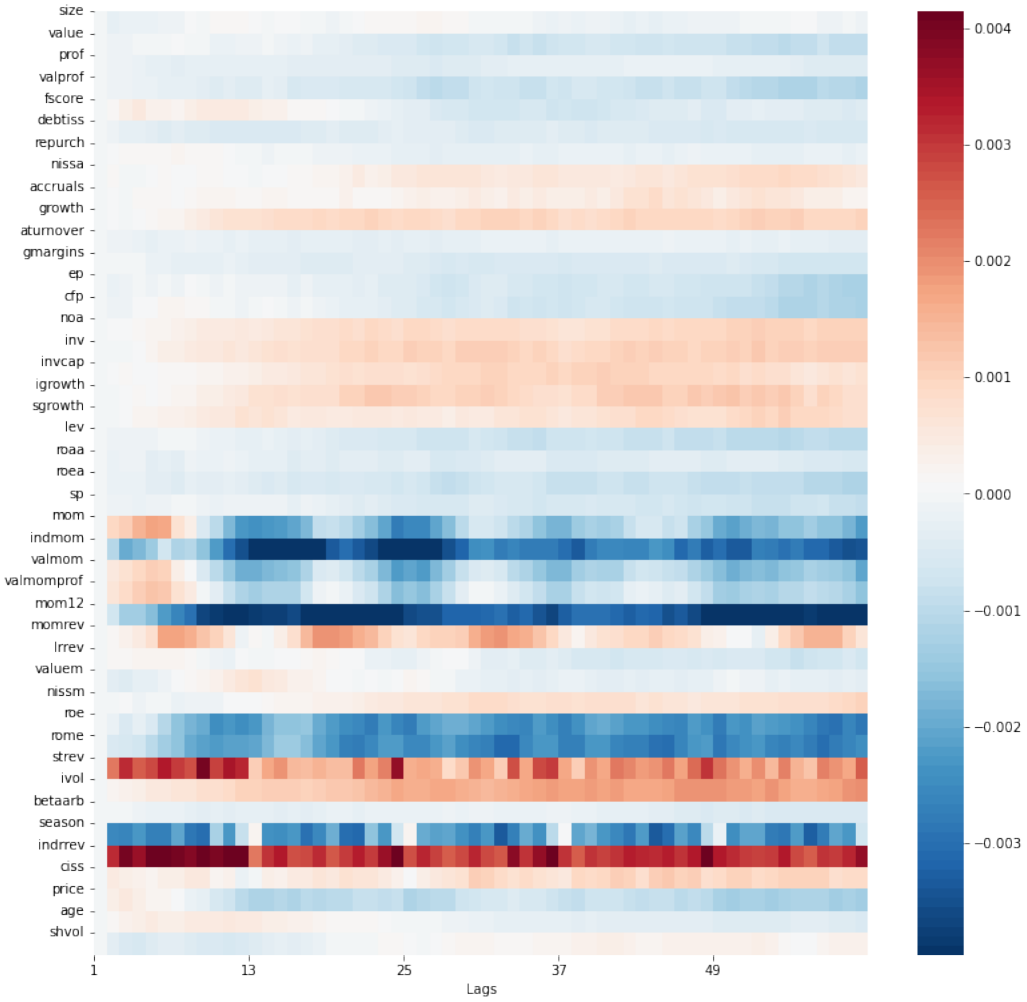

Figure 68: Term structure of alpha (dynamic) with rank 5

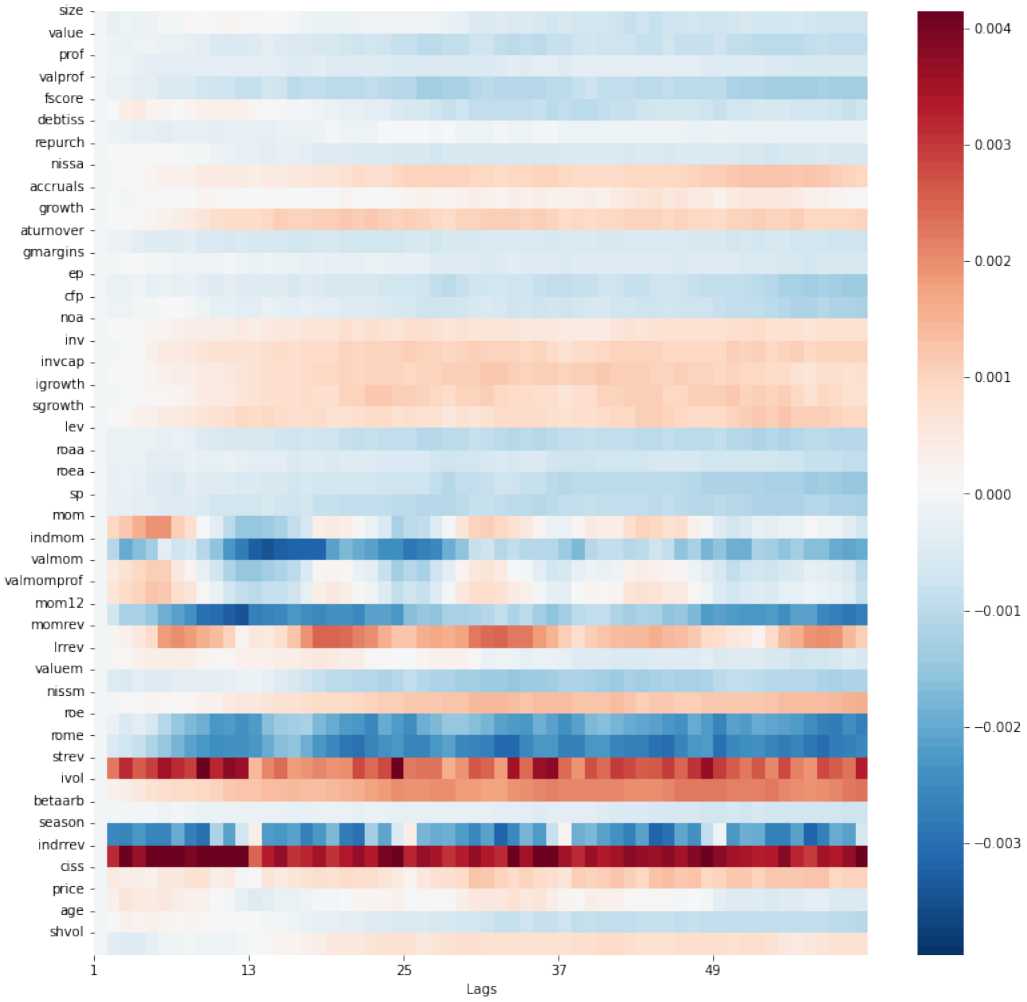

