to Optimization

Assignment 2

Upload solutions to LumiNUS website.

1. Toyz is a discount toy store in Woodland Mall. During the summer and fall, the store must build up its inventory to have enough stock for the Christmas season. To purchase and build up its stock during the months when the revenue is low, the store borrows money from a bank.

   Following is the stores projected revenue and liabilities schedule for July through December, where revenues of last month and bills of the coming month are paid at the first of each month.

Month	Revenues	Liabilities
July	\$20,000	\$ 50,000
August	30,000	60,000
September	40,000	50,000
October	50,000	60,000
November	80,000	50,000
December	100,000	30,000

   At beginning of July, the store can take out a 6-month loan at 12% interest rate and must be paid back at the beginning of next January. The store can not pay back this loan early. The store can also borrow monthly loan at a rate of 4% interest per month. The store wants to borrow enough money to meet its cash flow needs while minimizing its interest cost. Formulate and solve the LP model for this problem. Provide the code in Python.

2. The manager of a department store is attempting to decide on the types of ad- vertising the store should use. He has invited representatives from the local radio station, television station, and newspaper to make presentations in which they describe the audiences. The television station representative indicates that a TV commercial, which costs $\$15,000$, would reach 25,000 potential customers. The newspaper representative claims to be able to provide an audience of 10,000 potential costumers at a cost of $\$4,000$ per ad, while the radio station representative says that the audience

for one of the stations commercials, which costs $\$6,000$, is 15,000 customers. The breakdown of the audience of the three medias are as follows:

TV	Male	Female
Senior	5,000	5,000
Young	5,000	10,000

Newspaper	Male	Female
Senior	4,000	3,000
Young	2,000	1,000

Radio	Male	Female
Senior	1,500	1,500
Young	4,500	7,500

The store has the following advertisement policy:

(a) Use at least twice as many radio commercials as newspaper ads.

(b) Reach as least 100,000 customers.

(c) Reach at least twice as many young people as senior citizens.

(d) Make sure at least 60% of the audience is female.

Available space limits the number of newspapers ads to seven. The store wants to make the optimal number of each type of advertisement to purchase to minimize total cost. Formulate and solve the LP model for this problem. Provide the code in Python.

3. **(Optimal currency conversion)**: Suppose that there are N available currencies, and assume that one unit of currency i can be exchanged for r_{ij} units of currency j. (Naturally, we assume that $r_{ij} > 0$.) There are also certain regulations that impose a limit u_i on the total amount of currency i that can be exchanged on any given day. Suppose that we start with B units of currency 1 and that we would like to maximize the number of units of currency N that we end up with at the end of the day, through a sequence of currency transactions. Provide a linear programming formulation of this problem. Assume that for any sequence $i_1, \cdots, i_k$ of currencies, we have $r_{i_1 i_2} r_{i_2 i_3} \ldots r_{i_{k-1} i_k} r_{i_k i_1} \leq 1$, which means that wealth cannot be multiplied by going through a cycle of currencies.

4. **(Sorting using Linear Programming)**: Suppose you are given n numbers $a_1, \ldots, a_n$. Construct linear optimization problems to

   (a) Find the smallest number

   (b) Sort the numbers from smallest to largest.