Figure 69: Term structure of alpha (dynamic) with rank 10

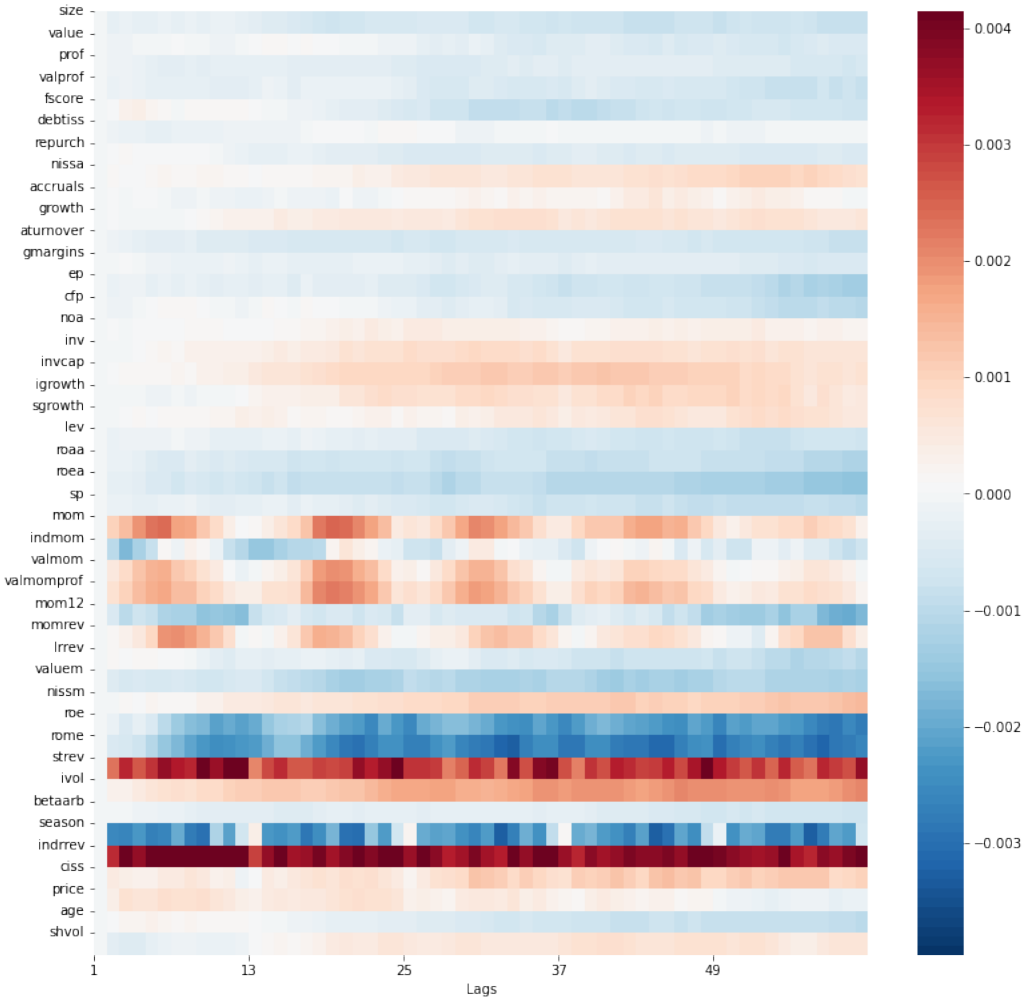

Figure 70: Term structure of alpha (dynamic) with rank 20